

FAULT TREE ANALYSIS

Assignment Test Submission One –
The Success of Making a Cup of Tea

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Abstract

Fault Tree Analysis (FTA) is a traditionally used method for analysing system faults and undesired events. In this report, it has been utilised to determine the probability of successfully making a cup of tea with milk and sugar. The overall probability is calculated to be 0.203, providing a 20% chance of success.

The events leading to the top event of making the cup of tea are:

- Supply Hot Water with a success probability of 0.428
- Supply a Cup with a success probability of 0.700
- Supply Ingredients with a success probability of 0.667

The impact of human factors has been considered by exploring the effects of fatigue on the probabilities of success. Under fatigue, the probability of successfully making a cup of tea is reduced to 0.102, which equates to a 10% chance of success.

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1. Introduction

Fault Tree Analysis (FTA) is a method used in system and hazard analysis to identify root causes and assess the likelihood of an event occurring [1]. Although FTA is typically used to analyse an undesired event or a system fault, it can be utilised to determine the probability of success. This report analyses the probability of success of a person making a cup of tea.

2. Background Theory

Fault Tree Analysis (FTA) utilises a graphical representation of possible events leading to a singular top event, in the form of a Fault Tree (FT), to determine the root cause and probability of a specific event [1, 2]. A series of logic gates is used to illustrate the combination of events. Figure 2-1 illustrates a simple FT, while Table 2-1 provides a description of the common logic gates and event symbols used in FTA [3]. In the example given, for the top event (Q) to arise, events A and B need to transpire, while event A requires either event C or D to occur.

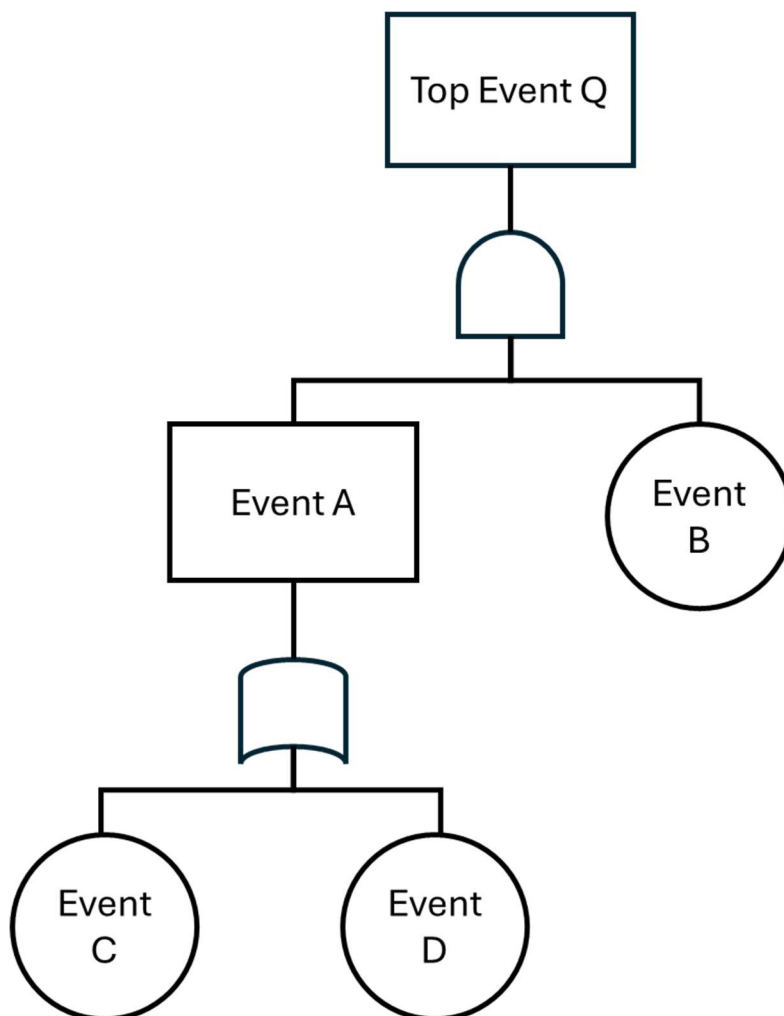
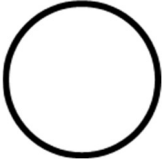


