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Torsion of Shafts

Mechanics and Dynamics (ACHA 4026)

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1. Introduction

This report investigates the properties of torsion of circular shafts using a controlled and provided apparatus to calculate the shear modulus of different materials, by applying torque to sample materials and measuring the resulting effects of such loads.

Using mathematical calculations properties of each material can be obtained, analysed then compared to known values, allowing for a comparison and validation of results as well as considerations on how to increase accuracy of results and reduce errors.

2. Experimental Procedure

The required apparatus to conduct the experiment is provided in the Torsion of circular sections Kit ES5 by TecQuipment Ltd, the first stage of the process is to follow the guidance in the ES5 guidance notes to assemble the kit ensuring that the kit is set level and all components are secure and true in their given place, in an effort to keep the procedure consistent and reduce errors throughout; part of the assembly process can be seen in figure 1.

The materials to test in the kit are, Brass, Aluminium and Steel, fit the test material into the rotating chuck and secure firmly ensuring the material will not be able to slip but not so tight the material deforms changing the cross section at the clamping points, all rods are 0.34 meters.

Measure the diameter (D) of the material that is being tested noting it down, using this value the polar second moment of inertia (J) can be calculated using the following:

$$J = \frac{\pi i * D^4}{32}$$

Equation 1 – Polar Second Moment of Inertia

Zero the rotating chuck ensuring there is no rotational load on the rod being measured, then clock in the angle gauge that can be seen in figure 2 setting it to 0 degrees.

Start the procedure by loading the torque arm with 100 grams of weight on the hanger in the case of the ES5 kit this is the hanger itself, noting the angle of rotation given on the angle finder on the rotating chuck.

Repeat this process for each material increasing in 100-gram increments until 500 grams is reached, using the weights provided, do this for each materials remembering to clock in the angle finder each time you change material. Keep a close watch to ensure there is no slipping of the rod in the chucks as this will invalidate results.

Only load the Aluminium rod up to 400 grams to prevent twisting the aluminium beyond its limit of elasticity.

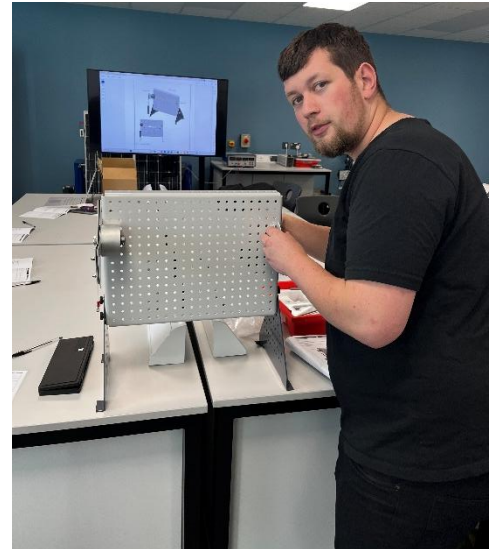


Figure 1- ES5 Torsion Kit being Assembled

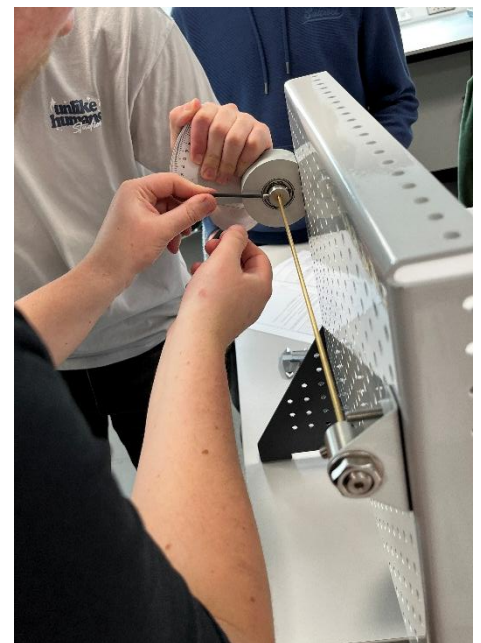


Figure 2 -Assembled Kit with Brass Rod

2.1 Calculations

A plethora of calculations were used to find a variety of values for each material in their various dynamic situations.

The weight load (w) is calculated in Newtons by the following:

$$w = \text{Mass}(g) * \text{Gravity}$$

Equation 2 - Weight Load Equation

Mass being the known weight at the time and gravity being taken as 9.81 m/s^2 .

