



Complete, fold and seal
(see note 2 below)

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Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
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1	Student Number	TAV19852707
2	Title of Examination Paper	Mathematics TEST 1
3	Module Code	GOEN4001
4	Date and Time of Examination	23/1/22 13:30
5	Examination Venue	ward 2 68960 Gower college
6	Seat Number	11
7	Number of Answer Books handed in	1

List the numbers of the questions you have attempted	Office Use
Q1a	23
b	6
c	4
d	2
Q2a	10
b	4
c	6
d	4
e	7
Q3a	16
b	4
c	4
d	4
Total	94%

Question 1

Q1a23/23

a) i. $y = 6x^4 + 5x^3 + 3x^2 + 6x + 7$

$$\frac{dy}{dx} = 24x^3 + 15x^2 + 6x + 6 \quad 2 //$$

ii. $y = \sin 5x + 5 \cos 4x + 7 \tan 3x$

$$\frac{dy}{dx} = 5 \cos 5x - 20 \sin 4x + 21 \sec^2(3x) \quad 2 //$$

iii. $y = 3 + \frac{1}{x^5} + \sqrt[4]{x}$

$$y = 3 + x^{-5} + x^{1/4}$$

$$\frac{dy}{dx} = -5x^{-6} + \frac{1}{4}x^{-3/4} \quad 2 //$$

iv. $y = \sin 2x e^{5x}$

$$\sin 2x = u \quad e^{5x} = v \quad 3 //$$

$$2 \cos 2x = \frac{du}{dx} \quad 5e^{5x} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \sin 2x (5e^{5x}) + e^{5x} (2 \cos 2x) = u \left(\frac{dv}{dx} \right) + v \left(\frac{du}{dx} \right)$$

v. $y = 15(2x^3 + 3)^4$

$$\frac{dy}{dx} = 60(2x^3 + 3)^3 \times 6x^2 \quad 3 //$$

vi. $y = \sqrt{7x^3 + 5x}$

$$y = (7x^3 + 5x)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(7x^3 + 5x)^{-1/2} \times (21x^2 + 5) \quad 3 //$$

a) vii. $y = 4x^5 \ln x$

$$u = 4x^5 \quad v = \ln x$$

$$\frac{du}{dx} = 20x^4 \quad \frac{dv}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = 4x^5 \left(\frac{1}{x}\right) + \ln x (20x^4) = u \left(\frac{dv}{dx}\right) + v \left(\frac{du}{dx}\right)$$

viii $y = \frac{4 \cos 4x}{e^{3x}} \quad v = e^{3x} \quad u = 4 \cos 4x$

$$\frac{dv}{dx} = 3e^{3x} \quad \frac{du}{dx} = -16 \sin 4x$$

$$\frac{dy}{dx} = \frac{v \left(\frac{du}{dx}\right) - u \left(\frac{dv}{dx}\right)}{v^2} = \frac{e^{3x} (-16 \sin 4x) - 4 \cos 4x (3e^{3x})}{(e^{3x})^2}$$

b) $w = 4x^4 + 3x^3y^3 + 2y^2 + 25$

$$\frac{dw}{dx} = 16x^3 + 9x^2y^3$$

$$\frac{dw}{dy} = 9x^3y^2 + 4y$$

i) $\frac{d^2w}{dx^2} = 48x^2 + 18xy^3$

ii) $\frac{d^2w}{dy^2} = 18x^3y + 4$

(iii) $\frac{d^2w}{dx dy} = 27x^2y^2$

(iv) $\frac{d^2w}{dx dy} = 27x^2y^2$

c) i) $y = 8x^2 + 6x - 15$ show that

$$3 \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - 32x - 60 = 0$$

$$3(16) + 2(16x + 6) - 32x - 60 = 0$$

$$48 + 32x + 12 - 60 = 0$$

$$48 + 32x + 12 - 60 = 0$$

$$48 + 32x + 12 - 32x - 60 = 0 \quad \text{True}$$

$$\frac{dy}{dx} = 16x + 6$$

$$\frac{d^2y}{dx^2} = 16$$

Question 1

d) $y = 2 \cos 3x$

$$\frac{dy}{dx} = -6 \sin 3x \quad \checkmark$$

$$\frac{d^2y}{dx^2} = -18 \cos 3x \quad \checkmark$$

$$\frac{d^3y}{dx^3} = 54 \sin 3x \quad \checkmark$$

$$\frac{d^4y}{dx^4} = 162 \cos 3x \quad \checkmark$$

Q1d

2/2

Question 2

a) $\frac{2x+5}{(x-2)(x+1)} = \frac{A}{(x-2)} + \frac{B}{(x+1)}$ $\times (x-2)(x+1)$

