

EXAMINATION SOLUTIONS

(to accompany the examination paper sent to the external examiner
but NOT to be seen by students)

Programme:	BEng Motorsport & Automotive Engineering	Level:	6
Module:	Advanced Vehicle Dynamics		
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Q	Question	Marks
1 a)	Students should mention • Caster angle results in vertical wheel displacement relative to chassis when wheel is steered. These displacements are in opposite directions each side of the vehicle • The result is that, with a positive caster angle, the inside wheel load is increased and the outside wheel load is decreased • This results in a diagonal load transfer, reducing load transfer at front, increasing at rear • Result is vehicle tends to experience reduced levels of understeer especially at mid corner (maximum steer) • Caster angle also results in camber change with steer which is generally a positive effect.	1 1 1 1 1



b) $m = 5 \text{ kg}$
 $c = 2 \text{ Ns/m}$
 $k = 10 \text{ N/m}$

i) $m \frac{\partial^2 x}{\partial t^2} + c \frac{\partial x}{\partial t} + kx = 0$

$$5 \frac{\partial^2 x}{\partial t^2} + 2 \frac{\partial x}{\partial t} + 10x = 0 \quad \checkmark$$

ii) $x = e^{rt} \quad \frac{\partial x}{\partial t} = r e^{rt} \quad \frac{\partial^2 x}{\partial t^2} = r^2 e^{rt}$

$$\therefore e^{rt} [5r^2 + 2r + 10] = 0 \quad \checkmark$$

$$r = \frac{-2 \pm \sqrt{4^2 - (4 \times 5 \times 10)}}{2 \times 5}$$

$$r = \frac{-2 \pm \sqrt{-184}}{10}$$

$$r = -\frac{1}{5} \pm i \frac{\sqrt{184}}{10} \quad \alpha = -\frac{1}{5} \quad \beta = \frac{\sqrt{184}}{10} \quad \checkmark$$

$$\therefore x = \left[A \cos\left(\frac{\sqrt{184}}{10} t\right) + B \sin\left(\frac{\sqrt{184}}{10} t\right) \right] e^{-\frac{1}{5}t} \quad \checkmark$$

When $t=0, x=0, \frac{\partial x}{\partial t} = 4$

$$0 = [A(1) + B(0)](1) \quad \therefore \underline{A=0} \quad \checkmark$$

$$x = \left[B \sin\left(\frac{\sqrt{184}}{10} t\right) \right] e^{-\frac{1}{5}t}$$

$$\frac{\partial x}{\partial t} = e^{-\frac{1}{5}t} \left[B \frac{\sqrt{184}}{10} \cos\left(\frac{\sqrt{184}}{10} t\right) \right] - \frac{1}{5} e^{-\frac{1}{5}t} \left[B \sin\left(\frac{\sqrt{184}}{10} t\right) \right] \quad \checkmark$$



when $t=0$, $\frac{\partial x}{\partial t} = 4$

$$4 = (1) \left[B \frac{\sqrt{184}}{10} (1) \right] - \frac{1}{5} [B (0)]$$

$$4 = B \frac{\sqrt{184}}{10}$$

$$B = \frac{40}{\sqrt{184}}$$

$$x = \left[\frac{40}{\sqrt{184}} \sin \left(\frac{\sqrt{184}}{10} t \right) \right] e^{-\frac{1}{5}t}$$

iii)

iii)

$$\zeta = 1.0$$

$$\zeta = \frac{c}{2\sqrt{mk}} = 1.0$$

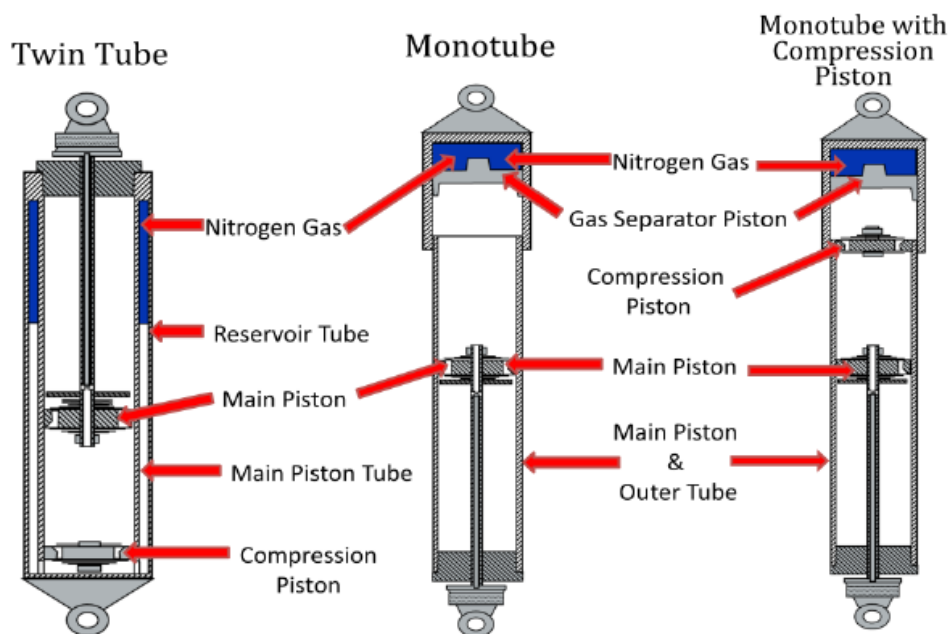
$$c = 2\sqrt{mk} = 2\sqrt{50} = 14.14 \text{ Ns/m.}$$



