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Nomenclature

ARB	Anti Roll Bar
CoG	Centre of Gravity
SMD	Spring-Mass-Damper
DOF	Degrees-of-Freedom
F	Force
m	Mass where subscripts T total, u unsprung, s sprung, f front, r rear, 1 left 2 right
A	Acceleration
K	Spring Rate where subscripts t is tyre, f front, r rear.
K_{au}	Anti-Roll Bar Stiffness
K_{AS}	Chassis/sprung mass stiffness
x	Spring Extension
C	Damping Coefficient where subscripts t is tyre, f front, r rear.
v	Velocity
Z	Vertical Displacement of Mass
φ	Roll Angle
θ	Pitch Angle
a	Distance from CoG to Front Axle
b	Distance from CoG to Rear Axle
α	Distance from CoG to Vehicle Tyres Right
β	Distance from CoG to Vehicle Tyres Left
Z	Vertical Displacement of s sprung, u unsprung, f front, r rear, 1 left, 2 right,
Z_a	Anti Roll Bar displacement unsprung mass.
Z_A	Vehicle body displacement from anti roll bar forces.
ζ	Damping Ratio
C_{crit}	Critical Damping Coefficient
$RRCH$	Rear Roll Centre Height
$FRCH$	Front Roll Centre Height
MR	Motion Ratio
RF	Ride Frequency
w	Weight
g	Gravitational Acceleration assumed 9.81 m/s/s.

1 Introduction

This report analyses the transient response of a Formula Ford vehicle using a 7-DOF “Full Ride Model” in Simulink. Initial tests on an Automobilista simulator included adjustments to parameters such as spring rates, damping coefficients, and anti-roll bars. The simulator acceleration data is then used as torque inputs to Simulink to calculate tyre load, ride frequency, and damping ratios, providing a suitable platform for a detailed vehicle dynamics analysis of ride comfort, controllability, safety, and handling.

2 Background Research

The vehicle dynamics industry follows ISO 8855:2011 and SAE J670 (SAE, 2022) standards for clear communication of terms in road vehicle dynamics (ISO, 2011). The Simulink model adheres to these standards and demonstrates axis positive rotation in figure 1. When theta is positive, the front of the vehicle's sprung mass lowers; when Phi is positive, the right (driver) side lowers.

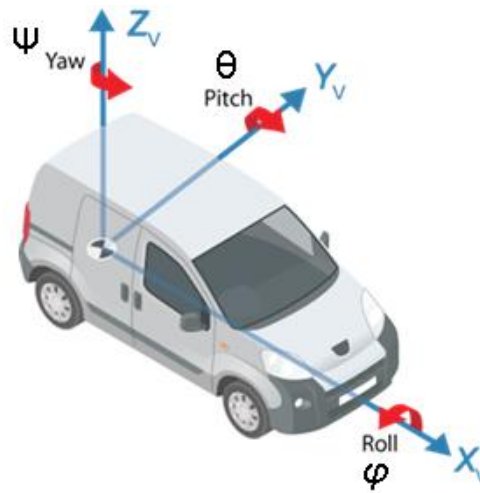


Figure 1: ISO 8855:2011 and Simulink Vehicle Coordinate System and Rotation Convention (MathWorks, n.d.).

3 Methodology

3.1 Equations of Motion

Spring-mass-damper (SMD) systems model vehicles using springs, sprung and unsprung masses, and dampers. Various degrees of freedom are possible, depending on the desired results. A 7-DOF model is used here to represent roll, pitch, and movement of sprung/unsprung masses, suitable for analysing lateral and longitudinal acceleration inputs (Guglielmino, et al., 2008). The SMD system relies on Newton's second law (equation 1), Hooke's law for spring rate (equation 2), and the damping coefficient formula (equation 3). Combining these (equation 4a) and expressing acceleration 'A' for the sprung mass as the second derivative of displacement 'Z_s' (equation 4b/c) forms the principle SMD equations, with unsprung masses covered in equations 5-8.

$$F = mA \quad (1)$$

$$F_{Spring} = -Kx \quad (2)$$

$$F_{Damper} = -Cv \quad (3)$$

$$mA = F_{Spring} + F_{Damper} \quad (4a)$$

$$m\ddot{Z}_{Sprung} = -Kx - Cv \quad (4b)$$

$$-m\ddot{Z}_{ij} = Kx + Cv \quad (4c)$$

For unsprung mass:

$$-M_{f1}\ddot{Z}_{f1} = K_{tf1}(Z_{f1} - y_{f1}) - K_{f1}(Z_s - Z_{f1} - a\theta + \beta\varphi) \quad (5)$$

$$-M_{f2}\ddot{Z}_{f2} = K_{tf2}(Z_{f2} - y_{f2}) - K_{f2}(Z_s - Z_{f2} - a\theta - \alpha\varphi) \quad (6)$$

$$-M_{r1}\ddot{Z}_{r1} = K_{tr1}(Z_{r1} - y_{r1}) - K_{r1}(Z_s - Z_{r1} + b\theta + \beta\varphi) \quad (7)$$

$$-M_{r2}\ddot{Z}_{r2} = K_{tr2}(Z_{r2} - y_{r2}) - K_{r2}(Z_s - Z_{r2} + b\theta - \alpha\varphi) \quad (8)$$

To apply lateral acceleration data to the Simulink 7-DOF model, an additional term is added to the roll equation (equation 9), resulting in a torque, equation 10 (excluding dampers). The same principle applies to pitch (equation 11), where negative theta causes a positive displacement of Zf1 and Zf2. Sprung mass is shown in equation 12.

$$-I_{xx}\ddot{\phi} = Kx + Cv - (m_{sprung}A \times h_{Diff}) \quad (9)$$

For sprung mass:

$$-I_{xx}\ddot{\phi} = \beta K_{f1}(Z_s - Z_{f1} - a\theta + \beta\varphi) + \beta K_{r1}(Z_s - Z_{r1} + b\theta + \beta\varphi) - \alpha K_{f2}(Z_s - Z_{f2} - a\theta - \alpha\varphi) - \alpha K_{r2}(Z_s - Z_{r2} + b\theta - \alpha\varphi) - (m_{sprung}A \times h_{Diff}) \quad (10)$$

$$-I_{yy}\ddot{\theta} = bK_{r1}(Z_s - Z_{r1} + b\theta + \beta\varphi) + bK_{r2}(Z_s - Z_{r2} + b\theta - \alpha\varphi) - aK_{f1}(Z_s - Z_{f1} - a\theta + \beta\varphi) - aK_{f2}(Z_s - Z_{f2} - a\theta - \alpha\varphi) + (m_{sprung}A \times h) \quad (11)$$

$$-M_s\ddot{Z}_s = K_{f1}(Z_s - Z_{f1} - a\theta + \beta\varphi) + K_{f2}(Z_s - Z_{f2} - a\theta - \alpha\varphi) + K_{r1}(Z_s - Z_{r1} + b\theta + \beta\varphi) + K_{r2}(Z_s - Z_{r2} + b\theta - \alpha\varphi) \quad (12)$$

3.2 Parameters and Assumptions

The Simulink system uses essential parameters such as damping coefficients, spring rates, masses, and inertia to approximate vehicle dynamics (table 1). Fine-tuning values and adding accurate equations/models can improve accuracy with extra output values. However, conducting real-world testing on the Formula Ford for accurate data to input into the Simulink system for calibration is the best option. Additional considerations are mentioned in Appendix A.

3.2.1 Aerodynamic Forces

When modelling the Formula Ford in Simulink, aerodynamic forces (components effected by aerodynamics: ride height, rake angle and tyre loads) are neglected due to simplicity and lack of data (frontal area, drag and lift coefficients) (Segers, 2014). The 7-DOF ride model does not include longitudinal and lateral velocities but integrating acceleration data can approximate these values.

3.2.2 Yaw

The 7-DOF ride model (this Simulink system is based on) typically excludes yaw, this is due to the exclusion of tyre rotation, steering angles, lateral and longitudinal motion (Azhar, et al., 2014). To model yaw, a 7-DOF vehicle handling (figure 2, right) model can be integrated into a 7-DOF ride (figure 2, left) model to create a 14-DOF vehicle ride and handling model (figure 3).

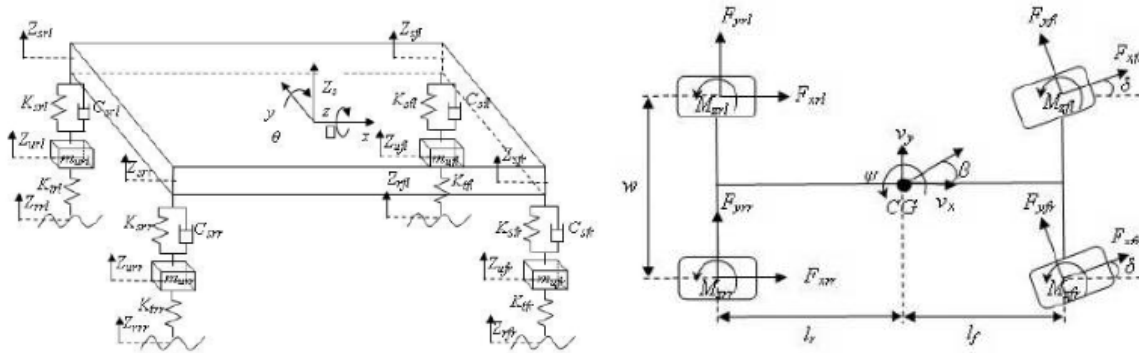


Figure 2: Vehicle Ride Model (Left), Vehicle Handling Model (Right) (Setiawan, et al., 2009).

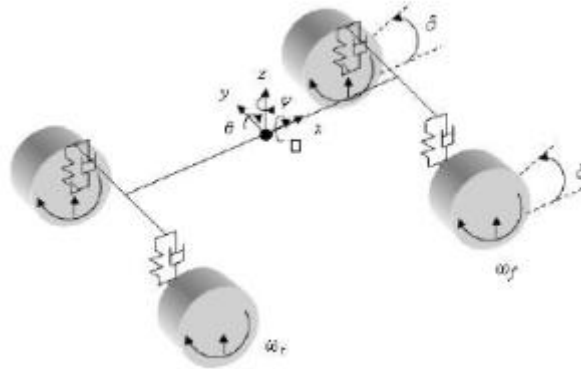


Figure 3: 14-DOF Vehicle Ride and Handling Model (Setiawan, et al., 2009).

3.2.3 Tyre Parameters

Each tyre in the 7-DOF SMD system is modelled using a damper and spring (C_t and K_t), with research providing a spring rate of 160,000 N/m (Serné & PTZ, 2022) and a damping coefficient of 50 N/m/s (Ganji et al., 2023). While this is a simplistic representation, the 'Pacejka Magic Formula' would improve accuracy but requires specific and unknown tyre parameters.

3.2.4 Roll and Pitch Inertia

Roll and pitch inertia significantly impacts vehicle dynamics. Dongchan (2018) suggests estimating pitch (I_{yy}), roll (I_{xx}), and yaw inertia using vehicle weight functions (equations 13, 14, 15, 16). This method, tested on passenger cars, is applied to the Simulink model despite questionable validity for racing vehicles. Gillespie (2021) proposes calculating vehicle inertia by summing individual component inertias (equation 17, figure 4). More accurate methods include a detailed CAD model or physical testing on an inertia rig.

$$I_{xx} = 0.28M_T - 71.6 \quad (13)$$

$$I_{xx} = 0.28(514) - 71.6 = 72.32 \text{ kg.m}^2 \quad (14)$$

$$I_{yy} = 2.56M_T - 1103 \quad (15)$$

$$I_{yy} = 2.56(514) - 1103 = 212.84 \text{ kg.m}^2 \quad (16)$$

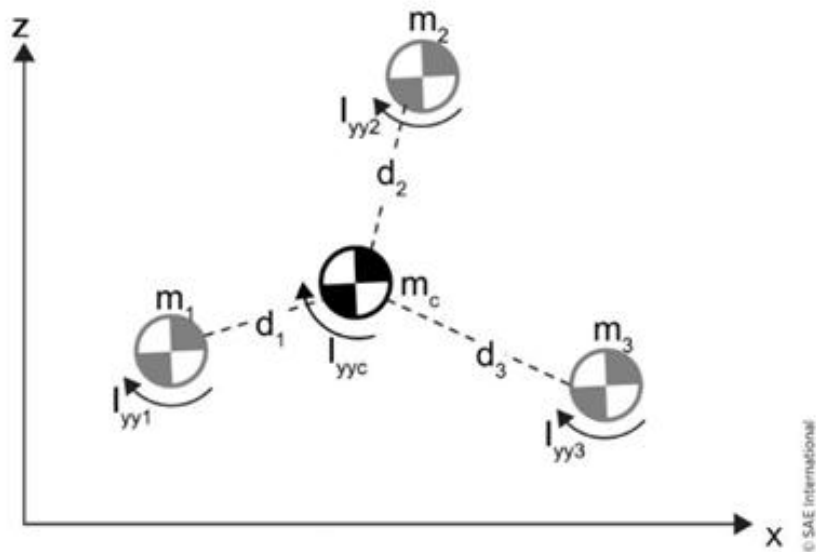


Figure 4: Calculating Vehicle Inertia by Individual Masses (Gillespie, 2021).

$$I_{yyc} = \sum_{i=1}^n (I_{yyi} + m_i d_i^2) \quad (17)$$

3.2.5 Roll Axis Height Difference

To apply simulator lateral acceleration data to the Simulink model, the height difference between the roll axis and CoG is needed. The CoG position is calculated by multiplying the wheelbase by the weight distribution percentage (equations 18 and 19). Figure 5 uses trigonometry to calculate height difference (hDiff) as shown in equations 20-23 (Segers, 2014).

$$a = 2.443 \times 0.55 = 1.34365 \text{ m} \quad (18)$$

$$b = 2.443 \times 0.45 = 1.09935 \text{ m} \quad (19)$$

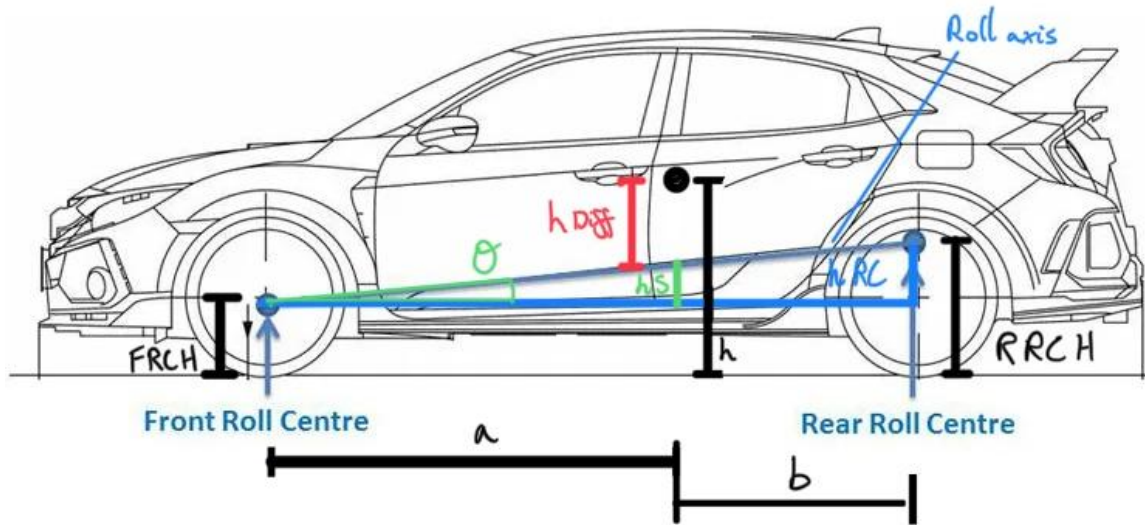


Figure 5: Roll Axis and Height Diagram, Adapted from (Suspension Secrets, n.d.).

Where:

$$FRCH = 0.9 \times 10^{-3} \text{ m}$$

$$RRCH = 59 \times 10^{-3} \text{ m}$$

$$h = 0.277 \text{ m}$$

$$hRC = (59 \times 10^{-3}) - (0.9 \times 10^{-3}) = 0.0581 \text{ m} \quad (20)$$

$$\theta = \tan^{-1} \left(\frac{0.0581}{2.443} \right) = 1.36236^\circ \quad (21)$$

$$hS = \sin(1.36236) \times 1.34365 = 0.031955 \text{ m} \quad (22)$$

$$hDiff = 0.277 - 0.031955 - (0.9 \times 10^{-3}) = 0.244 \text{ m} \quad (23)$$

Table 1: Parameters and Values.

Parameter	Description	Value	Units
MT	Vehicle Total Mass	514	kg
Ms	Vehicle Sprung Mass	426	Kg
Mf1	Front Left Corner Unsprung Mass	22	Kg
Mf2	Front Right Corner Unsprung Mass	22	Kg
Mr1	Rear Left Corner Unsprung Mass	22	kg
Mr2	Rear Right Corner Unsprung Mass	22	Kg
a	Distance from CoG to front axle	1.34365	m
b	Distance from CoG to rear axle	1.09935	m
Ctf	Front tyre damping coefficient	50	N/m/s
Ctr	Rear tyre damping coefficient	50	N/m/s
α_{Front}	Front distance from centre to vehicle right side	0.810	m
α_{Rear}	Rear distance from centre of vehicle right side	0.785	m
β_{Front}	Front distance from centre to vehicle left side	0.810	m
β_{Rear}	Rear distance from centre to vehicle left side	0.785	m
h	Distance from CoG height to instantaneous centre	0.277	m
hDiff	Height distance from roll axis to CoG	0.244145	m
Kf	Front spring rate (standard) this value changes for testing	18000	N/m
Kr	Rear spring rate (standard) this value changes for testing	16000	N/m
Ktf	Front tyre spring rate	160000	N/m
Ktr	Rear tyre spring rate	160000	N/m
Cfsb	Front damper slow velocity compression damping coefficient (standard)	1654	N/m/s
Cffb	Front damper fast velocity compression damping coefficient (standard)	533.25	N/m/s
Cfsr	Front damper slow velocity rebound damping coefficient (standard)	1936.25	N/m/s
Cffr	Front damper fast velocity rebound damping coefficient (standard)	511	N/m/s
Crspb	Rear damper slow velocity compression damping coefficient (standard)	1507.75	N/m/s
Crfb	Rear damper fast velocity compression damping coefficient (standard)	552.75	N/m/s
Crshr	Rear damper slow velocity rebound damping coefficient (standard)	1609	N/m/s
Crfr	Rear damper fast velocity rebound damping coefficient (standard)	552.75	N/m/s
Ixx	Sprung mass roll inertia	72.32	Kg.m ²
Iyy	Sprung mass pitch inertia	212.84	Kg.m ²

3.3 Simulation Scenarios

Numerous simulator test runs were conducted to validate the Simulink model, adjusting spring rates, damping coefficients, and anti-roll bars. Table 2 shows changes to input parameters for 18 tests. Each simulation was performed on a flat proving track with two manoeuvres: double lane change (figure 6 Left) and slalom (figure 6 Right). These manoeuvres provided data for transient vehicle response modelling. While springs and dampers were adjusted independently, research suggests they should be adjusted together after determining key parameters such as ride frequency and damping ratio to prevent excessive oscillations and absorb shocks (Smith, 2013).



Figure 6: Simulator Manoeuvres: (Left) Double Lane Change, (Right) Slalom (OXTS, 2012).

Table 2: Simulation Test Data.