$$2x+5 = A(x+1) + B(x-2) \quad \checkmark$$

if $x = -1$ Then

$$2(-1)+5 = 0 + B(-1-2)$$

$$\frac{2(-1)+5}{(-1-2)} = B = -1 \quad \checkmark$$

if $x = 2$ Then

$$2(2)+5 = A(2+1) + 0$$

$$\frac{2(2)+5}{2+1} = A = 3 \quad \checkmark$$

$$\frac{2x+5}{(x-2)(x+1)} = \frac{3}{(x-2)} + \frac{-1}{(x+1)} \quad \checkmark$$

Q2a

10/10

Question 2

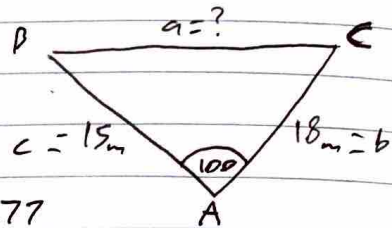
b)

i. use cosine rule ✓

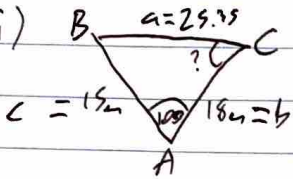
$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

$$18^2 + 15^2 - 2(18)(15) \cos(100) = 642.77$$

$$\sqrt{642.77} = 25.35 \text{ m} = BC$$

Q2b4/4

ii)



$$\text{Sine rule} = \frac{a}{\sin(A)} = \frac{c}{\sin(C)}$$

$$\frac{25.35}{\sin(100)} = \frac{15}{\sin(C)}$$

$$15(\sin(100)) = 25.35 \times \sin(C)$$

$$14.77 = 25.35 \times \sin(C)$$

$$\frac{14.77}{25.35} = \sin C = 0.5826$$

$$\sin^{-1}(0.5826) = C = \angle BCA = 35.6^\circ$$

c)

$$x^2 - 9x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$x = 3 \text{ or } 2 \Rightarrow \tan x = 3 \text{ or } 2$$

$$\tan^{-1}(3) = 71.57^\circ = x$$

$$\tan^{-1}(2) = 63.43^\circ = x$$

$$180 + 71.57 = 251.57^\circ = x$$

$$180 + 63.43 = 243.43^\circ = x$$

d)

$$\text{if } y = x^2 - 6x + 5 \text{ and } y = x - 7$$

$$\text{then } x^2 - 6x + 5 = x - 7$$

$$x^2 - 7x + 8 = 0$$

$$a = 1$$

$$b = -7$$

$$c = 8$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2(a)}$$

$$\rightarrow \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(8)}}{2(1)}$$