c)

Students should draw three main damper constructions, twin tube, monotube with compression piston, monotube without compression piston.

1 Mark for each correct illustration with labels.



3

Recognise Nitrogen gas allows for rod displacement. It has an effect of preloading damper and as the gas is pressurised it helps to prevent cavitation.

1

Explain that as twin tubes don't separate oil and gas they need to be installed in the upright position to prevent gas/oil mixing.

1

Q 2	Question	Marks
2.a	Calculate the distance from the front axle to the Centre of Gravity : distance 'a'. The weight spit is 58% rear and 42% front. So the centre of gravity will be closer to the rear. Distance to front = wheelbase x rear weight bias = 2200 mm x 0.58 = 1276 mm	2

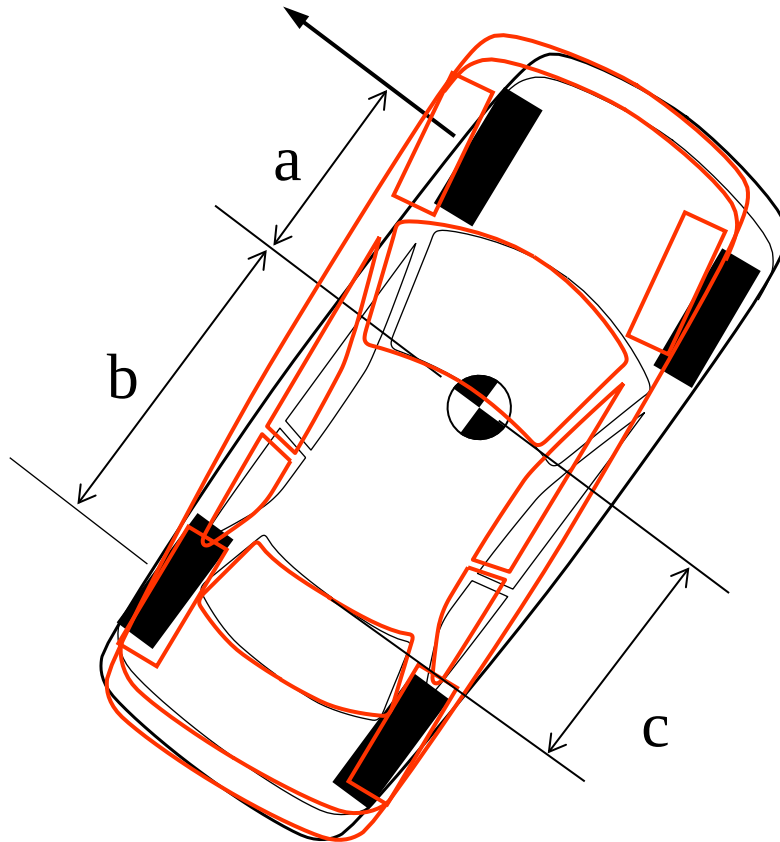


2.b	Calculate the rotational inertia of the vehicle I_{zz} $I_{zz} = (M1 \times r1^2) + (M2 \times r2^2)$ $I_{zz} = (429 \text{ kgs} \times 0.9\text{m}^2) + (580 \text{ kgs} \times 0.8\text{m}^2)$ $I_{zz} = 340.2 \text{ kg.m}^2 + 371 \text{ kg.m}^2$ $I_{zz} = 711.4 \text{ kg.m}^2$	2
2.c	Calculate the distance from the Centre of Gravity to the Inertia Conjugate. Which is distance 'c' on the diagram. Distance of the Centre of Percussion behind the Center of Gravity, (C) Where : $C = I_{zz} / m \times a \quad (\text{where } a \text{ is the distance from front axle to C of G})$ $C = 711.4 \text{ kg.m}^2 / (1000\text{kg} \times 1.276\text{m})$ $C = 0.557 \text{ m}$	2
2.d	Calculate the Dynamic Index (DI) for the vehicle and comment on your findings $DI = \text{Inertia} / (\text{Mass} \times a \times b)$ $DI = 711.4 \text{ kg.m}^2 / (1000 \text{ kgs} \times 1.276\text{m} \times (2.2\text{m} - 1.276\text{m}))$ $DI = 0.6 \quad (\text{which is very low})$ A DI of one is when the centre of percussion is on the rear axle A DI > 1 = a very stable vehicle that is reluctant to 'turn in'. A DI < 1 = a stable vehicle that turns in ok If the DI < 0.8 the vehicle is unstable, turns in too quickly and is difficult to balance/control, especially as the speed increases.	2