Setup number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
front spring rate (N/m)	18000	22000	26000	14000	9000	26000	9000	18000	18000	18000	18000	18000	18000	18000	18000	18000	18000	18000
rear spring rate (N/m)	16000	20000	24000	12000	8000	8000	24000	16000	16000	16000	16000	16000	16000	16000	16000	16000	16000	16000
front slow bump (N/m/s)	1645	1645	1645	1645	1645	1645	1645	1645	1645	1245	2045	1645	1645	1645	1645	1645	1645	1645
front fast bump (N/m/s)	533.25	533.25	533.25	533.25	533.25	533.25	533.25	383.25	683.25	533.25	533.25	533.25	533.25	533.25	533.25	533.25	533.25	533.25
front slow rebound (N/m/s)	1936.3	1936.3	1936.3	1936.3	1936.3	1936.3	1936.3	1936.3	1936.3	1936.3	1936.3	1936.3	1936.3	1436.3	2436.3	1936.3	1936.3	1936.3
front fast rebound (N/m/s)	511	511	511	511	511	511	511	511	511	511	511	383.25	638.75	511	511	511	511	511
rear slow bump (N/m/s)	1507.8	1507.8	1507.8	1507.8	1507.8	1507.8	1507.8	1507.8	1507.8	1107.8	1907.8	1507.8	1507.8	1507.8	1507.8	1507.8	1507.8	1507.8
rear fast bump (N/m/s)	552.75	552.75	552.75	552.75	552.75	552.75	552.75	402.75	702.75	552.75	552.75	552.75	552.75	552.75	552.75	552.75	552.75	552.75
rear slow rebound (N/m/s)	1609	1609	1609	1609	1609	1609	1609	1609	1609	1609	1609	1609	1609	1209	2009	1609	1609	1609
rear fast rebound (N/m/s)	552.75	552.75	552.75	552.75	552.75	552.75	552.75	552.75	552.75	552.75	552.75	402.75	702.75	552.75	552.75	552.75	552.75	552.75
ARB front (N/m)	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	8000	8000	N/A
ARB rear (N/m)	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	8000	N/A	8000

3.4 Model Development

The flow chart in figure 7 provides an overview of model development from standard (figure 8), summarizing data calculations, advancements, and improvements. Other calculations are shown in Appendix B, with the final Simulink model in Appendix C.

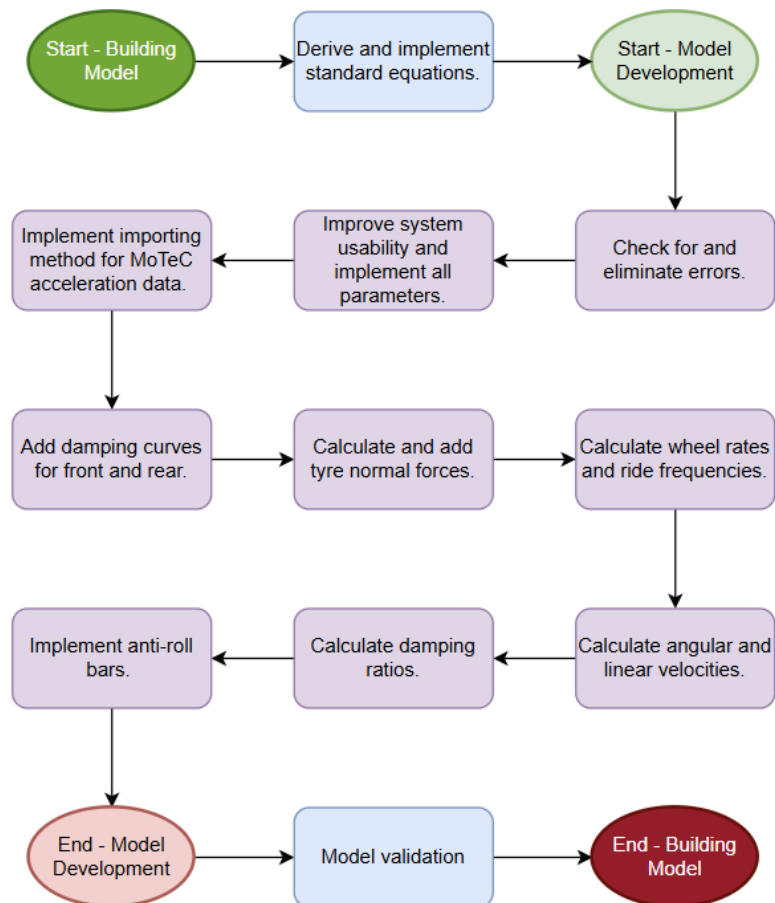


Figure 7: Simulink Model Construction and Development Flowchart.

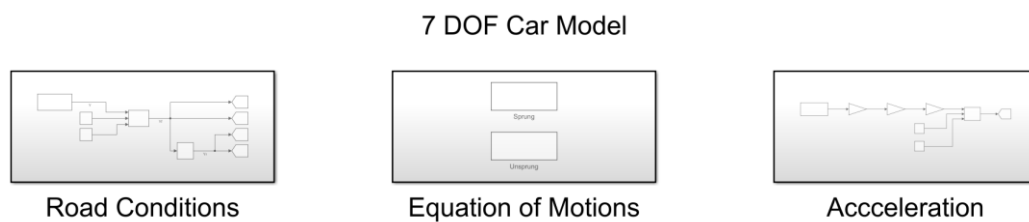


Figure 8: Simulink Base Model.

3.4.1 MoTeC Acceleration Data

To represent vehicle movements from the Automobilista simulator in Simulink, lateral and longitudinal acceleration data are applied as torques (figure 9). Lateral torque is subtracted in the roll equation, and longitudinal torque is added in the pitch equation, following ISO axis rotation standards and positive Phi and theta directions (figure 1). Lateral and longitudinal velocities are estimated by integrating MoTeC accelerations.

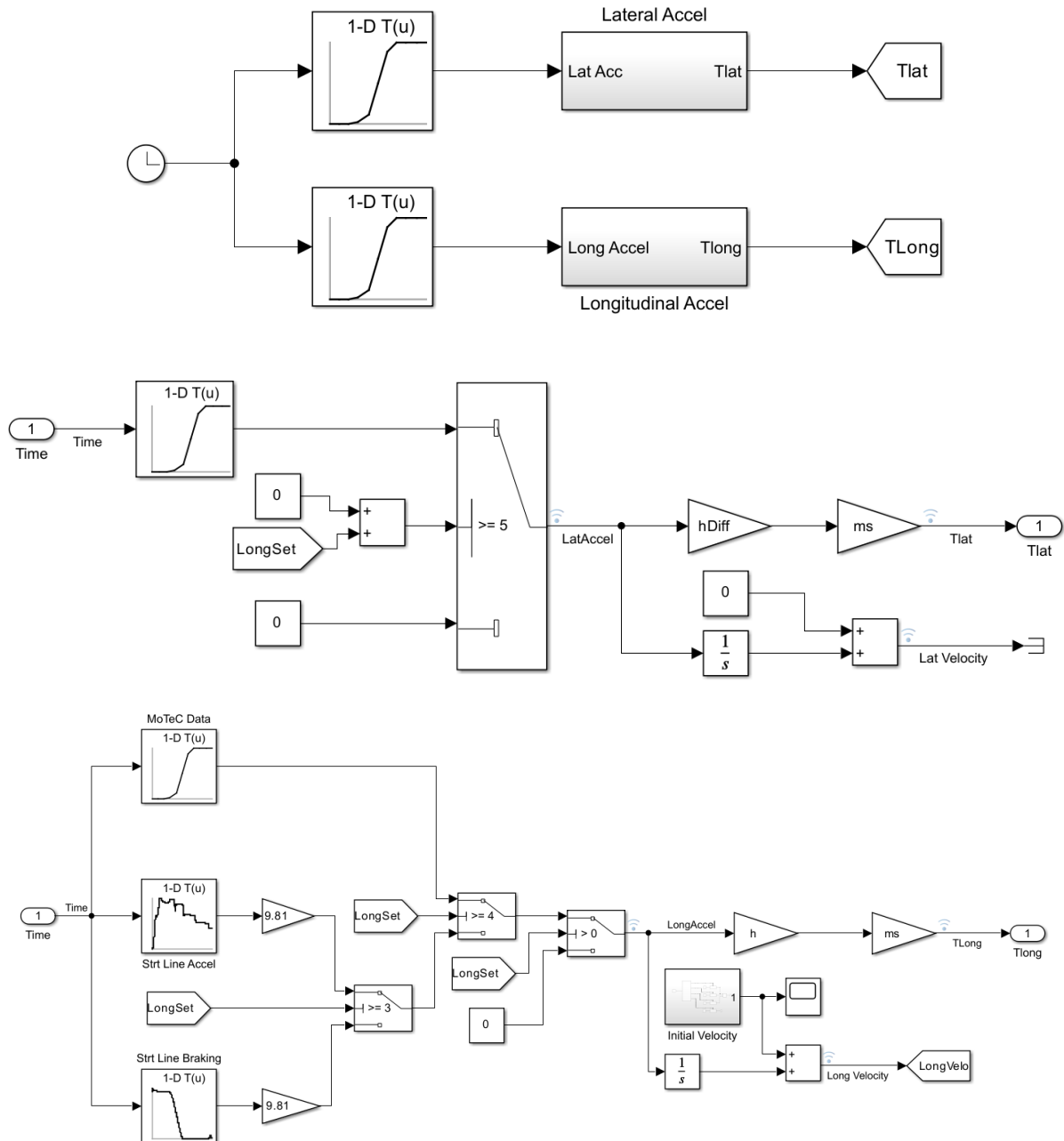


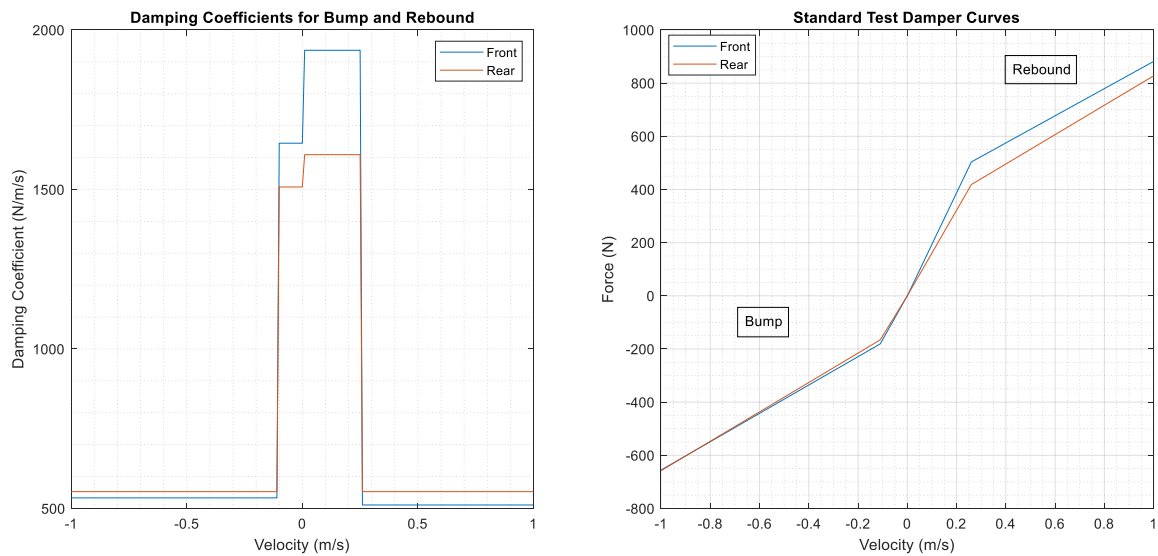
Figure 9: (Top) MoTeC Data Lookup Table from Code, (Middle) Lateral Torque Calculations, (Bottom) Longitudinal Torque Calculation.

3.4.2 Implementing Damper Curves

A lookup table is used to represent damper curves for each simulator test. Table 2 shows parameter values such as spring rates, damper coefficients, and anti-roll bars. Based on these values, table 3 data, and damper curve assumptions (appendix B), a Simulink lookup table is created with velocity as input and force as output (MathWorks, n.d.). Graph 1 (left) shows damper coefficients used to calculate Graph 1 (right), illustrating standard test damper curves for front and rear dampers. Positive damper velocity indicates extension (rebound), and negative velocity indicates compression (bump). Due to hysteresis, real damper curves are not typically linear (Guglielmino, et al., 2008), but a linear straight-line assumption is made for simplicity.

Table 3: High Speed and Low Speed Damping (Brennan, 2020)

Damping Mode	Function	Influences
Low Speed Damping < 0.15m/s (approx.)	- Chassis Movements - Weight Transfer	- Tyre load rates - Lateral load transfer & load distribution - Transient cornering period - Driver feel & confidence
High Speed Damping > 0.15m/s (approx.)	- Response to high frequency road inputs - Controlling energy transfer into and out of the unsprung mass	- Minimising variation (RMS) of contact patch pressure. - Absorbing high amplitude inputs i.e. curb strikes



Graph 1: (Left) Standard Test Damping Coefficients, (Right) Standard Test Damper Curves.

3.4.3 Tyre Normal Forces

To model changes to tyre normal forces in Simulink, basic calculations of stationary tyre loads are required. For the Formula Ford on a flat road, a 50:50 left-right weight distribution (%) can be assumed. Front-rear weight distribution (%) is 45:55, as shown in equations 24-38 (Jazar, 2013). In Simulink (figure 10), tyre forces from lateral and longitudinal load transfer can be subtracted from values in equations 29 and 30. Resultant normal loads were compared to MoTeC simulator test results with an error of approximately 1.9% (10-20N) per wheel, assumed due to calculation assumptions.

$$W = m \times g \quad (24)$$

$$MS_{Front} = 426 \times 0.45 = 191.7 \text{ N} \quad (25)$$

$$MS_{Rear} = 426 \times 0.55 = 234.3 \text{ N} \quad (26)$$

$$F_{NormFront} = (191.7 + 44) \times 9.81 = 2312.217 \text{ N} \quad (27)$$

$$F_{NormRear} = (234.3 + 44) \times 9.81 = 2730.123 \text{ N} \quad (28)$$

$$F_{FrontTyreNorm} = \frac{2312.217}{2} = 1156.1085 \text{ N} \quad (29)$$

$$F_{RearTyreNorm} = \frac{2730.123}{2} = 1365.0615 \text{ N} \quad (30)$$

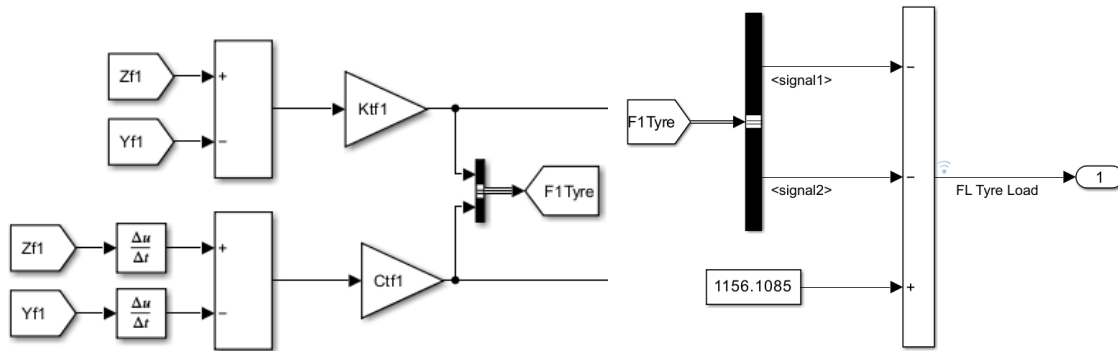


Figure 10: Front Left Tyre Normal Force Calculation Simulink Subsystems.

3.4.4 Wheel Rate and Ride Frequency Calculations

The ride frequency is useful for selecting suitable spring rates and predicting vehicle handling. Smith (2013) presents two theories: a higher rear ride frequency for smoothing out rides on uneven surfaces in automotive vehicles, and a higher front ride frequency for better turn-in and faster corner exits in motorsport. The formula for calculating optimal spring rate (equation 31) is rearranged for ride frequency (equation 32 and figure 11). The Formula Ford vehicle has a 1:1 motion ratio. Another useful calculation is the wheel rate (equation 33 simplified), which evaluates wheel displacements relative to spring rates non-linearly (Smith, 2013). A more accurate version with suspension geometry data is provided by Smith (2001).

$$K = 4\pi^2 \times Rf^2 \times Ms \times MR^2 \quad (31)$$

$$Rf = \sqrt{\frac{K}{4\pi^2 \times Ms \times MR^2}} \quad (32)$$

$$WR = \frac{K}{MR^2} \quad (33)$$

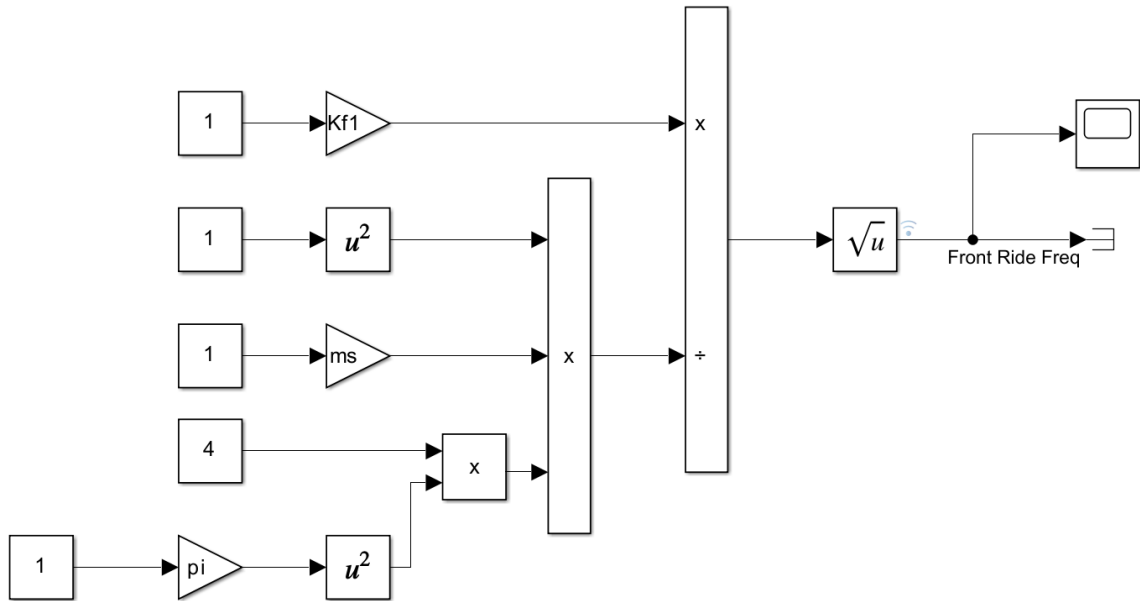


Figure 11: Simulink Front Ride Frequency.

3.4.5 Anti-Roll Bars

To model ARBs in Simulink, Do et al. (2024) provides a breakdown of forces and moments on ARBs, suspension, and vehicle body (figure 12). The equations represent forces on sprung (equation 35) and unsprung mass (equation 34) when vertical displacements of left and right wheels differ. Assumptions are made based on track width and length due to unknown suspension geometry. Rotational stiffness values are from simulator data, and chassis stiffness from Castaneda et al. (2024). Validation testing is conducted later due to numerous assumptions.