FIGURE 2-1 - A REPRESENTATION OF A FAULT TREE DIAGRAM

TABLE 2-1 - LOGIC GATES AND EVENT SYMBOLS USED IN FAULT TREE ANALYSIS

SYMBOL	TYPE	DESCRIPTION
LOGIC GATES		
	AND Gate	The output only occurs if all the inputs occur together.
	OR Gate	The output only occurs if at least one of the inputs occur.
	Priority AND Gate	The output only occurs if all inputs occur in a given order from left to right.
	Exclusive OR Gate	The output only occurs when one, but not both inputs occur.
EVENT SYMBOLS		
	Primary or Basic Event	An initiating event where sufficient data is available.
	Event	A condition that has happened and can represent either a Top Event or an Intermediate Event.
	Undeveloped Event	A secondary failure or an underdeveloped event which can be explored further.
	Conditioning Event	A specific condition or restriction that can be applied to any gate.

FTA can be conducted qualitatively to present the combination of different possible events leading to the top event, while highlighting the interrelationships between different events [1]. By using it to determine the probability of occurrence, the subsequent evaluation can be used to

generate outputs for decision making related to the acceptability risk and required preventative measures of an event [1]:

1. The probability of occurrence of the event
2. The probability and significance of events causing the top event
3. The risk significance or importance of components

2.1 Probability and Fault Trees

The probability of an event from a Fault Tree (FT) is determined by Boolean algebra, logic and probability formula [1, 4]. Each logic gate used in a FT will have a specified formula to determine the probability of an output event.

2.1.1 AND Gate Probability Formula

When the input events for an AND gate are independent of each other the probability of the output gate is the product of the probability of the input events. Using Figure 2-1 the formula can be expressed as [1, 4]:

$$P(Q) = P(A) \cdot P(B) \quad 2-1$$

2.1.2 OR Gate Probability Formula

The probability of an OR gate is dependent on the relationship between the input events. If the events are mutually exclusive where both cannot exist at the same time, the probability of event A in Figure 2-1 would be determined by [4]:

$$P(A) = P(C) + P(D) \quad 2-2$$

For events which are not mutually exclusive where both could exist at the same time, the probability of event A would be calculated using [4, 1]:

$$P(A) = P(C) + P(D) - P(C) \cdot P(D) \quad 2-3$$

When the input events are dependent on other events, for example event D is dependent on event C, whenever event C occurs, D also occurs, the probability of A is determined by [4]:

$$P(A) = P(C) + P(D) - P(C) = P(D) \quad 2-4$$

3. Calculation of Probabilities

The Fault Tree for a person successfully making a cup of tea is given in Figure 3-1 and a list of the event probabilities is provided in Table 3-1.

TABLE 3-1 - EVENT PROBABILITIES FOR MAKING A CUP OF TEA

REFERENCE	EVENT	PROBABILITY
A1.1	Chance of Water Supply	0.99
A1.2	Chance of No Leaks in Kettle	0.9
A2.1	Plug In Operation Reliability	0.7
A2.2	Switch on Reliability	0.7
A2.3	Mains Supply Reliability	0.99
A2.4	Kettle Reliability	0.99
B1	Cup Reliability	0.7
B2	Cup Availability	1.0
C1.1	Tea Availability	0.8
C1.2	Coffee Availability	0.9
C2.1	Sugar Availability	0.6
C2.2	Sweetener Availability	0.9
C3.1	Milk Availability	0.9
C3.2	Milk Freshness	0.8

To calculate the probability of successfully making a cup of tea with milk and sugar, the events leading up to it need to be considered and evaluated. These are:

- Event A – Supply of Hot Water
- Event B – Supply of a Cup
- Event C – Supply of Ingredients

Each event will have its own probability of success which need to be calculated. In this scenario the input events for the OR gates are not mutually exclusive therefore the probability is calculated using equation 2-3.

