



Prifysgol Cymru
Y Drindod Dewi Sant
University of Wales
Trinity Saint David

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Research Proposal: Development of 2024 Hybrid IndyCar Model and
Optimisation of Energy Deployment and Recovery

Tharish Ravin Ranjit (2208284)

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Table of Contents

- 1. Introduction 1
- 2. Literature Review 2
 - 2.1. Case Studies 2
 - 2.2. Vehicle Dynamics Modelling 3
 - 2.3. Powertrain Modelling 4
 - 2.4. Track Modelling 5
 - 2.5. Optimisation 5
- 3. Methods 6
 - 3.1. Data Gathering 6
 - 3.2. Modelling 7
 - 3.3. Simulation and Analysis 7
- 4. Results 8
- 5. Evaluation 10
- 6. References 11
- 7. Appendix 14

List of Figures

Figure 2-1: Predefined Racing line (Limebeer, Perantoni, & Rao, 2014)	2
Figure 2-2: Variable Racing Lines (Xuze, Fotouhi, & Auger, 2020).....	2
Figure 2-3: Speed comparison for the three cases studied (Limebeer, Perantoni, & Rao, 2014)	3
Figure 2-4: Speed profiles with different energy consumption limitations (Xuze, Fotouhi, & Auger, 2020)	3
Figure 2-5: Curvilinear-coordinate-based description of a track segment (Limebeer, Perantoni, & Rao, 2014)	5
Figure 3-1: Proposed Project Gantt Chart.....	6
Figure 3-2: V-Model of Methodology	7
Figure 4-1: Speed vs Distance graph (Hybrid - Purple, Standard - Orange).....	8
Figure 4-2: Vehicle Speed track map	9
Figure 5-1: Motorsport Development Steps (Xuze, Fotouhi, & Auger, 2020)	10
Figure 7-1: Optimum Lap Vehicle Characteristics	14
Figure 7-2: Optimum Lap Engine Torque and Power vs Engine Speed.....	15
Figure 7-3: Optimum Lap Tractive Force at Wheel	15
Figure 7-4: Optimum Lap Traction Model.....	16
Figure 7-5: Forza Motorsport Engine Torque and Power vs Engine Speed (Microsoft, 2017)	16

List of Tables

Table 3-1: Energy Modes 7

1. Introduction

For many years, motor racing has been a source of inspiration for consumer automobiles. When a breakthrough in automobile racing occurs, it is nearly always transferrable to mass-produced vehicles in some manner. As a result, race technology has affected numerous automotive components, including engine design, ignition location, and even the rear-view mirror. For example, air intakes (designed to boost air intake in race cars) have resulted in vents and scoops on various road vehicles. Racing serves as a testing ground for new automotive technology. Road vehicle development is often driven by high-end series with large budgets and extensive manufacturer influence. For example, Audi, Porsche, and Jaguar Land Rover benefited from the all-electric Formula E (Shanahan, 2022).

According to heyca data for 2022, pure electric car sales increased by 40% yearly, while self-charging hybrid car sales increased by 28%, but plug-in hybrid vehicle sales decreased by 12% (Powell, 2023). Exposure to and familiarity with hybrid technology in motor racing may have led to this shift in consumer behaviour. Hybrid powertrains have long been used in top-tier motorsport, including Formula 1, Formula E, the World Endurance Championship, and the World Rally Championship (Evbenata, 2021).

IndyCar is one top-level racing series yet to experience this change. IndyCar is a type of open-wheel racing predominantly in the United States. For any given weekend, between 24 and 33 drivers participate. Powering these drivers is an engine that either Honda or Chevrolet supplies. To keep costs down and racing close, all cars use the same chassis, aero kit, and tyres, among other components. This means that for teams, optimising their package is paramount to success. IndyCar has some of the best track diversity of any top-level racing series, with three main types of racetracks. Ovals and Road Courses use purpose-built tracks, while, as the name suggests, Street Races utilise closed-off public roads (Cassie, 2023).

Due to factors like the USA's relatively slow adoption of electric/hybrid road vehicles (Birol, 2021), the impact of the COVID-19 pandemic on development (Fingas, 2020) and ongoing global supply chain challenges (Benyon, 2022), the hybridisation of IndyCar has taken longer than expected. IndyCar has decided not to introduce its planned new 2.4-litre engine for 2024 and has elected to add the new hybrid unit to the current 2.2-litre engine instead. The hybrid system utilises a Skeleton supercapacitor energy storage system and a driver-activated Empel electric motor generator unit (Honda, 2023), providing an estimated extra 100hp to the push-to-pass system on road courses.

Optimally controlling the deployment and regeneration of this energy is crucial in pursuing minimum lap times. Simulations have been vital in motorsport performance and strategy analysis due to testing limitations in IndyCar and other series. This research project intends to investigate the Optimal Control of the 2024 IndyCar Hybrid Energy Deployment and Recovery. Limebeer et al., Liu X et al., and Bertuccio J et al. have conducted similar work concerning Formula One, Formula E and the WEC, respectively. Their works, among others, are discussed in the literature review. The review is followed by the proposed approach used for this study, expected results and an evaluation of the potential impact of the proposed research.