Where:

$$K_{au} = 8000 \text{ Nm/rad}$$

$$K_{AS} = 1500 \text{ Nm/deg}$$

$$F_{aul} = -F_{aur} = K_{au}(\Delta Z_{ar} - \Delta Z_{al}) \quad (34)$$

$$F_{ASL} = -F_{ASR} = F_{AUL} \frac{t_A}{t_B} = K_{AS}(\Delta Z_{Ar} - \Delta Z_{Al}) \quad (35)$$

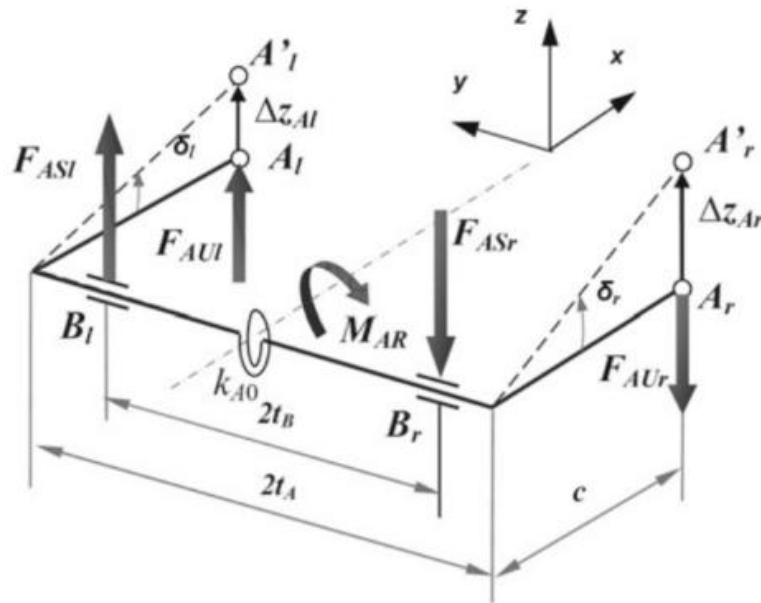
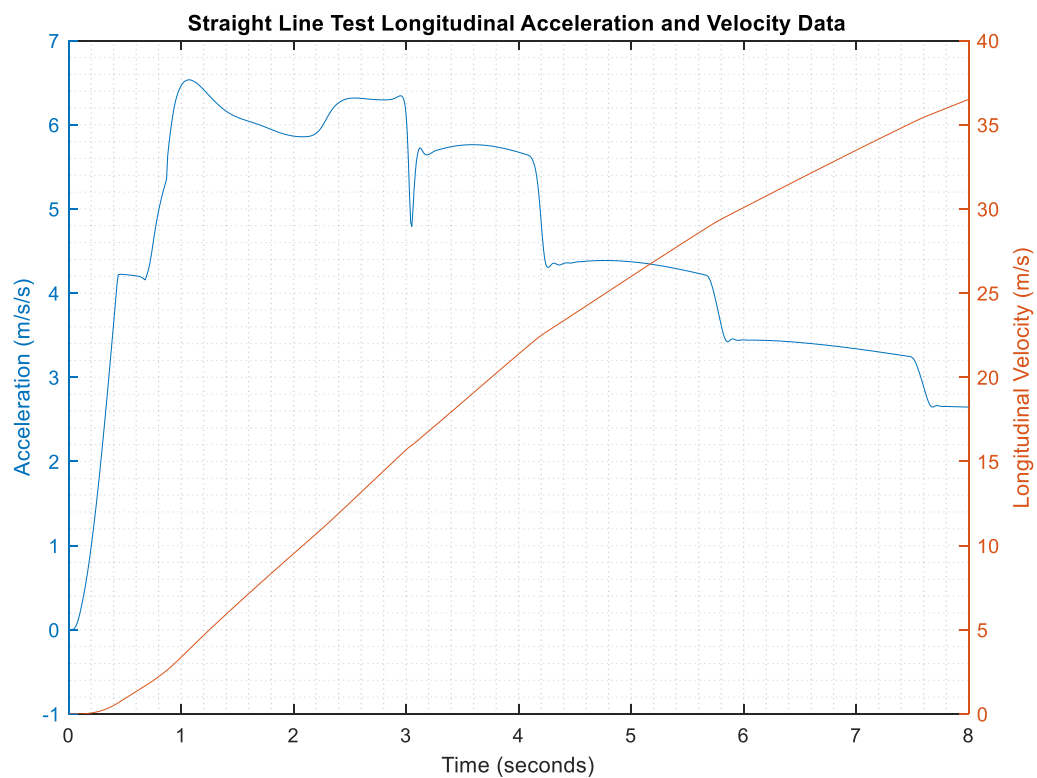


Figure 12: Vehicle Dynamics Modelling of ARBs (Do, et al., 2024).

4 Analysis

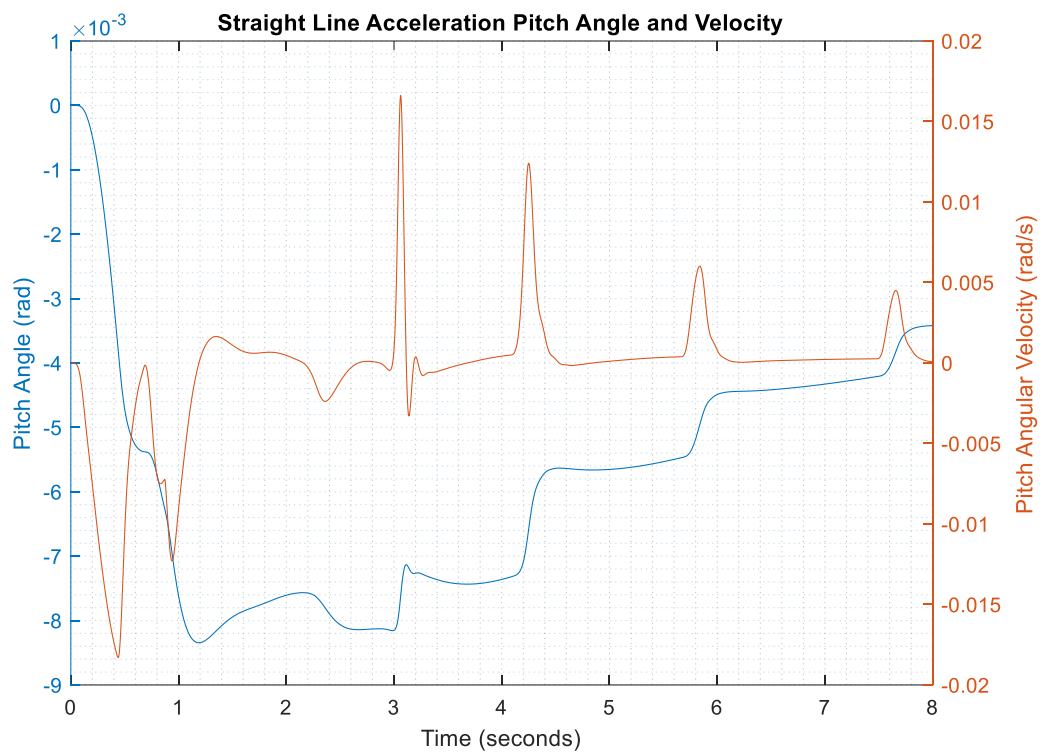
4.1 Straight Line Acceleration

The acceleration input data (graph 2) from VI-Grade's race car model during a straight-line acceleration test was used to validate the model and applied to the Simulink model for the Formula Ford setup. Vehicle velocity is calculated by integrating acceleration in the Simulink 7-DOF model. The motion starts stationary, followed by rapid acceleration with decreasing rates as speed increases. Unsettlements at 0.5, 3, 4.2, 5.8, and 7.6 seconds are due to gear changes, causing temporary reductions in longitudinal acceleration, torque and pitch angle. External forces such as aerodynamic drag, rolling resistance, and mechanical properties cause acceleration reduction. The setup parameters are detailed in table 1 and graph 1.



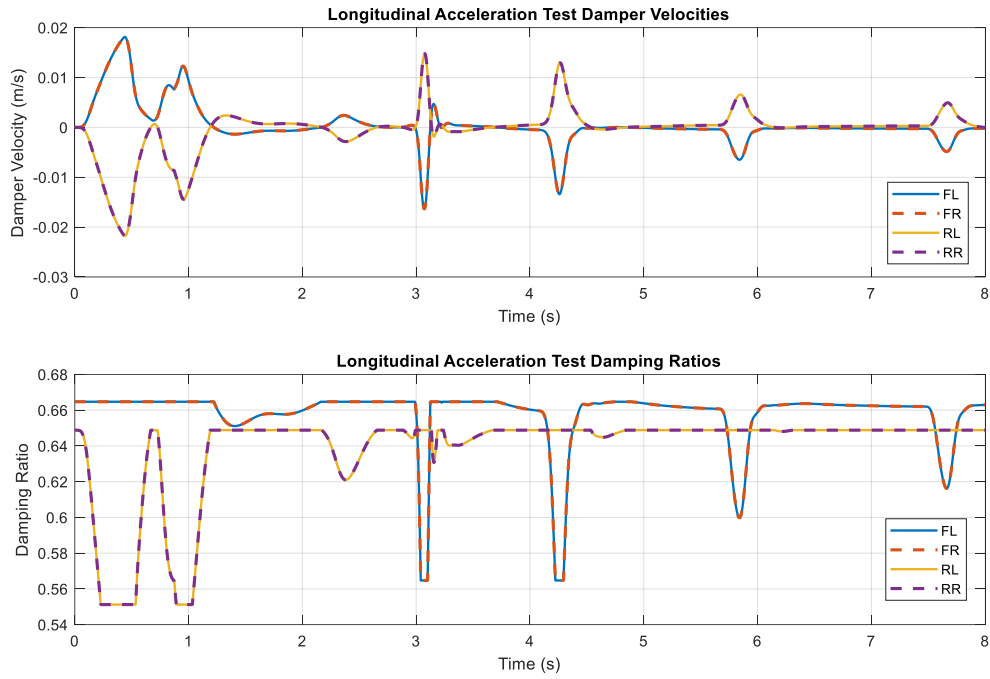
Graph 2: Straight Line Longitudinal Chassis Acceleration and Velocity Data.

The Simulink results show that the longitudinal acceleration vehicle dynamics maneuver behaved as expected. Graph 3 indicates high input torque on the vehicle's sprung mass in an anticlockwise direction (negative pitch angle, θ in figure 1) causes squatting under acceleration (front lift and rear squat). Higher acceleration results in a larger pitch angle. As acceleration rate decreases (3-8 seconds), the vehicle levels out. Pitch angle and angular velocity suggest quick vehicle response to longitudinal acceleration, supported by front (1.035 Hz) and rear (0.975 Hz) ride frequencies similar to passenger cars (Smith, 2013).



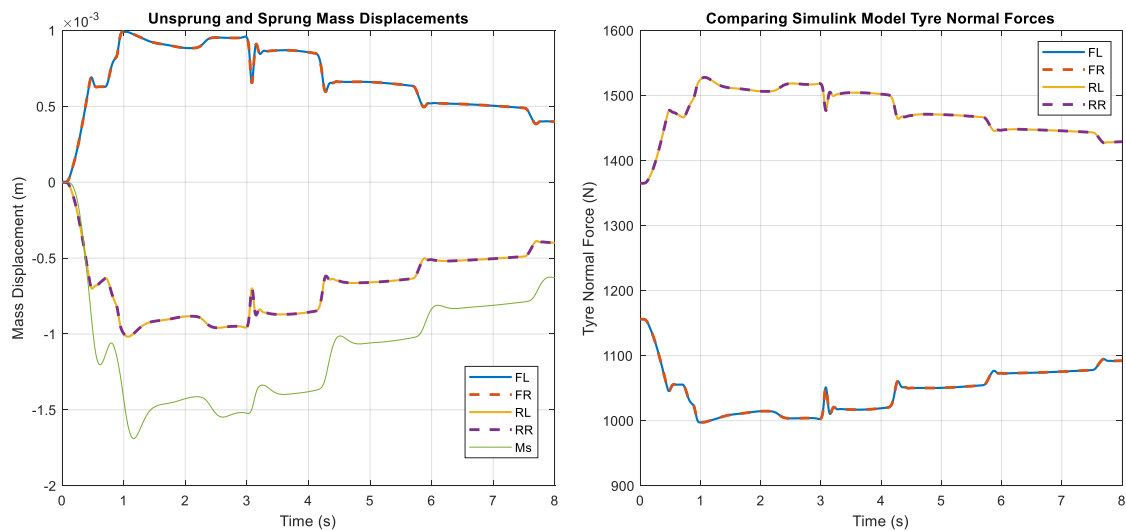
Graph 3: Straight Line Acceleration Pitch Results.

The quick changes and control of oscillations in angular velocity suggest medium spring stiffness and damping coefficients, stabilizing the vehicle effectively. Damping ratios between 0.55 and 0.67 fall in the 0.6-1 racing category (Rouelle, 2021), indicating a middle-range setup. This is verified by changes in rebound and compression damping coefficients (graph 4 damping ratios) seen at the start and gear change intervals. These changes, noticeable in graphs 3 and 5, increase oscillations (at 3 seconds) for unsprung mass movements, pitch angle, and angular velocity, extending the time taken to return to stable conditions.



Graph 4: Longitudinal Acceleration Test Damper Data.

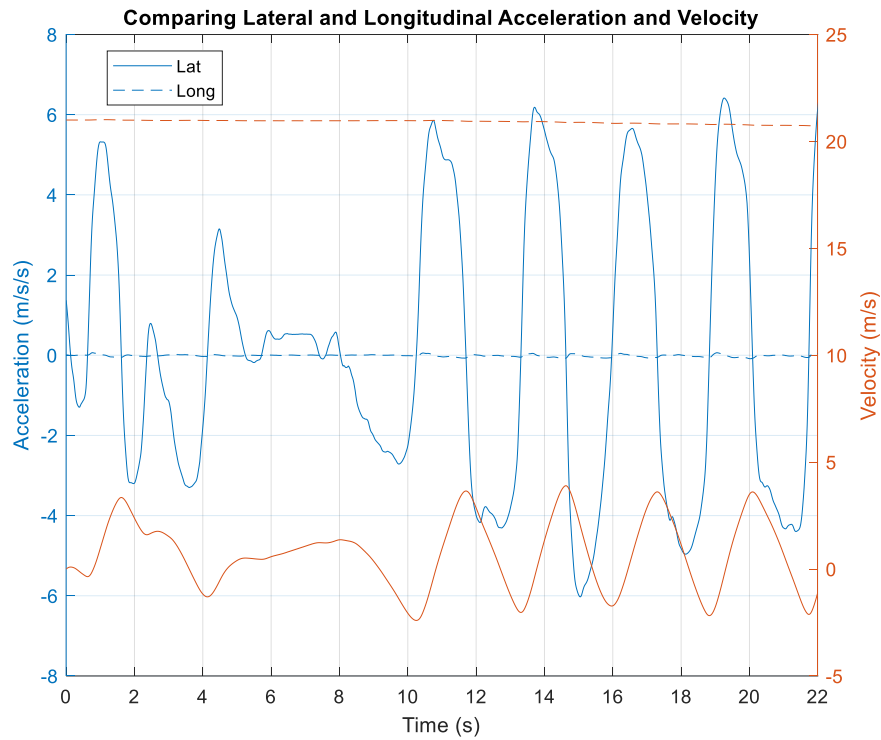
Due to the CoG being closer to the rear axle and softer rear springs and dampers, acceleration causes rear compression and front lift (Heisler, 2002). This affects the ride height, causing the sprung mass to squat. Input torque also causes load transfer, influenced by suspension geometry, spring rates and damper coefficients. Graph 5 shows increased rear tyre normal forces and decreased front forces.



Graph 5: Longitudinal Acceleration Test Mass Displacement and Tyre Normal Forces.

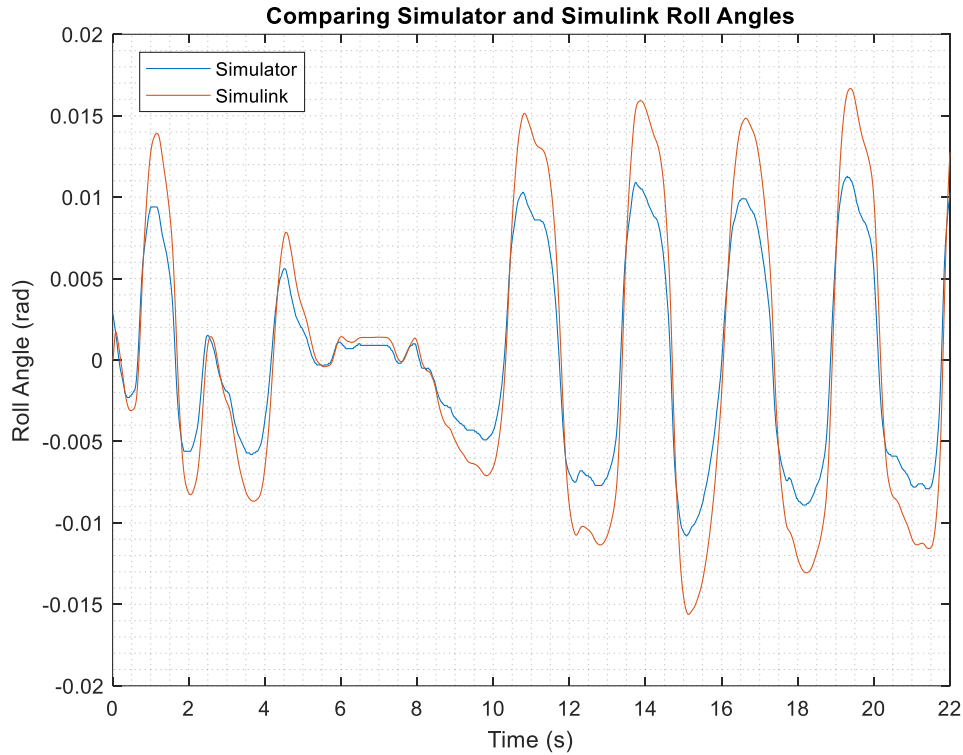
4.2 Double Lane Change and Slalom Standard Results

By importing standard lateral and longitudinal acceleration data into the Simulink model, velocities can be calculated by integrating the acceleration (graph 6). To ensure accurate lateral modelling, standard test results from the simulator and Simulink are plotted and compared. In graph 6, 0-7 seconds covers the double lane change maneuver and 7-22 seconds the slalom test. Longitudinal velocity remains consistent at 21m/s, slightly reducing as lateral velocity increases during slalom tests.



Graph 6: Standard Longitudinal and Lateral Acceleration Data with Calculated Velocities.

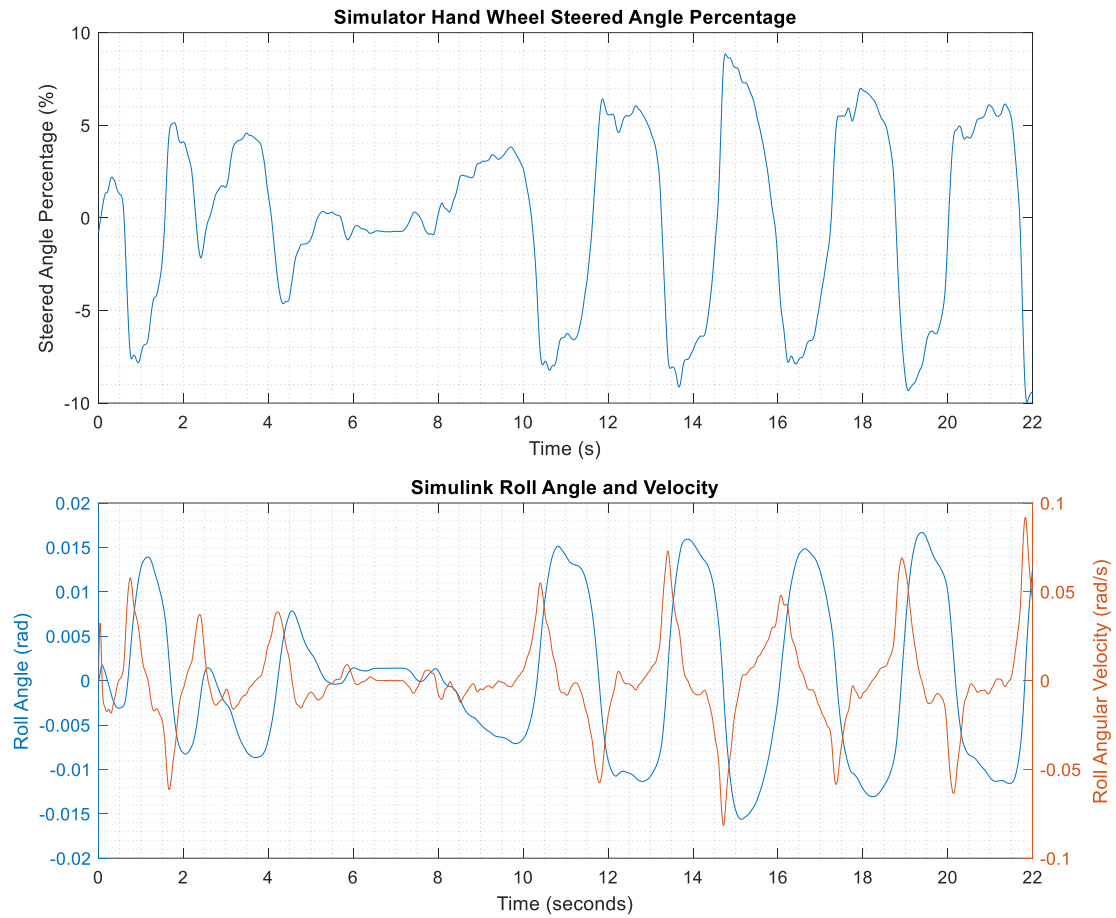
From this input acceleration, roll angle data is calculated and shown in graph 7. The Simulink model's 7-DOF system produces similar trendlines to the simulator, but overestimates roll angle. Potential causes include incorrect inertia values, neglected tyre parameters, and exclusion of forces like aerodynamics and yaw, which are critical during cornering dynamics. Comparing both systems to real-world applications might show different results where the body is modelled as rigid (Azadi, et al., 2021). Despite this, the Simulink model reliably approximates vehicle response trendlines, though actual values may be less reliable.



