

UWTSD

Module

Task 2

Alternative Fuels Combustion Modelling

ACHA7002

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1 Bisection

A function was created based on equation 1.1 to solve roots using the bisection method

$$x_2 = (x_1 + x_3)/2 \quad \text{Equation 1-1}$$

1.1 $f(x) = \sin(x) \sinh(x)$

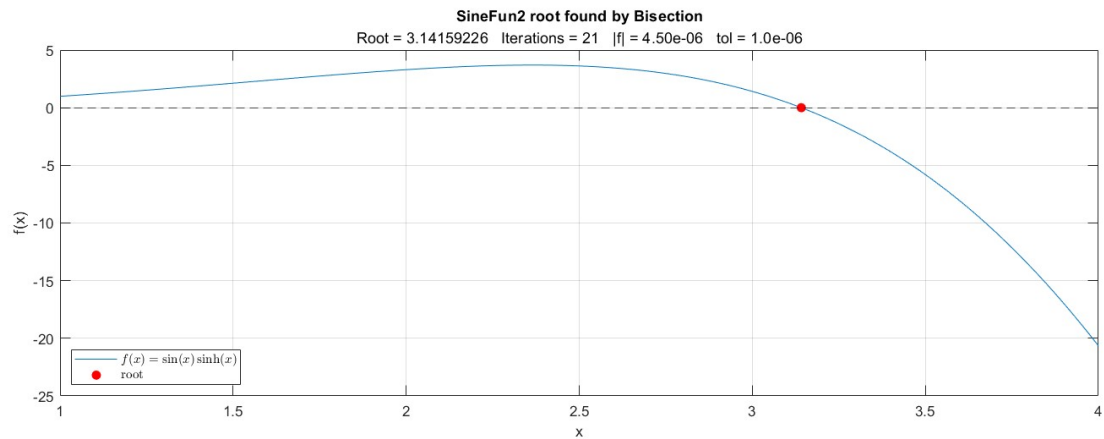


Figure 1-1 this shows the function labelled in the legend and the root

1.2 $f(x) = (R^2 - x^2)^{1/2} - x \tan(x)$

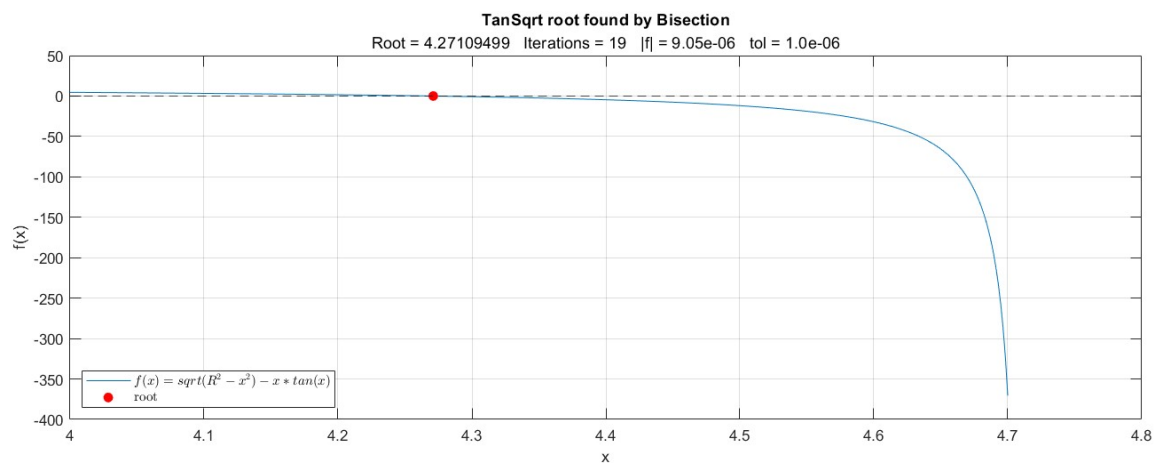


Figure 1-2 this shows the function labelled in the legend and the root

2 Newton's method

A function was made from equation 2.1 to evaluate roots with the newton method to use this method, functions and their derivative functions were created.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{Equation 2-1}$$

