

b.

$$y = \frac{1}{x} + \cos 2x \quad \text{find } \frac{d^5y}{dx^5}$$

$$\frac{dy}{dx} = \frac{-1}{x^2} + 2\sin 2x$$

$$\frac{1}{x} = -\frac{1}{x^2}$$

$$\cos \rightarrow -\sin$$

$$\sin \rightarrow \cos$$

$$\frac{d^2y}{dx^2} = \frac{2}{x^3} + 4\cos 2x$$

$$\frac{-1}{x^2} = -x^{-2} \rightarrow +2x^{-3}$$

$$\frac{d^3y}{dx^3} = -6x^{-4} - 8\sin 2x$$

$$\frac{d^4y}{dx^4} = +24x^{-5} - 16\cos 2x$$

$$\frac{d^5y}{dx^5} = -120x^{-6} + 32\sin 2x$$

$$\text{Simplify} = -15x^{-6} + 4\sin 2x$$

c.

$$f(x) = \ln(\operatorname{cosec} x - \cot x) \quad \text{Show } f'(x) = \operatorname{cosec} x$$

$$\frac{d}{dx} \frac{1}{\operatorname{cosec} x - \cot x} \cdot \frac{d}{dx} (\operatorname{cosec} x - \cot x)$$

$$= \frac{\frac{d}{dx} \operatorname{cosec} x - \frac{d}{dx} \cot x}{\operatorname{cosec} x - \cot x}$$

$$= \frac{(-\cot x) \operatorname{cosec} x - (-\operatorname{cosec}^2 x)}{\operatorname{cosec} x - \cot x}$$

$$= \frac{\operatorname{cosec}^2 x - \cot x \operatorname{cosec} x}{\operatorname{cosec} x - \cot x} = \operatorname{cosec} x$$