Graph 7: Simulink and Simulator Standard Test Roll Angle Comparison.

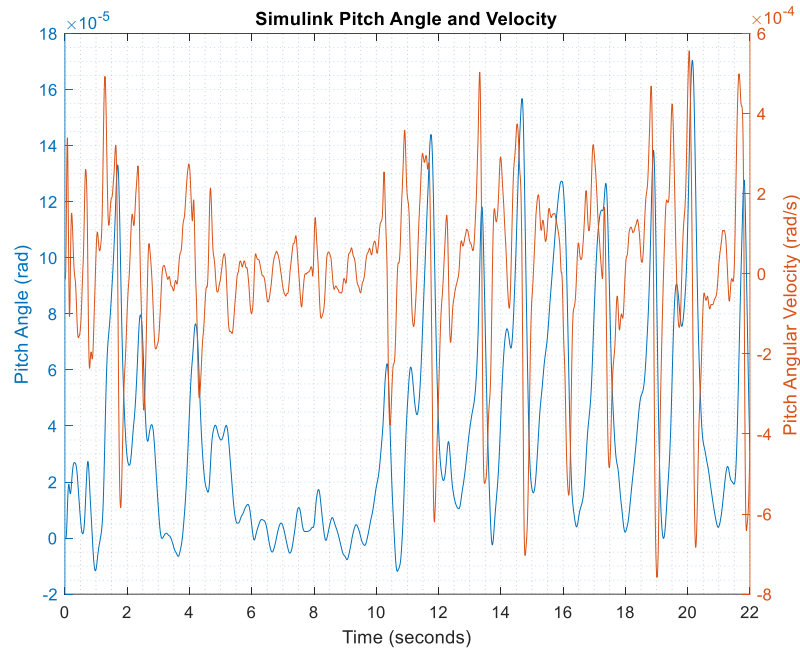
In graph 8, positive steering percentage indicates steering right, and negative indicates steering left. As the vehicle exits lane 1 (0.5 seconds), it is steered left (0.5-1.5 seconds), then right to align with lane 2 (1.5-2.25 seconds), then oversteered and corrected with left steering (2.25-2.75 seconds). Exiting lane 2 at 2.5 seconds, a right angle realigns with lane 1, then left at 4 seconds to rejoin lane 1. In the slalom section (7-22 seconds), handwheel angles alternate uniformly but with occasional corrections, shown by data spikes and the largest left angle at 15 seconds. Body roll angle increases slightly after handwheel inputs due to steering system delays, suspension reaction, roll inertia, and CoG height differences (Alexander, 2013).

Graph 8 indicates that during the first rotation in the double lane change, the roll angle increases while cornering. Peak body roll occurs with larger steering angles due to increased centripetal force from the tyres, causing greater lateral acceleration and body roll due to inertia (Nguyen & Nguyen, 2023). Angular velocity responds to changes in roll angle and is greatest during the slalom test due to rapid and constant steering angle changes.

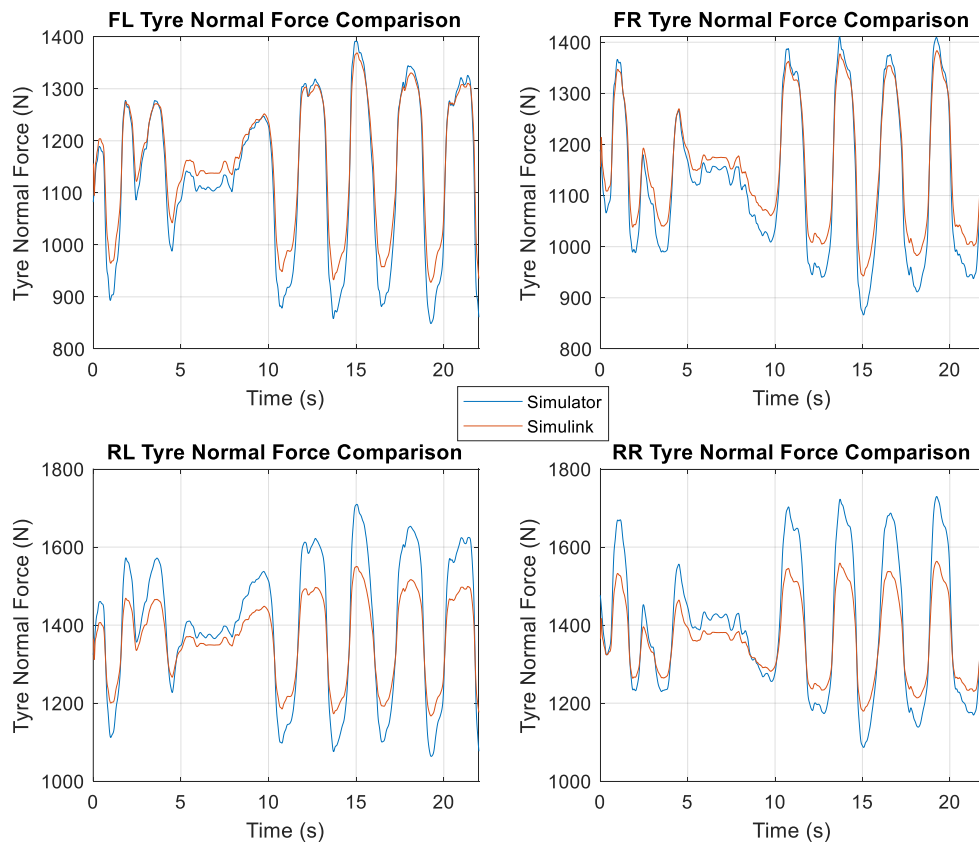


Graph 8: Standard Test Hand Wheel Angle Percentages, Roll Angles and Velocities.

Graph 9 shows pitch angle and pitch angular velocity. Changes in these values are minimal due to the constant longitudinal velocity and minimal longitudinal acceleration inputs (graph 6). The pitch angle is not zero when stationary because of the positive rake angle in the vehicle suspension setup. This parameter was neglected in the Simulink 7-DOF model for simplicity. Reasonable accuracy is shown in graph 10 when comparing different simulations, whilst load transfer seems appropriate where outer wheels have higher loads during cornering.



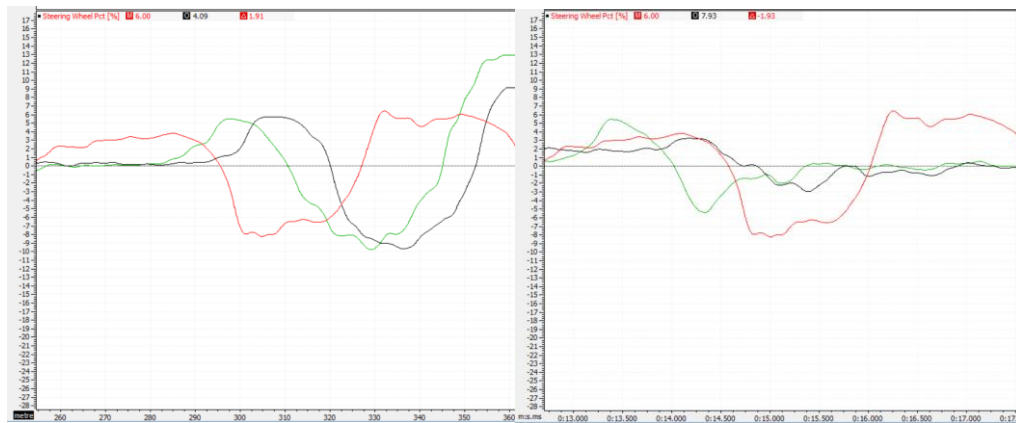
Graph 9: Standard Test Pitch Angle and Velocity.



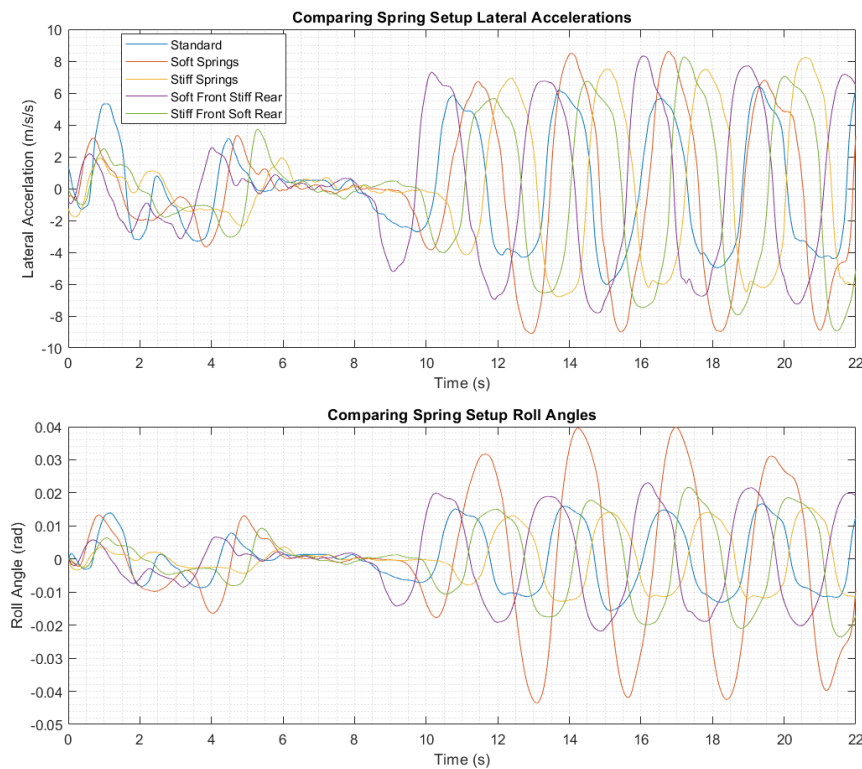
Graph 10: Standard Test Tyre Normal Force Comparison Between Simulator and Simulink.

4.3 Comparing Different Setups

Misaligned acceleration data for different setups complicates comparisons (graph 11 for same data section with left time and right distance). Lateral accelerations vary with suspension setups (graph 12), affecting roll and load transfer. To ensure a fair test, standard test lateral and longitudinal acceleration data was applied to all other setups. Despite this, stiff springs reduce body roll the most, followed by standard setup. Different front and rear spring configurations yield indistinct results, while soft springs cause more roll.



Graph 11: (Left) Distance Against Steering Angle, (Right) Time Against Steering Angle.



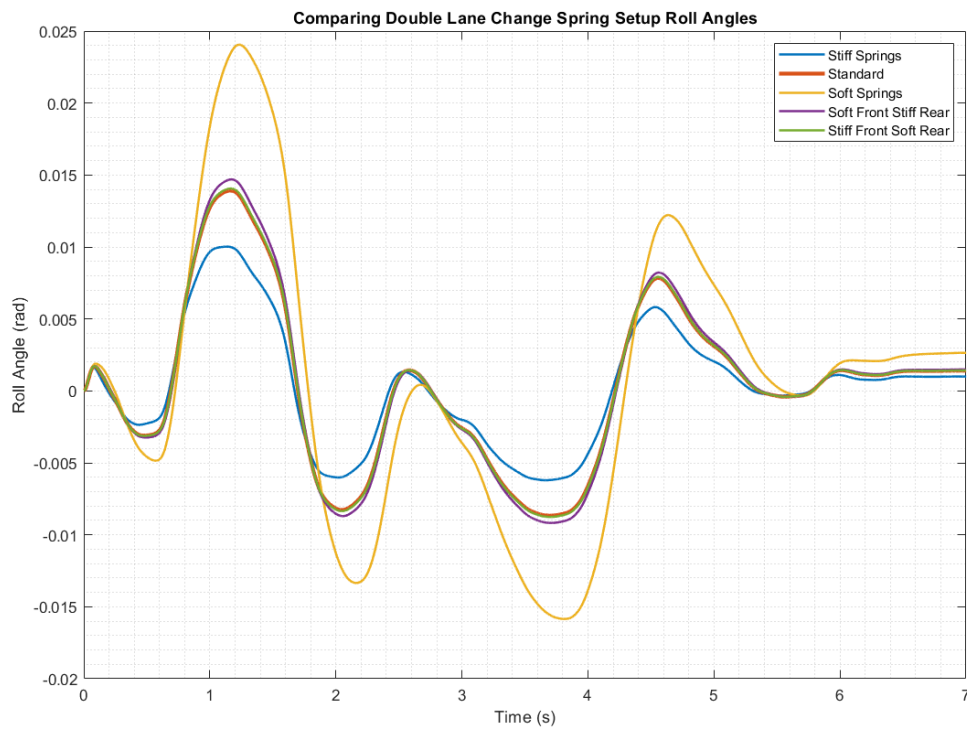
Graph 12: Comparing Spring Setup Lateral Accelerations and Roll Angles.

4.3.1 Double Lane Change Springs

The first comparison of parameters was conducted on springs for the double lane change maneuver. Spring rates for the front and rear of the vehicle in Simulink were adapted to the parameters shown in table 4, with test numbers referring to setup numbers in table 2. The Simulink model estimates the front and rear ride frequency for each setup. Initial interpretations suggest that stiffer springs, standard, and 'front stiff rear soft' setups improve handling by reducing body roll, while a higher front ride frequency enhances cornering stability (Smith, 2013). Conversely, soft springs and 'front soft rear stiff' setups improve ride comfort but increase body roll. Graph 13 shows that the stiff spring suspension vehicle experiences half the body roll of a soft spring setup, with other setup roll angles aligning with predictions based on ride frequency.

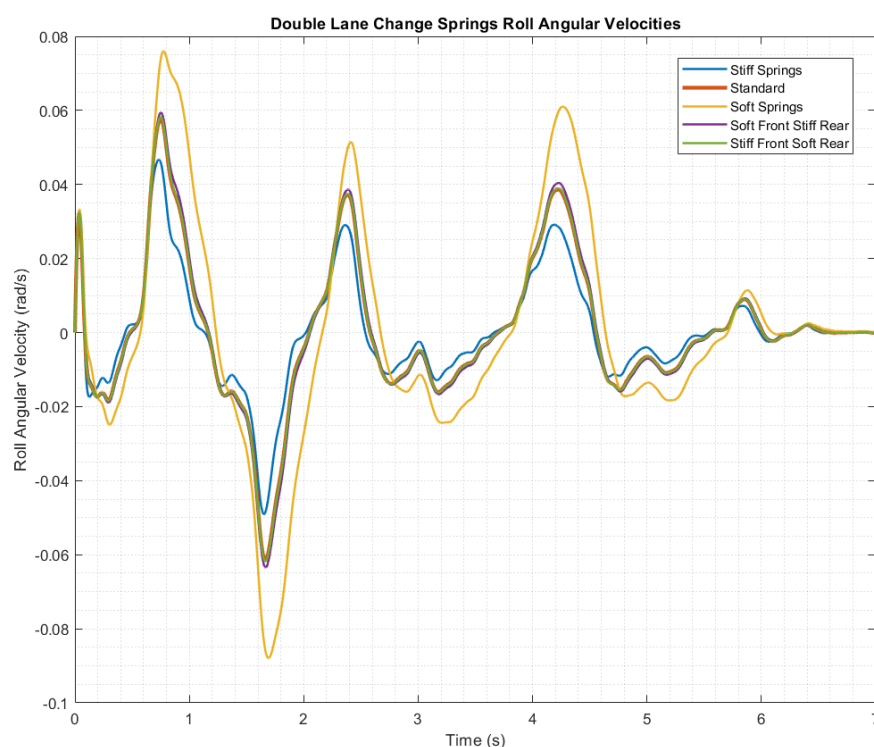
Table 4: Double Lane Change Spring Parameters.

Spring Parameters	Standard	Stiff Springs	Soft Springs	Front Stiff Rear Soft	Front Soft Rear Stiff
Test Number	1	3	5	6	7
Front Spring Rate (N/m)	18000.00	26000.00	9000.00	26000.00	9000.00
Rear Spring Rate (N/m)	16000.00	24000.00	8000.00	8000.00	24000.00
Front Ride Frequency (Hz)	1.03	1.24	0.73	1.24	0.73
Rear Ride Frequency (Hz)	0.98	1.19	0.69	0.69	1.19



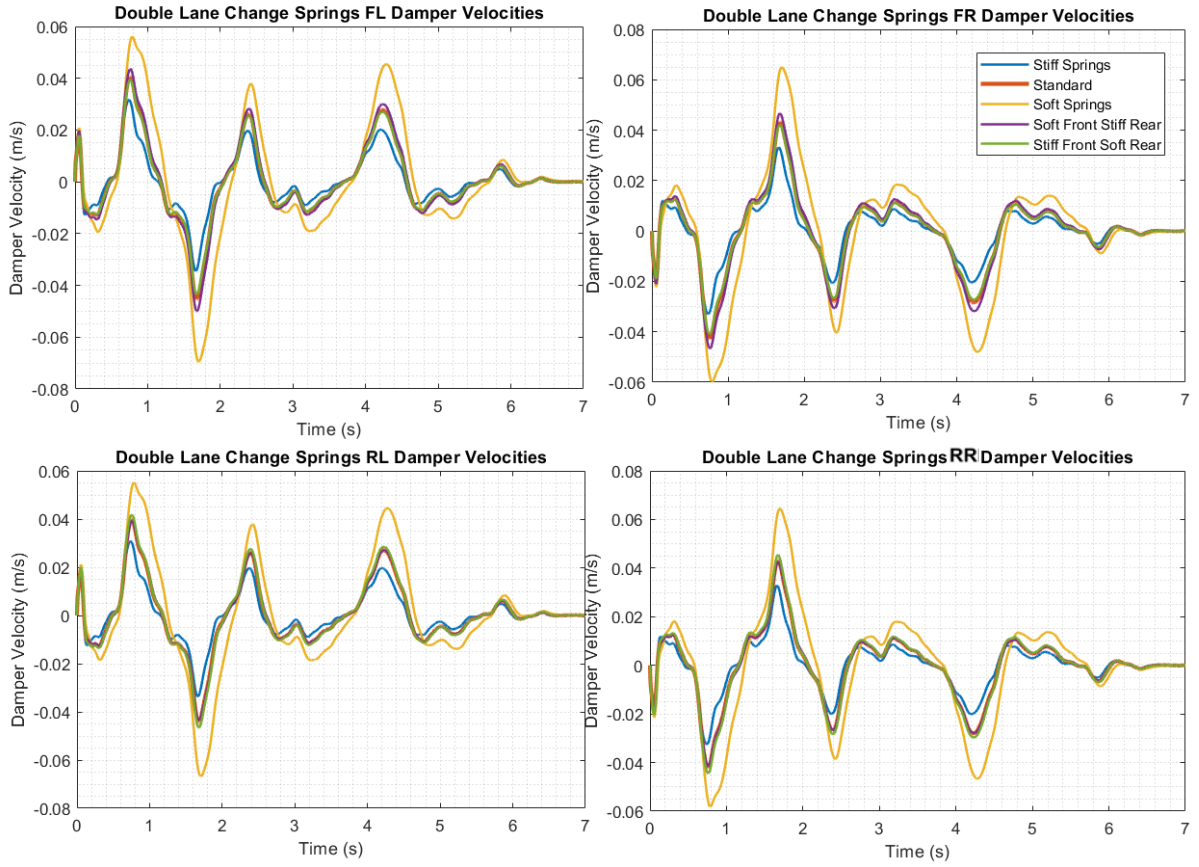
Graph 13: Double Lane Change Roll Angles for Spring Setups.

