

University of Wales Trinity Saint David
School of Engineering

Advanced Engine Design
M6X00410

Samuel Treleaven
1702453

07/02/20

*Calculation on Piston, Rod and Crankshaft
Assemblies*

Contents

1	Introduction.....	3
2	Data Collected.....	3
3	Gudgeon Pin.....	3
4	Connecting Rod.....	9
4.1	Point A.....	10
4.2	Point B.....	11
4.3	Point C.....	13
5	Connecting Rod Bolt.....	15
5.1	Approximate Ratio of Stiffness.....	15
5.2	Bolt and Cap Stiffness.....	16
5.3	Alternating Load.....	17
5.4	Required Tightening Torque.....	17
5.5	Mean and Range of Stress.....	19
5.6	Goodman Diagram.....	19
5.7	Modified Engine.....	20
5.8	Standard and Up-rated Bolt Designs.....	22
6	Crankshaft.....	23
6.1	Point A.....	25
6.2	Point B.....	26
7	Limitations and Improvements.....	28
8	Conclusion.....	28
	Bibliography.....	29
	Appendices.....	30

Appendices

Appendix A – Question One Formula View

Appendix B – Question Two Formula View

Appendix C – Question Three Formula View

Appendix D – Question Four Formula View

1 Introduction

The purpose of this assignment is to investigate the fatigue safety factors, FSF, for the standard engine's components on a standard duty cycle using a range of calculations. The engine used was the same Ford Sigma 1.6 engine used in the previous assignment. Four engine components were investigated. These were the gudgeon pin, the connecting rod, the connecting rod cap and bolts and the crankshaft.

2 Data Collected

The first part of this assignment was to collect the data for the engine. The masses and dimensions of the connecting rod and piston were collected from the data tables in assignment one of this module. The new data required was measured from the gudgeon pin, the connecting rod bolt, the connecting rod and cap interference face area and the crankshaft. For the connecting rod, the cross-sectional area at three points was measured. The values for the gas forces were obtained from the previous assignment; for the standard engine this occurred at 5700RPM and peak inertia was at the maximum 6000RPM.

3 Gudgeon Pin

To calculate the fatigue safety factor, FSF, for the gudgeon pin the mean and range of stresses need to be calculated so a Goodman diagram can be created.

To calculate the bending stress, the second moment of area needed to be calculated using the equation 3.1 where D_1 was the external diameter of the gudgeon pin and D_2 was the internal diameter of the gudgeon pin, for this gudgeon pin the values were 20mm and 12mm respectively.

$$I = \frac{\pi(D_1^4 - D_2^4)}{64}$$

Equ. 3.1 - Second moment of area

The cross-sectional area of the gudgeon pin was then calculated using equation 3.2, this would be used in the calculation of shear stress.

$$A = \frac{\pi(D_1^2 - D_2^2)}{4}$$

Equ. 3.2 Cross-sectional area

The inertia caused by the gudgeon pin was calculated as it would affect the overall net force on the pin. The inertia was calculated using equation 3.3 where the first square bracket was the exposed mass and the second was the acceleration.

$$F_{inertia\ pin} = [l \times \pi(R_1^2 - R_2^2) \times \rho] \times \left[R\omega^2 \cos \theta + \frac{R}{L} R\omega^2 \cos 2\theta \right]$$

Equ. 3.3 Gudgeon pin inertia tensile & comp

For the mass, l was taken to be 18mm, R_1 and R_2 was half of the internal and external diameters and ρ was the density, which is 7800kg/m³ (Hearn, 1999).

For the acceleration, R was the crank radius which was 40.7mm and ω was the angular velocity which needed to be converted from RPM to radians per second.

For the compressive force inertia, this was at max RPM at top dead centre and the tensile force was at peak net power which was at 5700RPM. The angle, θ , was the crank angle and L the connecting rod length, which was 134mm.

Table 3.1 contains all of these calculated values. The value for the maximum net compressive force was taken from the previous assignment which was the inertia force at maximum RPM at TDC on the exhaust stroke. The reason the force is negative is that it puts the pin into compression. The maximum net tensile force was also taken from the previous assignment where the gas force minus the inertia force was at its largest, this was at 5700RPM and is a positive value as it stretches the pin.

Maximum Net Compressive Force	-7340.17 N
Maximum Net Tensile Force	40953.84 N
Second Moment of Area, I	6655.559 mm ⁴
Cross Sectional Area, A	191.4408 mm ²
Gudgeon Pin Mass	0.026878 kg
Gudgeon Pin Inertia Comp	563.0457 N
Gudgeon Pin Inertia Tensile	508.149 N

Table 3.1

From these calculated values the bending and shear stress for both the compressive and tensile forces can be calculated.

To calculate the bending moment, equation 3.4 was used for both the compressive and tensile bending moment. L_1 and L_2 where the same for both calculations which were the distances at which each force acts relative to the point of high stress, these were 15.8mm and 10.5mm. however, the net force used was the compressive and tensile

respectively, likewise with the inertia force by the pin. The reason the net force was halved was due to only one side of the pin being investigated so only half the force would be exerted here.

$$M_B = \frac{F_{net}}{2} \times L_1 - F_{inertia\ pin} \times L_2$$

Equ. 3.4 Bending moment

Equation 3.5 Calculates the bending stresses. The value of D_1 and I was the same for tensile and compressive calculations.

$$\sigma_B = \frac{M_B \times \frac{D_1}{2}}{I}$$

Equ. 3.5 Bending stress

The shear force calculation was similar to the bending moment calculation however the distance at which the force acts was not used, see equation 3.6.

$$F_{shear} = \frac{F_{net}}{2} - F_{inertia\ pin}$$

Equ. 3.6 Shear force

The shear stress was calculated by dividing the shear force by the cross-sectional area.

$$\tau = \frac{F_{shear}}{A}$$

Equ. 3.7 Shear stress

To calculate the resultant force for both maximum stress, the tensile, and the minimum stress, the compressive, equation 3.8 was used. This equation was different from the von mises stress as the shear stress was only multiplied by two and not three.

$$\sigma_{res} = \sqrt{\sigma_B^2 + (2 \times \tau)^2}$$

Equ. 3.8 Resultant stress

These values calculated can be seen in table 3.2 where the calculations were carried out in mm.

	Compressive	Tensile
Bending Moment	-63899.3 Nmm	318199.7715 Nmm
Bending Stress	-96.0089 N/mm ²	478.096227 N/mm ²
Shear Force	-4233.13 N	19968.771 N
Shear Stress	-22.112 N/mm ²	104.3078109 N/mm ²
Resultant Stress	-105.705 N/mm ²	521.6286801 N/mm ²

Table 3.2

The mean stress and the range of stresses were calculated using equations 3.9 and 3.10. The mean would then be used in plotting a goodman diagram and the range used in the calculation for the fatigue safety factor.

$$\sigma_{mean} = \frac{\sigma_{max\ res} + \sigma_{min\ res}}{2}$$

Equ. 3.9 Mean stress

$$\sigma_{range} = \sigma_{max\ res} - \sigma_{min\ res}$$

Equ. 3.10 Range of stresses

The values for the mean stress and ranges of stresses can be seen in table 3.3.