Torque (T) is calculated in Newton Meters (Nm) by the following:

$$T = \text{Load Force (N)} * \text{Distance (m)}$$

Equation 3 - Weight Load Equation

The length of the torque arm in the ES5 Kit is 0.1 meters.

Polar second moment of inertia (J) is calculated by

$$J = \frac{\pi * D^4}{32}$$

Equation 1 - Polar Second Moment of Inertia Equation

Conversion of Angle to radians is calculated from:

$$\frac{\pi}{180}$$

Equation 4 - Radians Conversion

3. Results as Raw Lab Data

All Results from following the methodology given in section 2 are displayed in digital tables for legibility purposes.

Table 1 - Brass Raw Lab Data

Load (m) Grams	Twist angle (Degrees)
100	6
200	11.75
300	17.8
400	23.6
500	29.5

Table 2 - Aluminium Raw Lab Data

Load (m) Grams	Twist angle (Degrees)
100	7.5
200	15.25
300	22.75
400	29.5

Table 3 - Steel Raw Lab Data

Load (m) Grams	Twist angle (Degrees)
100	2.5
200	5
300	7.5
400	10
500	12.5

4. Analysis of Lab Data

All lab data was then processed within Microsoft Excel, as this allows the data to be visually represented and all calculations can be done within the software.

4.1 Brass Calculations and Graph

Table 4 - Brass Data Analysis

Shaft Modulus Brass						
Load (m) g	Load (W) N	Torque (T) Nm	Twist Angle (deg)	Twist Angle Rads	T*L	J θ
100	0.98	0.098	6	0.10472	0.03332	9.49457E-13
200	1.962	0.1962	11.75	0.205076	0.066708	1.85935E-12
300	2.943	0.2943	17.8	0.310669	0.100062	2.81672E-12
400	3.924	0.3924	23.5	0.410152	0.133416	3.71871E-12
500	4.905	0.4905	29.5	0.514872	0.16677	4.66816E-12
Diameter	0.0031		Length (m)	0.34	J	9.06665E-12

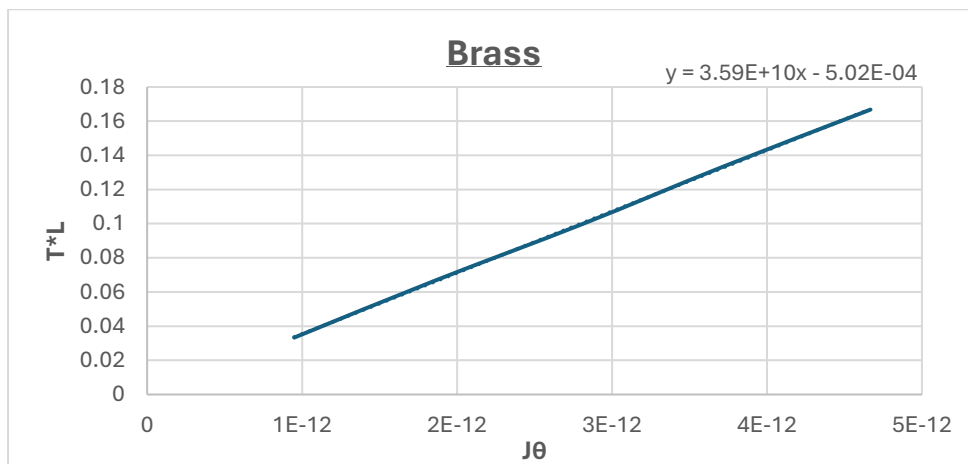


Figure 3 - Brass Plot of $T \cdot L$ vs $J \cdot \theta$

4.2 Aluminium Calculations and Graph

Table 5 – Aluminium Data Analysis

Shaft Modulus Aluminium						
Load (m) g	Load (W) N	Torque (T) Nm	Twist Angle (deg)	Twist Angle Rads	T*L	J θ
100	0.98	0.098	7.5	0.1309	0.03332	1.18682E-12
200	1.962	0.1962	15.25	0.266163	0.066708	2.4132E-12
300	2.943	0.2943	22.75	0.397062	0.100062	3.60002E-12
400	3.924	0.3924	30.5	0.532325	0.133416	4.82641E-12
Diameter	0.0031		Length (m)	0.34	J	9.06665E-12