$$x = 5.56 \text{ or } 1.44 = \text{sand}$$

Q2c6/6Q2d4/6

Question 2

e) i. $\sin x = 0.85$

$$\sin^{-1}(0.85) = 58.21^\circ \checkmark$$

$$180 - 58.21 = 121.79^\circ \checkmark$$

2

Q2e

7/7

ii. $5 \cos x = 4$

$$\cos x = \frac{4}{5} \checkmark$$

$$\cos^{-1}\left(\frac{4}{5}\right) = 36.87^\circ \checkmark$$

$$360 - 36.87 = 323.13^\circ \checkmark$$

3

iii. $\tan x = -0.8$

$$\tan^{-1}(0.8) = 38.66^\circ \checkmark$$

$$180 - 38.66 = 141.34^\circ \checkmark$$

$$360 - 38.66 = 321.34^\circ \checkmark$$

2

Question 3

a) i. $\ln x + \ln(x-2) = \ln(x+10) \checkmark$

$$\ln x(x-2) = \ln(x+10) \checkmark$$

$$x(x-2) = (x+10) \checkmark$$

$$x^2 - 2x = x + 10 \checkmark$$

←

$$x^2 - 3x - 10 = 0 \checkmark$$

$$a=1$$

$$b=-3$$

$$c=-10$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

↓

$$\frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)}$$

$$x = 5 \text{ or } -2 \text{ not a solution}$$

ii. $\log(3x+2) + \log(x+1) = \log 12 \checkmark$

$$\log(3x+2)(x+1) = \log 12 \checkmark$$

$$(3x+2)(x+1) = 12 \checkmark$$

$$3x^2 + 2x + 3x + 2 = 12 \checkmark$$

←

$$3x^2 + 5x - 10 = 0 \checkmark$$

3

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a=3$$

$$b=5$$

$$c=-10$$

$$\frac{-5 \pm \sqrt{5^2 - 4(3)(-10)}}{2(3)}$$

$$x = 1.17 \text{ or } -2.84$$

$$\text{or } -2.84$$

not a solution

Q3a

16/18

Question 3

Q1) iii. $4^{(1-2x)} \times 6^{(3x+4)} = 12^{(5x)}$

$$\log(4^{(1-2x)} \times 6^{(3x+4)}) = \log 12^{(5x)} \quad \checkmark$$

$$\log 4^{(1-2x)} + \log 6^{(3x+4)} = (5x) \log 12 \quad \checkmark$$

$$(1-2x) 0.6 + (3x+4) 0.78 = (5x) 1.08 \quad \checkmark$$

$$0.6 - 1.2x + 2.34x + 3.12 = 5.4x \quad \checkmark$$

$$1.14x + 3.72 = 5.4x \quad \checkmark$$

4

$$3.72 = 5.4x - 1.14x \quad \checkmark$$

$$3.72 = 4.26x \quad \checkmark$$

$$\frac{3.72}{4.26} = x = 0.87 \quad \checkmark$$

iv.

$$\frac{3x-2}{3x+9} = \frac{4}{3} \quad \checkmark$$

$$3(3x-2) = 4(3x+9) \quad \checkmark$$

$$9x-6 = 12x+36 \quad \checkmark$$

$$9x-12x = 36+6 \quad \checkmark$$

$$-3x = 42 \quad \checkmark$$

$$x = \frac{42}{-3} = -14 \quad \checkmark$$

3

v.

$$\frac{(x+4)}{9} + \frac{(2x-1)}{7} = 12 \quad \checkmark$$

$$\times (9 \times 7) \quad \checkmark$$

$$63 \frac{(x+4)}{9} + 63 \frac{(2x-1)}{7} = 63 \times 12 \quad \checkmark$$

3

$$7(x+4) + 9(2x-1) = 756 \quad \checkmark$$

$$7x + 28 + 18x - 9 = 756 \quad \checkmark$$

$$25x = 737 \quad \checkmark$$

$$x = \frac{737}{25} = 29.48 \quad \checkmark$$

Question 3

b) i. $z^2 - 9z + 14$
 $(z-7)(z-2)$ ✓ 2

14
1 14
2 7

Q3b

4/6

ii. $q^2 - 81$
 $(q+9)(q-9)$ ✓ 2

iii. $12xu - 3xv + 28yu - 7yv$
 $-3x(-4u-v) + 7y(-4u-v)$ ✗ 0
 $(-3x-7y)(-4u-v)$ ✗

c) i. $8m^2 - 2m - 21 = 0$ $8x - 21 = -168$
 $8m^2 + 12m - 14m - 21 = 0$ $-14 + 12$

$4m(2m+3) - 7(2m+3) = 0$
 $(2m+3)(4m-7) = 0$

$2m = -3$ ✓ $4m = 7$ ✓
 $m = -\frac{3}{2}$ ✓ $m = \frac{7}{4}$ ✓
 $m = -1.5$ ✓ or $m = 1.75$ ✓ 2

Q3c

4/4

ii. $w^2 + 13w + 40 = 0$ ✓
 $(w+5)(w+8) = 0$ ✓ 2
 $w = -5$ or -8 ✓

40
1 x 40
2 x 20
4 x 10
5 x 8

d) $6x^2 - 8x - 30 = 0$ ✓

$a=6$ ✓ $-b \pm \sqrt{b^2 - 4ac}$ → $-(-8) \pm \sqrt{(-8)^2 - 4(6)(-30)}$
 $b=-8$ ✓ 2a ✓ $2(6)$
 $c=-30$ ✓

$x = 3$ or -1.67
✓ ✓

Q3d

4/4