Mean Stress	207.962 N/mm ²
Range of Stresses	627.3333 N/mm ²

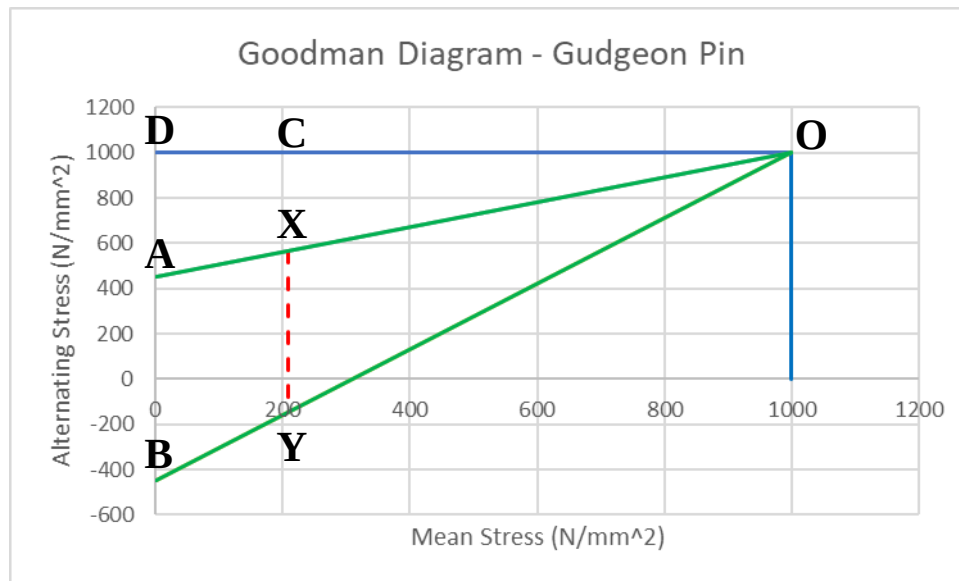
Table 3.3

The final calculation to be carried out before the goodman diagram could be produced was the endurance limit. This was calculated using equation 3.11 where the UTS is the ultimate tensile strength which was 1000N/mm². (Bosch, 2011)

$$EL = 0.45 \times UTS$$

Equ. 3.11 Endurance limit

A Goodman diagram was produced where the blue lines are at the UTS, as the stress can not exceed that. The green lines intercept the positive and negative endurance limit, at mean stress of zero, and the point at which mean stress and alternating stress could be at a maximum. The red dashed line is a vertical line that goes through mean stress and intercepts both green lines at point X and Y.



Graph 3-1

The distance XY is the allowable range which is what needs to be calculated to find the fatigue safety factor. To find this value, triangles are visualised and relationships can be produced. Firstly, looking at triangle OXY and OAB we can produce the relationship $\frac{XY}{AB} = \frac{OX}{OA}$ however, the values of OX and OA are unknown. By looking at triangles OCX and ODA the relationship $\frac{OX}{OA} = \frac{OC}{OD}$ can be produced. Subsequently, the relationship $\frac{XY}{AB} = \frac{OC}{OD}$ is produced from substitution. We know the value of OC, the UTS minus the mean, OD, the UTS, and AB, 2 multiplied by the endurance limit. From this, equation 3.12 is produced to calculate the allowable range at the mean stress.

$$Allowable\ Range = 2 \times EL \times \frac{UTS - \sigma_{mean}}{UTS}$$

Equ. 3.12 Allowable range at mean

To calculate the working safety factor for infinite fatigue life on the gudgeon pin equation 3.13 is used.

$$FSF = \frac{Allowable\ Range}{\sigma_{range}}$$

Equ. 3.13 Fatigue safety factor

The values calculate for the endurance limit, allowable range and the fatigue safety factor are in table 3.4.

Endurance Limit	450 N/mm ²
Allowable Range at mean	712.8342 N/mm ²
FSF	1.136293

Table 3.4

The fatigue safety factor is a value which corresponds to the number of cycles that a component can experience before it breaks due to fatigue. A Wöhler diagram can be used to compare the number of cycles to the alternating stress. A fatigue safety factor of one corresponds to 10^7 cycles. Anything above 10^7 cycles are deemed to have an infinite life (Bozdana, 2007) whereas anything less than 10^3 is deemed to have low cycle fatigue. This also means anything with a fatigue safety factor of less than one has a finite life. For the gudgeon pin, the FSF was 1.14, this means that the component should have an infinite fatigue life so will only break under non-standard forces acting on it.

4 Connecting Rod

For the connecting rod, the fatigue safety factors are calculated at three different locations. These points, A, B and C and can be seen in figure 4.1. The values for the compressive and tensile forces were taken from the standard engine for these calculations.

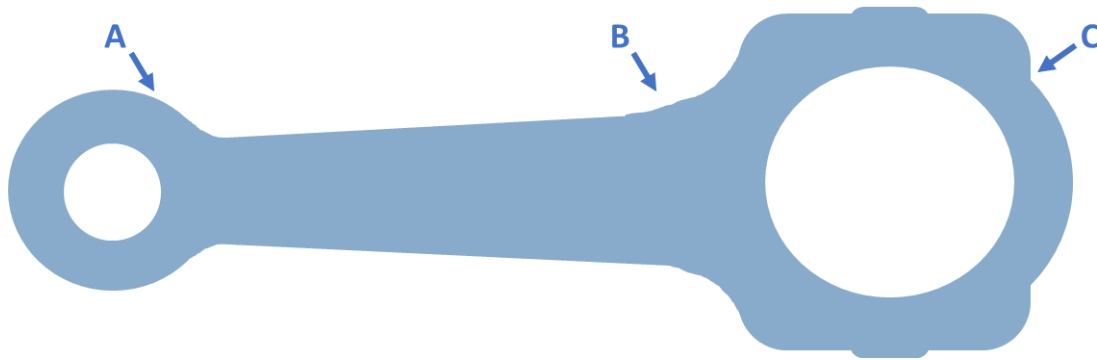


Figure 4.1 – connecting rod

The endurance limit used in these three points were modified by dividing the polished endurance limit, equation 3.11, by a stress concentration factor. The UTS for the connecting rod was 750 N/mm² (Bosch, 2011). However, the stress concentration factor was different for each location.

$$Final\ EL = \frac{EL}{K_t}$$

Equ. 4.1

The stress concentration factors were recorded from a graph first created by Peterson (1951). This graph can be seen below in figure 4.

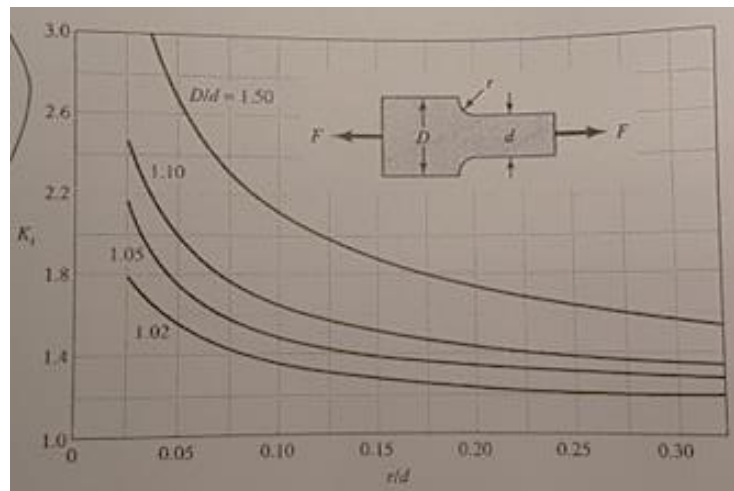


Figure 4.2 (Peterson, 1951)

4.1 Point A

To calculate the compressive and tensile stresses the maximum net compressive force and tensile force were obtained. These were the force calculated in the previous assignment where the compressive force is the net force under firing and the tensile force is the force at top dead centre on the exhaust stroke. The stress concentration factor calculated was 1.5 (Peterson, 1951) and the cross-sectional area was measured and calculated to be 83.67mm². These values can be seen in table 4.1.