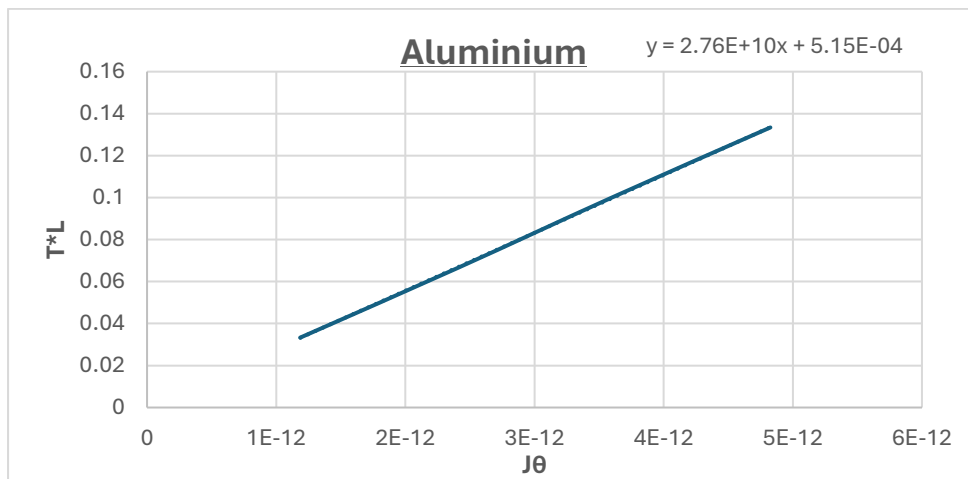


Figure 4 - Aluminium Plot of $T \cdot L$ vs $J \cdot \theta$

4.3 Steel Calculations and Graph

Table 6 – Steel Data Analysis

Shaft Modulus Steel						
Load (m) g	Load (W) N	Torque (T) Nm	Twist Angle (deg)	Twist Angle Rads	T*L	J θ
100	0.98	0.098	2.5	0.043633	0.03332	3.95607E-13
200	1.962	0.1962	5	0.087266	0.066708	7.91214E-13
300	2.943	0.2943	7.5	0.1309	0.100062	1.18682E-12
400	3.924	0.3924	10	0.174533	0.133416	1.58243E-12
500	4.905	0.4905	12.5	0.218166	0.16677	1.97804E-12
Diameter	0.0031		Length (m)	0.34	J	9.06665E-12

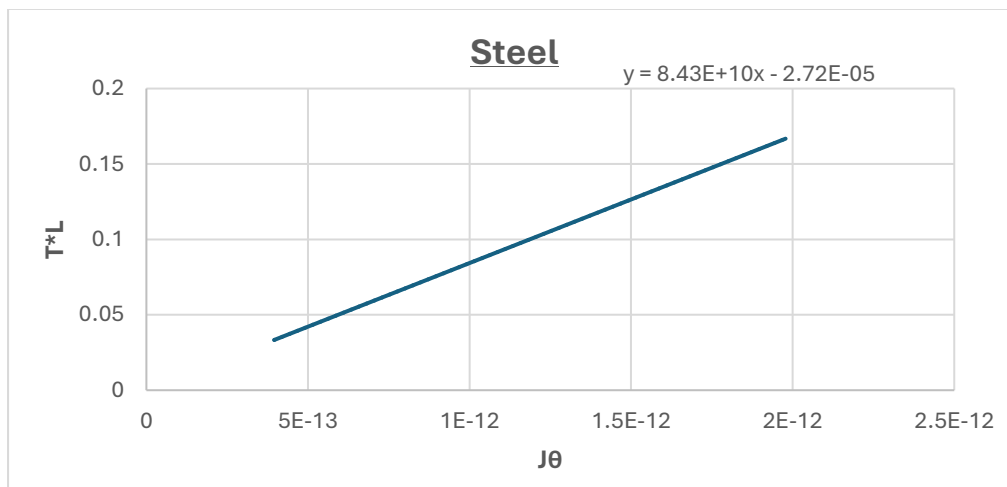


Figure 5 - Steel Plot of T*L vs J*Theta

5.0 Conclusions – Comparing the Shear Modulus

Shear modulus also called the modulus of rigidity represented by the symbol G , it is a ratio of a materials shear stress to its shear strain, fundamentally force per unit area against displacement at the edge, shear occurs in a material when a force is applied parallel to a materials surface, the shear modulus is a value that represents a materials resistance to the shearing force, giving an indication of a of a materials rigidity hence modulus or rigidity.

(McClements and Lichtig, 2023; Chin, 2025)

5.1 Calculating the Shear modulus (G)

Calculating the shear modulus from the collected data is possible by plotting Torque * Length (TL) on the y Axis and the polar second moment of inertia * twist angle in radians ($J*\theta$) on the x axis, these are components of the general torsion of a circular shaft equation which when re-arranged to $y = mx$ format means that the value of the shear modulus (G) should be the gradient of the graph.