2. Literature Review

This section of the report looks at case studies similar to the proposed study. This is followed by an overview of literature and background information relevant to the project in modelling vehicle dynamics, powertrains and tracks and optimal control or optimisation methods used in motorsport.

2.1. Case Studies

Similar energy management work has been conducted on Formula 1, E, and WEC cars. The F1 and Formula E studies utilised optimal control techniques to achieve minimal lap times. They used a General-Purpose MATLAB Software for Solving Optimal Control Problems. Using a fixed racing line was a limitation of the F1 model (Limebeer, Perantoni, & Rao, 2014). Figure 2-1 shows the predefined racing line the car model was instructed to follow. The Formula E (Xuze, Fotouhi, & Auger, 2020) study accounted for different racing lines, Figure 2-2; however, they did not include the effects of aerodynamics. As the 2023 Formula E season has progressed, it has been seen as a significant factor for energy management.

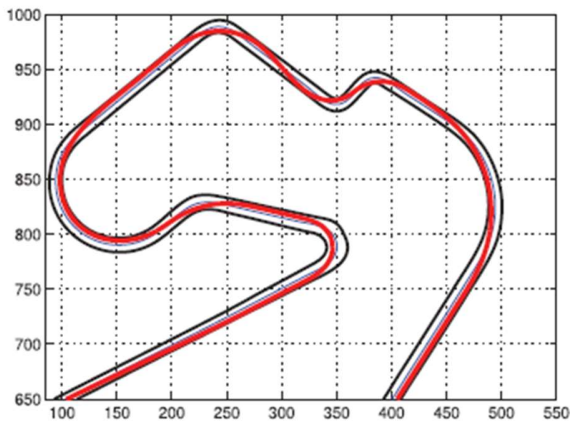


Figure 2-1: Predefined Racing line (Limebeer, Perantoni, & Rao, 2014)

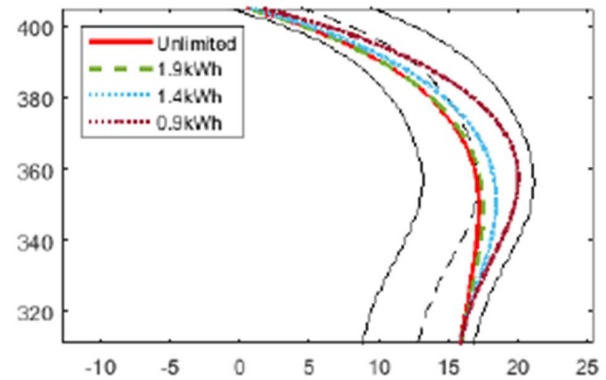


Figure 2-2: Variable Racing Lines (Xuze, Fotouhi, & Auger, 2020)

Figures 2-3 and 2-4 show comparisons of speed profiles with differing energy consumption constraints. The F1 and Formula E studies were conducted using Circuit de Barcelona-Catalunya and Circuit International Automobile Moulay El Hassan, respectively. Figure 2-3 shows speed comparisons relating to the three F1 investigated cases. The (red) curve is a qualifying lap, the (cyan) curve is the racing lap with 100 kg of fuel, and the (blue) curve is the racing lap with 75 kg of fuel (Limebeer, Perantoni, & Rao, 2014). Figure 2-4 shows speed profiles with different Formula E energy constraints. Maximum vehicle speed and minimum lap times are directly proportional to the energy expelled.

The WEC study used VI-CarRealTime to model the vehicle, Simulink to model the powertrain, and MATLAB for optimisation. This study aimed to optimise energy deployment to minimise fuel use over a single lap. Strict WEC deployment rules helped simplify the optimisation, and the analysis evaluated the relative sizing of energy storage systems (Bertuccio, 2022). Multi-objective optimisation utilised an empirical testing approach.

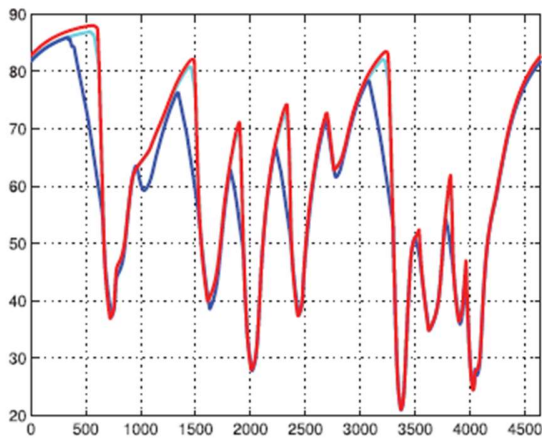


Figure 2-3: Speed comparison for the three cases studied (Limebeer, Perantoni, & Rao, 2014)