Maximum Net Compressive Force	-40066.4 N
Maximum Net Tensile Force	7340.17 N
Stress Concentration Factor, Kt	1.5
Area per side, A	83.67 mm ²

Table 4.1

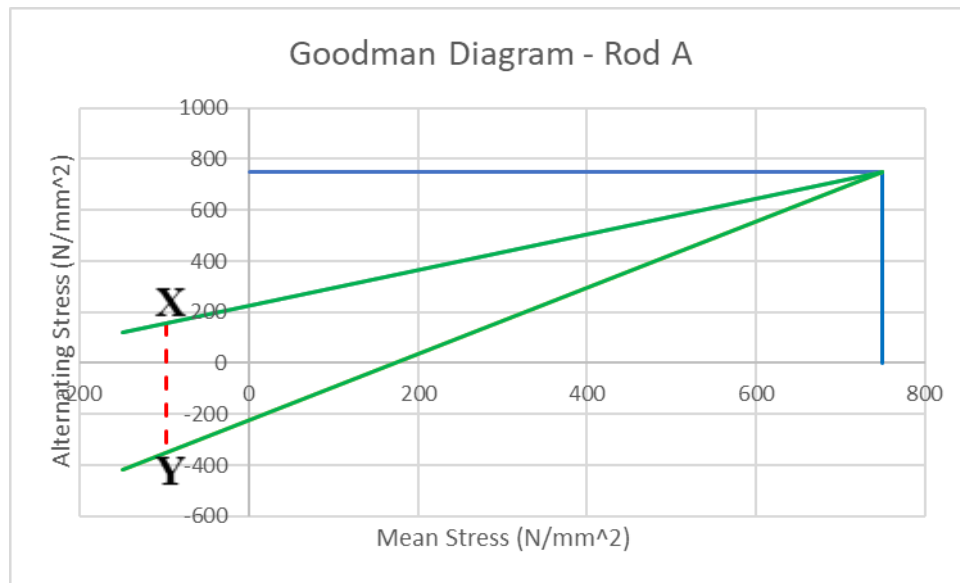
From these values, the compressive stress and tensile stress were calculated by dividing half the respective force by the cross-sectional area. The reason only half the force was used is that only one side of the connecting rod is being investigated so the force exerted would be equally split between the two sides. The mean and range were also calculated. These can be seen in table 4.2.

Compressive Stress	-239.431 N/mm ²
Tensile Stress	43.86381 N/mm ²
Mean Stress	-97.7835 N/mm ²
Range of Stresses	283.2946 N/mm ²

Table 4.2

The polished and final endurance limits were calculated and so a goodman diagram was plotted for the stress at point A. This diagram could then be used to calculate the

allowable range like explained in section 3, and so the fatigue safety factor could be calculated.



Graph 4-1

The values calculated can be seen in table 4.3. The fatigue safety factor is 1.80 which is much larger than one so the connecting rod at location A has an infinite fatigue life. (Bozdana, 2007)

Polished Endurance Limit	337.5 N/mm ²
Final Endurance Limit	225 N/mm ²
Allowable Range at mean	508.6701 N/mm ²
FSF	1.795552

Table 4.3

4.2 Point B

For point B, new compressive force and tensile force were calculated these forces has a change in inertia to previously as there was more reciprocating mass than previously. This extra mass was a third of the connecting rod mass. The new forces were calculated as can be seen in equations 4.2 and 4.3.

$$F_{compressive} = F_{gas} - \left(\frac{F_{inertia @ 5700RPM \text{ at point A}}}{m_{piston} + m_{pin}} \times \left(m_{piston} + m_{pin} + \frac{m_{rod}}{3} \right) \right)$$

Equ. 4.2 Compressive force

$$F_{tensile} = \frac{F_{inertia @ 6000RPM \text{ at point A}}}{m_{piston} + m_{pin}} \times \left(m_{piston} + m_{pin} + \frac{m_{rod}}{3} \right)$$

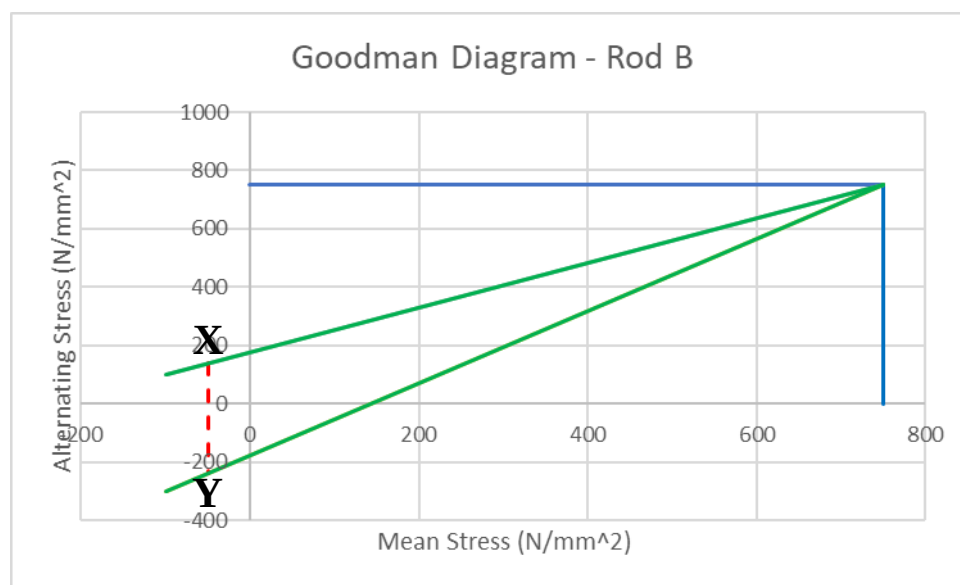
Equ. 4.3 Tensile force

The stress concentration factor for point B was calculated to be 1.9 (Peterson, 1951) and the cross-sectional area was measured and calculated as 277.95mm^2 . The reason this is a lot larger than point A is that this is across the entire connecting rod at point B and not just one side. The Compressive and tensile stresses were calculated similarly to point A however the entire force was used as it is all acting on the area at point B. The mean and range of stresses were also produced. The values can be seen in table 4.4.

Maximum Net Compressive Force	-37505.3 N
Maximum Net Tensile Force	10177.92 N
Stress Concentration Factor, K_t	1.9
Cross Sectional Area, A	277.95 mm^2
Compressive Stress	-134.935 N/mm^2
Tensile Stress	36.61782 N/mm^2
Mean Stress	-49.1588 N/mm^2
Range of Stresses	171.5532 N/mm^2

Table 4.4

The final endurance limit was calculated using a stress concentration factor of 1.9 and a Goodman diagram was produced for point B.



Graph 4-2

The fatigue safety factor and allowable range were calculated and can be seen in table 4.5. The FSF was calculated to be 2.21, this means that the connecting rod and point B has been over-engineered as it will definitely have an infinite fatigue life. (Bozdana, 2007) This means that the rod could be reduced in size at this point and still be able to retain an infinite fatigue life, however it may not be a possible thing to do due to the shape of the connecting rod needing to get larger for the big end.

Polished Endurance Limit	337.5 N/mm ²
Final Endurance Limit	177.6316 N/mm ²
Allowable Range at mean	378.5489 N/mm ²
FSF	2.206598

Table 4.5

4.3 Point C

Point C occurs below the big end, this means that no compression force is applied at this point, this means the compressive force and stress are zero. However, since there is, even more, moving mass the tensile force increases again. This time the mass acting at point C includes an extra third connecting rod mass than at point B, so two thirds more than point A. This calculation can be seen in equation 4.4

$$F_{tensile} = \frac{F_{inertia @ 6000RPM \text{ at point A}}}{m_{piston} + m_{pin}} \times \left(m_{piston} + m_{pin} + \frac{2 \times m_{rod}}{3} \right)$$

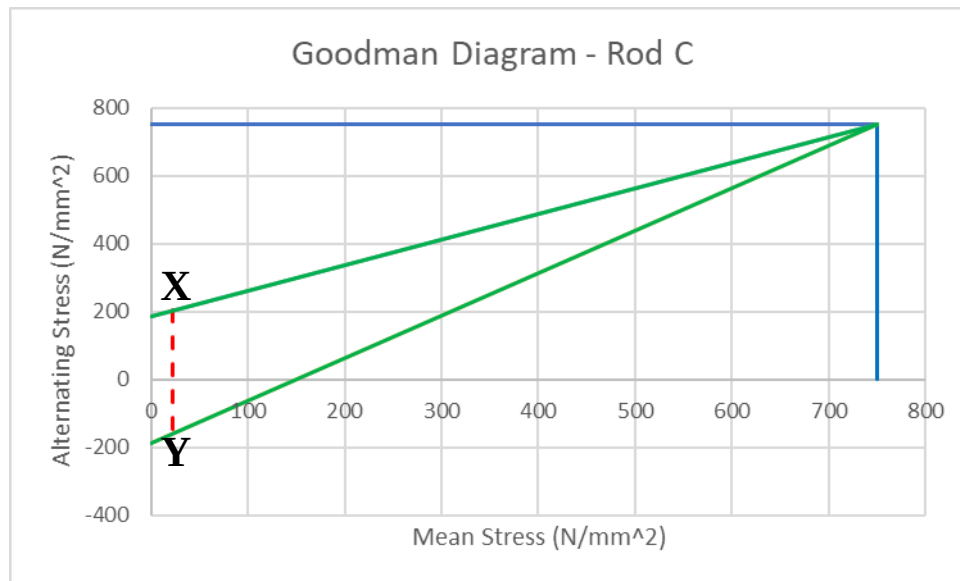
Equ. 4.4 Tensile force

The area at point C was measured and calculated to be 142.8mm² per side. This means the tensile stress used half of the tensile force in the calculation. The values for the stresses can be seen in table 4.6.