2.1 Newton b $f(x) = e^x - (1 + x + (x^2/2))$

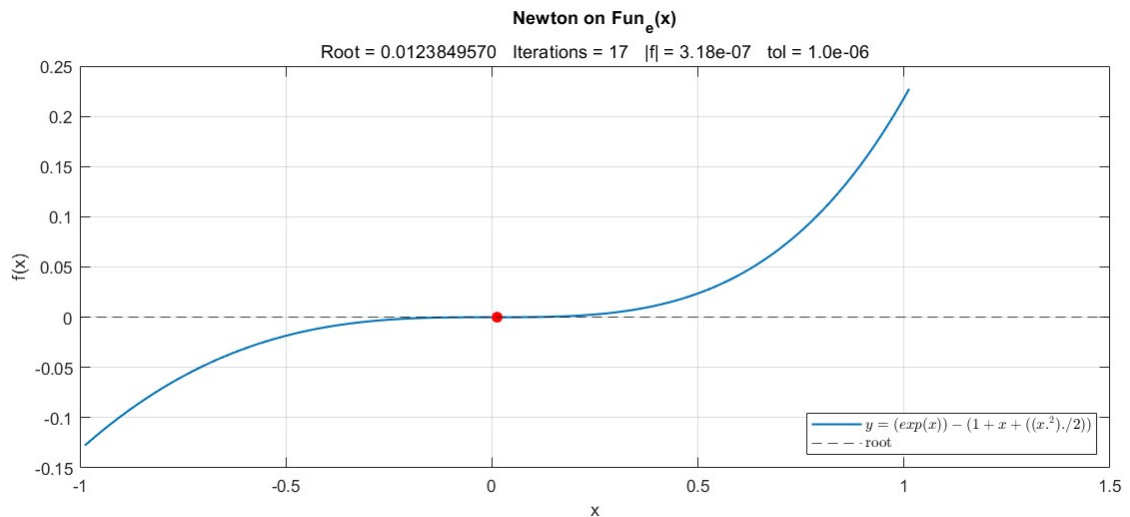


Figure 2-1 this shows the function labelled in the legend and the root

2.2 Newton c $f(x) = 10 \log(x) - x$

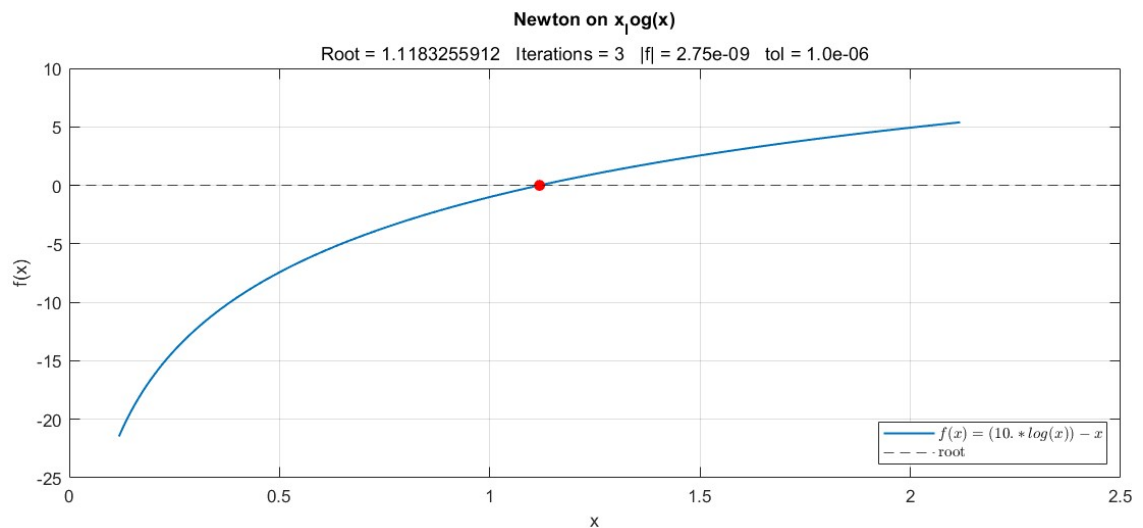


Figure 2-2 this shows the function labelled in the legend and the root

3 Intersect plot $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

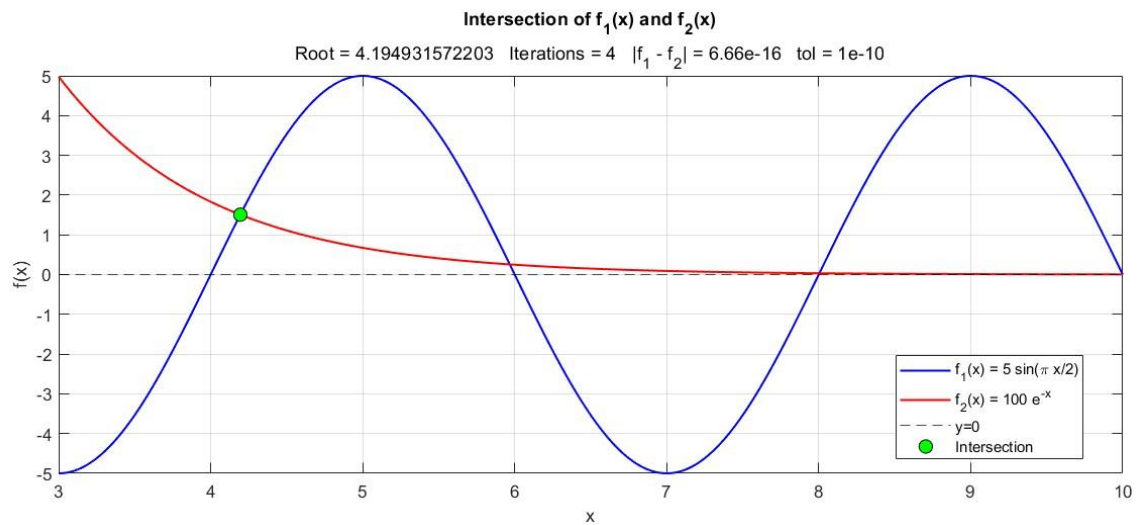


Figure 3-1 plot of function intersection

4 Bracketing

4.1 Bisect $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

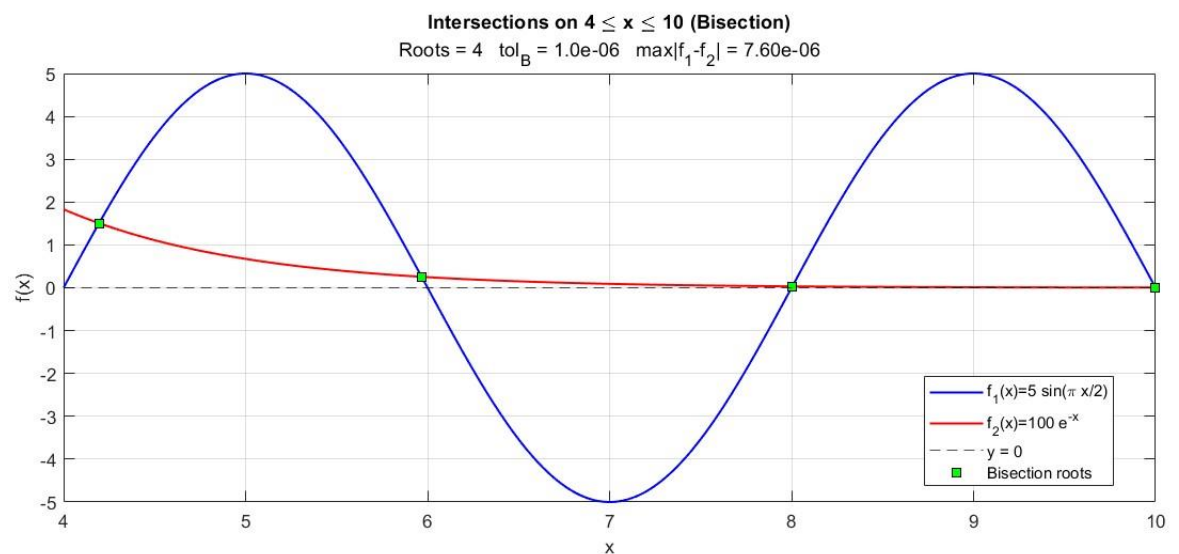


Figure 4-1 multiple roots Bisect for functions shown in legend

4.2 Newton $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

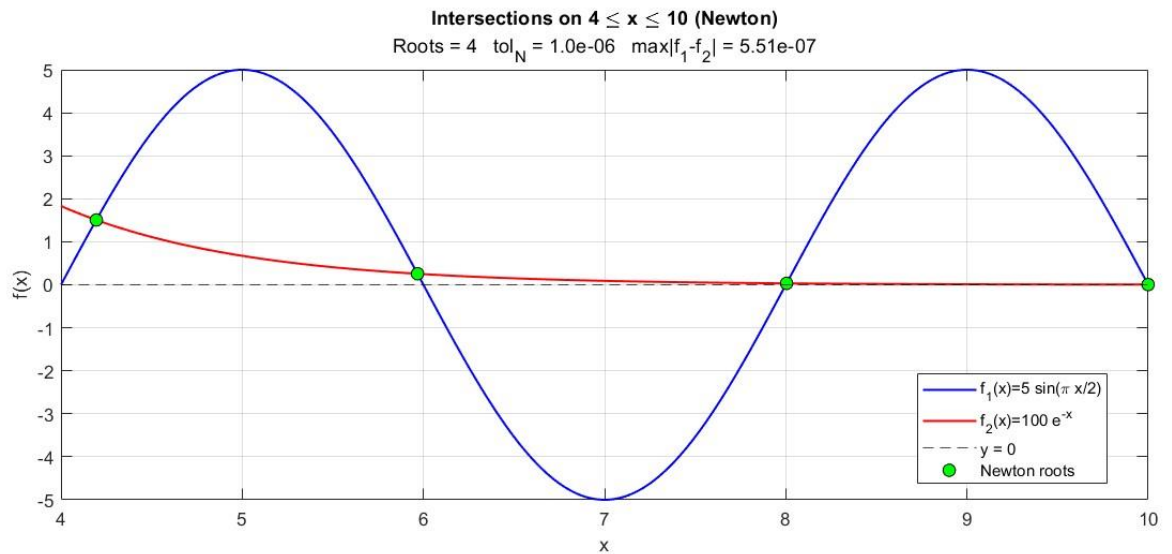


Figure 4-2 multiple roots Newton for functions shown in legend

5 Appendices

5.1 Convergence table Bisection $f(x) = \sin(x) \sinh(x)$

ITERATION NUMBER, K	CURRENT MIDPOINT, X_k	FUNCTION VALUE, $F(X_k)$	ABSOLUTE VALUE, $ F(X_k) $
1	3.2500000000	-1.3931e+00	1.3931e+00
2	2.8750000000	2.3274e+00	2.3274e+00
3	3.0625000000	8.4281e-01	8.4281e-01
4	3.1562500000	-1.7178e-01	1.7178e-01
5	3.1093750000	3.6017e-01	3.6017e-01