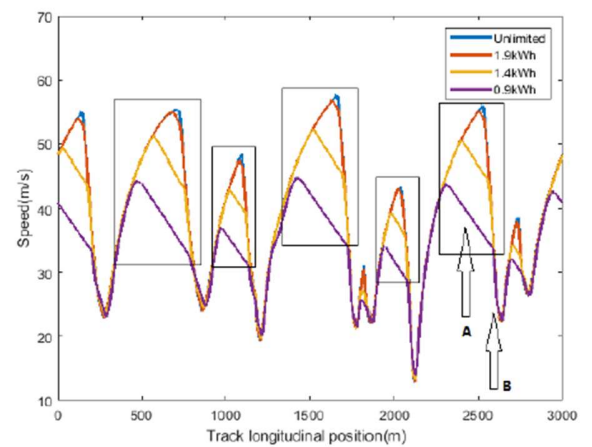


Figure 2-4: Speed profiles with different energy consumption limitations (Xuze, Fotouhi, & Auger, 2020)

An IndyCar Vehicle set-up study has been conducted. It used a 7DOF Simulink model and an Artificial Neural Network to predict the optimal set-up parameters of the car, reducing time and costs for race teams (D'Andrea, et al., 2021). This study relied on actual accelerometer data from the Huston Circuit for the simulation and focused body movement and weight transfer to improve car performance. High costs and the lack of available data make this test difficult to replicate and modify.

2.2. Vehicle Dynamics Modelling

Simulation methods are indispensable for analysing vehicles' performance and handling dynamics. These methods can be broadly categorised into three types: static and quasi-static, transient, and hardware-in-the-loop (HIL). While static and quasi-static simulation models are time-independent, the transient analysis considers the time response of the material. HIL systems involve computer simulations of the vehicle and its surroundings, including an active steering system with a real-time controller (Skill-Lync, 2020).

Automated Dynamic Analysis of Mechanical Systems (ADAMS) is widely used among the standard simulation tools for automotive applications; however, obtaining all the necessary data to define and execute computational models can be challenging. In such cases, simplification through methods such as the lumped mass vehicle model is necessary. Despite its simplification of the actual vehicle architecture, this model can analyse vehicle vibration characteristics and the effects of structural parameters on vehicle performance (Na, Jang, Yoo, & Kim, 2015).

The lumped mass vehicle model consists of mass, spring, and damper elements that define the model's degrees of freedom. It requires a minimum of 7 degrees of freedom and is typically three-dimensional to analyse the vehicle's vertical, pitch, and roll vibrations (Kim & Ro, 2002). One advantage of the lumped mass models is their simplicity and ability to obtain analytic descriptions and design active or semi-active controls. This model is widely used to study vehicle handling dynamics undergoing emergency collision avoidance, and MathWorks provides a reference application based on the ISO 3888-2 standard (MathWorks, 2022), (ISO, 2011).

2.3. Powertrain Modelling

The growing interest in the development of electric vehicles has led to increased research on modelling electric vehicle powertrains. Powertrain modelling is creating a mathematical model of a vehicle's powertrain system. The model includes all the components required to produce the forces to propel the vehicle and the other resistive forces acting on the vehicle (Ahamed, 2023). The mathematical model plays a crucial role in designing the powertrain for any vehicle. F1 and Fe studies utilised mathematical equations to simulate the vehicle powertrain, and the WEC study used MATLAB and Simulink.

The paper "Review of Modern Vehicle Powertrains and Their Modelling and Simulation in MATLAB/Simulink" reviews the drive systems of current modern vehicles and the types and characteristics of their major components. It also reviews the available models and computer programs for their simulation, focusing mainly on MATLAB/Simulink applications (SZÁNTÓ & SZIKI, 2020).

(Fadul, Aris, Mison, Halin, & Iqbal, 2018) discusses the modelling of hybrid electric vehicle powertrains. The review focuses on modelling the powertrain system of a hybrid electric vehicle using a combination of analytical and numerical methods. The review also discusses the use of simulation tools such as MATLAB/Simulink and ADVISOR for modelling hybrid electric vehicle powertrains. A shortcoming of the work was a lack of the inclusion of thermal effects on performance.

The Formula E study by Cranfield University incorporates a thermodynamics model as the battery's temperature must be carefully managed to prevent power derating during high-temperature races. The model describes the battery's thermal behaviour using a lumped model that considers reversible heat and irreversible heat. The battery temperature is described by an equation that includes the heat generated, conductive heat transfer rate, and radiative heat transfer rate. The heat generated comprises an irreversible heat generation component and a reversible heat generation component. The irreversible heat is dominant in high current applications, such as Formula E, while the reversible heat component significantly affects the battery temperature during discharging/charging cycles due to the change of current direction, which happens very often on Formula E cars due to regenerative brakes (Xuze, Fotouhi, & Auger, 2020).

(Chanhua, et al., 2021) discusses the simulation of a hybrid electric energy storage system (HESS) for a hybrid electric vehicle (HEV). HESS combines different energy storage systems, such as batteries and supercapacitors. The HESS aims to improve the EV's power and energy density by combining the advantages of both energy storage devices. The research proposes a driving condition-adaptive rule-based energy management strategy (EMS) for the HESS. It optimises the power distribution between the battery and the supercapacitor according to different driving modes and protects them from overcharging or over-discharging. The effectiveness of the HESS and EMS is validated by computer simulation and semi-physical rapid control prototype (RCP) test bench experiments. The results show that compared to a single battery case, the HESS can stabilise the working status of the battery, save its energy consumption, and prolong its lifetime. The work also evaluates the life cycle cost of the HESS and shows that it is economically feasible compared to a single battery case. Incorporating more robust and adaptable simulation models with fewer assumptions would improve results and versatility.