Point C	
Maximum Net Compressive Force	0 N
Maximum Net Tensile Force	13015.67 N
Stress Concentration Factor, Kt	1.8
Area per side, A	142.8 mm ²
Compressive Stress	0 N/mm ²
Tensile Stress	45.57309 N/mm ²
Mean Stress	22.78654 N/mm ²
Range of Stresses	45.57309 N/mm ²

Table 4.6

The final endurance limit was calculated using a stress concentration factor of 1.8 (Peterson, 1951) and then a goodman diagram was produced to calculate the allowable range.



Graph 4-3

The allowable range calculated from graph 4-3 allowed the fatigue safety factor at point C to be calculated. This came out to be a really large value of 7.98, this can be seen in table 4.7, which means this part of the connecting rod has an infinite fatigue life.

(Bozdana, 2007)

Polished Endurance Limit	337.5 N/mm ²
Final Endurance Limit	187.5 N/mm ²
Allowable Range at mean	363.6067 N/mm ²
FSF	7.97854

Table 4.7

The FSF at point C is so large due to the fact that there is a huge amount of cross-sectional area at this point. This is caused due to the connecting rod bolts needing a wide face to seat onto. Also at point C, no compressive stress occurs which means that the mean stress is half of the maximum stress so very low stress is used in the calculation of the allowable range at the mean.

5 Connecting Rod Bolt

The fatigue safety factor of the standard connecting rod bolts is investigated in this section. Also, an investigation on how the bolts or engine may need to be modified to retain an acceptable fatigue safety factor for the modified engine which has a maximum RPM of 8000.

5.1 Approximate Ratio of Stiffness

To calculate an approximation for the ration of stiffness, the area of the cap face and the bolt hole needs to be calculated. The cap face was measured and calculated using equation 5.1 where l and w are the length and width of the face.

$$A_{cap} = l \times w - A_{bolt}$$

Equ. 5.1 Cap area

The bolt area was calculated using the average diameter taken from the base of thread and the tip of the thread, which was measured and calculated to be 7.065mm.

$$A_{bolt} = \frac{\pi \times D_{average}^2}{4}$$

Equ. 5.2 Bolt area

An approximation of the stiffness ratio is calculated by dividing the cap area by the bolt area. This approximation assumes that the young modulus for both the bolt and the cap are the same, where in fact they are slightly different.

$$k_{ratio \ approx} = \frac{A_{cap}}{A_{bolt}}$$

Equ. 5.3 Approximate ratio of stiffness

The values calculated for the areas and ratio of stiffness can be seen in table 5.1.

Cap Area (Ac)	113.96	mm^2
Bolt Area (Ab)	39.20254	mm^2
Ratio of Stiffness (Ac/Ab)	2.906953	

Table 5.1

5.2 Bolt and Cap Stiffness

The actual bolt and cap stiffness were calculated using equations 5.4 and 5.5. They use the area calculated in section 5.1 and use the same length, L , which is the stretched length of the bolt or the compressed length of the cap. This value is the measured length from the base of the head of the bolt to the second engaged thread, which was 25.5mm. The youngs modulus for the bolt was $210 \times 10^3 \text{ N/mm}^2$, (American Society for Metals, 1990), and for the connecting rod cap it was $170 \times 10^3 \text{ N/mm}^2$ (American Society for Metals, 1985).

$$k_{cap} = \frac{E_{cap} \times A_{cap}}{L}$$

Equ. 5.4 Cap stiffness

$$k_{bolt} = \frac{E_{bolt} \times A_{bolt}}{L}$$

Equ. 5.5 Bolt stiffness

The ratio of stiffness is calculated by dividing the cap stiffness by the bolt stiffness. This ratio is important in designing the bolt as this needs to be as large as possible to reduce the amount of stress experienced in the bolt. This could be done by changing the material of the cap to increase the youngs modulus of it, so the ratio would increase. Similarly, you could change the bolt material to reduce the youngs modulus, so the ratio would increase.

$$k_{ratio} = \frac{k_{cap}}{k_{bolt}}$$

Equ. 5.6 Ratio of stiffness

The values for the stiffness can be seen in table 5.2. The previous approximation for the ratio of stiffness was 2.91. This is larger than the actual value of 2.35 due to the youngs modulus of the cap being smaller than the youngs modulus of the bolt. However, it was still quite a good approximation to use if the material specification for the bolt and cap were unknown.

Bolt Stiffness (Kb)	322844.5 N/mm
Cap Stiffness (Kc)	759733.1 N/mm
Actual Ratio of Stiffness (Kc/Kb)	2.353248

Table 5.2

5.3 Alternating Load

The maximum inertia force on the cap occurs at top dead centre on the exhaust stroke at maximum RPM. This inertia calculation does not include the cap mass as part of the rotational mass. The RPM used in this calculation is the max of 8000RPM plus 500RPM, so omega is 8500RPM in radians per second. This has been used to account for any fluctuations in inertia forces received during the standard duty cycle of the engine.

$$F_{inertia\ max} = \left(\frac{2 \times m_{rod}}{3} - m_{cap} \right) R\omega^2 + \left(m_{piston} + m_{pin} + \frac{m_{rod}}{3} \right) R\omega^2 \left(1 + \frac{R}{L} \right)$$

Equ. 5.7 Max inertia load

The maximum alternating load on the bolt is calculated using equation 5.8.

$$F_{alt\ load\ on\ bolt} = \frac{1}{1 + \frac{K_{cap}}{K_{bolt}}} \times \frac{F_{inertia\ max}}{2}$$

Equ. 5.8 Max alternating load on the bolt

This equation is a rearrangement of the principle that the extra force in the bolt is the stretch in the bolt multiplied by the bolt stiffness. The stretch is the inertia force divided by the combined stiffness of the cap and bolt. The reason only half the inertia is used is that only one bolt is being investigated and the force would be evenly distributed across both bolts.

The values for the total maximum inertia force on the entire cap and the alternating load on the bolt can be seen in table 5.3.

Max Inertia Load	15096.61 N
Max Alternating Load on Bolt	2251.042 N

Table 5.3

5.4 Required Tightening Torque

To calculate the required tightening torque the axial force in the bolt must be calculated first. This is carried out using equation 5.9. The reason the UTS is multiplied by 0.9 is that the yield is 90% of the UTS value, which is 1100 N/mm² for the bolt (Bosch, 2011). The axial stress for the bolt occurs at 60% yield, so that's why its multiplied by 0.6. This axial stress is then multiplied by the area of the bolt to give the axial force.

$$F_{axial} = A_{bolt} \times 0.6 \times (UTS \times 0.9)$$

Equ. 5.9 Static axial bolt force

The resisting torque at the threads is the force required to overcome the thread friction multiplied by the average radius of the threads. The coefficient of friction for the threads was researched to be 0.18.

$$T_{thread} = F_{axial} \times \mu_{thread} \times \frac{D_{average\ thread}}{2}$$

Equ. 5.10 Torque to overcome thread

The torque needed to overcome the head friction can be seen in equation 5.11. The coefficient of friction for the head was researched to be 0.12 and the average seating face was the thread diameter, 7.065mm, plus the bolt head diameter, 12.9mm (Bosch, 2011), divided by two. This value was divided by two again to get the average radius.

$$T_{head} = F_{axial} \times \mu_{head} \times \frac{D_{average\ seating\ face}}{2}$$

Equ. 5.11 Torque to overcome head

The total torque required to tighten the bolt into 90% yield was calculated using equation 5.12.

$$T_{total} = T_{thread} + T_{head}$$

Equ. 5.12 Required tightening torque

The values for the axial force and torques can be seen in table 5.4. The value of 28.8Nm was needed to tighten the bolt into 90% yield. This value is a value which would be expected for a connecting rod bolt so this confirms that no significant errors were made or accumulated in the measuring and calculating of the torque.