6	3.1328125000	1.0051e-01	1.0051e-01
7	3.1445312500	-3.4037e-02	3.4037e-02
8	3.1386718750	3.3633e-02	3.3633e-02
9	3.1416015625	-1.0289e-04	1.0289e-04
10	3.1401367188	1.6790e-02	1.6790e-02

11	3.1408691406	8.3496e-03	8.3496e-03
12	3.1412353516	4.1249e-03	4.1249e-03
13	3.1414184570	2.0114e-03	2.0114e-03
14	3.1415100098	9.5435e-04	9.5435e-04
15	3.1415557861	4.2576e-04	4.2576e-04
16	3.1415786743	1.6144e-04	1.6144e-04
17	3.1415901184	2.9278e-05	2.9278e-05
18	3.1415958405	-3.6804e-05	3.6804e-05
19	3.1415929794	-3.7631e-06	3.7631e-06
20	3.1415915489	1.2758e-05	1.2758e-05
21	3.1415922642	4.4972e-06	4.4972e-06

5.2 Convergence table Bisection $f(x) = (R^2 - x^2)^{1/2} - x \tan(x)$

ITERATION NUMBER, K	CURRENT MIDPOINT, X_k	FUNCTION VALUE, $F(X_k)$	ABSOLUTE VALUE, $ F(X_k) $
1	4.1750000000	2.0804e+00	2.0804e+00
2	4.2625000000	2.1951e-01	2.1951e-01
3	4.3062500000	-9.8810e-01	9.8810e-01
4	4.2843750000	-3.5533e-01	3.5533e-01
5	4.2734375000	-6.1206e-02	6.1206e-02
6	4.2679687500	8.0764e-02	8.0764e-02
7	4.2707031250	1.0190e-02	1.0190e-02
8	4.2720703125	-2.5404e-02	2.5404e-02
9	4.2713867187	-7.5813e-03	7.5813e-03
10	4.2710449219	1.3108e-03	1.3108e-03

