

FACTORISING
SOLUTION TO EXERCISES

EXERCISE 1

1. Factorise: $a^2 - 9b^2$

$$a^2 - 9b^2 = a^2 - (3b)^2 = (a + 3b)(a - 3b)$$

2. Factorise: $c^3 - 25c$

$$c^3 - 25c = c(c^2 - 25) = c(c^2 - 5^2) = c(c + 5)(c - 5)$$

3. Factorise: $d^2 - 14d + 49$

$$d^2 - 14d + 49 = d^2 - 2 \times d \times 7 + 7^2 = (d - 7)^2$$

4. Factorise: $4e^2 + 12ef + 9f^2$

$$4e^2 + 12ef + 9f^2 = (2e)^2 + 2 \times 2e \times 3f + (3f)^2 = (2e + 3f)^2$$

5. Factorise: $18g^2 - 12g + 2$

$$18g^2 - 12g + 2 = 2(9g^2 - 6g + 1) = 2[(3g)^2 - 2 \times 3g \times 1 + 1^2] = 2(3g - 1)^2$$

6. Factorise: $h^4 - 18h^2 + 81$

$$\begin{aligned} h^4 - 18h^2 + 81 &= (h^2)^2 - 2 \times h^2 \times 9 + 9^2 = (h^2 - 9)^2 \\ &= (h^2 - 3^2)^2 \\ &= ((h + 3)(h - 3))^2 \\ &= (h + 3)^2(h - 3)^2 \end{aligned}$$

7. Factorise: $j^3 - 125$

$$\begin{aligned} j^3 - 125 &= j^3 - 5^3 = (j - 5)(j^2 + j \times 5 + 5^2) \\ &= (j - 5)(j^2 + 5j + 25) \end{aligned}$$

8. Factorise: $27k^3 + 8$

$$\begin{aligned} 27k^3 + 8 &= (3k)^3 + 2^3 = (3k + 2)[(3k)^2 - 3k \times 2 + 2^2] \\ &= (3k + 2)[9k^2 - 6k + 4] \end{aligned}$$

9. Factorise: $64 - l^6$

$$\begin{aligned} 64 - l^6 &= 8^2 - (l^3)^2 = (8 + l^3)(8 - l^3) \\ &= (2^3 + l^3)(2^3 - l^3) \\ &= (2 + l)(2^2 - 2 \times l + l^2)(2 - l)(2^2 + 2 \times l + l^2) \\ &= (2 + l)(2 - l)(4 + 2l + l^2)(4 - 2l + l^2) \end{aligned}$$

10. Factorise: $2 - 20m^2 + 50m^4$

$$\begin{aligned} 2 - 20m^2 + 50m^4 &= 2(1 - 10m^2 + 25m^4) \\ &= 2[1^2 - 2 \times 1 \times 5m^2 + (5m^2)^2] \\ &= 2(1 - 5m^2)^2 \end{aligned}$$

11. Factorise: $(n + 2)^2 - 6(n + 2) + 9$

$$\begin{aligned} (n + 2)^2 - 6(n + 2) + 9 &= (n + 2)^2 - 2 \times (n + 2) \times 3 + 3^2 \\ &= [(n + 2) - 3]^2 \\ &= (n - 1)^2 \end{aligned}$$

12. Factorise: $4 - 4(1 - 3p) + (1 - 3p)^2$

$$\begin{aligned} 4 - 4(1 - 3p) + (1 - 3p)^2 &= 2^2 - 2 \times 2 \times (1 - 3p) + (1 - 3p)^2 \\ &= [2 - (1 - 3p)]^2 \\ &= (1 + 3p)^2 \end{aligned}$$

13. Factorise: $(3q - 2r)^2 - (2r - s)^2$

$$\begin{aligned} (3q - 2r)^2 - (2r - s)^2 &= [(3q - 2r) + (2r - s)][(3q - 2r) - (2r - s)] \\ &= (3q - s)(3q - 4r + s) \end{aligned}$$

14. Factorise: $(t + 3u)^2 - (2u + v)^2$

$$\begin{aligned} (t + 3u)^2 - (2u + v)^2 &= [(t + 3u) + (2u + v)][(t + 3u) - (2u + v)] \\ &= (t + 5u + v)(t + u - v) \end{aligned}$$