Static Axial Bolt Force @ 90% yield	23286.31	N
Torque to overcome Thread	14806.6	Nmm
Torque to overcome Head	13947.33	Nmm
Required Tightening Torque	28753.93	Nmm
Required Tightening Torque	28.75393	Nm

Table 5.4

5.5 Mean and Range of Stress

To calculate the maximum and minimum stress for the connecting rod a new approach had to be taken than in the previous sections. The range of stresses was calculated as it is the extra nominal stress at the root of the thread. This is the alternating load on the bolt divided by the bolt area which gives the stress which is the range for the bolt as the minimum stress comes from the initial tightening stress.

$$\sigma_{range} = \frac{F_{alt \text{ load on bolt}}}{A_{bolt}}$$

Equ. 5.13 range of stress

The minimum stress, as previously stated, comes from the initial tightening stress which is 90% of yield. The yield like discussed in equation 5.14 is 90% of the UTS.

$$\sigma_{min} = (UTS \times 0.9) \times 0.9$$

Equ. 5.14 Min Stress

The maximum stress can then be calculated by adding the range of stresses to the minimum stress, see equation 5.15.

$$\sigma_{max} = \sigma_{min} + \sigma_{range}$$

Equ. 5.15 Max stress

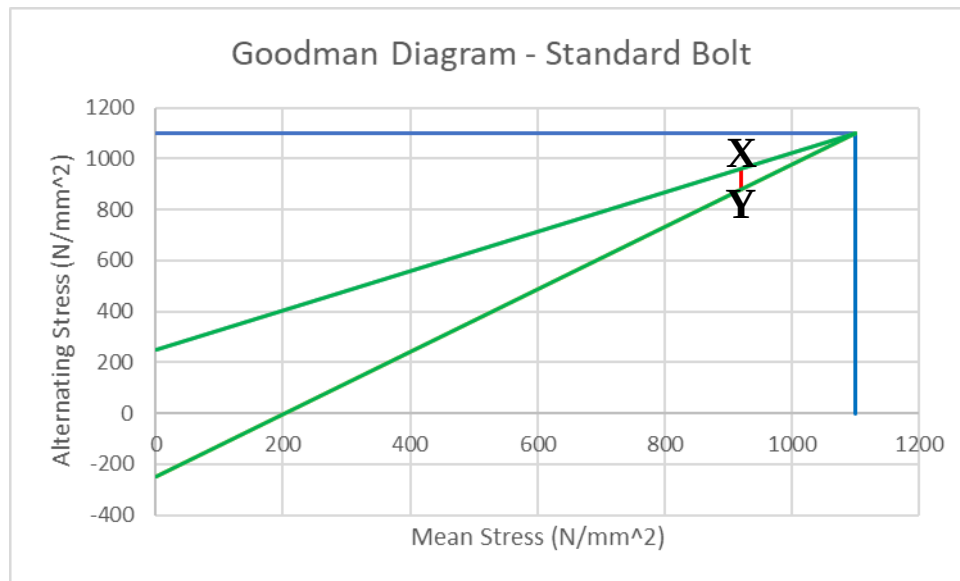
The values for the stresses can be seen in table 5.5. The mean stress is a very large figure close to the UTS. This is expected as the bolt is tightened into yield which as known from Hooke's law (Engineers Edge, 2020), is close to the UTS.

Max Stress	948.4208 N/mm ²
Min Stress	891 N/mm ²
Mean Stress	919.7104 N/mm ²
Range of Stresses	57.42083 N/mm ²

Table 5.5

5.6 Goodman Diagram

The final endurance limit was then calculated using a stress concentration factor of 2 which occurs at the thread root. A goodman diagram was produced as seen in graph 5-1.



Graph 5-1

The allowable range at the mean was calculated to be 81.13 N/mm^2 which resulted in a fatigue safety factor of 1.41, as can be seen in table 5.6. This FSF means the bolts should have an infinite fatigue life. (Bozdana, 2007) The value is close to one which means if the engine was modified, so higher forces were exerted on the bolts, modifications would have to be made to the bolt to keep the FSF at approximately 1.41.

Polished Endurance Limit	495 N/mm^2
Final Endurance Limit	247.5 N/mm^2
Allowable Range at mean	81.13031 N/mm^2
FSF	1.412907

Table 5.6

5.7 Modified Engine

The modified engine produces an increase of over 10,000N of inertia load, see table 5.7. This means a much higher load will be exerted on the connecting rod bolts which would cause them to be more susceptible to breakage.

Max Inertia Load @ 6500 RPM	15096.61 N
Max Inertia Load @ 8500RPM	25816.09 N

Table 5.7

The fatigue safety factor for the standard bolts at the increased load is 0.73, see table 5.8.

Max Stress	989.193 N/mm ²
Min Stress	891 N/mm ²
Mean Stress	940.0965 N/mm ²
Range of Stresses	98.19301 N/mm ²
Polished Endurance Limit	495 N/mm ²
Final Endurance Limit	247.5 N/mm ²
Allowable Range at mean	71.95657 N/mm ²
FSF	0.732807

Table 5.8

This FSF means that the bolts will have a finite fatigue life and last about 10^5 cycles before breakage occur due to fatigue. (Bozdana, 2007) This would be acceptable if the engine was used in a Hillclimb car that would have an engine rebuild every run, however, this is not a normal occurrence.

This means the engine is going to have to be modified. The first area that could be modified is the mass of the piston, gudgeon pin and rod could be reduced. However to achieve the same inertia the mass would have to be reduced by 255kg, which is over two-thirds of the piston mass. This means that it is not practical nor achievable to reduce the engines component mass. This leaves the only option to redesign the bolts.

When redesigning the bolts the same fatigue safety factor would be desired. To accomplish this the same axial stress as a percentage of the UTS would need to be the same. This relationship can be seen in equation 5.16, where the same force is applied in each bolt.

$$\frac{\left(\frac{F}{\pi \frac{D_{average\ old}^2}{4}} \right)}{UTS_{old}} = \frac{\left(\frac{F}{\pi \frac{D_{average\ new}^2}{4}} \right)}{UTS_{new}}$$

Equ. 5.16

This equation can be simplified to equation 5.17 and rearranged to form equation 5.18.

$$\frac{\left(\frac{1}{D_{average\ old}^2} \right)}{UTS_{old}} = \frac{\left(\frac{1}{D_{average\ new}^2} \right)}{UTS_{new}}$$

Equ. 5.17

$$D_{average\ new} = \sqrt{D_{average\ old}^2 \times \frac{UTS_{old}}{UTS_{new}}}$$

Equ. 5.18

The uprated bolts could be made out 8740 chrome moly as this has a 200,000 PSI UTS, which is 1378.95 N/mm². (Burton Power, 2020) Using equation 5.18, where the old diameter is 7.065mm and the UTS is 1100 N/mm², a new average diameter of 6.31mm. This means a waisted shank can be added with a diameter of 5.55mm to achieve the same fatigue safety factor of 1.41.

5.8 Standard and Uprated Bolt Designs

The standard and uprated design, with the new, waisted shank, can be seen in figure 5.1, where the blue arrow is the standard bolts average diameter of 7.065mm and the red arrow is the uprated bolts shank diameter of 5.55mm.