11	4.2712158203	-3.1337e-03	3.1337e-03
12	4.2711303711	-9.1103e-04	9.1103e-04
13	4.2710876465	1.9999e-04	1.9999e-04
14	4.2711090088	-3.5550e-04	3.5550e-04
15	4.2710983276	-7.7748e-05	7.7748e-05
16	4.2710929871	6.1121e-05	6.1121e-05
17	4.2710956573	-8.3134e-06	8.3134e-06
18	4.2710943222	2.6404e-05	2.6404e-05
19	4.2710949898	9.0452e-06	9.0452e-06

5.3 Convergence table Newton b $f(x) = e^x - (1 + x + (x^2/2))$

ITERATION NUMBER, K	NEWTON ESTIMATE, X_k	FUNCTION VALUE, $F(X_k)$	ABSOLUTE VALUE, $ F(X_k) $
1	5.0000000000	1.2991e+02	1.2991e+02
2	4.0877727878	4.6164e+01	4.6164e+01
3	3.2410204713	1.6067e+01	1.6067e+01
4	2.4873811478	5.4488e+00	5.4488e+00
5	1.8495218507	1.7969e+00	1.7969e+00
6	1.3371865174	5.7709e-01	5.7709e-01
7	0.9449068047	1.8124e-01	1.8124e-01
8	0.6561510939	5.5942e-02	5.5942e-02
9	0.4498834254	1.7048e-02	1.7048e-02
10	0.3057059899	5.1490e-03	5.1490e-03
11	0.2064514591	1.5455e-03	1.5455e-03

12	0.1388342573	4.6192e-04	4.6192e-04
13	0.0930964832	1.3767e-04	1.3767e-04
14	0.0623065532	4.0949e-05	4.0949e-05
15	0.0416459840	1.2165e-05	1.2165e-05
16	0.0278123001	3.6107e-06	3.6107e-06
17	0.0185630599	1.0711e-06	1.0711e-06
18	0.0123849570	3.1760e-07	3.1760e-07

5.4 Convergence table Newton c $f(x) = 10 \log(x) - x$

ITERATION NUMBER, K	NEWTON ESTIMATE, X_k	FUNCTION VALUE, $F(X_k)$	ABSOLUTE VALUE, $ F(X_k) $
1	1.0000000000	-1.0000e+00	1.0000e+00
2	1.1111111111	-5.7506e-02	5.7506e-02
3	1.1182993554	-2.0837e-04	2.0837e-04
4	1.1183255912	-2.7519e-09	2.7519e-09

5.5 Convergence table Bisect $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

5.5.1 Root 1, bracket [4.1818, 4.2424]

ITERATION NUMBER, K	CURRENT MIDPOINT, X_k	FUNCTION VALUE, $F(X_k)$	ABSOLUTE VALUE, $ F(X_k) $
1	4.1969696970	1.8324e-02	1.8324e-02
2	4.1893939394	-4.9895e-02	4.9895e-02
3	4.1931818182	-1.5748e-02	1.5748e-02
4	4.1950757576	1.2970e-03	1.2970e-03

5	4.1941287879	-7.2234e-03	7.2234e-03
6	4.1946022727	-2.9626e-03	2.9626e-03
7	4.1948390152	-8.3265e-04	8.3265e-04
8	4.1949573864	2.3222e-04	2.3222e-04
9	4.1948982008	-3.0021e-04	3.0021e-04
10	4.1949277936	-3.3992e-05	3.3992e-05
11	4.1949425900	9.9114e-05	9.9114e-05
12	4.1949351918	3.2561e-05	3.2561e-05
13	4.1949314927	-7.1555e-07	7.1555e-07
14	4.1949333422	1.5923e-05	1.5923e-05
15	4.1949324174	7.6036e-06	7.6036e-06

5.5.2 Root 2, bracket [5.9394, 6.0000]

ITERATION NUMBER, K	CURRENT MIDPOINT, x_k	FUNCTION VALUE, $F(x_k)$	ABSOLUTE VALUE, $ F(x_k) $
1	5.9545454545	9.7294e-02	9.7294e-02
2	5.9621212121	3.9879e-02	3.9879e-02
3	5.9659090909	1.1150e-02	1.1150e-02
4	5.9678030303	-3.2193e-03	3.2193e-03
5	5.9668560606	3.9659e-03	3.9659e-03
6	5.9673295455	3.7339e-04	3.7339e-04
7	5.9675662879	-1.4229e-03	1.4229e-03
8	5.9674479167	-5.2477e-04	5.2477e-04
9	5.9673887311	-7.5689e-05	7.5689e-05
10	5.9673591383	1.4885e-04	1.4885e-04
11	5.9673739347	3.6580e-05	3.6580e-05

12	5.9673813329	-1.9555e-05	1.9555e-05
13	5.9673776338	8.5127e-06	8.5127e-06
14	5.9673794833	-5.5209e-06	5.5209e-06
15	5.9673785585	1.4959e-06	1.4959e-06

5.5.3 Root 3, bracket [8.0000, 8.0606]

ITERATION NUMBER, K	CURRENT MIDPOINT, X_k	FUNCTION VALUE, $F(X_k)$	ABSOLUTE VALUE, $ F(X_k) $
1	8.0151515152	8.5947e-02	8.5947e-02
2	8.0075757576	2.6205e-02	2.6205e-02
3	8.0037878788	-3.6697e-03	3.6697e-03
4	8.0056818182	1.1268e-02	1.1268e-02
5	8.0047348485	3.7993e-03	3.7993e-03
6	8.0042613636	6.4808e-05	6.4808e-05
7	8.0040246212	-1.8024e-03	1.8024e-03
8	8.0041429924	-8.6881e-04	8.6881e-04
9	8.0042021780	-4.0200e-04	4.0200e-04
10	8.0042317708	-1.6860e-04	1.6860e-04
11	8.0042465672	-5.1895e-05	5.1895e-05
12	8.0042539654	6.4565e-06	6.4565e-06
13	8.0042502663	-2.2719e-05	2.2719e-05
14	8.0042521159	-8.1313e-06	8.1313e-06
15	8.0042530407	-8.3737e-07	8.3737e-07