EXERCISE 2

1. Factorise: $a^2 - 5a + 6$

$$\begin{aligned} a^2 - 5a + 6 &= \left(a - \frac{-(-5) + \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1}\right) \left(a - \frac{-(-5) - \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1}\right) \\ &= \left(a - \frac{5 + 1}{2}\right) \left(a - \frac{5 - 1}{2}\right) \\ &= (a - 3)(a - 2) \end{aligned}$$

2. Factorise: $b^2 + 10b + 9$

$$\begin{aligned} b^2 + 10b + 9 &= \left(b - \frac{-10 + \sqrt{10^2 - 4 \times 1 \times 9}}{2 \times 1}\right) \left(b - \frac{-10 - \sqrt{10^2 - 4 \times 1 \times 9}}{2 \times 1}\right) \\ &= \left(b - \frac{-10 + 8}{2}\right) \left(b - \frac{-10 - 8}{2}\right) \\ &= (b + 1)(b + 9) \end{aligned}$$

3. Factorise: $c^2 + 3c - 10$

$$\begin{aligned}c^2 + 3c - 10 &= \left(c - \frac{-3 + \sqrt{3^2 - 4 \times 1 \times (-10)}}{2 \times 1}\right) \left(c - \frac{-3 - \sqrt{3^2 - 4 \times 1 \times (-10)}}{2 \times 1}\right) \\&= \left(c - \frac{-3 + 7}{2}\right) \left(c - \frac{-3 - 7}{2}\right) \\&= (c - 2)(c + 5)\end{aligned}$$

4. Factorise: $d^2 - 4d - 12$

$$\begin{aligned}d^2 - 4d - 12 &= \left(d - \frac{-(-4) + \sqrt{(-4)^2 - 4 \times 1 \times (-12)}}{2 \times 1}\right) \left(d - \frac{-(-4) - \sqrt{(-4)^2 - 4 \times 1 \times (-12)}}{2 \times 1}\right) \\&= \left(d - \frac{4 + 8}{2}\right) \left(d - \frac{4 - 8}{2}\right) \\&= (d - 6)(d + 2)\end{aligned}$$

5. Factorise: $1 + 2e - 3e^2$

$$\begin{aligned}1 + 2e - 3e^2 &= -3 \left(e - \frac{-2 + \sqrt{2^2 - 4 \times (-3) \times 1}}{2 \times (-3)}\right) \left(e - \frac{-2 - \sqrt{2^2 - 4 \times (-3) \times 1}}{2 \times (-3)}\right) \\&= -3 \left(e - \frac{-2 + 4}{-6}\right) \left(e - \frac{-2 - 4}{-6}\right) \\&= -3 \left(e + \frac{1}{3}\right) (e - 1) \\&= (3e + 1)(1 - e)\end{aligned}$$

6. Factorise: $1 - 6f + 5f^2$

$$\begin{aligned}1 - 6f + 5f^2 &= 5 \left(f - \frac{-(-6) + \sqrt{(-6)^2 - 4 \times 5 \times 1}}{2 \times 5}\right) \left(f - \frac{-(-6) - \sqrt{(-6)^2 - 4 \times 5 \times 1}}{2 \times 5}\right) \\&= 5 \left(f - \frac{6 + 4}{10}\right) \left(f - \frac{6 - 4}{10}\right) \\&= 5(f - 1) \left(f - \frac{1}{5}\right) \\&= (f - 1)(5f - 1)\end{aligned}$$

EXERCISE 3

1. Factorise: $ac + bc - ad - bd$

$$\begin{aligned}ac + bc - ad - bd &= c(a + b) - d(a + b) \\&= (a + b)(c - d)\end{aligned}$$

2. Factorise: $pr + 3ps + 2qr + 6qs$

$$\begin{aligned}pr + 3ps + 2qr + 6qs &= p(r + 3s) + 2q(r + 3s) \\&= (r + 3s)(p + 2q)\end{aligned}$$

3. Factorise: $x^2 + xy - 2xz - 2yz$

$$\begin{aligned}x^2 + xy - 2xz - 2yz &= x(x + y) - 2z(x + y) \\&= (x + y)(x - 2z)\end{aligned}$$

4. Factorise: $2ef - eg - 4fg + 2g^2$

$$\begin{aligned}2ef - eg - 4fg + 2g^2 &= 2f(e - 2g) - g(e - 2g) \\&= (e - 2g)(2f - g)\end{aligned}$$