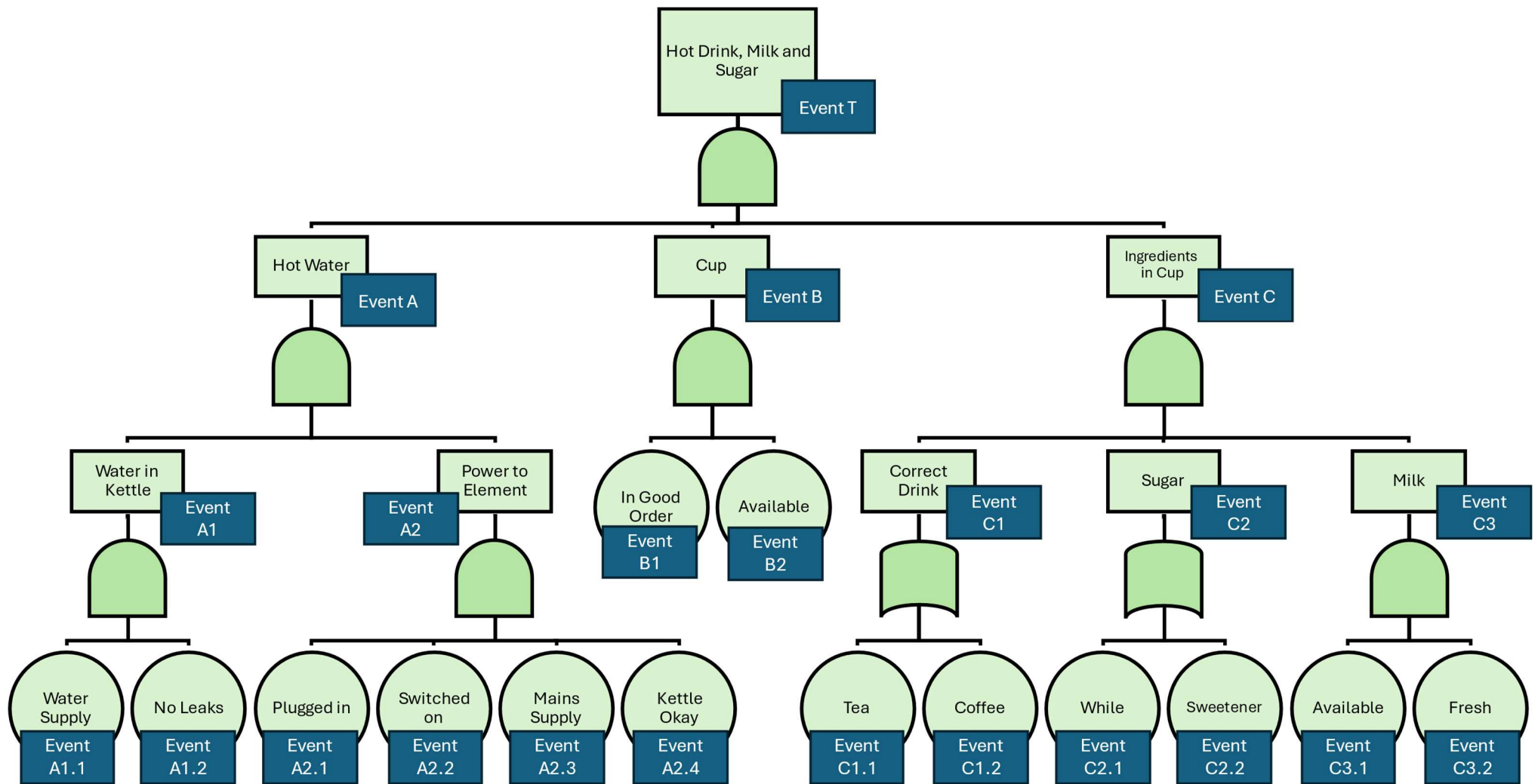


FIGURE 3-1 - FAULT TREE FOR MAKING A CUP OF TEA

3.1 Event A – Supply of Hot Water

The supply of hot water requires water to be in the kettle and power to the element. Using equation 2-1 the probability of success can be calculated.

$$P(A) = P(A1) \cdot P(A2) \quad 3-1$$

$$P(A) = (P(A1.1) \cdot P(A1.2)) \cdot (P(A2.1) \cdot P(A2.2) \cdot P(A2.3) \cdot P(A2.4))$$

$$P(A) = (0.99 \cdot 0.9) \cdot (0.7 \cdot 0.7 \cdot 0.99 \cdot 0.99)$$

$$P(A) = 0.891 \cdot 0.480$$

$$P(A) = 0.428 \quad 3-2$$

3.2 Event B – Supply of a Cup

The supply of a cup requires a cup to be available and to be in good order. Using equation 2-1 the probability of success can be calculated.