5.5.4 Root 4, bracket [9.9394, 10.0000]

ITERATION NUMBER, K	CURRENT MIDPOINT, X_k	FUNCTION VALUE, $F(X_k)$	ABSOLUTE VALUE, $ F(X_k) $
1	9.9848484848	1.1438e-01	1.1438e-01
2	9.9924242424	5.4924e-02	5.4924e-02
3	9.9962121212	2.5193e-02	2.5193e-02
4	9.9981060606	1.0326e-02	1.0326e-02
5	9.9990530303	2.8932e-03	2.8932e-03
6	9.9995265152	-8.2340e-04	8.2340e-04
7	9.9992897727	1.0349e-03	1.0349e-03
8	9.9994081439	1.0575e-04	1.0575e-04
9	9.9994673295	-3.5883e-04	3.5883e-04
10	9.9994377367	-1.2654e-04	1.2654e-04
11	9.9994229403	-1.0398e-05	1.0398e-05
12	9.9994155421	4.7673e-05	4.7673e-05
13	9.9994192412	1.8638e-05	1.8638e-05
14	9.9994210908	4.1197e-06	4.1197e-06
15	9.9994220156	-3.1393e-06	3.1393e-06

5.6 Convergence table Newton $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

5.6.1 Root 1, initial guess $x_0 = 4.181818$

ITERATION NUMBER, K	NEWTON ESTIMATE, X_k	FUNCTION VALUE, $F(X_k)$	ABSOLUTE VALUE, $ F(X_k) $
1	4.1818181818	-1.1841e-01	1.1841e-01
2	4.1948834034	-4.3332e-04	4.3332e-04

3	4.1949315715	-6.0619e-09	6.0619e-09
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5.6.2 Root 2, initial guess $x_0 = 5.939394$

ITERATION NUMBER, K	NEWTON ESTIMATE, X_k	FUNCTION VALUE, $F(X_k)$	ABSOLUTE VALUE, $ F(X_k) $
1	5.9393939394	2.1192e-01	2.1192e-01
2	5.9674437243	-4.9296e-04	4.9296e-04
3	5.9673787559	-1.8722e-09	1.8722e-09

5.6.3 Root 3, initial guess $x_0 = 8.000000$

ITERATION NUMBER, K	NEWTON ESTIMATE, X_k	FUNCTION VALUE, $F(X_k)$	ABSOLUTE VALUE, $ F(X_k) $
1	8.0000000000	-3.3546e-02	3.3546e-02
2	8.0042530769	-5.5145e-07	5.5145e-07

5.6.4 Root 4, initial guess $x_0 = 10.000000$

ITERATION NUMBER, K	NEWTON ESTIMATE, X_k	FUNCTION VALUE, $F(X_k)$	ABSOLUTE VALUE, $ F(X_k) $
1	10.0000000000	-4.5400e-03	4.5400e-03
2	9.9994216158	-1.3845e-09	1.3845e-09

5.7 Your code and functions

5.7.1 Lab 2 (main script)

```
clear all
format short g

%% Bisection Method

[root_x1, d1] = bisection('SineFun2', 1, 4, 50, 1e-6);
[root_x2, d2] = bisection('TanSqrt', 4, 4.7, 50, 1e-6);
```

```

iter_x1 = numel(d1);
iter_x2 = numel(d2);
err_x1 = abs(SineFun2(root_x1));
err_x2 = abs(TanSqrt(root_x2));

% f(x)=sin(x)*sinh(x) on [1,4]
xx = linspace(1,4,400);
yy = SineFun2(xx);
figure;
plot(xx,yy,'-'); hold on;
plot(root_x1, SineFun2(root_x1), 'ro', 'MarkerFaceColor','r');
yline(0,'k--');
grid on;
xlabel('x'); ylabel('f(x)');
legend('$f(x) = \sin(x)\sinh(x)$','root','Interpreter','latex','location','southwest')
title('SineFun2 root found by Bisection');
subtitle(sprintf('Root = %.8f Iterations = %d |f| = %.2e tol = %.1e', ...
    root_x1, iter_x1, err_x1, 1e-6));
hold off;

% f(x)=sqrt(R^2-x^2)-x*tan(x) on [4,4.7]
xx = linspace(4,4.7,400);
yy = TanSqrt(xx);
figure;
plot(xx,yy,'-'); hold on;
plot(root_x2, TanSqrt(root_x2), 'ro', 'MarkerFaceColor','r');
yline(0,'k--');
grid on;
xlabel('x'); ylabel('f(x)');
legend('$f(x)=\sqrt{R^2-x^2}-x\tan(x)$','root','Interpreter','latex','location','southwest')
title('TanSqrt root found by Bisection');
subtitle(sprintf('Root = %.8f Iterations = %d |f| = %.2e tol = %.1e', ...
    root_x2, iter_x2, err_x2, 1e-6));
hold off;

%% Convergence table Bisection, f(x) = sin(x)sinh(x)

fprintf('\nAppendix 5.1: Bisection on f(x) = sin(x)sinh(x)\n');
fprintf('-----\n');
fprintf('%5s %14s %14s %14s\n','Iter','x_k','f(x_k)','|f(x_k)|');

for k = 1:numel(d1)
    xk = d1(k);
    fx = SineFun2(xk);
    fprintf('%5d % .10f % .4e % .4e\n', k, xk, fx, abs(fx));
end
%% Convergence table Bisection, f(x) = (R^2 - x^2)^{1/2} - x tan(x)

fprintf('\nAppendix 5.2: Bisection on f(x) = (R^2 - x^2)^{1/2} - x tan(x)\n');
fprintf('-----\n');
fprintf('%5s %14s %14s %14s\n','Iter','x_k','f(x_k)','|f(x_k)|');

for k = 1:numel(d2)
    xk = d2(k);
    fx = TanSqrt(xk);
    fprintf('%5d % .10f % .4e % .4e\n', k, xk, fx, abs(fx));
end