Springs in extension exert a returning force on a vehicle's sprung mass. Stiffer springs, with higher spring rates, offer greater resistance to vertical displacement compared to softer springs, reducing CoG height changes and minimizing body roll (Jiang, 2019). Graph 14 demonstrates stiff springs cause sharp changes in angular velocity, focusing on handling over ride comfort. In contrast, softer suspension provides smoother responses but higher roll angular velocity due to larger roll angles. Stiffer setups exhibit higher ride frequencies with more vibrations. Safety implications include reduced rollover chances with stiff springs, but possible jacking and an uncontrollable vehicle, while softer springs offer predictability but less cornering performance and a higher rollover possibility (improbable for the Formula Ford).

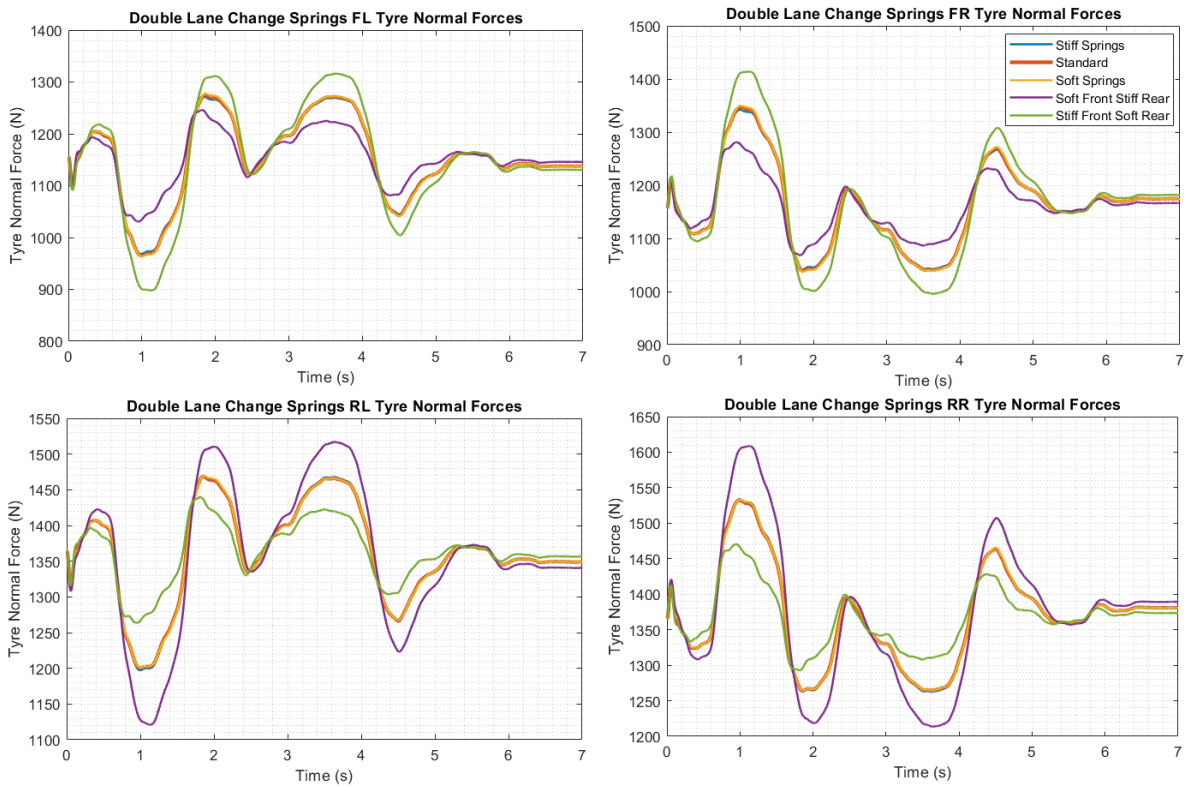


Graph 14: Double Lane Change Spring Setups Roll Angular Velocities.

Graphs 15 and 16 show that when the suspension compresses due to load transfer during cornering, the damper acts in a bump situation (negative), and the tyre normal load increases. Increased spring stiffness can result in higher amounts of load transfer, potentially reducing cornering performance by decreasing inside wheel grip. The standard suspension setup maintains a more consistent normal force on each tyre.



Graph 15: Double Lane Change Spring Setups Damper Velocities.



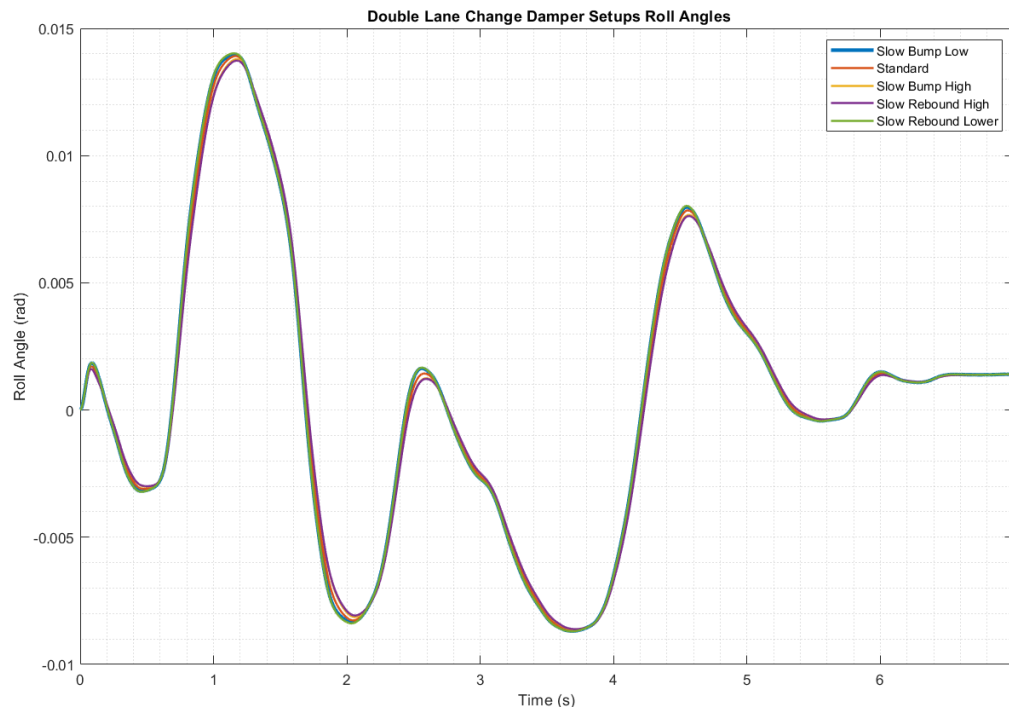
Graph 16: Double Lane Change Spring Setups Tyre Normal Forces.

4.3.2 Double Lane Change Dampers

Dampers are crucial for altering a vehicle's response and handling. Though dampers and springs are typically adjusted together (Zuijdijk, 2013), this analysis evaluates dampers independently. Table 5 presents damping coefficients, fitted into a damper curve (graph 1) and applied in Simulink with a lookup table. Graph 17 shows that higher slow velocity rebound coefficients reduce body roll the most, while reducing coefficients increase it. Increasing damping coefficients generally reduces vehicle roll.

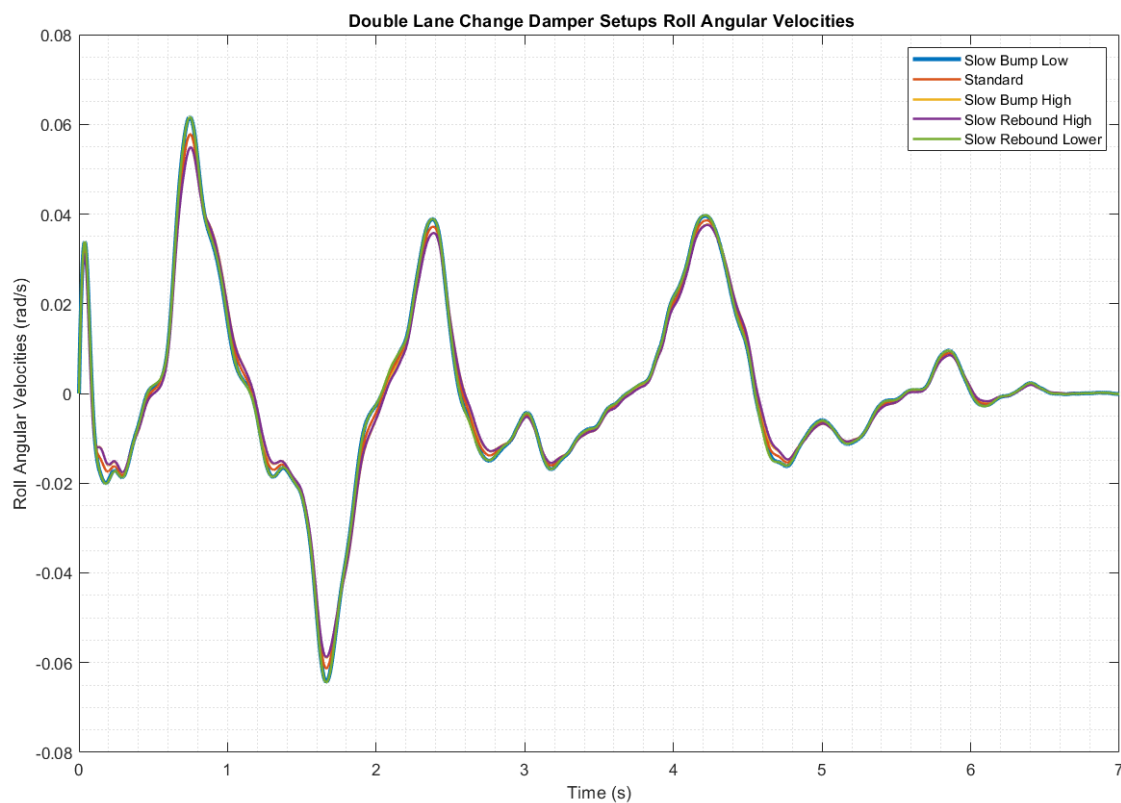
Table 5: Double Lane Change Damper Coefficient Setups.

Damping Coefficients	Standard	Slow Bump Lower	Slow Bump Higher	Slow Rebound Lower	Slow Rebound Higher
Test Number	1	10	11	14	15
front slow bump (N/m/s)	1645	1245	2045	1645	1645
front fast bump (N/m/s)	533.25	533.25	533.25	533.25	533.25
front slow rebound (N/m/s)	1936.25	1936.25	1936.25	1436.25	2436.25
front fast rebound (N/m/s)	511	511	511	511	511
rear slow bump (N/m/s)	1507.75	1107.75	1907.75	1507.75	1507.75
rear fast bump (N/m/s)	552.75	552.75	552.75	552.75	552.75
rear slow rebound (N/m/s)	1609	1609	1609	1209	2009
rear fast rebound (N/m/s)	552.75	552.75	552.75	552.75	552.75

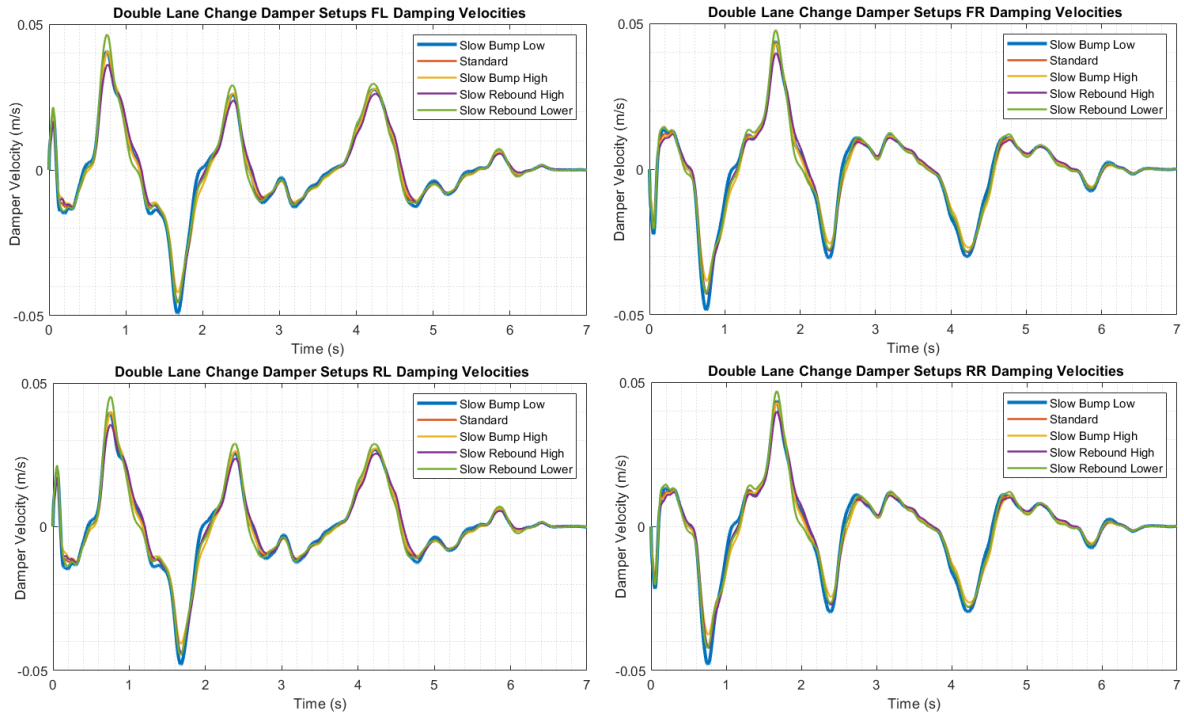


Graph 17: Double Lane Change Damper Setups Roll Angles.

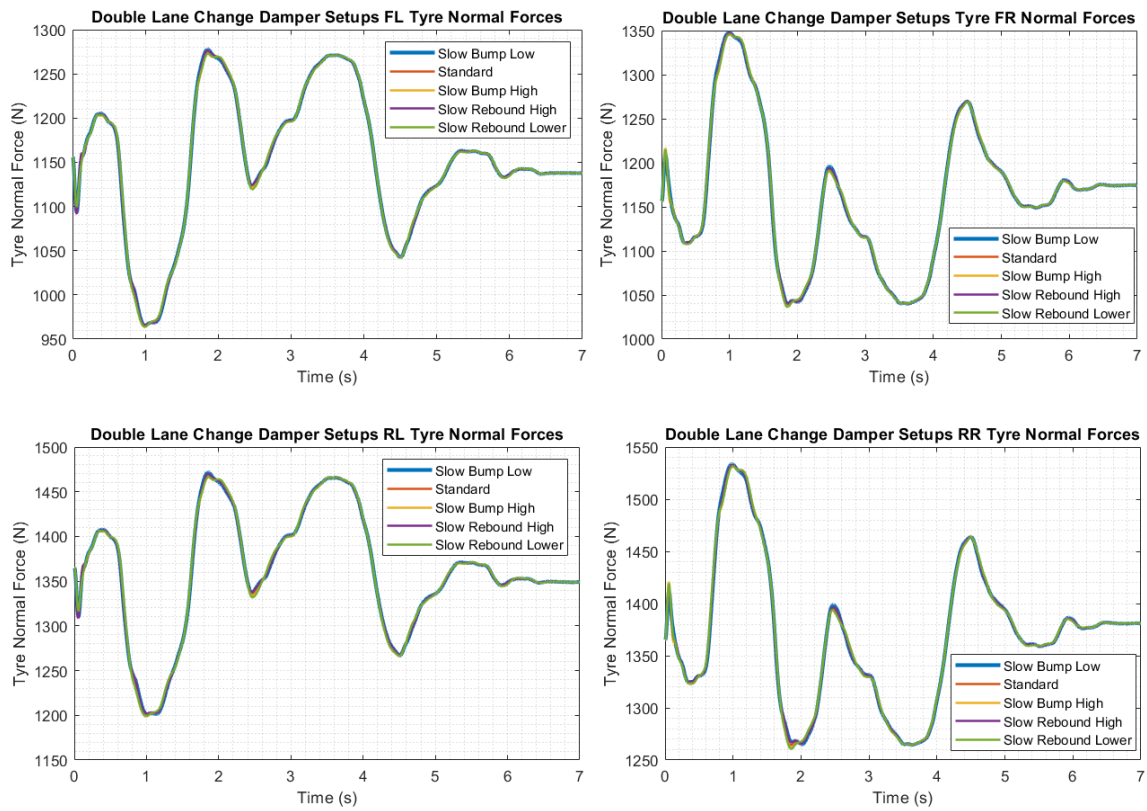
Roll angular velocities (graph 18) show reduced maximum roll velocities with higher rebound damping and increased with lower rebound damping coefficients, indicating the importance of rebound damping during cornering, by prevent excessive rolling. However, excessive rebound damping can lead to jacking in high cornering events (Zuijdijk, 2013). In racing, bump damping is prioritized to control ride height and reduce roll with stiffer springs. Graph 19 shows changes in damping coefficients affect maximum damper velocity, with increased bump and rebound damping decreasing maximum velocities. Graph 20 indicates minimal differences in load transfer due to damping coefficient changes, as cornering forces were not significant enough and spring rates too high to demonstrate significant effects when adjusting dampers (Zuijdijk, 2013).



Graph 18: Double Lane Change Damper Setups Roll Angular Velocity.



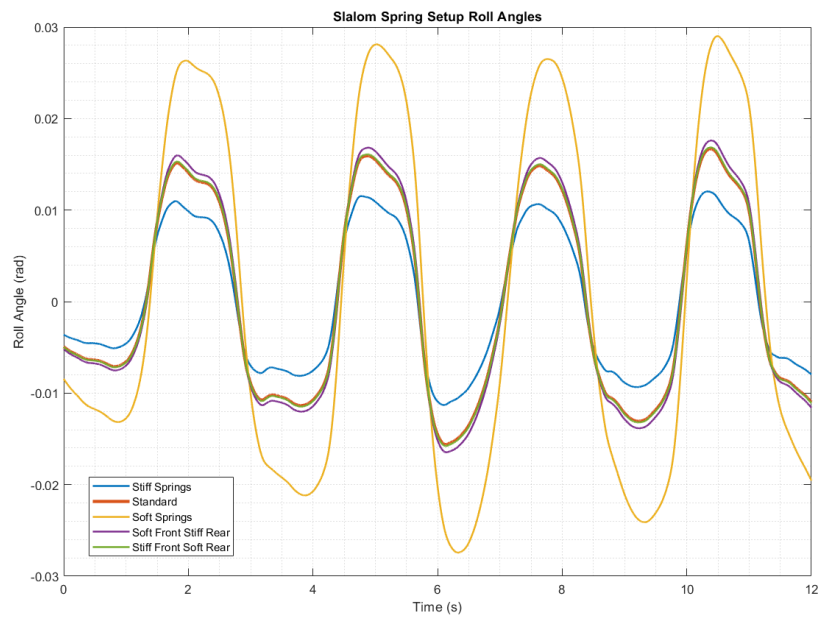
Graph 19: Double Lane Change Damper Setups Damper Velocities.