5. Factorise: $3ab + 4cd - cb - 12ad$

$$\begin{aligned}3ab + 4cd - cb - 12ad &= b(3a - c) - 4d(3a - c) \\&= (3a - c)(b - 4d)\end{aligned}$$

6. Factorise: $10mq - pr - 2pq + 5mr$

$$\begin{aligned}10mq - pr - 2pq + 5mr &= 5m(2q + r) - p(r + 2q) \\&= (2q + r)(5m - p)\end{aligned}$$

7. Factorise: $2st - 3t - 4s + 6$

$$\begin{aligned}2st - 3t - 4s + 6 &= 2s(t - 2) - 3(t - 2) \\&= (t - 2)(2s - 3)\end{aligned}$$

8. Factorise: $8vw - 12w + 6v - 9$

$$\begin{aligned}8vw - 12w + 6v - 9 &= 2v(4w + 3) - 3(4w + 3) \\&= (4w + 3)(2v - 3)\end{aligned}$$

9. Factorise: $15x^2 - 10xy - 3xz + 2yz$

$$\begin{aligned}15x^2 - 10xy - 3xz + 2yz &= 3x(5x - z) - 2y(5x - z) \\&= (5x - z)(3x - 2y)\end{aligned}$$

10. Factorise: $10ps - 2pq + 15qs - 3q^2$

$$\begin{aligned}10ps - 2pq + 15qs - 3q^2 &= 2p(5s - q) + 3q(5s - q) \\&= (5s - q)(2p + 3q)\end{aligned}$$

11. Factorise: $abx + 3abd - 5xc - 15cd$

$$abx + 3abd - 5xc - 15cd = x(ab - 5c) + 3d(ab - 5c)$$

$$= (ab - 5c)(x + 3d)$$

12. Factorise: $pqx + 2pqy - 4ry - 2rx$

$$pqx + 2pqy - 4ry - 2rx = pq(x + 2y) - 2r(x + 2y)$$

$$= (x + 2y)(pq - 2r)$$

13. Factorise: $x^2y^2 - 4y^2 + 5x^2 - 20$

$$x^2y^2 - 4y^2 + 5x^2 - 20 = x^2(y^2 + 5) - 4(y^2 + 5)$$

$$= (y^2 + 5)(x^2 - 4)$$

$$= (y^2 + 5)(x + 2)(x - 2)$$

14. Factorise: $2 - 18a^2 - 3b + 27a^2b$

$$2 - 18a^2 - 3b + 27a^2b = 2(1 - 9a^2) - 3b(1 - 9a^2)$$

$$= (1 - 9a^2)(2 - 3b)$$

$$= (1 + 3a)(1 - 3a)(2 - 3b)$$

15. Factorise: $x^2y^2 - x^2 - 9y^2 + 9$

$$x^2y^2 - x^2 - 9y^2 + 9 = x^2(y^2 - 1) - 9(y^2 - 1)$$

$$= (y^2 - 1)(x^2 - 9)$$

$$= (y + 1)(y - 1)(x + 3)(x - 3)$$

16. Factorise: $3ab - 21a + 9b - 63$

$$3ab - 21a + 9b - 63 = 3b(a + 3) - 21(a + 3)$$

$$= (a + 3)(3b - 21)$$

$$= 3(a + 3)(b - 7)$$

17. Factorise: $4cd + 8d^2 - 8cf - 16df$

$$4cd + 8d^2 - 8cf - 16df = 4d(c + 2d) - 8f(c + 2d)$$

$$= (c + 2d)(4d - 8f)$$

$$= 4(c + 2d)(d - 2f)$$

18. Factorise: $x^3 + 3x^2y - 5x^2z - 15xyz$

$$x^3 + 3x^2y - 5x^2z - 15xyz = x^2(x - 5z) + 3xy(x - 5z)$$

$$= (x - 5z)(x^2 + 3xy)$$

$$= x(x - 5z)(x + 3y)$$

19. Factorise: $ax + ay - az - bx - by + bz$

$$\begin{aligned} ax + ay - az - bx - by + bz &= a(x + y - z) - b(x + y - z) \\ &= (x + y - z)(a - b) \end{aligned}$$

20. Factorise: $cd - 3c - 2d - 2f + cf + 6$

$$\begin{aligned} cd - 3c - 2d - 2f + cf + 6 &= c(d - 3 + f) - 2(d - 3 + f) \\ &= (d - 3 + f)(c - 2) \end{aligned}$$