```

```

%% Newton's Method

x_o_e = 5;
x_o_ln = 1;
tol = 1e-6;
imax = 20;

[root_ln, it_ln] = newton(x_o_ln, 'x_log', 'x_log_diff', imax, tol);
[root_e, it_e] = newton(x_o_e, 'Fun_e', 'Fun_e_diff', imax, tol);

xplot = linspace(root_ln-1, root_ln+1, 400);
figure;
plot(xplot, x_log(xplot), 'LineWidth', 1.2); hold on;
yline(0, 'k--');
plot(root_ln, x_log(root_ln), 'ro', 'MarkerFaceColor', 'r');
grid on;
xlabel('x'); ylabel('f(x)');
legend('$f(x) = (10.*log(x)) - x$', 'root', 'Interpreter', 'latex', 'location', 'southeast')
title('Newton on x_log(x)');
subtitle(sprintf('Root = %.10f Iterations = %d |f| = %.2e tol = %.1e', ...
    root_ln, it_ln, abs(x_log(root_ln)), tol));
hold off;

xplot = linspace(root_e-1, root_e+1, 400);
figure;
plot(xplot, Fun_e(xplot), 'LineWidth', 1.2); hold on;
yline(0, 'k--');
plot(root_e, Fun_e(root_e), 'ro', 'MarkerFaceColor', 'r');
grid on;
xlabel('x'); ylabel('f(x)');
legend('$y = (exp(x)) - (1+x+(x.^2)./2)$', 'root', 'Interpreter', 'latex', 'location', 'southeast')
title('Newton on Fun_e(x)');
subtitle(sprintf('Root = %.10f Iterations = %d |f| = %.2e tol = %.1e', ...
    root_e, it_e, abs(Fun_e(root_e)), tol));
hold off;
%% Convergence table Newton b, f(x) = e^x - (1 + x + x^2/2)

fprintf('\nAppendix 5.3: Newton b, f(x) = e^x - (1 + x + x^2/2)\n');
fprintf('-----\n');
fprintf('%5s %14s %14s %14s\n', 'Iter', 'x_k', 'f(x_k)', '|f(x_k)|');

xk = x_o_e;
for k = 1:imax
    fx = Fun_e(xk);
    dfx = Fun_e_diff(xk);

    fprintf('%5d % .10f % .4e % .4e\n', k, xk, fx, abs(fx));

    if abs(fx) < tol || dfx == 0
        break;
    end

    xk = xk - fx/dfx;
end
%% Convergence table Newton c, f(x) = 10 log(x) - x

```

```

fprintf('\nAppendix 5.4: Newton c, f(x) = 10 log(x) - x\n');
fprintf('-----\n');
fprintf('%5s    %14s    %14s    %14s\n','Iter','x_k','f(x_k)','|f(x_k)|');

xk = x_o_ln;
for k = 1:imax
    fx = x_log(xk);
    dfx = x_log_diff(xk);

    fprintf('%5d    %.10f    %.4e    %.4e\n', k, xk, fx, abs(fx));

    if abs(fx) < tol || dfx == 0
        break;
    end

    xk = xk - fx/dfx;
end

%% Newton's Method - Intersection of two functions

x_sect = linspace(3, 10, 1000);
y_Sin = Sin_Five(x_sect);
y_Exp = Exp_Inv(x_sect);
y_Inter = Intersect('Sin_Five','Exp_Inv',x_sect);

% Zoomed comparison
figure;
plot(x_sect,y_Sin); hold on;
plot(x_sect,y_Exp);
plot(x_sect,y_Inter);
yline(0,'r--');
xline(4.19493,'r--');
plot(4.19493,0,'ro');
grid on;
ylim([-2, 2]);
xlim([4, 4.2]);
xlabel('x'); ylabel('f(x)');
title('Intersection comparison (zoom near x \approx 4.19)');
legend('sine','e inverse','difference f_1-f_2','y = 0','x \approx 4.19','Root','Location','best');
hold off;

[Intersection, it_int] =
newton_Inter(4,'Sin_Five','Sin_Five_Diff','Exp_Inv','Exp_Inv_Diff',imax,1e-10);
fprintf('Intersection near 4: %.12f found in %d iterations (tol = 1e-10)\n\n',
Intersection, it_int);

% plot of both functions with the root marked
xw = linspace(3,10,800);
figure;
plot(xw, Sin_Five(xw), 'b-', 'LineWidth',1.2); hold on;
plot(xw, Exp_Inv(xw), 'r-', 'LineWidth',1.2);
yline(0,'k--');
plot(Intersection, Sin_Five(Intersection),
'ko','MarkerFaceColor','g','MarkerSize',8);
grid on;
xlabel('x'); ylabel('f(x)');