$$P(B) = P(B1) \cdot P(B2) \quad 3-3$$

$$P(B) = 0.7 \cdot 1.0$$

$$P(B) = 0.700 \quad 3-4$$

3.3 Event C – Supply of Ingredients

To make a successful cup of tea, a supply of tea, milk, and sugar are required. To calculate the overall success of selecting the ingredients using equation 2-1 and events C1, C2 and C3; first the probability of events C1 and C2 need to be determined by equation 2-3 .

Event C1

$$P(C1) = P(C1.1) + P(C1.2) - P(C1.1) \cdot P(C1.2) \quad 3-5$$

$$P(C1) = (0.8 + 0.9) - (0.8 \cdot 0.9)$$

$$P(C1) = 1.7 - 0.72$$

$$P(C1) = 0.980 \quad 3-6$$

Event C2

$$P(C2) = P(C2.1) + P(C2.2) - P(C2.1) \cdot P(C2.2) \quad 3-7$$

$$P(C2) = (0.6 + 0.9) - (0.6 \cdot 0.9)$$

$$P(C2) = 1.5 - 0.54$$

$$P(C2) = 0.960 \quad 3-8$$

Event C

$$P(C) = P(C1) \cdot P(C2) \cdot P(C3) \quad 3-9$$

$$P(C) = P(C1) \cdot P(C2) \cdot (P(C3.1) \cdot P(C3.2))$$

$$P(C) = 0.98 \cdot 0.96 \cdot (0.9 \cdot 0.8)$$

$$P(C) = 0.98 \cdot 0.96 \cdot 0.72$$

$$P(C) = 0.677 \quad 3-10$$

3.4 Event T – Making A Cup of Tea

Using the probability of success for events A, B, and C the probability of the Top Event (T) can be calculated. Figure 4-1 showcases a visual representation of the probabilities leading to the Top Event.

$$P(T) = P(A) \cdot P(B) \cdot P(C) \quad 3-11$$

$$P(T) = 0.428 \cdot 0.700 \cdot 0.677$$

$$P(T) = 0.203 \quad 3-12$$

This provides a **20%** chance of successful making a cup of tea with milk and sugar.

4. Application of Human Factors

The process of making a cup of tea requires human input and under certain conditions the success factors may decrease or increase. When considering a person making a cup of tea under fatigue, the ability to plug in and switch on the kettle drops by 25% and the ability to select an appropriate cup drops by 10%. The new probability of these events is provided in Table 4-1

TABLE 4-1 – PROBABILITIES FOR EVENTS UNDER FATIGUE

Reference	EVENT	PROBABILITY
A2.1	Plug In Operation Reliability	0.525
A2.2	Switch on Reliability	0.525
B2	Cup Availability	0.900

The change in probabilities will alter the chance of success for events A and B. As shown in Figure 4-2, Event A's probability becomes 0.241 and Event B's probability becomes 0.630. This results in a probability for the top event of 0.102 equating to a **10%** chance of success.