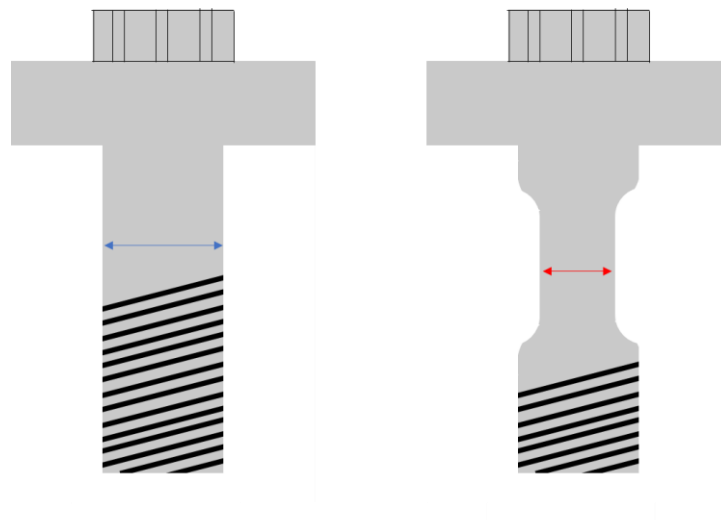


Figure 5.1

The uprated bolt has a higher UTS so a waist was added to reduce the stiffness of the new bolt. This caused a reduction in the ratio of stiffness between the cap and the bolt, which is what is desired by any designer. The reduction in stiffness reduced the extra force that would be exerted on the threads if no waist was added. The reason the waist was also added was to keep the same clamping force on the bolt so that no modification would have to be made to the big end tunnel.

6 Crankshaft

For the crankshaft two areas were looked at for the fatigue safety factors, the first was point A and the second was point B. These locations can be seen in figure 6.1. Point B is the main area of interest as the force is acting further away at that point than from A so the bending moment will be greater.

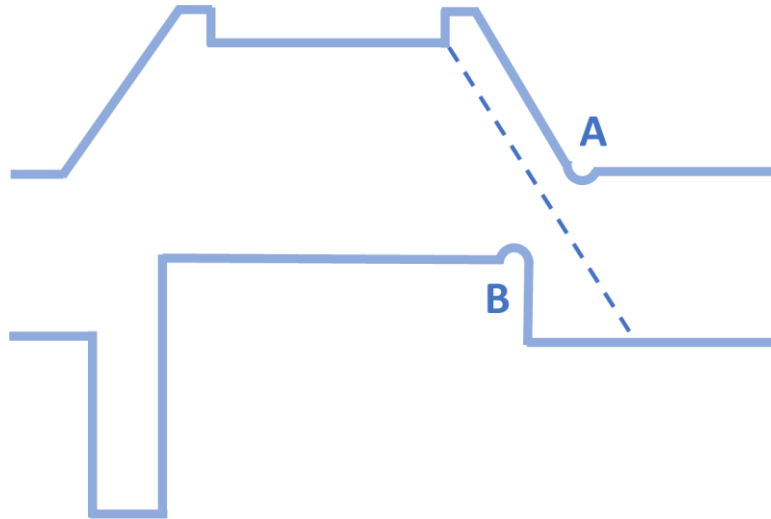


Figure 6.1 – Crankshaft

The compressive and tensile forces, as well as the stress concentration factor, effective width of the beam and the second moment of area, are the same for both point A and point B. These values can be seen in table 6.1.

Maximum Net Compressive Force	33576.44 N
Maximum Net Tensile Force	-14531.2 N
Stress Concentration Factor, K_t	1.6
Effective Width of Beam	43 mm
Second Moment of Area	24578.08 mm ⁴

Table 6.1

The stress concentration factor used was 1.6 (Peterson, 1951) and the effective width of the beam was calculated to be the average of the main journal, 46mm, and crankpin, 40mm.

The tensile force is calculated using equation 6.1. Where ω is 5700RPM calculated in radians per second as this is where the peak net force is under firing. This assumes that the peak pressure occurs at top dead centre however this is not true but including this in the equation does not affect the result by much.

$$F_{tensile} = F_{gas} - \left[\frac{2 \times m_{rod}}{3} R \omega^2 + \left(m_{piston} + m_{pin} + \frac{m_{rod}}{3} \right) R \omega^2 \left(1 + \frac{R}{L} \right) \right]$$

Equ. 6.1 Net tensile force

The compressive force is caused by the inertia at top dead centre on the exhaust stroke and in equation 6.2 omega is 6000RPM in radians per second.

$$F_{compressive} = \frac{2 \times m_{rod}}{3} R \omega^2 + \left(m_{piston} + m_{pin} + \frac{m_{rod}}{3} \right) R \omega^2 \left(1 + \frac{R}{L} \right)$$

Equ. 6.2 Net compressive force

The second moment of area is calculated using equation 6.3 where b is the effective width of the beam and D is the distance between point A and B, which was measured to be 19mm.

$$I = \frac{b \times D^3}{12}$$

Equ. 6.3 - Second moment of area

The bending moment is calculated using only half of the net force as the force exerted on the main journal is split evenly between each side of the crankpin. The distance L is how far the force is acting from each point. For point A this distance was measured to be 13mm and 39mm for point B.

$$M_B = \frac{F_{net}}{2} \times L$$

Equ. 6.4 Bending moment

The bending stress is similar to the equation in section 3 however, rather than a radius being used, it was half of distance AB.

$$\sigma_B = \frac{M_B \times \frac{D}{2}}{I}$$

Equ. 6.5 Bending stress

The shear stress was calculated using equation 6.6. And the mean and range of stresses were calculated the same as previously, using equations in section 3. Likewise, the

polished and final endurance limit were calculated the same as in the previous sections but with a UTS of 750 N/mm² (Bosch, 2011).

$$\tau = \frac{\frac{F_{net}}{2}}{A}$$

Equ. 6.6 Shear stress

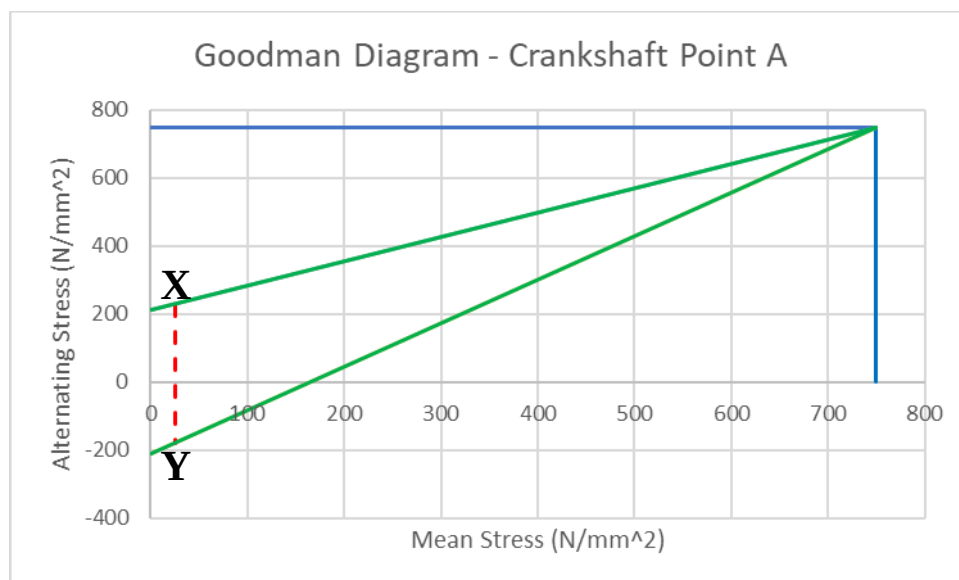
6.1 Point A

The values calculated at point A can be seen in table 6.2.

	Point A	
	Tensile	Compressive
Bending Moment	218246.9 Nmm	-94452.9 Nmm
Bending Stress	84.35748 N/mm ²	-36.5082 N/mm ²
Shear Force	16788.22 N	-7265.6 N
Shear Stress	12.7159 N/mm ²	-5.50319 N/mm ²
Resultant Stress	88.10767 N/mm ²	-38.1312 N/mm ²
Mean Stress	24.98822 N/mm ²	
Range of Stresses	126.2389 N/mm ²	

Table 6.2

These values were then used to plot a Goodman diagram. From this diagram, the allowable range was calculated.