2.4. Track Modelling

Studies (F1 and Formula E) similar to the proposed work model tracks using a curvilinear coordinate system that follows the vehicle using the track centre-line position as the curvilinear abscissa from (Cossalter, 1999). Figure 2-5 shows a segment of track from (Limebeer, Perantoni, & Rao, 2014) where the location of the vehicle COM in terms of the curvilinear abscissa is described as $s(t)$ and the vector $n(s(t))$. This notation facilitates optimal control problem-solving.

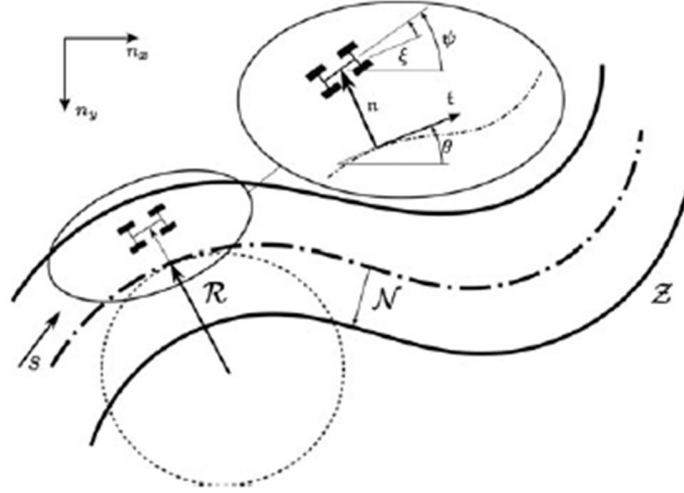


Figure 2-5: Curvilinear-coordinate-based description of a track segment (Limebeer, Perantoni, & Rao, 2014)

2.5. Optimisation

Dynamic optimisation is a fundamental technique utilised in scientific research for finding optimal solutions in various fields such as economics, engineering, and physics. Dynamic optimisation can be performed in discrete or continuous time depending on the modelling of the time variable. The discrete-time approach involves solving the problem by determining a sequence of values for the control and state variables that fulfil the optimality conditions. The continuous time method seeks to obtain a function of the control variable that satisfies the optimality conditions over time. The choice between the two approaches depends on the specific problem being addressed and the research objectives (Wilde, 2013).

GPOPS-II is a software package written in MATLAB for solving continuous optimal control problems using hp-adaptive Gaussian quadrature collocation and sparse nonlinear programming. Optimal control problems refer to problems where the goal is to optimise systems described by differential-algebraic equations while adhering to various constraints. GPOPS-II can solve multiple-phase problems where the dynamics or constraints of the system may vary at specific points in time. The software employs a Legendre-Gauss-Radau quadrature orthogonal collocation method where the continuous-time optimal control problem is transcribed to a large sparse nonlinear programming problem (NLP). The LGR quadrature rule has high accuracy and efficiency for integrating smooth functions (Eide, Hager, & Rao, 2021). The software also incorporates the latest developments in mesh refinement, costate estimation, and integral constraints (Patterson & Rao, 2014).

3. Methods

To explain the methodology of the project, a Gantt chart (Figure 3-1) is used. The project is split into five main sections: Planning, Learning, Modelling, Simulation and Analysis and Reporting. The planning stage comprised a basic literature review and a viability discussion with a likely supervisor. The Learning, Modelling, Simulation and Analysis stages are touched on in this section of the report, and the reporting section is time allocated for report writing.