2e	a) Draw a diagram to show the effect of the calculated DI on rear tyre slip angle. Analyse the effect of the rear tyre slip angle on vehicle dynamic behaviour. The student should explain that when the DI is less than 1; when the vehicle turns in the rear wheels will move outward building a slip angle and generating lateral grip. Whilst the vehicle is in the corner any additional steering angle application	



will result in rear wheels moving outward building a slip angle. If the rear tyres are on the limit and the grip is saturated this additional slip angle will reduce lateral grip. With a DI of 0.6 the vehicle would respond very quickly, probably too quickly and become ‘undrivable’ on the limit of grip.

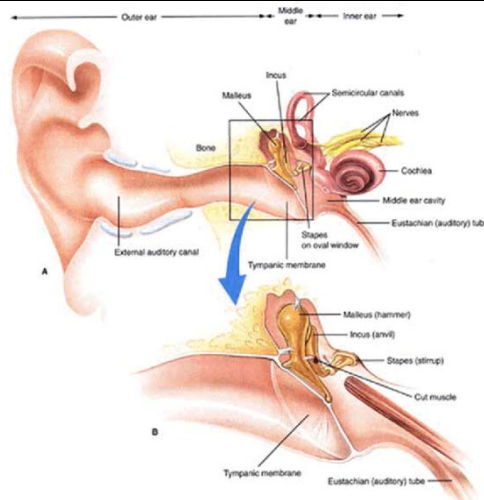
4



2f

Discuss the physiological limitations affecting vehicle driver's perception and response times.

- ☐ Inner ear inputs are decoded along with muscle tensions to unscramble the motion of the support platform – the so-called “seat of the pants”
- ☐ Although less complex than visual processing, there is still a delay around 300 msec associated with this processing route



2

Consequently there are limits to the speed that humans can:
Perceive changing external environment
Process the meaning of the changes
Plan a strategy to respond to the information
Effect changes to the body (steering hand wheel angle and right foot activity)

2g

The vehicle engineers wish to increase the Dynamic Index to 0.8 by placing additional mass in the front and rear bumpers (3.2m apart). The vehicle inertia will increase from 711,4kg.m² to 1095.4kg.m²

1. What will be the new radius of gyration (k) ?

$$DI = \text{mass} \times k^2 / (\text{mass} \times a \times b) = k^2 / a \times b$$

$$k^2 = DI \times a \times b$$

$$k = \sqrt{DI \cdot a \cdot b}$$

$$k^2 = 0.8 \times 1.276\text{m} \times (2.2\text{m} - 1.276\text{m})$$

$$k^2 = 0.8 \times 1.276\text{m} \times 0.924\text{m} = 0.948 \text{ m}$$

$$k = \sqrt{0.943\text{m}}$$

$$k = 0.971\text{m}$$

2. Assume the mass is distributed evenly between the front and rear bumpers. What mass should be placed in each bumper?

Inertia needs to increase to 1095.4 kg.m²
Extra inertia required = 1095.4 kg.m² - 711.4 kg.m² = 384 kg.m²