Graph 6-1

The fatigue safety factor was calculated to be 3.23 at point A, see table 6.3. It is expected to be a large value at A as there is a smaller bending moment acting, therefore, a smaller

stress maximum stress. This means a smaller actual range is calculated therefore a much larger fatigue safety factor is expected.

Polished Endurance Limit	337.5	N/mm ²
Final Endurance Limit	210.9375	N/mm ²
Allowable Range at mean	407.8191	N/mm ²
FSF	3.230535	

Table 6.3

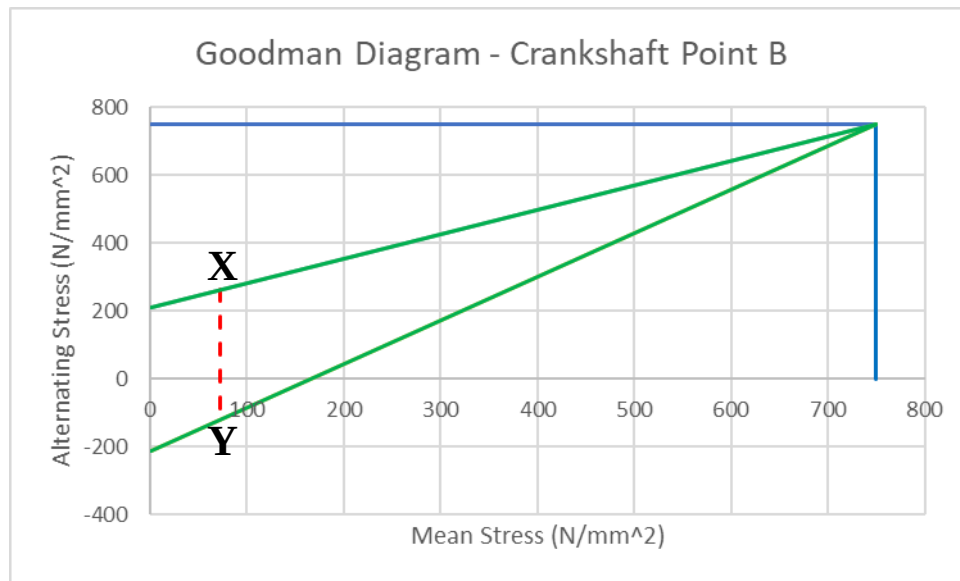
6.2 Point B

The values that were calculated at point B can be seen in table 6.4. The values for the bending moment are much larger, as discussed previously, due to the increased distance between the point and the force. However the shear force and stress are the same and this is due to the distance where the force is acting is irrelevant and just the magnitude is used, and for both point A and B they both experience the same magnitude of shear force.

	Point B	
	Tensile	Compressive
Bending Moment	654740.6 Nmm	-283359 Nmm
Bending Stress	253.0724 N/mm ²	-109.525 N/mm ²
Shear Force	16788.22 N	-7265.6 N
Shear Stress	12.7159 N/mm ²	-5.50319 N/mm ²
Resultant Stress	254.3471 N/mm ²	-110.076 N/mm ²
Mean Stress	72.13539 N/mm ²	
Range of Stresses	364.4234 N/mm ²	

Table 6.4

The final endurance limit is the same as the previous point as they have been calculated to have the same UTS and stress concentration factor. These values were used to plot a goodman diagram. It looks very similar to the previous diagram however the mean is slightly larger at point B with a value of 72.14 N/mm² to point A's 24.99 N/mm². This means the allowable range, XY, will be smaller at point B as the green lines will be closer together.



Graph 6-2

The allowable range was calculated to be smaller which means that the fatigue safety factor is also smaller with a value of 1.05, see table 6.5.

Polished Endurance Limit	337.5 N/mm ²
Final Endurance Limit	210.9375 N/mm ²
Allowable Range at mean	381.2988 N/mm ²
FSF	1.046307

Table 6.5

This value is very close to one which means it only is just deemed to have an infinite fatigue safety factor. (Bozdana, 2007) However, for an area of such importance, the fatigue safety factor would be expected to be slightly larger. This value could have been incorrectly calculated by measuring the crankpin and main journal dimensions incorrectly. To get an improved value the crankshaft could be cut along the neutral axis so that more accurate measurements could be made which could result in an expected larger fatigue factor of safety to be calculated.

7 Limitations and Improvements

The limitation of this investigation was the accuracy of the measurement. For example, when investigating the various points on the connecting rod, measurements were taken using vernier callipers and rulers which would have user errors in the recorded dimensions. To improve this issue the connecting rod could be cut at the various points so that the cross-sectional area could be seen as a two-dimensional image and then measured. Another limitation is the values obtained for the young's modulus and UTS of the materials are fixed values from Ford where in fact each value would be within tolerance. To improve the calculations the values for the maximum and minimum tolerances could be used so a range of fatigue safety factors could be calculated.

Another limitation is the accuracy of one of the methods used. The Goodman diagram was used however this diagram makes a vital assumption that the ultimate tensile strength is the limit of fatigue strength as the alternating stress approaches zero (Palmer, 2014). Also in an investigation, the Goodman diagram predicted survival of one million cycles when in fact the component failed between 250,000 and 300,000 cycles (Palmer, 2014).

8 Conclusion

To conclude, the investigation into the various working safety factors for the infinite fatigue life of a standard Ford Stiga engine proved that all the components are made to last for life. This would be expected in its automotive discipline. However, if this was to be used in the motorsport industry many of these parts could be reduced in mass for the standard power, or increased power for the standard parts. As the engines will be rebuilt regularly they are not needed to be deemed able to last forever. Certain components such as connecting rod, its bolts and cap and the gudgeon pin will be changed more regularly than the crankpin (Bowen, 2009). This means the crankpin still needs a relatively high fatigue safety factor, close to one, whereas the other components could be less than one and still function as desired.

Bibliography

- American Society for Metals, 1985. *Metals Handbooks: Desk Edition*. Desk ed. Ohio: ASM International.
- American Society for Metals, 1990. Properties and Selections Irons, Steels and High-Performance Alloys. In: G. F. Vander Voort, ed. *Metals Handbook*. Ohio: ASM International.
- Bosch, R., 2011. *Automotive Handbook*. 7th ed. New Jersey: Wiley.
- Bowen, R., 2009. *Rebuild it right: Renovation & Restoration: Grassroots Motorsport*. [Online]
Available at: <https://grassrootsmotorsports.com/articles/rebuild-it-right/?fbclid=IwAR0EW4pLvZWRAkQ0uMa8Awkd1A4siNAeERvPGcF5ySqV1iA0nBW8oDwb8PQ>
[Accessed 2 February 2020].
- Bozdana, D. A. T., 2007. *Engineering Materials*. Gaziantep: Mechanical Engineering - University of Gaziantep.
- Burton Power, 2020. *ARP con rod bolt set*. [Online]
Available at: https://www.burtonpower.com/arp-con-rod-bolt-set-ford-zetec-e-1-6-1-8-2-0-arpbe05.html?utm_source=google&utm_medium=prodlisad&utm_campaign=gb_feed?WbRf=ProdListAd&gclid=CjwKCAiA-P7xBRAvEiwAow-VaZuVJpYGNCPERhxV69zI1hgeklLYkt_DHsCa738hia3stJjM5qVIORoChnYQA
[Accessed 05 02 2020].
- Engineers Edge, 2020. *Yield Strength: Material Science: Engineers Edge*. [Online]
Available at: https://www.engineersedge.com/material_science/yield_strength.htm
[Accessed 2 February 2020].
- Hearn, E. J., 1999. Rings, Discs and Cylinders Subjected to Thermal Gradients. In: *Mechanics of Materials 2: The Mechanics of Elastic and Plastic Deformation of Solids and Structural Materials*. 3rd ed. Oxford: Butterworth-Heinemann, p. 138.
- Palmer, D., 2014. *The Goodman Diagram: Design News*. [Online]
Available at: <https://www.designnews.com/electronics-test/why-goodman-diagram-not-so-good/172851655733023?fbclid=IwAR37w5ib3KGjHb2vhYkj66g4uVowDELe6qVPkc96ogHb5S7NiwYZhOJBQYw>
[Accessed 2 February 2020].
- Peterson, R. E., 1951. *Design Factors for Stress Concentration*. 23 ed. New York: Penton Media Inc..