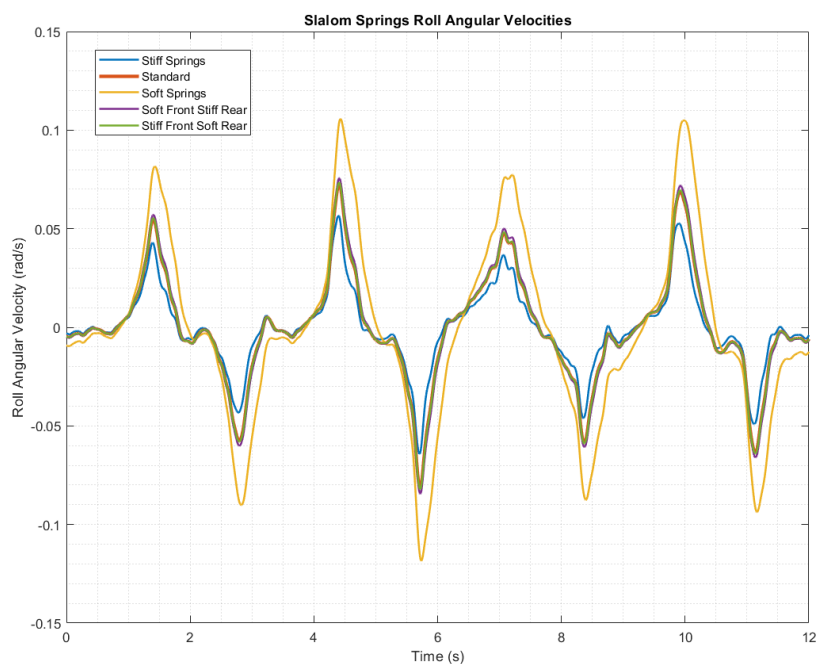
Graph 20: Double Lane Change Damper Setups Tyre Normal Forces.

4.3.3 Slalom Springs

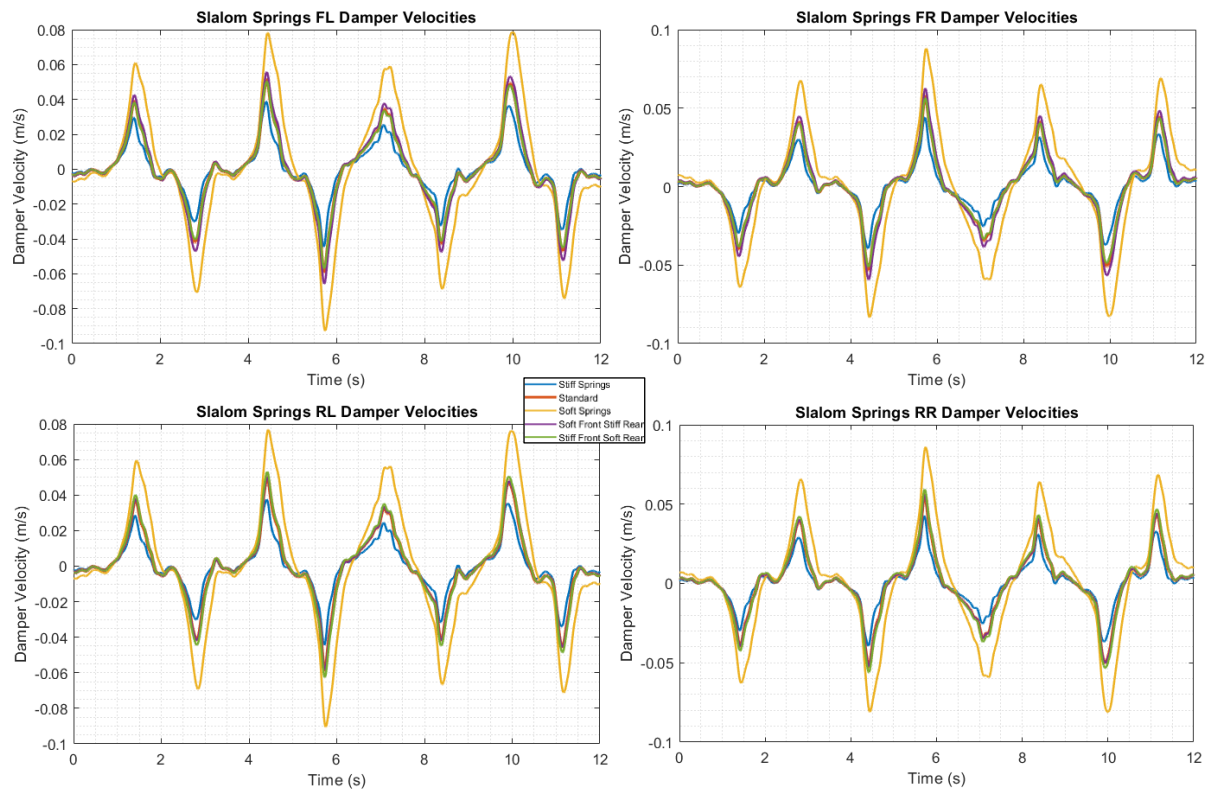
Stiffer springs in the slalom test lead to an uncomfortable ride (graphs 21, 22, 23, and 24), with noticeable changes at 3-4 seconds in graph 21, aligning with graph 6's lateral acceleration data. The best setups for this maneuver are stiff springs for reduced body roll and balanced tyre normal forces. The 'stiff front soft rear' setup offers responsive steering and higher front tyre grip but sacrifices ride comfort and increases oversteer risk (Smith, 2013).



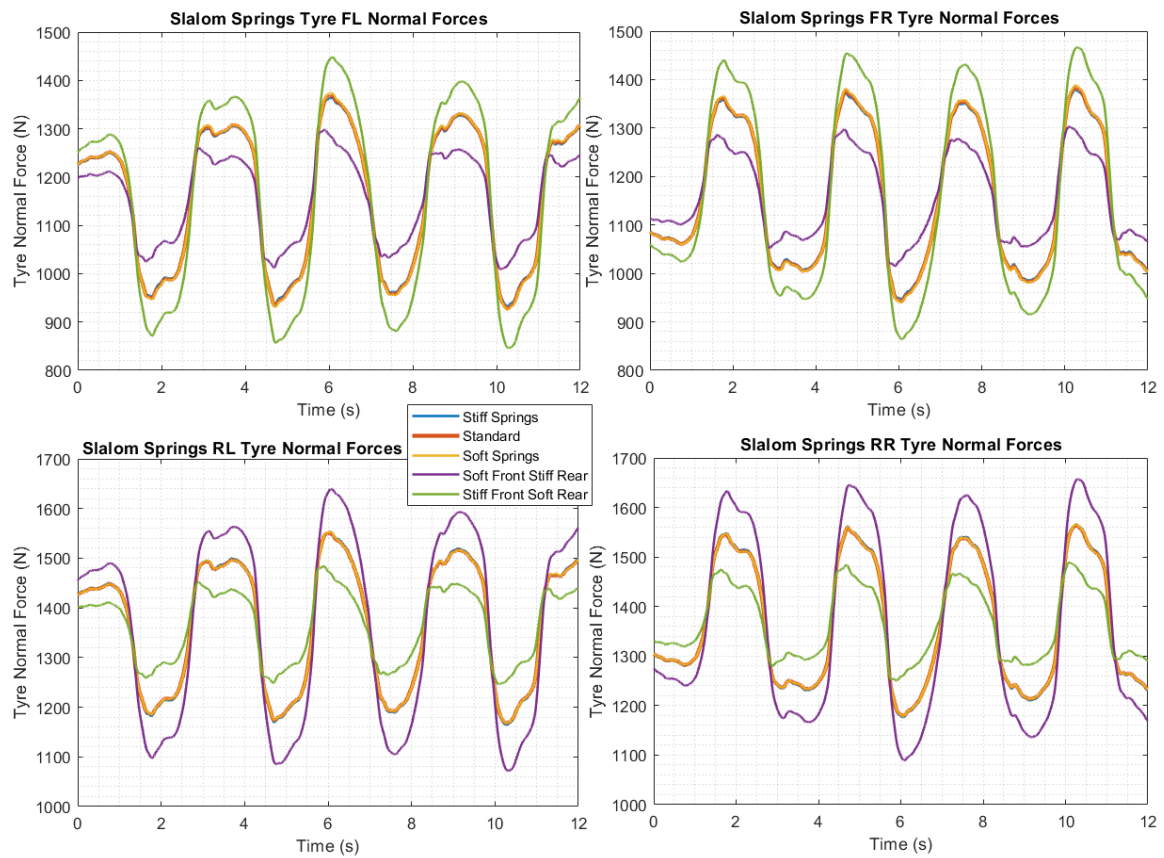
Graph 21: Slalom Spring Setups Roll Angles.



Graph 22: Slalom Spring Setups Roll Angular Velocities.



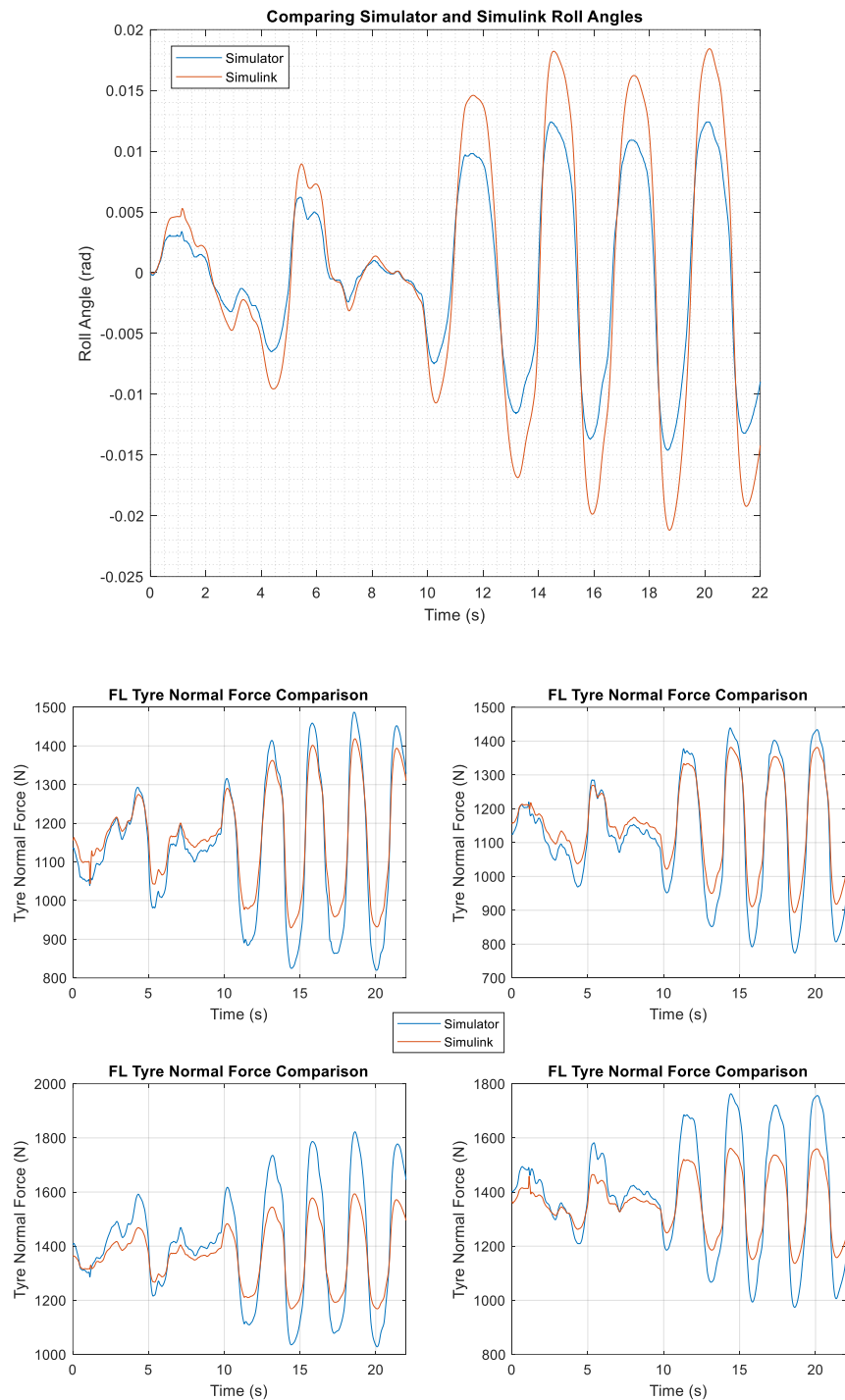
Graph 23: Slalom Spring Setups Damper Velocities.



Graph 24: Slalom Spring Setups Tyre Normal Forces.

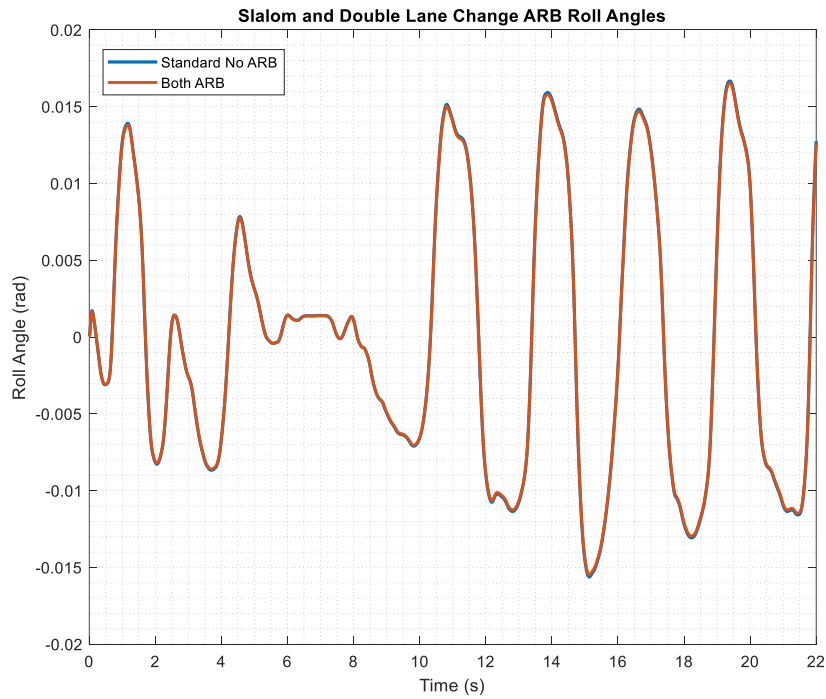
4.3.4 Slalom and Double Lane Change Anti Roll Bars

Simulink pitch angle results were compared to simulator results (graph 25) to validate ARBs. The system showed similar trendlines but overestimated roll angles and underestimated tyre normal forces. Factors causing this are detailed in parameters, assumptions, and model validity. Due to reliable trendlines, ARB testing continued to better understand vehicle corner stability benefits.

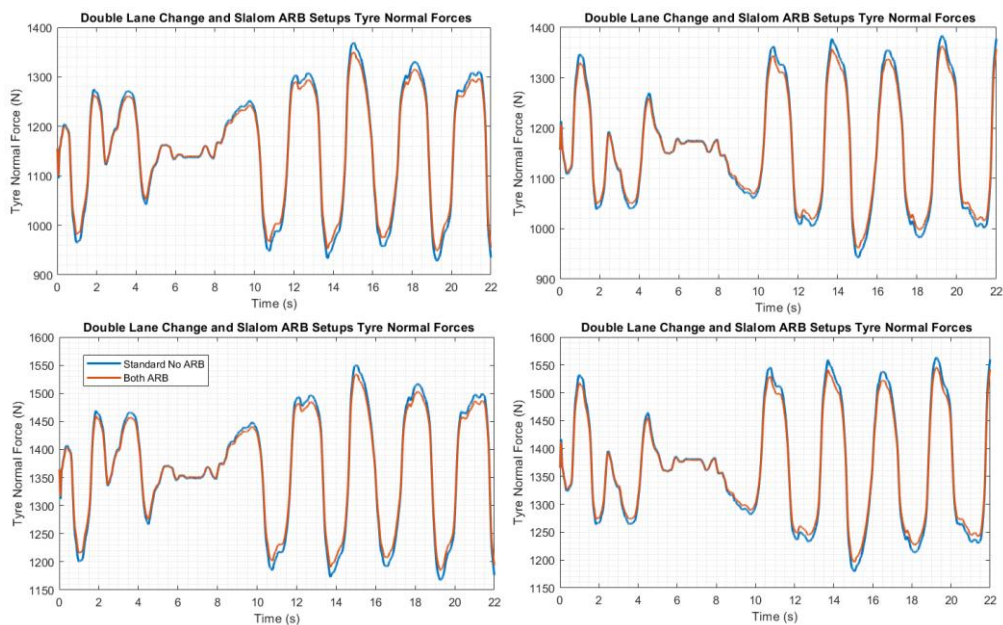


Graph 25: Simulink ARBs Validation Against Simulator (Top) Roll Angle, (Bottom) Tyre Normal Forces.

After validating the ARB model, standard longitudinal and acceleration data was input with standard suspension data and front and rear ARBs. Graph 26 shows small differences in roll angles with and without ARBs. ARBs reduce body roll by connecting left and right suspension components as a torsional spring (Do, et al., 2024). The ARB used was relatively soft, with small impacts on body roll and angular velocity due to low cornering forces. Graph 27 shows more equalized tyre forces between left and right sides during lateral acceleration changes. Stiffer ARBs could reduce body roll and maintain optimal contact patch area for all four tyres.



Graph 26: Slalom and Double Lane Change Effects on Roll Angle due to ARBs.



Graph 27: Slalom and Double Lane Change Effects on Tyre Normal Forces due to ARBs.

5 Model Validation

Result accuracy relies on correct roll and pitch inertias in the Simulink model, similar to Azhar et al. (2014), whose 7-DOF ride model depended on inertia values. These parameters are assumptions from previous studies and simulator data without real-world measurements. The lack of tyre modelling and yaw, impacts reliability and accuracy in values such as pitch angle and tyre forces. Trendlines show good correlation with simulator results, allowing accurate interpretation of ride quality and handling. Vi-Grade straight line braking scenario validated the longitudinal aspects. Though vehicle configurations differed, trendlines and pitch angle results correlated for ‘soft front stiff rear’ and ‘stiff front soft rear’ (figure 13 and Graph 28). Most results compare well with the simulator, with slight variations in graphs 29 and 30. The 7-DOF model provides valuable insights with varying accuracy. Fine-tuning parameters would help reduce error. Adjusting dampers improved accuracy over springs, possibly due to linear springs in Simulink not matching the simulator, and neglected spring extension/compression range in SMD system (Palli et al., 2017). See appendix C for acceleration data and other output graphs.