```

```

title('Intersection of f_1(x) and f_2(x)');
subtitle(sprintf('Root = %.12f    Iterations = %d    |f_1 - f_2| = %.2e    tol = 1e-10', ...
    Intersection, it_int, abs(Sin_Five(Intersection) - Exp_Inv(Intersection))));
legend('f_1(x) = 5 sin(\pi x/2)', 'f_2(x) = 100 e^{-x}', ...
    'y=0', 'Intersection', 'Location', 'best');
hold off;

%% Bracketing (finding multiple roots)

Range = linspace(4,10,100);
[brackets, EST] = Bracketing('Sin_Five', 'Exp_Inv', Range);

imax = 50;
tolN = 1e-6;
NR_roots = NaN(numel(EST),1);
for k = 1:numel(EST)
    NR_roots(k) = newton_Inter(EST(k), 'Sin_Five', 'Sin_Five_Diff', ...
        'Exp_Inv', 'Exp_Inv_Diff', imax, tolN);
end

g = @(xx) Intersect('Sin_Five', 'Exp_Inv', xx);
tolB = 1e-6;
BI_roots = NaN(size(brackets,1),1);
BI_hist = cell(size(brackets,1),1);

for k = 1:size(brackets,1)
    [BI_roots(k), BI_hist{k}] = bisect(g, brackets(k,1), brackets(k,2), imax,
    tolB);
end

% comparison table
fprintf('\n%-6s %-17s %-16s %-16s\n', '#', 'Bracket [a,b]', 'Newton root', 'Bisection
root');
for k = 1:numel(NR_roots)
    fprintf('%-6d [%7.4f,%7.4f] %14.8f %14.8f\n', ...
        k, brackets(k,1), brackets(k,2), NR_roots(k), BI_roots(k));
end

%% Convergence table Bisection, f(x) = 100e^{-x} - 5sin(\pi x/2)

fprintf('\nAppendix 5.5: Bisection on f(x) = 100e^{-x} - 5sin(\pi x/2)\n');
fprintf('-----\n');

for r = 1:numel(BI_hist)
    fprintf('Root #%d, bracket [%.4f, %.4f]\n', r, brackets(r,1), brackets(r,2));
    fprintf('%5s %14s %14s %14s\n', 'Iter', 'x_k', 'f(x_k)', '|f(x_k)|');

    dvec = BI_hist{r};
    for k = 1:numel(dvec)
        xk = dvec(k);
        fx = g(xk);
        fprintf('%5d % .10f % .4e % .4e\n', k, xk, fx, abs(fx));
    end
    fprintf('\n');
end
end

```

```

%% Convergence table Newton,  $f(x) = 100e^{-x} - 5\sin(\pi x/2)$ 

fprintf('\nAppendix 5.6: Newton on  $f(x) = 100e^{-x} - 5\sin(\pi x/2)$ \n');
fprintf('-----\n');

for r = 1:numel(EST)
    fprintf('Root # %d, initial guess  $x_0 = %.6f$ \n', r, EST(r));
    fprintf('%5s %14s %14s %14s\n', 'Iter', 'x_k', 'f(x_k)', '|f(x_k)|');

    xk = EST(r);
    for k = 1:imax
        fx = g(xk);

        dfx = Sin_Five_Diff(xk) - Exp_Inv_Diff(xk);

        fprintf('%5d % .10f % .4e % .4e\n', k, xk, fx, abs(fx));

        if abs(fx) < tolN || dfx == 0
            break;
        end

        xk = xk - fx/dfx;
    end
    fprintf('\n');
end

%% Multiple intersections (Bisection only)

xx = linspace(4,10,1200);

figure;
plot(xx, Sin_Five(xx), 'b-', 'LineWidth',1.2); hold on;
plot(xx, Exp_Inv(xx), 'r-', 'LineWidth',1.2);
yline(0,'k--'); grid on;

plot(BI_roots, Sin_Five(BI_roots), 'ks', 'MarkerSize',7, 'MarkerFaceColor','g');

resB = abs(Sin_Five(BI_roots) - Exp_Inv(BI_roots));
maxRes_B = max(resB);
nroot_B = numel(BI_roots);

xlabel('x'); ylabel('f(x)');
title('Intersections on  $4 \leq x \leq 10$  (Bisection)');
subtitle(sprintf('Roots = %d tol_B = %.1e max|f_1-f_2| = %.2e', ...
    nroot_B, tolB, maxRes_B));

legend('f_1(x)=5 sin(\pi x/2)', ...
    'f_2(x)=100 e^{-x}', ...
    'y = 0', ...
    'Bisection roots', ...
    'Location','best');
hold off;

%% Multiple intersections (Newton only)

figure;
plot(xx, Sin_Five(xx), 'b-', 'LineWidth',1.2); hold on;

```

```

plot(xx, Exp_Inv(xx), 'r-', 'LineWidth',1.2);
yline(0,'k--'); grid on;

plot(NR_roots, Sin_Five(NR_roots), 'ko', 'MarkerSize',7, 'MarkerFaceColor','g');

resN      = abs(Sin_Five(NR_roots) - Exp_Inv(NR_roots));
maxRes_N  = max(resN);
nroot_N   = numel(NR_roots);

xlabel('x'); ylabel('f(x)');
title('Intersections on  $4 \leq x \leq 10$  (Newton)');
subtitle(sprintf('Roots = %d   tol_N = %.1e   max|f_1-f_2| = %.2e', ...
    nroot_N, tolN, maxRes_N));

legend('f_1(x)=5 sin(\pi x/2)', ...
    'f_2(x)=100 e^{-x}', ...
    'y = 0', ...
    'Newton roots', ...
    'Location','best');
hold off;

%% diagnostic: TanSqrt over [4,4.7]

x = linspace(4,4.7,500);
figure;
plot(x, TanSqrt(x)); hold on; yline(0,'k--'); grid on;
xlabel('x'); ylabel('f(x)');
title('Diagnostic: TanSqrt on [4,4.7]');
subtitle('Shows behaviour near asymptote; root located by bisection. ');
hold off;

%% Final residual prints
fprintf('SineFun2(root_x1) = %.3e\n', SineFun2(root_x1));
fprintf('TanSqrt(root_x2) = %.3e\n', TanSqrt(root_x2));