Appendices

Appendix A – Question One Formula View

	A	B	C	D	E	F
3		Maximum Net Compressive Force	-7340.17	N		
4		Maximum Net Tensile Force	40953.84	N		
5		Second Moment of Area, I	$=(\pi()/64)*((20^4)-(12.5^4))$	mm ⁴		
6		Cross Sectinal Area, A	$=(\pi()/4)*((20^2)-(12.5^2))$	mm ²		
7		Gudgeon Pin Mass	$=\pi()*((0.01^2)-(0.00625^2))*7800*0.018$	kg		
8		Gudgeon Pin Inertia Comp	$=(-C3/0.3504)*C7$	N		
9		Gudgeon Pin Inertia Tensile	$=(6624.50695869117/0.3504)*C7$	N		
10						
11			Compressive		Tensile	
12		Bending Moment	$=((C3/2)*15.8)-(C8*10.5)$	Nmm	$=((C4/2)*15.8)-(C9*10.5)$	Nmm
13		Bending Stress	$=(C12*10)/C5$	N/mm ²	$=(E12*10)/C5$	N/mm ²
14		Shear Force	$=(C3/2)-(C8)$	N	$=(C4/2)-(C9)$	N
15		Shear Stress	$=C14/C6$	N/mm ²	$=E14/C6$	N/mm ²
16		Resultant Stress	$=\text{SQRT}((C13^2)+4*(C15^2))$	N/mm ²	$=\text{SQRT}((E13^2)+4*(E15^2))$	N/mm ²
17						
18						
19		Mean Stress	$=(C16+E16)/2$	N/mm ²		
20		Range of Stresses	$=E16-C16$	N/mm ²		
21						
22		Endurance Limit	$=0.45*1000$	N/mm ²		
23		Allowable Range at mean	$=2*C22*((1000-C19)/1000)$	N/mm ²		
24		FSF	$=C23/C20$			

Appendix B – Question Two Formula View

	C	D	E	F
1				
2		Point A		
3		Maximum Net Compressive Force	=-46690.856992166+6624.50695869117	N
4		Maximum Net Tensile Force	7340.17	N
5		Stress Concentration Factor, Kt	1.5	
6		Area per side, A	83.67	mm^2
7				
8		Compressive Stress	=E3/(2*E6)	N/mm^2
9		Tensile Stress	=E4/(2*E6)	N/mm^2
10		Mean Stress	=(E9+E8)/2	N/mm^2
11		Range of Stresses	=E9-E8	N/mm^2
12				
13		Polished Endurance Limit	=0.45*750	N/mm^2
14		Final Endurance Limit	=E13/E5	N/mm^2
15		Allowable Range at mean	=2*E14*((750-E10)/750)	N/mm^2
16		FSF	=E15/E11	

	I	J	K	L
1				
2		Point B		
3		Maximum Net Compressive Force	=-46690.856992166+((6624.50695869117/0.3504)*(0.3504+((1/3)*0.4064)))	N
4		Maximum Net Tensile Force	=(E4/0.3504)*(0.3504+((1/3)*0.4064))	N
5		Stress Concentration Factor, Kt	1.9	
6		Cross Sectional Area, A	277.95	mm^2
7				
8		Compressive Stress	=K3/K6	N/mm^2
9		Tensile Stress	=K4/K6	N/mm^2
10		Mean Stress	=(K9+K8)/2	N/mm^2
11		Range of Stresses	=K9-K8	N/mm^2
12				
13		Polished Endurance Limit	=0.45*750	N/mm^2
14		Final Endurance Limit	=K13/K5	N/mm^2
15		Allowable Range at mean	=2*K14*((750-K10)/750)	N/mm^2
16		FSF	=K15/K11	

	L	O	P	Q	R
1					
2			Point C		
3	N		Maximum Net Compressive Force	0	N
4	N		Maximum Net Tensile Force	=(E4/0.3504)*(0.3504+((2/3)*0.4064))	N
5			Stress Concentration Factor, Kt	1.8	
6	mm^2		Area per side, A	142.8	mm^2
7					
8	N/mm^2		Compressive Stress	=Q3/(2*Q6)	N/mm^2
9	N/mm^2		Tensile Stress	=Q4/(2*Q6)	N/mm^2
10	N/mm^2		Mean Stress	=(Q9+Q8)/2	N/mm^2
11	N/mm^2		Range of Stresses	=Q9-Q8	N/mm^2
12					
13	N/mm^2		Polished Endurance Limit	=0.45*750	N/mm^2
14	N/mm^2		Final Endurance Limit	=Q13/Q5	N/mm^2
15	N/mm^2		Allowable Range at mean	=2*Q14*((750-Q10)/750)	N/mm^2
16			FSF	=Q15/Q11	

Appendix C – Question Three Formula View

	A	B	C	D
1				
2				
3	a	Cap Area (Ac)	=153.1625-C4	mm^2
4		Bolt Area (Ab)	=(PI()*7.065^2)/4	mm^2
5		Ratio of Stiffness (Ac/Ab)	=C3/C4	
6				
7	b	Bolt Stiffness (Kb)	=((210*10^3)*C4)/25.5	N/mm
8		Cap Stiffness (Kc)	=((170*10^3)*C3)/25.5	N/mm
9		Actual Ratio of Stiffness (Kc/Kb)	=C8/C7	
10				
11				
12	c	Max Inertia Load	=((0.270933-0.1038)*0.0407+(0.3504+0.135467)*0.0407*(1+(0.0407/0.134)))*(2*PI()*((6500/60))^2	N
13		Max Alternating Load on Bolt	=(1/(1+C9))*(C12/2)	N
14				
15				
16	d	Static Axial Bolt Force @ 90% yield	=C4*((1100*0.9)*0.6)	N
17		Torque to overcome Thread	=(7.065/2)*0.18*C16	Nmm
18		Torque to overcome Head	=((7.065+12.9)/4)*0.12*C16	Nmm
19		Required Tightening Torque	=C18+C17	Nmm
20		Required Tightening Torque	=C19/1000	Nm
21				
22				
23	e	Max Stress	=C24+C26	N/mm^2
24		Min Stress	=(1100*0.9)*0.9	N/mm^2
25		Mean Stress	=(C24+C23)/2	N/mm^2
26		Range of Stresses	=C13/(PI()*((7.065^2/4))	N/mm^2
27				
28				
29	f	Polished Endurance Limit	=0.45*1100	N/mm^2
30		Final Endurance Limit	=C29/2	N/mm^2
31		Allowable Range at mean	=2*C30*((1100-C25)/1100)	N/mm^2
32		FSF	=C31/C26	
33				

Appendix D – Question Four Formula View

	A	B	C	D
1				
2		Crankshaft		
3		Maximum Net Compressive Force	$=-(0.270933*0.0407+(0.3504+0.135467)*0.0407*(1+(0.0407/0.134)))*(2*\pi()* (6000/60))^2$	N
4		Maximum Net Tensile Force	$=46690.856992166-(0.270933*0.0407+(0.3504+0.135467)*0.0407*(1+(0.0407/0.134)))*(2*\pi()* (5700/60))^2$	N
5		Stress Concentration Factor, Kt	1.6	
6		Effective Width of Beam	$=(40+46)/2$	mm
7		Second Moment of Area	$=(C6*(19^3))/12$	mm ⁴

	A	B	C	D	E	F	G	H	I	J
5		Stress Concentration Factor, Kt	1.6							
6		Effective Width of Beam	$=(40+46)/2$	mm						
7		Second Moment of Area	$=(C6*(19^3))/12$	mm ⁴						
8										
9										
10										
11										
12										
13										
14										
15										
16										
17										
18										
19										
20										
21										
22										
23										

	A	B	C	D	E	F	G	H	I	J
10										
11										
12										
13										
14										
15										
16										
17										
18										
19										
20										
21										
22										
23										