Masses to be placed in bumpers 3.2 m apart.
So k = 1.6 m
k² = 2.56 m

3



	$I = m.k^2 \quad m = I / k^2$ $\text{Mass} = 384 \text{ kg.m}^2 / 2.56\text{m} = 150 \text{ kg}$ $\text{Mass in each bumper} = 75 \text{ kg}$	3
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Q3	Question	Marks
3a	a) What tractive effort is required to maintain the constant 5 m/s vehicle speed in the snow? Rolling resistance = $1500\text{kg} \times 9.81 \times 0.1 = 1472 \text{ N}$ Aero drag = $0.5 \times c_d \times \text{area} \times \text{density of air} \times \text{velocity}^2$ $= 0.5 \times 0.5 \times 3 \times 1.3 \times 5 \times 5 = 23\text{N}$ Therefore total drive force required = $1472 + 23 = 1495\text{N}$	2
3b	What is the centripetal acceleration? At 5 m/s Centripetal accel = $\text{vel, vel/rad} = 5 \times 5 / 15 = 1.67 \text{ m/s/s}$	2
3c	What is the vehicle yaw rate in a steady state condition? Circumference = $\pi \times \text{dia} = 3.14 \times 30 = 94.2\text{m}$ $94.2\text{m}/5\text{m/s} = 18.84 \text{ sec/lap}$ Yaw rate = $360/18.84 = 19.1 \text{ degs/sec}$ or $2 \times \pi / 18.84 = 0.33 \text{ rads/sec}$	2
3d	Assuming the spring rate is constant front and rear. What is the weight on each wheel? Static weight on each wheel is $1500\text{kg} \times 9.81 / 4 = 3679\text{N}$ Weight transfer = $\text{CofG height} \times \text{mass}/2 \times \text{centripetal accel} / \text{track}$ $= 0.6\text{m} \times 1500\text{kg}/2 \times 1.67\text{m/s/s} / 1.65$ $= 455\text{N}$ on each axle Therefore: Left side outer wheels $3679\text{N} + 455\text{N} = 4134\text{N}$ each Right side inner wheels = $3679\text{N} - 455\text{N} = 3224 \text{ N}$	2
3e	What is the lateral force on each axle ? $1500\text{kg}/2 \times 1.67 = 1253\text{N}$	1



3f	What is the lateral force resistance on the inside and the outside wheels? Each wheel has the same amount of slip therefore the lat grip will be shared out in proportion to the vertical loading. Total vert force on one axle = $750\text{kg} \times 9.81 = 7358\text{N}$ Left side outer wheel loading percentage = $4134 / 7358 = 56.2\%$ Right side inner wheel = $3224 / 7358 = 43.8\%$ Therefore Lat force resistance on outside wheel = $56.2\% \times 1253\text{N} = 704\text{N}$ Lat force resistance on inside wheel = $43.8\% \times 1253 = 549\text{N}$	3
3g	The driver senses danger and lifts her foot from the accelerator pedal. What will be the rate of deceleration ? Rolling resistance from part a) was 1495N Force = Mass x Acceleration Acceleration = Force / Mass = $1495\text{N} / 1500 = 0.996 \text{ m/s}^2$ Draw and label two frictions circles with the longitudinal and lateral forces. One for the front outer wheel and one for the front inner wheel. The diagram shows two circles representing wheels. The left circle is smaller and contains a right-angled triangle with a hypotenuse of 967N, a horizontal base of 548N, and a vertical height of 704N. The right circle is larger and contains a right-angled triangle with a hypotenuse of 1240N, a horizontal base of 704N, and a vertical height of 1022N. Calculate the maximum possible braking force could be applied before the inside wheel forces exceed the available grip. Using Pythagoras $(1240\text{N}^2) - (704\text{N}^2) = \text{longitudinal brake force grip}^2$ Therefore available longitudinal brake force available from the outside wheel = 1022N	2



	Using Pythagoras $(967\text{N}^2) - (548\text{N}^2) = \text{longitudinal brake force grip}^2$ Therefore available longitudinal brake force available from the inside wheel = 797N	
3h	What initial deceleration could the vehicle achieve before locking the inside wheel? (assume no longitudinal weight transfer) Brakes will apply the same braking force to each wheel. Max force before lock occurs = 797N (from part e)) $797\text{ N} \times \text{four wheels} = 3188\text{ N}$ Deceleration (due to brakes) = force / mass = $3188\text{N} / 1500\text{Kg}$ = 2.13 m/s^2 Plus of course the rolling resistance deceleration. (from part g) So total deceleration = $2.13 + 0.996 = 3.12\text{ m/s}^2$ With all the wheels locked there would be little lateral grip and the vehicle would slide nearly directly forward.	1
3i	The all wheel Anti-lock braking system control system wishes to provide maximum deceleration. What is the maximum deceleration on the curve? Discuss limiting factors. The brake force is no longer symmetrical but can be asymmetrical. Inside wheel longitudinal braking effort available 797N Outside wheel longitudinal tractive effort available = 1022N Total deceleration effort from all four wheels. = $(1022\text{N} \times 2) + (797\text{N} \times 2) = 2044\text{N} + 1594\text{N}$ Total front braking effort = 3638N Potential new acceleration = $3638\text{N} / 1500\text{kg} = 0.243\text{ m/s}^2$ The driver is on the limit of lateral grip, which is low in the snow. Potential for more deceleration is limited. With front wheel drive only, the rear of the car has some spare lateral grip. Any additional braking effort would lead to potential lateral instability, as the friction circle grip is exceeded.	2
3j	What is the resulting yaw moment on the vehicle? Discuss system compromises and driver reaction requirements.	2