(TecQuipment Ltd, 2012)

$$\frac{T}{J} = \frac{G\theta}{L}$$

Equation 5 - General Torsion Equation

The General Torsion Equation can be rearranged to the $y = mx$ form.

$$(T * L) = G * (J * \theta)$$

Equation 6 - Rearranged torsion Equation

5.2 Brass Modulus of Rigidity

Looking at figure 3 the plot of TL against Jθ for the data collected for Brass, the gradient of the graph is displayed in the top right corner, this is automatically calculated in the software that the graph is plotted in, which is $y = 3.59\text{E}+10x + 0.0005$.

The gradient is 3.59×10^{10} Pascals or 35.9 GigaPascals, this can also be written as n/m^2 , to validate these values, comparing them to known values from academic resources was used, the value of 38 GN/m^2 was used from EJ Hearn's Mechanics of Materials 1 3rd edition, this is the same list value of 38.0 GPa given in the ES5 Kit guidance.

(Hearn, 1997; TecQuipment Ltd, 2012)

This value compares quite well, considering the basic set up and measuring used in the procedure, accuracy could be increased by using a consistent measuring apparatus for the angle, instead of reading off of an analogue gauge with coarse resolution, taking the absolute error of 2.1 GPa, the percentage error can be calculated from $\% \text{ err} = \frac{2.1}{38} * 100 = 5.53\%$, which is within the desired 10% discussed within lecture where the experiment took place.

(Penny, 2025)

5.3 Aluminium Modulus of Rigidity

The Gradient (G value) in figure 4 is shown as $2.76\text{E}+10x$, being 27.6 GPa, aluminum is commonly doped with other materials to become an aluminum alloy, cast and alloy versions of aluminum still tend to average around the same values, a typical cast aluminum value is 26.6 GPa, and for an alloy 26.9 GPa, the ES5 guidance suggest 25.5GPa but this is not specified if it is for a pure element or an alloy.

(Engineers Edge, accessed 12/25; Hearn, 1997; TecQuipment Ltd, 2012)

Calculating the percentage error against the value provided in the TecQuipment guidance, is the best validation option as it is the material provided with the kit that also provides a Shear modulus value.

The percentage error is $\left(\frac{27.6-25.5}{25.5} \right) * 100 = 8.2\%$, this shear modulus is above what the ES5 kit suggests in its documentation, still within the discussed 10%, using the higher alloy values would decrease this error such as if the 26.9 GPa were to be used the error would reduce to 2.6%.

5.4 Aluminium Modulus of Steel

Looking at figure 4 the gradient value is 8.43×10^4 , or 84.3 GPa, the value provided by the ES5 guidance is for mild steel at 79.6 GPa another value for low carbon mild steel is 80 GPa, this result being the second material that has tested above other tested values, could indicate that it is an issue in the testing methodology or data recording, the percentage error against the ES5 kit for the steel is $\frac{84.3-79.6}{79.6} = 5.9\%$, this again falls within the 10% acceptance level discussed.

6.0 Evaluation – Minimising errors

Minimising errors for the methodology given in chapter two, consisted of ensuring the degree gauge was zeroed before each test to ensure that the gauge did not over or under read throughout the experiment, another error minimising method was suggested in the ES5 guidance was to gently tap the apparatus each time a weight was added to overcome internal friction within the measuring apparatus and rotating assembly more commonly referred to as stiction. (TecQuipment Ltd, 2012)

The angle gauge should be read by the same personnel each time to reduce measuring accuracies and increase consistency, the vernier callipers used to measure the diameter of the rods were changed for a much higher quality pair that measure more accurately with a better resolution for more accurate results.

7.0 Further work and improvements

To improve accuracy of results if there were no financial constraints would be to use a digital and calibrated angle gauge to measure the twist of each rod, being digital this would remove inaccuracies caused by human error such as misreading the analogue gauge.

Slip tracking could also be implemented into the methodology, to ensure the samples do not slip in the chuck, so slipping errors would not be included in the results, therefore increasing accuracy.

Testing larger diameters of materials in an industrial spec test rig could give a more average values as there is a larger volume of material being tested, another way to do this would be to increase the sample sizes as testing only one specimen of each material, does not account for batch errors or anomalies in the materials.

Using test material from several source locations would be advisable too.

8.0 References

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