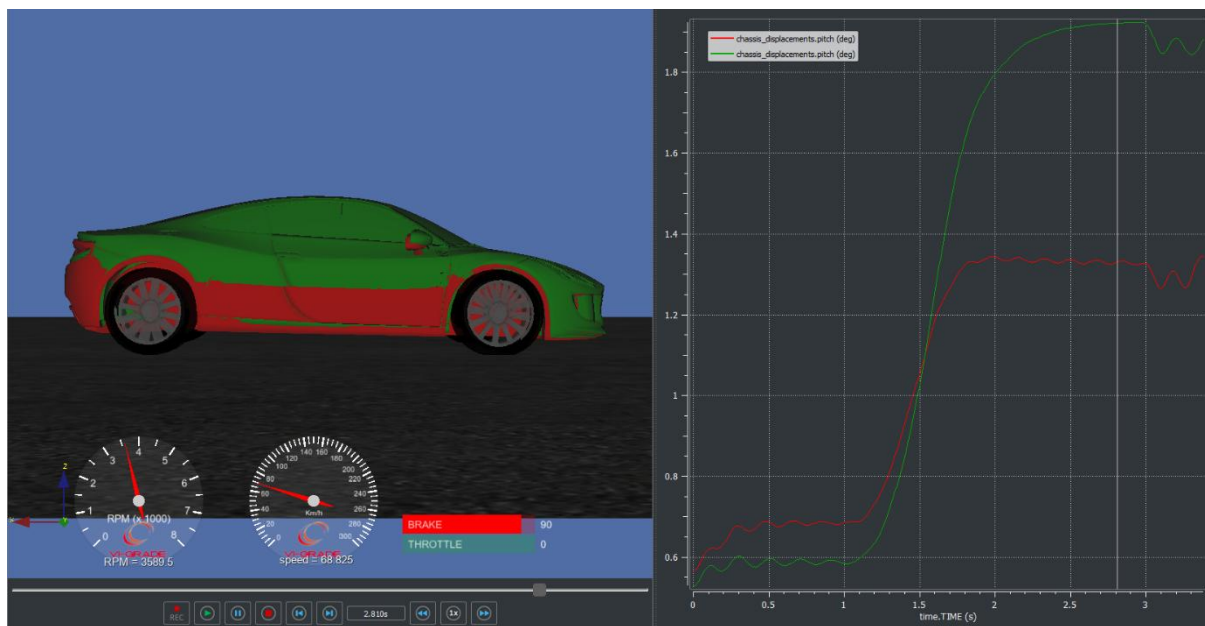
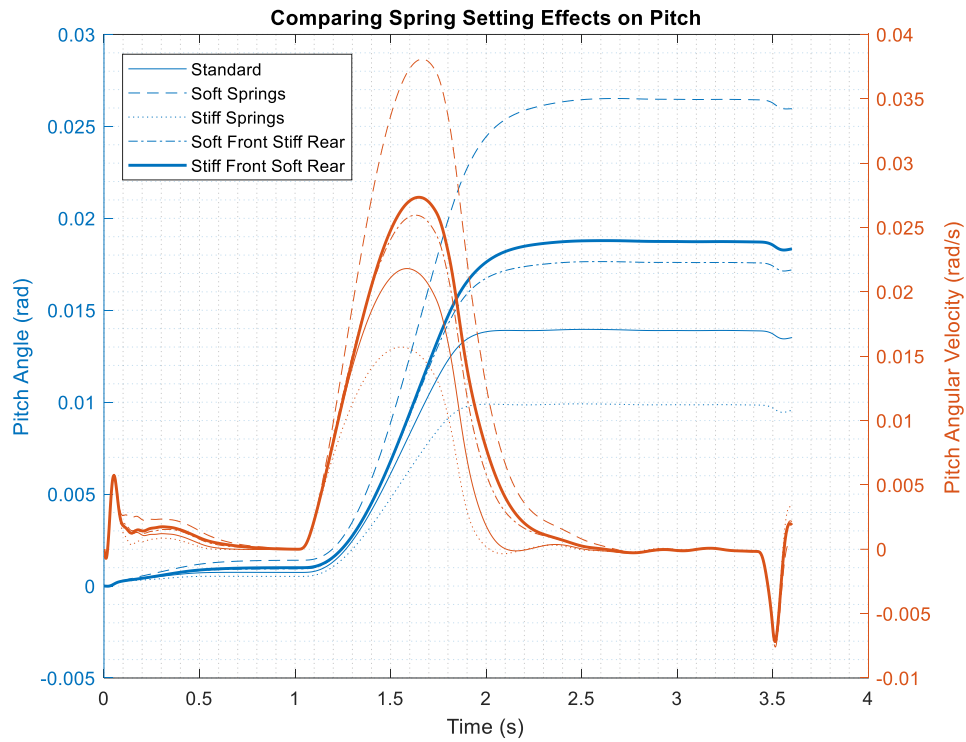
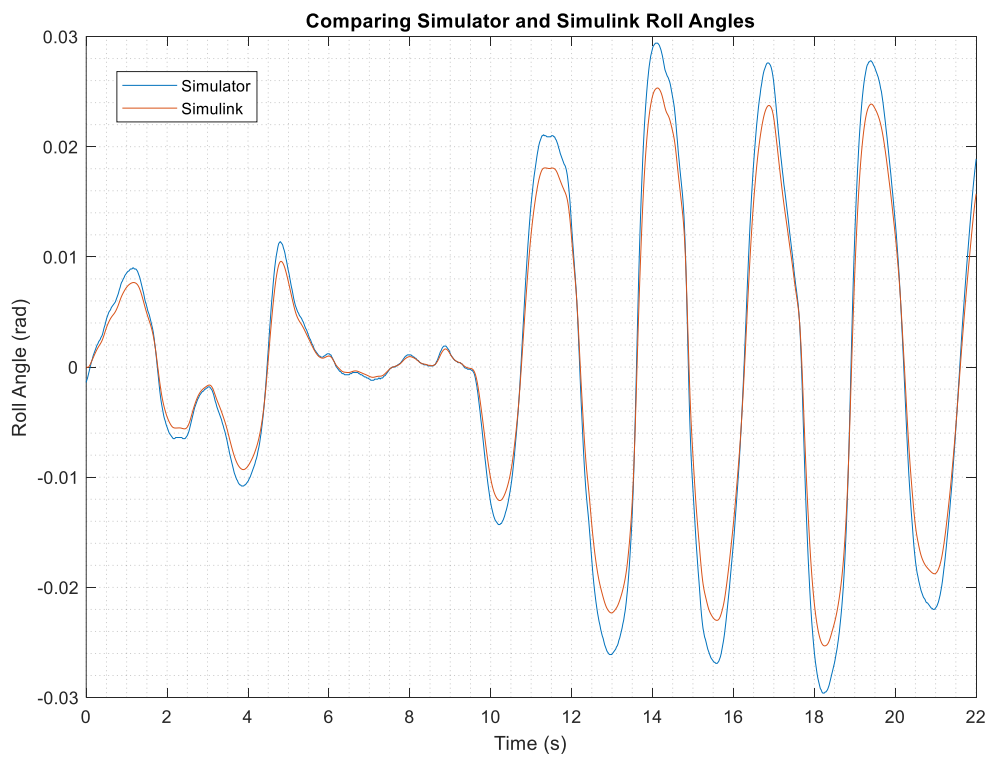


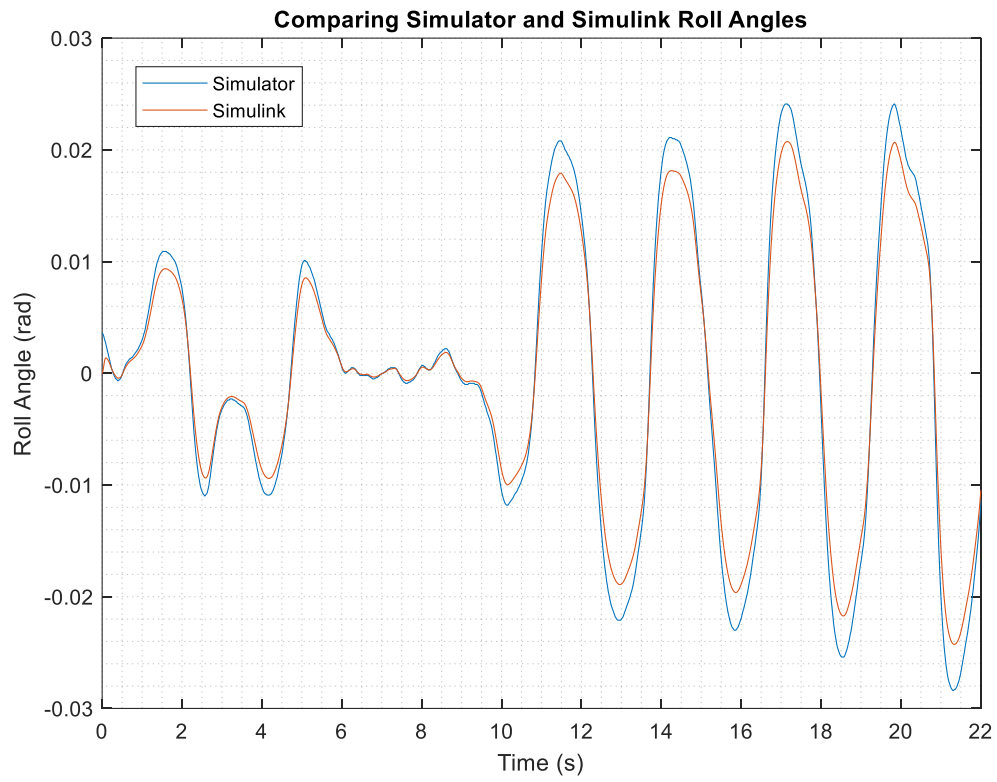
Figure 13: Vi-Grade Straight Line Braking (y- axis Pitch Angle in degrees), Green Line Stiff Front Soft Rear, Red Line Soft Front Stiff Rear.



Graph 28: Straight Line Braking Spring Setup Effects on Pitch.



Graph 29: Comparing Simulator and Simulink Roll Angles for Test Number 8 (Fast Bump Lower).



Graph 30: Comparing Simulator and Simulink Roll Angles for Test Number 12 (Fast Rebound Lower).

6 Conclusion

In conclusion, the 7-DOF Simulink model is adequate for vehicle dynamics analysis, showing good correlation with simulator testing and Vi-Grade vehicle dynamics software. Key findings include: increased ride frequency from stiffer spring rates results in firmer suspension, reducing body and pitch roll and improving cornering and responsiveness; reduced spring rates decrease cornering stability but improve ride quality and comfort. Combining stiffer and softer springs on different axles affects oversteering or understeering. Increased spring rates can increase load transfer and reduce handling performance with smaller contact patches on inner wheels. Increasing damping coefficients can lead to suspension not levelling out quick enough, or increasing ride stiffness to an uncontrollable point. Dampers and springs should be adjusted together to control oscillations and suspension response rates. Anti-roll bars reduce body roll and load transfer, maintaining optimum tyre contact patch area during cornering. The Simulink model could be improved with a 7-DOF vehicle handling model to account for yaw, tyre lateral and longitudinal forces, and rotational velocity. Refining inertia parameters would improve model accuracy.

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Appendix A

The Simulink 7-DOF model uses a single reference point for vehicle sprung mass displacement. In reality, ride height varies at each corner during dynamic events and uneven surfaces. Stationary vehicles often have a positive rake angle, causing front and rear ride height differences without external forces (Smith, 2013). This assumption reduces accuracy in sprung mass positioning, affecting aerodynamic forces, ground clearance, and suspension systems, leading to changes in tyre contact patches and available grip (Pauwelussen, 2014). Figure 14 (a) shows equal ride height, contact patches, and ground clearance for a stationary axle with double wishbone suspension on a flat surface. Figure 14 (b) shows increased ride height and reduced tyre contact patch on the left side during a left turn due to centrifugal forces on the sprung mass. Different suspension geometries and layouts have varying impacts.

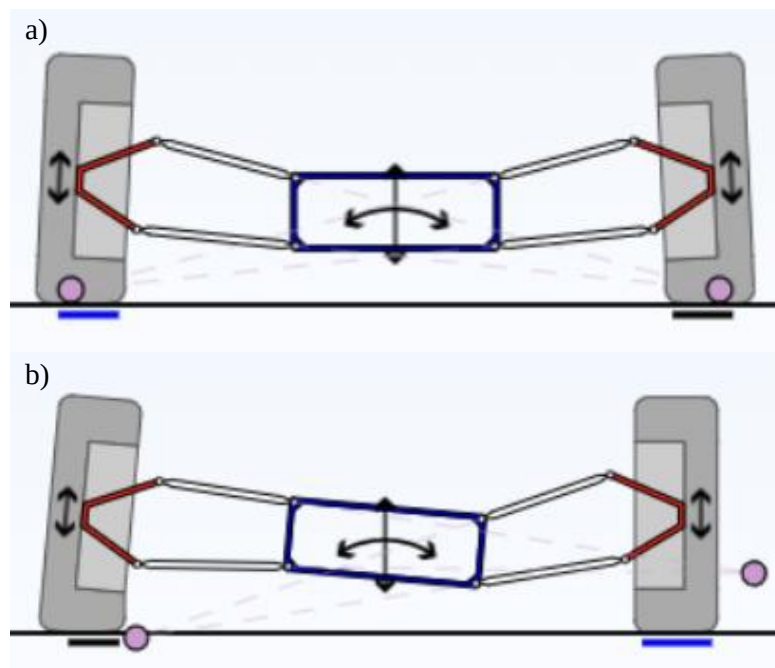


Figure 14: Suspension Positioning Effects (a) Stationary (b) Dynamic Left Turning (Racing Aspirations, n.d.)

Appendix B

Due to the unknown damper curve, the 'knee point' for fast and slow damping transitions is estimated: 0.01m/s for compression and 0.025m/s for rebound. These estimates are based on a Quantum 2-way adjustable racing damper test (figure 15) and a Nitron damper test for a lightweight racing car (figure 16) (Nitron, 2021). Though not precise for Formula Ford dampers, this allows the implementation of a 4-way damper system instead of a single damping coefficient. Accurate knee-points or damper curve data would improve this model.

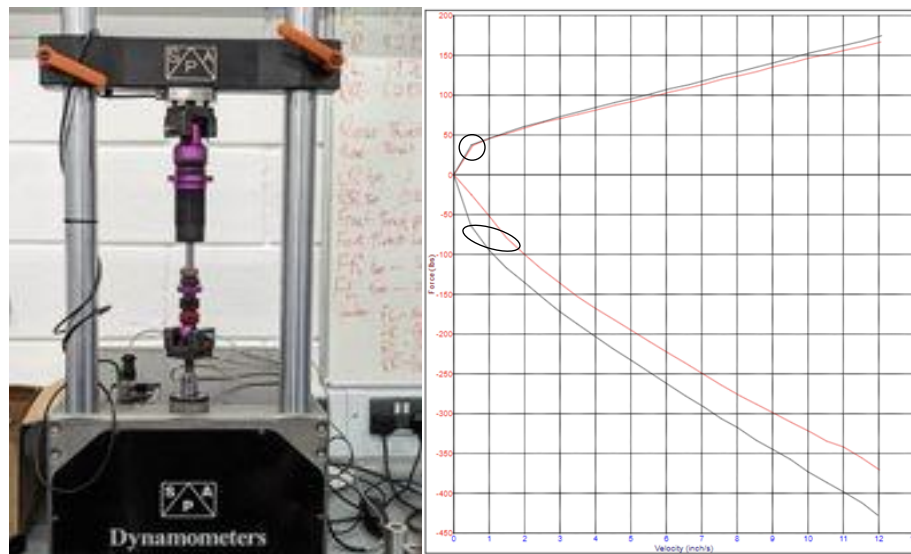


Figure 15: Damper Dynamometer and Damper Curve.

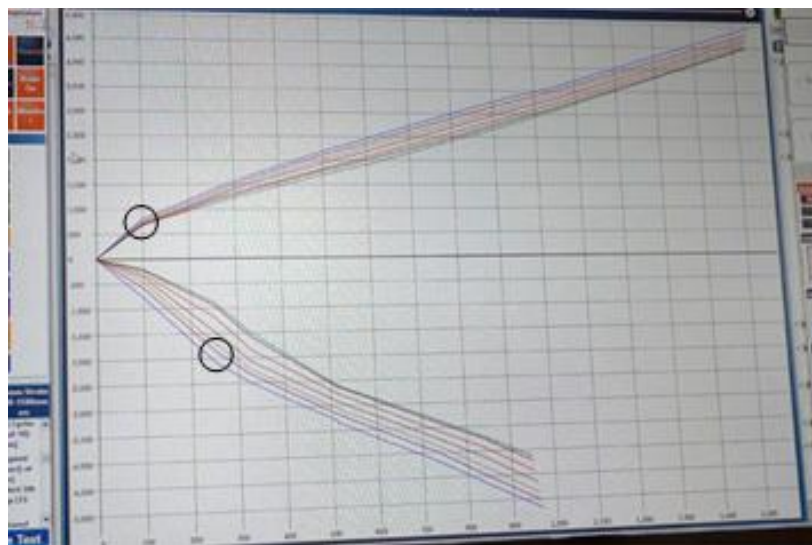


Figure 16: Nitron Damper Dynamometer Test Results (Nitron, 2021).

Damping ratios (equations 36 and 37) are crucial for vehicle comfort and handling. An underdamped system ($\zeta < 1$) returns to stationary after oscillations, a critically damped system ($\zeta = 1$) returns fastest with no oscillations, and an overdamped system ($\zeta > 1$) returns slower with no oscillations. Most vehicles use underdamped systems for better comfort and handling. The typical range is 0.3-0.5 for automotive and 0.6-1.0 for racing (Rouelle, 2021), ensuring quick suspension return/settling (figure 17).

$$C_{crit} = 2\sqrt{Km} \quad (36)$$

$$\zeta = \frac{c}{C_{crit}} \quad (37)$$

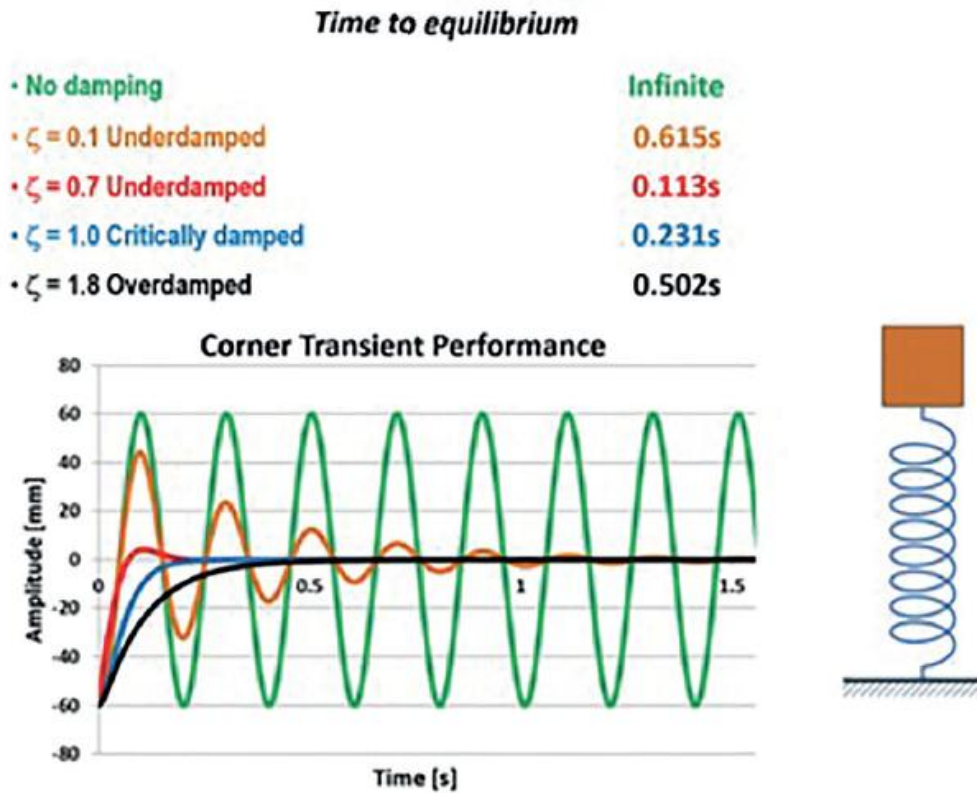


Figure 17: Damping Ratio Comparison (Rouelle, 2021).