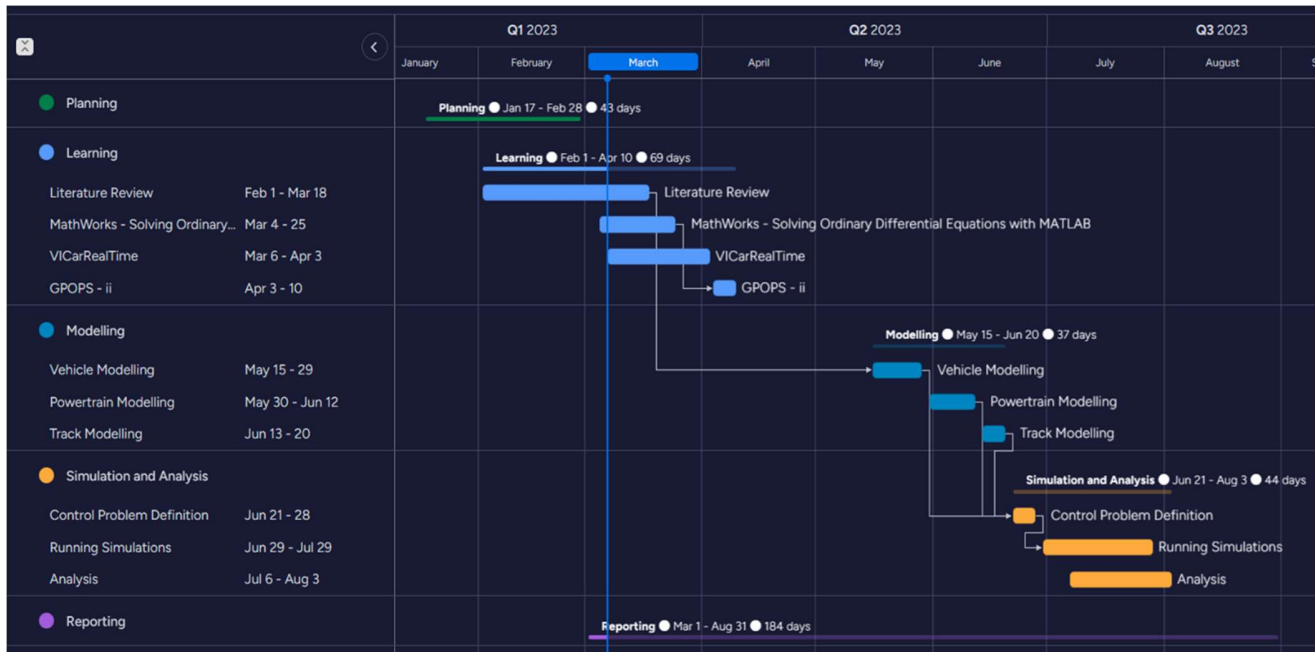


Figure 3-1: Proposed Project Gantt Chart

3.1. Data Gathering

Accurate analyses require accurate simulations. An accurately modelled vehicle is necessary for these simulations. This model relies on the accuracy of the input data. Gathering information regarding vehicle dynamics and the powertrain is necessary. This V-Model in Figure 3-2 describes the modelling methodology. The maximum charging and discharging rates and energy storage capacity are critical information. As this information is not in the public domain (at the time of writing), the first step for accruing the data is contacting IndyCar directly to obtain access to the engine rules for 2024. IndyCar is yet to respond to requests for technical regulations. IndyCar's partners in this project, MAHLE powertrains, have said they cannot share their customer's information but suggested contacting supercapacitor suppliers. A large amount of information has recently been made public by Honda Performance (Honda, 2023). As mentioned in the introduction, Honda stated the type (supercapacitor) and supplier (Skeleton) of the energy storage system. The supplier of the motor generator unit (Empel) was also revealed. If no further information is obtained, the use and extrapolation to modern-day standards of 2009 F1 rules and implemented systems will be necessary.

3.2. Modelling

Vehicle, powertrain, and track modelling will utilise MATLAB and Software for Solving Multiple-Phase Optimal Control Problems. VI-CarRealTime and recent literature will be used to validate the models.