	The front wheel drive is symmetrical. The braking force is asymmetrical. Outside = 2011N Inside = 1594N Resulting asymmetric drive = $2011\text{N} - 1594\text{N} = 417\text{N}$ Yaw moment = Force / half track = $417\text{N} / 1.65\text{m} / 2 = 126.4\text{ Nm}$ This will turn have the effect of causing an anti-clockwise moment. The effect will be to drag the vehicle towards the outside of the circle. The driver will have to apply additional steering lock to maintain the desired path.	
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Q4	Question	Marks
4a	What is the initial maximum rate of acceleration? Vertical force on the front of the vehicle is $750\text{kg} \times 9.81 = 7358\text{N}$ For each front wheel vertical load = 3679N Low μ side grip = $(3679\text{N}) \times 0.1 = 367.9\text{N}$ High μ side grip = $(3679 \times 0.85) = 3127\text{N}$ With an open differential left side traction = right side traction Total tractive effort = $2 \times 367.9\text{ N} = 735.8\text{N}$ acceleration = force / mass = $735.8\text{N} / 1250\text{kg} = 0.589\text{ m/sec}^2$	2
4b	What is the yaw moment generated ? The tractive drive forces are symmetrical. Therefore there is no resulting yaw moment	1
4c	The vehicle is now equipped with a geared type torque biasing differential. Calculate the resulting maximum acceleration Low μ side grip = $(3679\text{N}) \times 0.1 = 367.9\text{N}$ High μ side tractive effort = $(367.9\text{N} \times 3.0 \text{ (torque biasing differential value)}) = 1104\text{N}$ Total tractive force = $368\text{N} + 1104\text{N} = 1472\text{N}$ acceleration = force / mass = $1472\text{N} / 1250\text{kg} = 1.18\text{ m/sec}^2$	2
4d	What is the resulting yaw moment generated?	



	The tractive drive forces are now asymmetric. Differential force = $1104\text{N} - 368\text{N} = 736\text{N}$ The force tries to rotate the car about the centre of gravity. The C of G is in the centre of the car so the moment arm is half the track. Yaw moment = $736\text{N} \times 1.8\text{m} / 2 = 662\text{Nm}$	1
4e	What lateral force from the front steered wheels is required to counter the asymmetric tractive force rotational moment? Yaw moment from asymmetric braking was 662Nm Distance from front axle to centre of gravity = rear mass/total mass $\times$ wheel base = $500\text{kg}/1250\text{kg} \times 2.65\text{m} = 1.06\text{m}$ Torque = force $\times$ radius. Force = torque / radius = $662\text{ Nm} / 1.06\text{m} = 625\text{N}$	2
4f	The tyre on the high μ side provides a tyre cornering stiffness of $1000\text{N}/\text{degree}$. What steering handwheel angle would be required to counter the rotational moment ? The student would be expected to ignore the grip on the low μ side as it is so little. $625\text{N} / 1000\text{N}/\text{deg} = 0.625\text{ degs}$ at the road wheel $0.625\text{ degs} \times 15$ (steering ratio) = 9.4 degrees steering correction required.	3
4g	The vehicle is now equipped with an additional brake based traction control system. Draw a diagram to explain the torque transfer process across the vehicle	3
4h	What would be the maximum rate of acceleration? Now all the available grip can be utilised Low μ side grip = $(3679\text{N}) \times 0.1 = 367.9\text{N}$ High μ side grip = $(3679 \times 0.85) = 3127\text{N}$ Total available grip = $368\text{N} + 3127\text{ N} = 3495\text{N}$	2

	acceleration = force / mass = $3495\text{N} / 1250\text{kg} = 2.78 \text{ m/sec/sec}$	
4i	a) As the vehicle accelerates it reaches 30 kph. At this speed what is the rate of energy passing into the brake disc? $30 \text{ kph} = 30,000 / 60^2 = 8.33 \text{ m/s}$ Original low μ side produced grip of 367.9N Which provided tractive effort on the high μ side of 1104 N However there is 3127N available $3127 \text{ N} - 1104 \text{ N} = 2023\text{N}$ from the brake based system However braking force is multiplied by the torque biasing differential (x3). Therefore braking force = $2023 / 3 = 674 \text{ N}$ Power = force x velocity – $674\text{N} \times 8.33 \text{ m/s} = 5617 \text{ joules / sec or watts}$	3