Appendix C

This section demonstrates the final Simulink model (figure 18) as well as any additional subsystems mentioned or modelled that have not been included in the main report (figure 19). Auto in the ARB area sets the ARB settings based on test number as described in table 2.

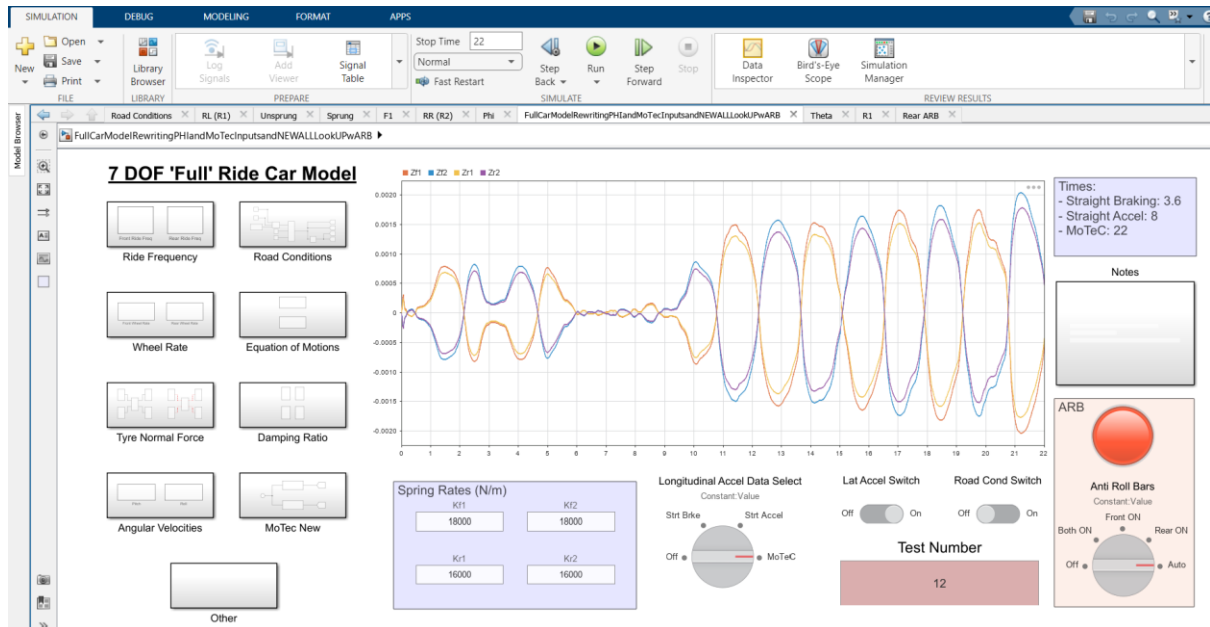


Figure 18: Simulink 7-DOF Vehicle Ride Handling Model System Overview.

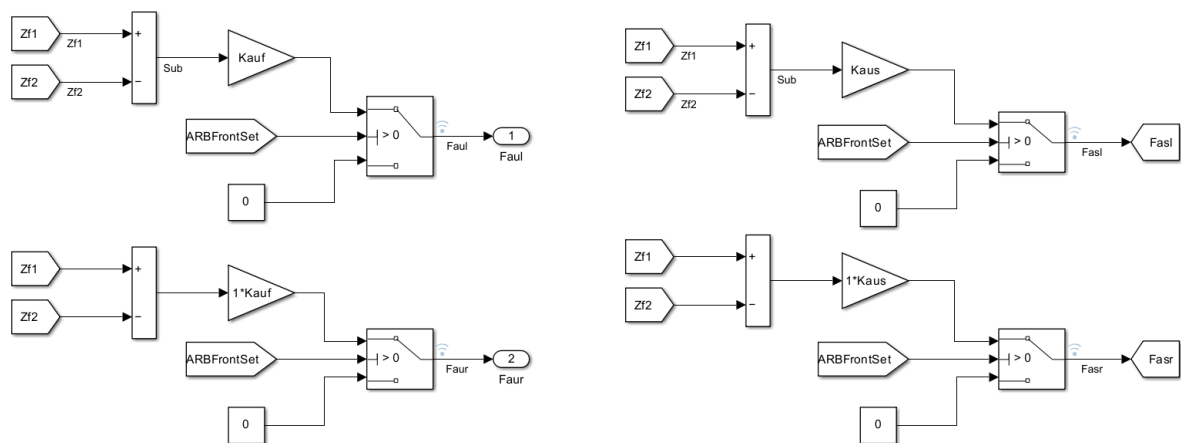
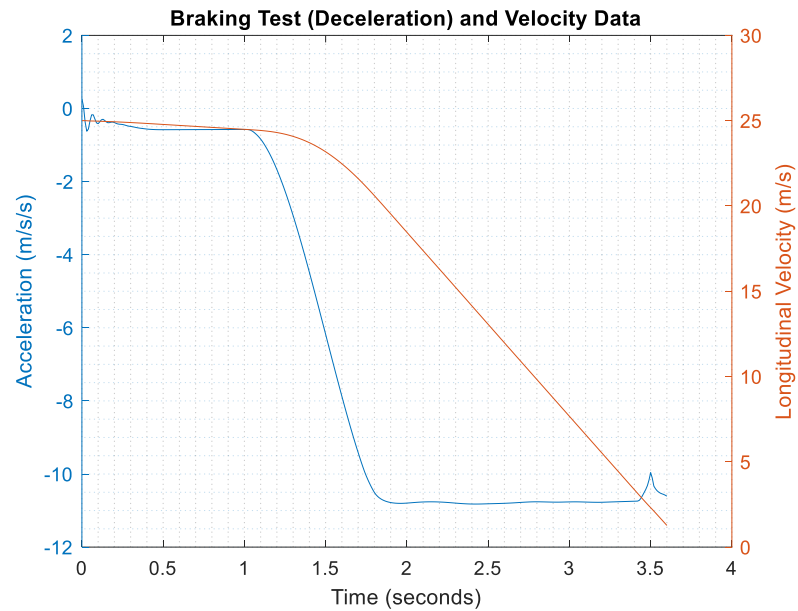


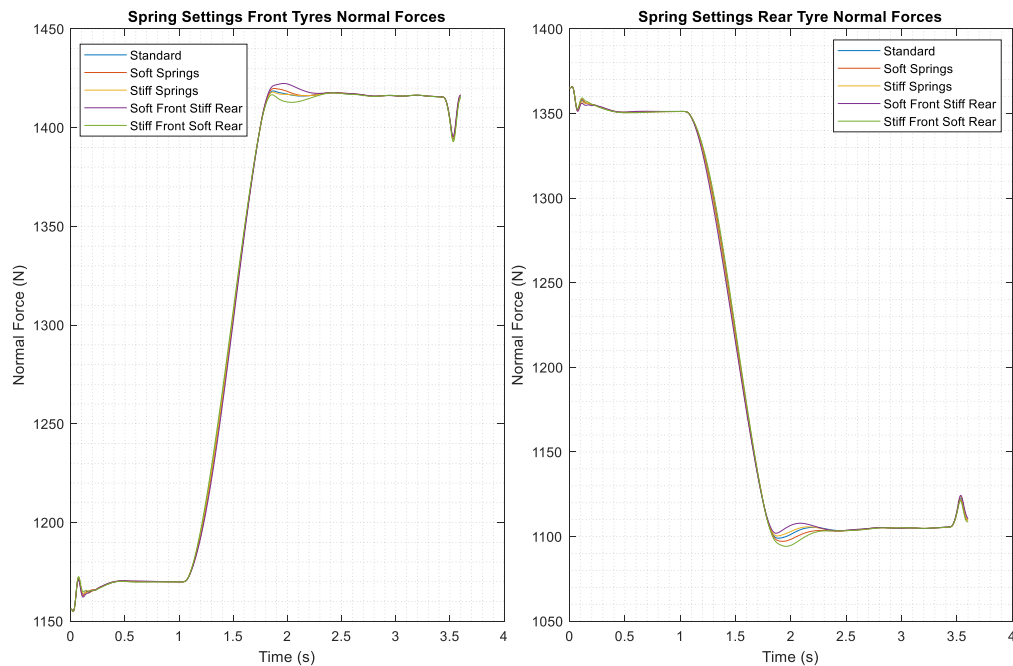
Figure 19: Simulink Front ARB Subsystem.

Appendix D

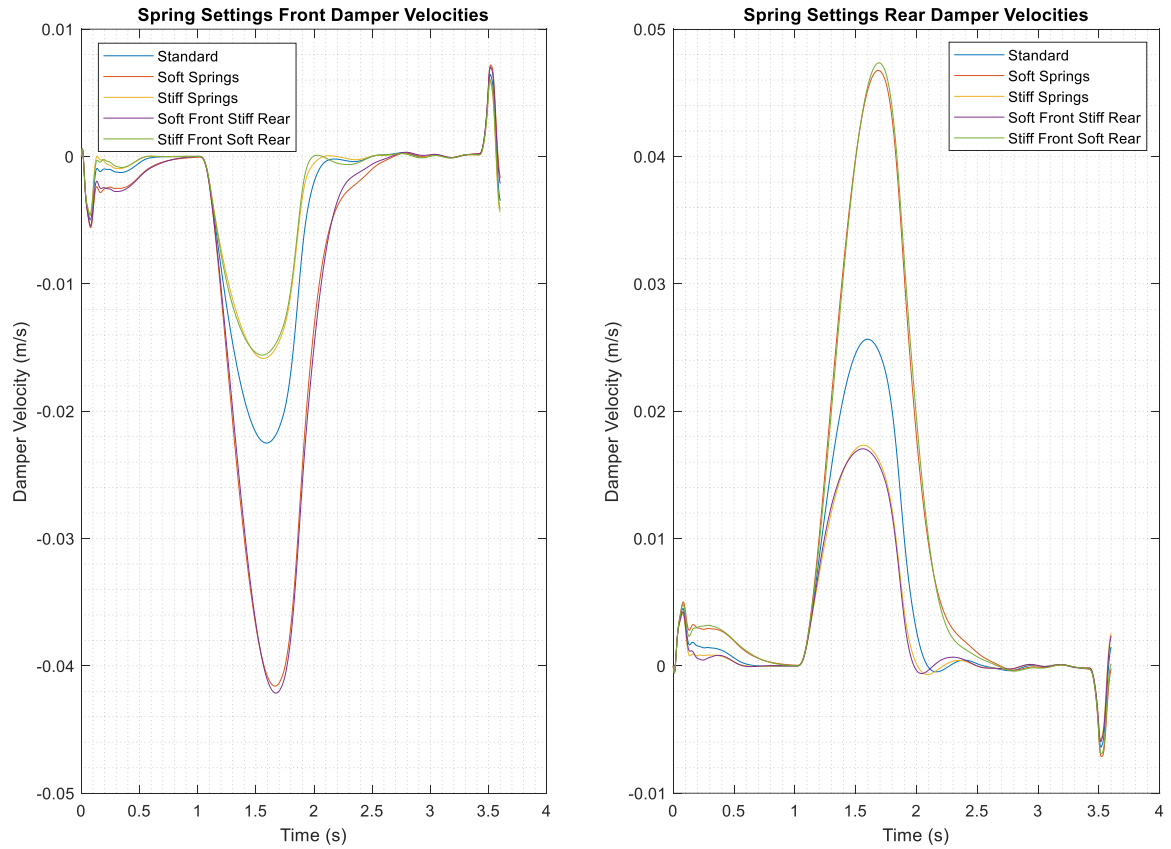
Additional graphs from the straight-line braking test are shown in graphs 31-36, the main vehicle dynamic effects in the longitudinal direction are discussed in straight line acceleration, where braking has for the most part opposite results, some slight variations are seen due to a 45:55 weight distribution.



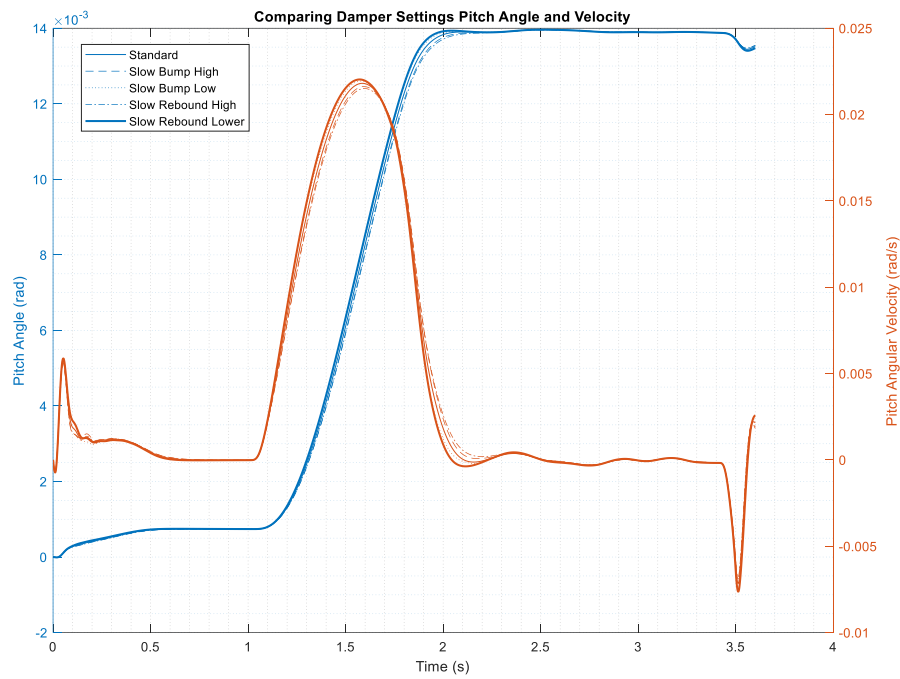
Graph 31: Straight-Line Braking Longitudinal Acceleration Input and Calculated Velocity.



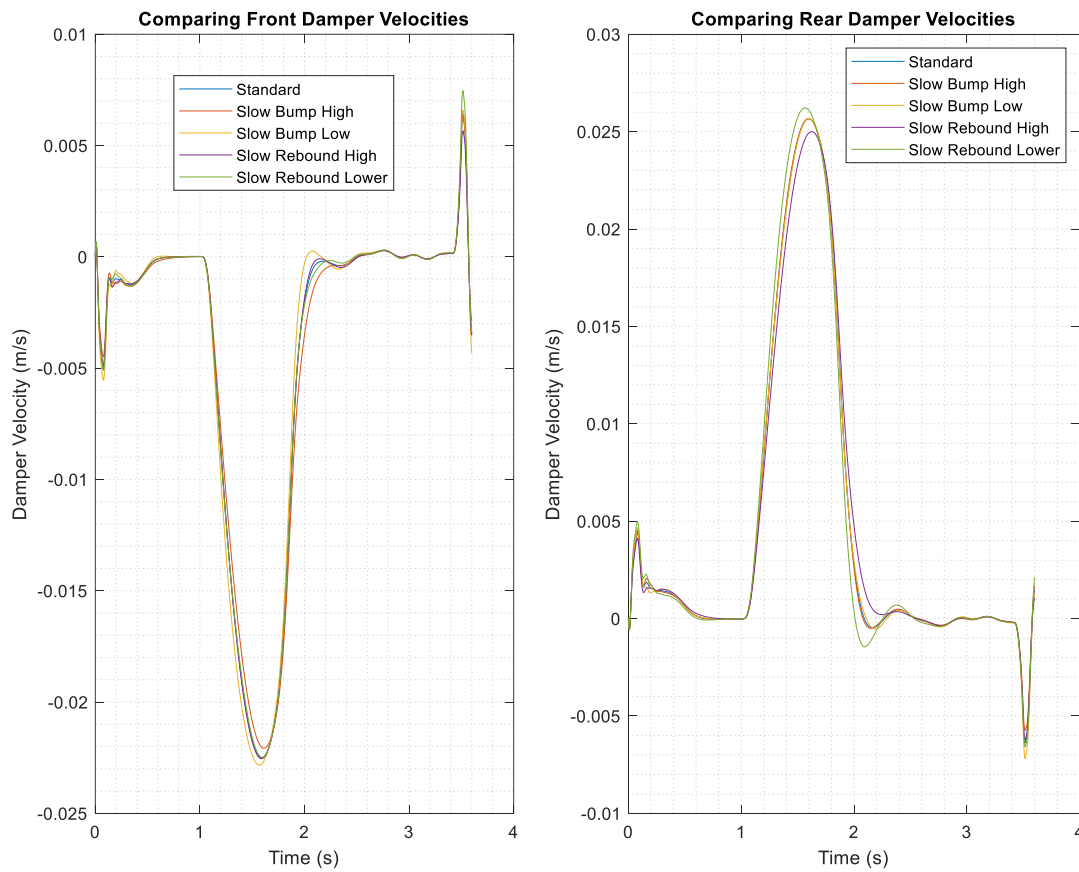
Graph 32: Straight-Line Braking Spring Setups Tyre Normal Forces.



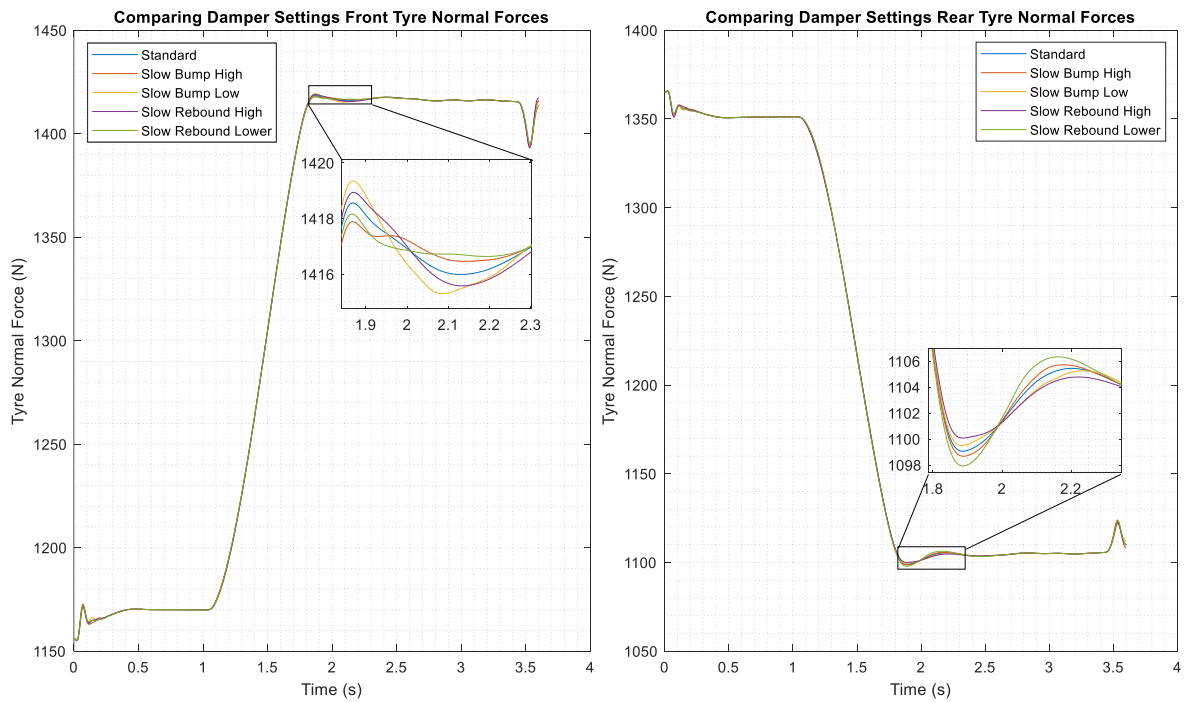
Graph 33: Straight-Line Braking Spring Setups Damper Velocities.



Graph 34: Straight-Line Braking Damper Setups Pitch Angles and Angular Velocities.



Graph 35: Straight-Line Braking Damper Setups Damper Velocities.



Graph 36: Straight-Line Braking Damper Setups Front and Rear Tyre Normal Forces.