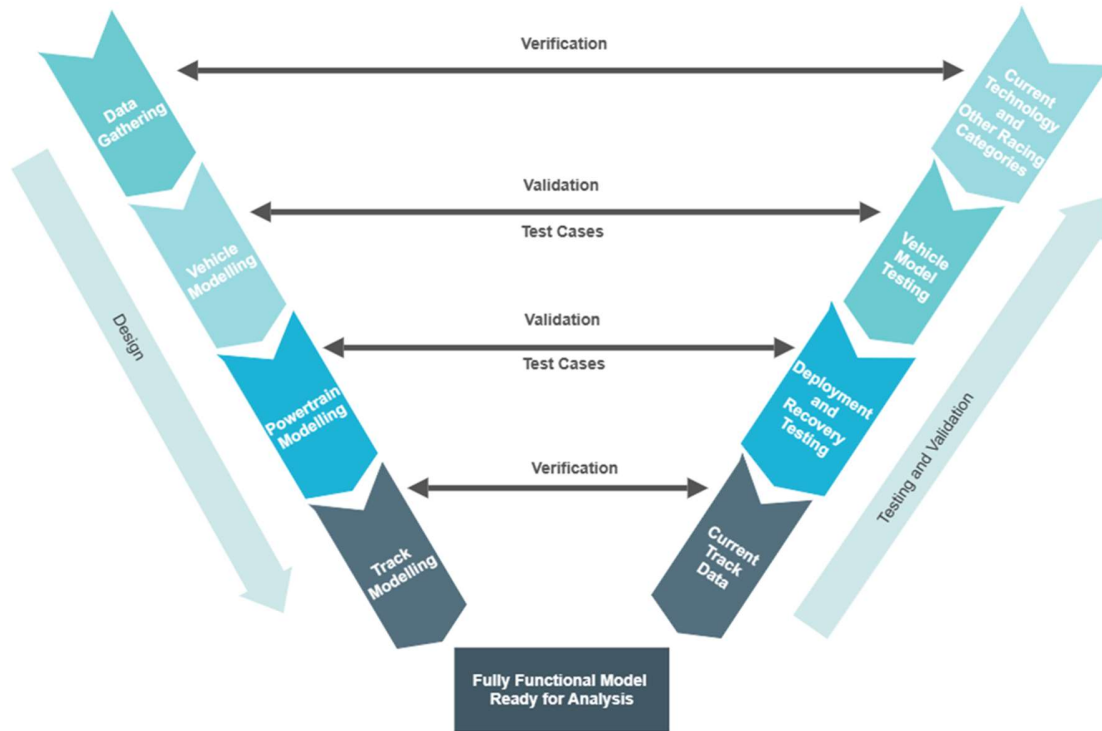


Figure 3-2: V-Model of Methodology

3.3. Simulation and Analysis

Table 3-1 shows a list of six energy modes that are to be investigated. Mode No. 1 is called Qualifying, allowing the driver to use the maximum amount of energy possible for a single lap. Overtake is mode No. 2, and it gives the driver a very high boost of energy to pass other cars. Mode No. 3 is called Deploy, meaning the car uses more energy for acceleration than it recovers from braking or coasting. Standard, Mode No. 4, balances the energy usage and recovery to maintain a constant state of charge over a lap. Modes 5 and 6 are recovery modes. No.5 reduces energy usage, while No.6 is constant maximum regeneration. Speed traces, the corresponding racing lines, and energy usage will be evaluated similarly to what has been done in previous studies of this nature.

Table 3-1: Energy Modes

Mode No.	Mode Name	Explanation
1	Qualifying	Maximum single lap deployment
2	Overtake	Extremely High Deployment
3	Deploy	Energy Deployment > Recovery
4	Standard	Energy Deployment = Recovery
5	Recover	Energy Deployment < Recovery
6	Harvest	Extremely High Recovery

4. Results

The proposed study is being done by modelling and simulation software such as MATLAB, GPOPS-ii, Simulink, Simscape, and VI-CarRealTime. Once the necessary data is obtained or estimated, MATLAB will be used to model the vehicle, powertrain, and track. Models will be validated against VI-CRT and Simulink models from recent literature. GPOPS-ii will be used to perform studies into energy modes listed in Table 3-1.

The freely available lap time simulator Optimum Lap was used to conduct a simple study comparing the impact of an additional 100hp on an IndyCar lap at the Indianapolis Motor Speedway. Figures 7-1 to 7-4 show the vehicle parameters and characteristics used. As IndyCar has seen little change since 2019, vehicle information from Microsoft's Forza Motorsport 7 (Microsoft, 2017) was used to calibrate the model. Figure 7-5 shows the torque and power curve used. These runs were then validated against 2019 IndyCar Indy 500 qualifying lap times (IndyCar, 2023).

Figure 4-1, the speed vs distance graph shows a significant disparity between a car at maximum vs standard power, with a 2% reduction in lap time. It is unlikely that a vehicle can use maximum power throughout a lap. Deciding when, where and how to use the electric power will significantly impact performance. Due to no changes to the aerodynamic characteristics, minimum cornering speeds for turns 1 and 3 are unchanged between models. The speed trace of the 100hp boosted car plateaus just above 380 kph due to unchanged gearing. The speed trace in Figure 4-2 overlays the vehicle speeds on a track map. The Optimum Lap simulator utilises a point mass model and relies on factors for analysis. The proposed study will utilise higher fidelity modelling methods to optimise energy usage and driving paths for varying input parameters. These results will be a vital component in strategy development.



Figure 4-1: Speed vs Distance graph (Hybrid - Purple, Standard - Orange)

Indianapolis Motor Speedway - Speed



Figure 4-2: Vehicle Speed track map

5. Evaluation

With the 2024 changes in IndyCar and the global growth of hybrid/electric vehicles, optimising energy usage is vital to improving performance and efficiencies. In terms of motor racing, understanding the intricacies of a rule change will allow teams to take a more proactive approach. An advantage over competitors, especially early in a rule change, will yield positive results and momentum for years.

The results may also be helpful for a sports governing body trying to improve the spectacle of racing. Findings will provide information about different energy deployment and recovery strategies. By utilising a mode method of deployment, IndyCar can ensure significant performance differences between modes such that teams cannot run at the optimal mode, creating more significant differences in the speeds of cars in different modes to produce more entertaining racing.

The proposed timescale of the research is described in Figure 3-1. The total funding required for the project is £140, as this is the cost of the GPOPS-ii software (GPOPS-ii, 2023). Steps 1 and 2 in Figure 5-1 show where this work fits into the motorsport development steps. By optimisation problem formulation, low-fidelity modelling and simulation is possible. Beyond this project's scope, future work (step 3) would be high-fidelity modelling and simulation in a driver-in-loop simulator. Following DIL testing and validation, real-world testing (step 4) would occur.