```

5.7.2 Functions

5.7.2.1 Newton-Raphson functions

```

function [root, iters] = newton(x0, f, df, imax, tol)

if ischar(f), f = str2func(f); end
if ischar(df), df = str2func(df); end

x  = x0;
fx = f(x);
cnt = 0;

while cnt < imax
    cnt = cnt + 1;

    dfx = df(x);
    if dfx == 0
        error('Derivative is zero - cannot continue. ');
    end

    xnew = x - fx/dfx;

```

```

    fx    = f(xnew);

    if abs(fx) < tol, x = xnew; break; end
    if abs(xnew - x) < tol, x = xnew; break; end

    x = xnew;
end

root = x;
iters = cnt;

fprintf('Newton: root=%.12g |f|=%.3e iters=%d tol=%.1e\n', root, abs(fx),
iters, tol);
end

function [Intersection, iters] = newton_Inter(x0, f1, df1, f2, df2, imax, tol)
% f3(x)=f1(x)-f2(x)=0

if ischar(f1), f1 = str2func(f1); end
if ischar(df1), df1 = str2func(df1); end
if ischar(f2), f2 = str2func(f2); end
if ischar(df2), df2 = str2func(df2); end

g = @(x) f1(x) - f2(x);
dg = @(x) df1(x) - df2(x);

x = x0;
gx = g(x);
cnt = 0;

while cnt < imax
    cnt = cnt + 1;

    dgx = dg(x);
    if dgx == 0
        error('Combined derivative is zero - cannot continue.');
```

```

    end

    xnew = x - gx/dgx;
    gx = g(xnew);

    if abs(gx) < tol, x = xnew; break; end
    if abs(xnew - x) < tol, x = xnew; break; end

    x = xnew;
end

Intersection = x;
iters = cnt;

fprintf('Newton-Inter: x=%.12g |f1-f2|=%.3e iters=%d tol=%.1e\n', Intersection,
abs(gx), iters, tol);
end

```

5.7.2.2 Intersection function

```
function [y] = Intersect(f1,f2,x)
```

```

y1 = feval(f1,x);
y2 = feval(f2,x);
y = y1 - y2;
end

```

5.7.2.3 $Y = (10\log(x)) - x$

5.7.2.3.1 $f(x)$

```

function [y] = x_log(x)
y = (10.*log(x))-x;
end

```

5.7.2.3.2 $f'(x)$

```

function [dy] = x_log_diff(x)
dy = (10./x)-1;
end

```

5.7.2.4 $f(x) = e^x - (1 + x + \frac{x^2}{2})$

5.7.2.4.1 $f(x)$

```

function [y] = Fun_e(x)
y = (exp(x)) - (1 + x + ((x.^2)./2));
end

```

5.7.2.4.2 $f'(x)$

```

function [dy] = Fun_e_diff(x)
dy = exp(x) - x - 1;
end

```

5.7.2.5 $100e^{-x}$

5.7.2.5.1 $f(x)$

```

function [y] = Exp_Inv(x)
y = 100.*(exp(-x));
end

```

5.7.2.5.2 $f'(x)$

```

function [dy] = Exp_Inv_Diff(x)
dy = (-100).*(exp(-x));
end

```

5.7.2.6 $f(x) = 5\sin(\frac{\pi x}{2})$

5.7.2.6.1 $f(x)$

```

function [y] = Sin_Five(x)
y = 5.*(sin((pi.*x)/2));
end

```

5.7.2.6.2 $f'(x)$

```

function [dy] = Sin_Five_Diff(x)
dy = ((5.*pi)./2).*(cos((pi.*x)./2));
end

```

5.7.2.7 $f(x) = \sqrt{R^2 - x^2} - x \tan(x)$

```

function [y] = TanSqrt(x)
R = 10;
check = R^2 - x.^2;
if any(check < 0)
    error('sqrt term imaginary: |x| > R');
end

```

```

end
y = sqrt(check) - (x.*tan(x));
end

```

5.7.2.8 $f(x) = \sin(x)\sinh(x)$

```

function [y] = SineFun2(x)
y = sin(x) .* sinh(x);
end

```

5.7.2.9 *Bisection*

```

function [root, d] = bisection(f, a, b, imax, tol)

```

```

if ischar(f), f = str2func(f); end

```

```

x1 = a; f1 = feval(f, x1);
x3 = b; f3 = feval(f, x3);
if f1 * f3 > 0
    error('No root in the specified interval');
end

```

```

d = NaN(1, imax);

```

```

for k = 1:imax
    x2 = (x1 + x3) / 2;
    f2 = feval(f, x2);

    if f1 * f2 < 0
        x3 = x2; f3 = f2;
    else
        x1 = x2; f1 = f2;
    end

    d(k) = (x1 + x3) / 2;

    if (x3 - x1) / 2 < tol
        d = d(1:k);
        break;
    end
end
end

```

```

d = d(:);
root = d(end);

```

```

end

```

5.7.2.10 *Bracketing*

```

function [brackets, EST] = Bracketing(fun_1, fun_2, x)

```

```

y = Intersect(fun_1, fun_2, x);

```

```

brackets = [];
EST = [];

```

```

for i = 1:length(x)-1
    if y(i) * y(i+1) < 0
        brackets = [brackets; x(i), x(i+1)];
    end
end

```

```
    [~, idx] = min(abs([y(i), y(i+1)]));
    if idx == 1
        EST_val = x(i);
    else
        EST_val = x(i+1);
    end
    EST = [EST; EST_val];
end
end

figure;
plot(x, y, '-'); hold on;
yline(0, 'k-');
for i = 1:size(brackets,1)
    xline(brackets(i,1), 'r-');
    xline(brackets(i,2), 'r-');
end
xlabel('x'); ylabel('f(x)');
title('Function and detected sign-change intervals');
grid on;
end
```