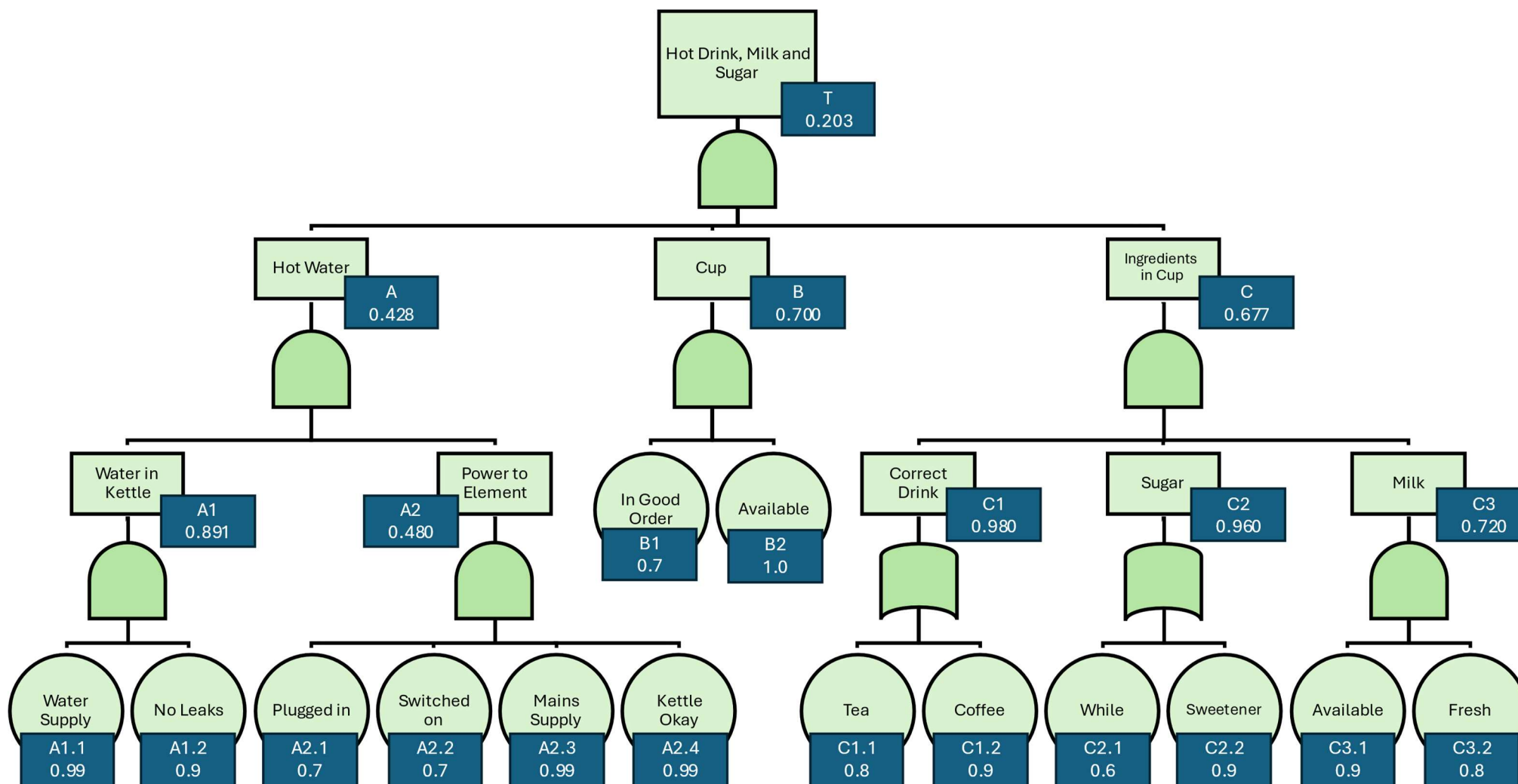


FIGURE 4-1 - FAULT TREE FOR MAKING A CUP OF TEA WITH PROBABILITY VALUES

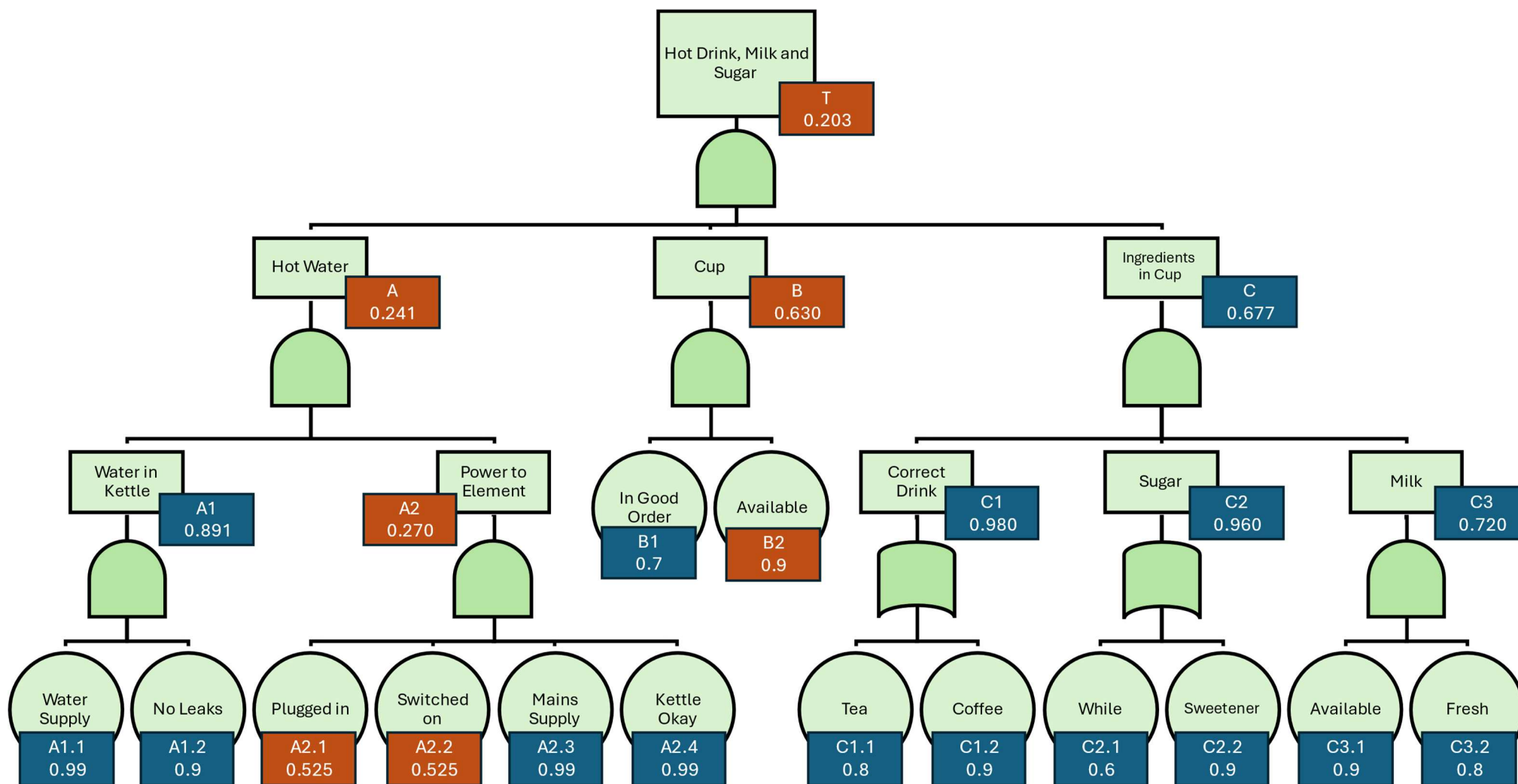


FIGURE 4-2 - FAULT TREE FOR MAKING A CUP OF TEA UNDER FATIGUE WITH PROBABILITY VALUES

5. Conclusions

This report analysed the probability of success of making a cup of tea with milk and sugar using Fault Tree Analysis. The analysis split the top event into three leading events:

- Event A - Supply of Hot Water with a success probability of 0.428
- Event B – Supply of a Cup with a success probability of 0.700
- Event C - Supply of Ingredients with a success probability of 0.667

Event A was the biggest contributing factor to the overall success, with the event of supplying power to the kettle (Event A2) only having a success probability of 0.480.

The overall findings conclude that there is a 20% chance of successfully making a cup of tea with milk and sugar under normal conditions. This rate of success decreases to 10% when the operator is under fatigue, highlighting the impact of human factors.

6. References

- [1] C. A. E. II, Hazard Analysis Techniques for System Safety, Hoboken, New Jersey: John Wiley & Sons Inc, 2005.
- [2] S. J. Shaefer and L. Theodore, Probability and Statistics Applications for Environmental Science, Boca Ration, Florida: CRC Press, 2007.
- [3] T. Hessian, "Fault Tree Analysis," November 2020. [Online]. Available: <https://sixsigmastudyguide.com/fault-tree-analysis/>. [Accessed 17 April 2025].
- [4] W. E. Vesely, F. F. Golfbery, N. H. Roberts and D. F. Haasl, U.S. Nuclear Regulatory Commission Fault Tree Handbook, 1981.