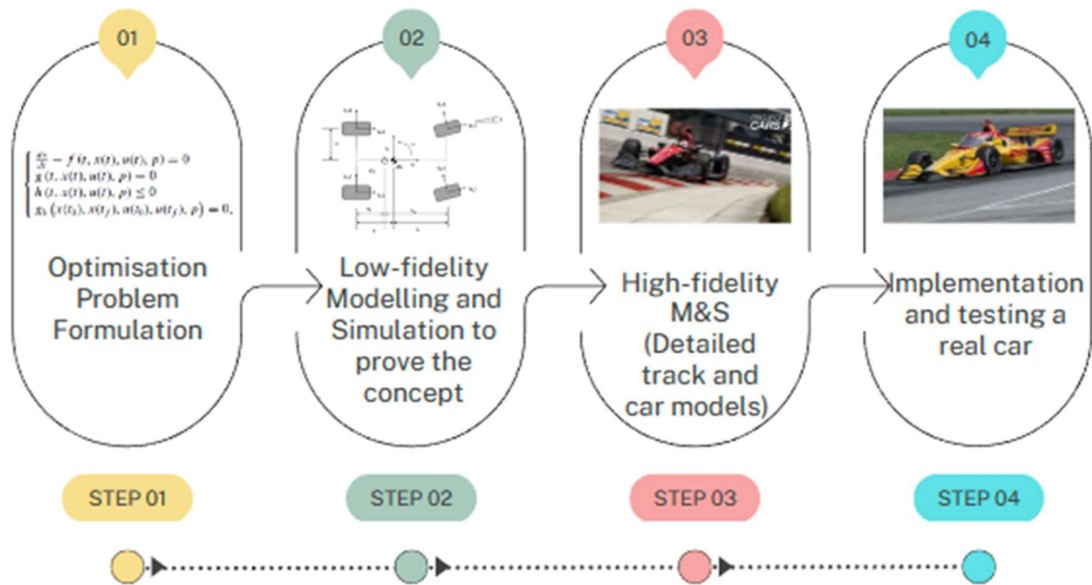


Figure 5-1: Motorsport Development Steps (Xuze, Fotouhi, & Auger, 2020)

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7. Appendix

IndyCar 2019 Forza

15 March 2023

Vehicle Configuration

	Parameter	Value
	Total Mass	709 kg
	Max Torque	400 N.m @ 12600 rpm
	Type of Fuel	Gasoline
	Type of Transmission	Sequential Gearbox
	Max Power	707.77 hp @ 12600 rpm
	Power Mass Ratio	1 hp/kg
	Downforce @ 100 km/h	694.46 N
	Drag @ 100 km/h	324.08 N

Performance Metrics

	Metric	Value
	Top Speed	378.43 km/h
	Time for 0 to 100 km/h	5.44 s
	Time for 100 to 0 km/h	1.7 s
	Lateral Acceleration - Skidpad 50 m	17.47 m/s ²

Figure 7-1: Optimum Lap Vehicle Characteristics

Charts Summary

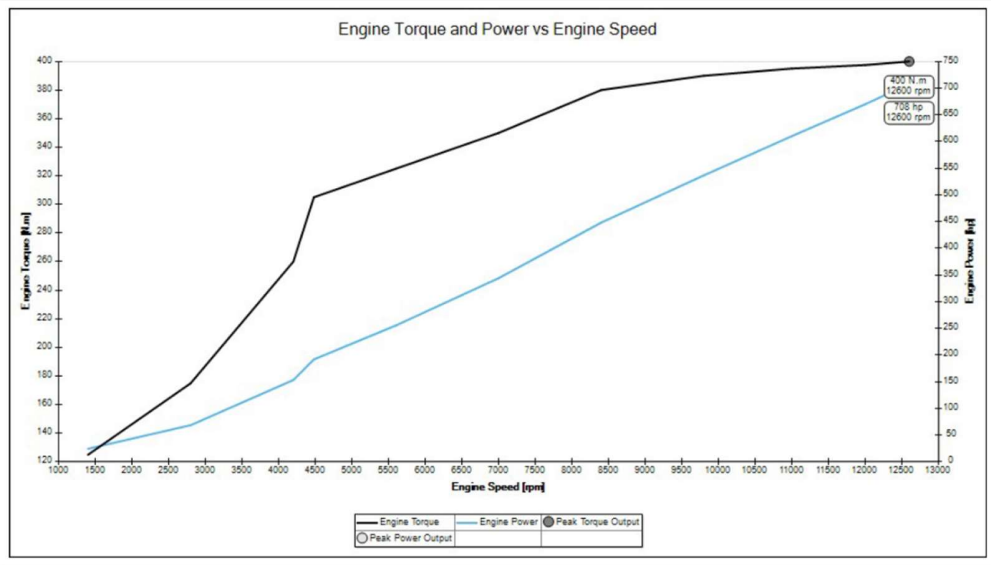


Figure 7-2: Optimum Lap Engine Torque and Power vs Engine Speed

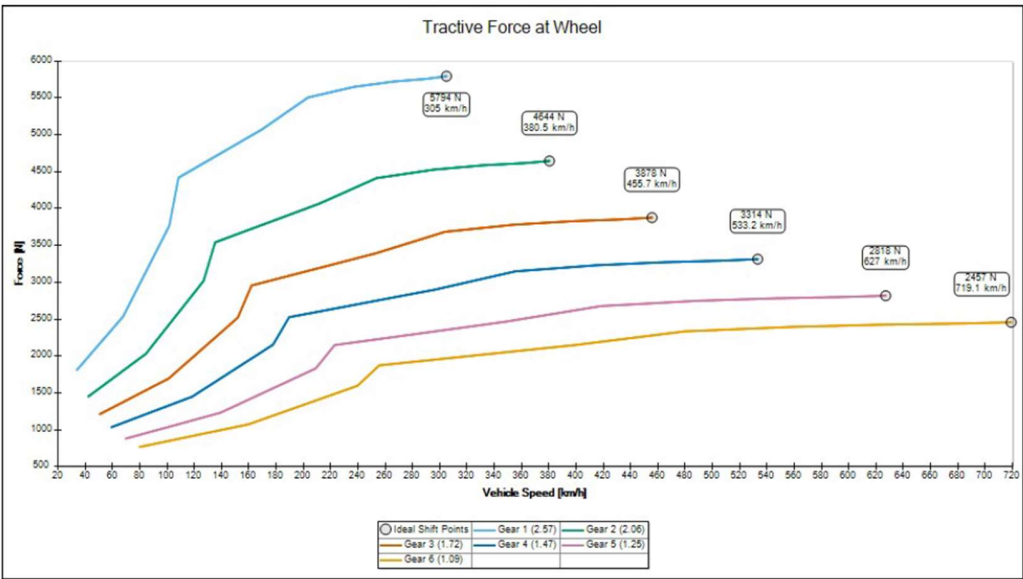


Figure 7-3: Optimum Lap Tractive Force at Wheel

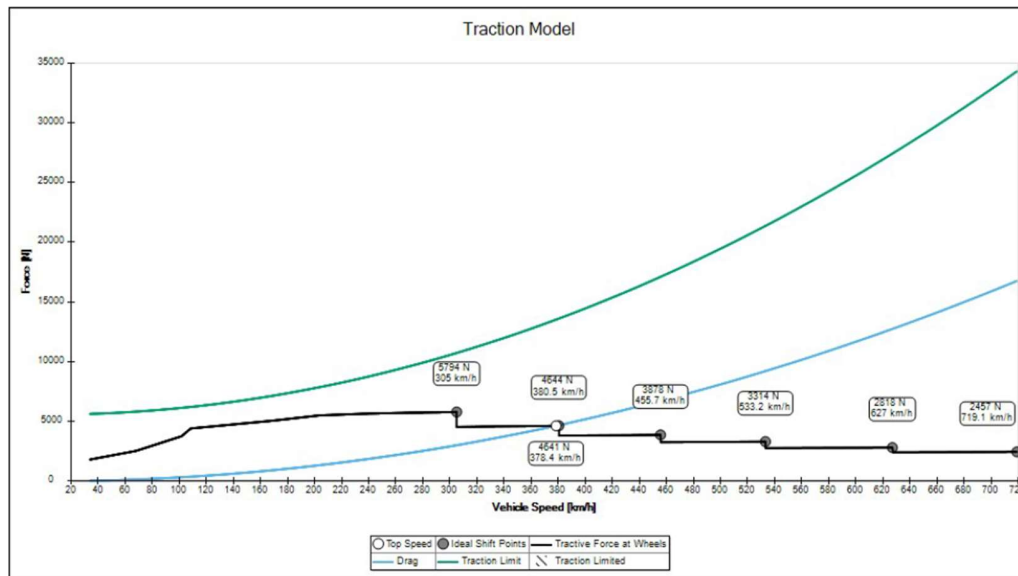


Figure 7-4: Optimum Lap Traction Model

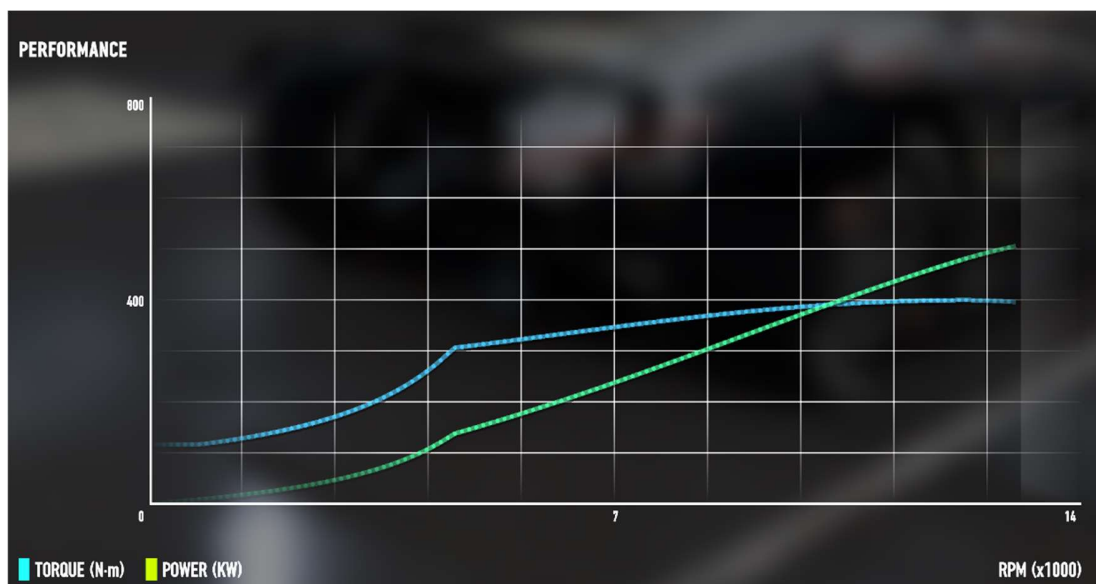


Figure 7-5: Forza Motorsport Engine Torque and Power vs Engine Speed (Microsoft, 2017)