



EXAMINATION SOLUTIONS: SESSION 2016/17

School:	ENGINEERING		
Programme:	EXTREME SPORTS ENGINEERING	Year:	L6
Module/Unit:	MATERIALS & MANUFACTURING FOR NICHE MARKETS (MEX01201)		
Author:	KELVIN LAKE	Page:	1 of 13

Question No	Question	Marks
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MBXD1201

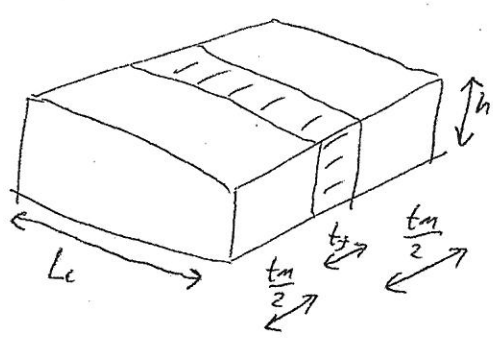
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Question No	Question	Marks
①	PHASES : MATRIX - PRIMARY REINFORCING AGENT - SECONDARY MATRIX IS USED TO CLASSIFY COMPOSITES.. METAL MATRIX COMPOSITES (MMC) CERAMIC MATRIX COMPOSITES (CMC) POLYMER MATRIX COMPOSITES (PMC) REINFORCING AGENT FIBRE - GENERALLY CIRCULAR, PRODUCES GREATEST OPPORTUNITY FOR STRENGTH ENHANCEMENT. CAN BE CONTINUOUS OR DIS-CONTINUOUS & ARRANGED IN 1D, 2D OR 3D. PARTICLES - CAN BE MICRO OR MACROSCOPIC. ARRANGED RANDOMLY LEADING TO ISOTROPIC MATERIAL PROPERTIES & THUS GENERALLY NOT AS STRONG AS FIBRE REINFORCEMENT. FLAKES - 2D PARTICLES - LOW COST.	② ③ ② ② ②

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Question No	Question	Marks
6	 $A_f = t_f h$ $A_m = t_m h$ $A_c = (t_m + t_f) h$ $F_c = F_f + F_m$ $\sigma_c A_c = \sigma_f A_f + \sigma_m A_m$ $E_c \epsilon_c = E_f \epsilon_f + E_m \epsilon_m$ $\therefore E_c = E_f \frac{A_f}{A_c} + E_m \frac{A_m}{A_c}$ $= E_f \frac{A_f L_c}{A_c L_c} + E_m \frac{A_m L_c}{A_c L_c}$ $= E_f \frac{V_f}{V_c} + E_m \frac{V_m}{V_c}$ $= E_f V_f + E_m V_m$	6

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Question No	Question	Marks
(E)	<u>ASSUMPTIONS</u> - BOND BETWEEN FIBRES & MATRIX IS PERFECT - ~~D~~ & SPACING BETWEEN FIBRES IS UNIFORM - FIBRES ARE CONTINUOUS & PARALLEL - FIBRES & MATRIX BEHAVE LINEARLY - FIBRES POSSES UNIFORM STRENGTH - COMPOSITE IS VOID FREE	(3)

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Question No	Question	Marks
(2)(a)	$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix}$ IF $\sigma_2 = \sigma_3 = \tau_{23} = \tau_{31} = \tau_{12} = 0$ & $\sigma_1 \neq 0$ $\therefore \epsilon_1 = S_{11} \sigma_1$ $\therefore S_{11} = \frac{\epsilon_1}{\sigma_1} = \underline{\underline{\frac{1}{E_1}}}$ ALSO $\epsilon_2 = S_{12} \sigma_1$ $\therefore S_{12} = \frac{\epsilon_2}{\sigma_1} = -\nu_{12} \frac{\epsilon_1}{\sigma_1} = \underline{\underline{-\frac{\nu_{12}}{E_1}}}$ ALSO $\epsilon_3 = S_{13} \sigma_1$ $\therefore S_{13} = \frac{\epsilon_3}{\sigma_1} = -\nu_{13} \frac{\epsilon_1}{\sigma_1} = \underline{\underline{-\frac{\nu_{13}}{E_1}}}$	(1) (1) (1)

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Question No	Question	Marks
(2a) (CONT)	IF $\sigma_1 = \sigma_3 = \tau_{23} = \tau_{31} = \tau_{12} = 0$ & $\sigma_2 \neq 0$ $\epsilon_2 = S_{22} \sigma_2$ $\therefore S_{22} = \frac{\epsilon_2}{\sigma_2} = \underline{\underline{\frac{1}{E_2}}}$ ALSO $\epsilon_3 = S_{23} \sigma_2$ $S_{23} = \frac{\epsilon_3}{\sigma_2} = -\nu_{23} \frac{\epsilon_2}{\sigma_2} = \underline{\underline{-\frac{\nu_{23}}{E_2}}}$ IF $\sigma_1 = \sigma_2 = \tau_{23} = \tau_{31} = \tau_{12} = 0$ & $\sigma_3 \neq 0$ $\epsilon_3 = S_{33} \sigma_3$ $\therefore S_{33} = \frac{\epsilon_3}{\sigma_3} = \underline{\underline{\frac{1}{E_3}}}$ IF $\sigma_1 = \sigma_2 = \sigma_3 = \tau_{31} = \tau_{12} = 0$ & $\tau_{23} \neq 0$ $\gamma_{23} = S_{44} \tau_{23}$ $\therefore S_{44} = \frac{\gamma_{23}}{\tau_{23}} = \underline{\underline{\frac{1}{G_{23}}}}$	① ① ① ①

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Question No	Question	Marks
(20) (CONT)	IF $\sigma_1 = \sigma_2 = \sigma_3 = \tau_{23} = \tau_{12} = 0$ & $\tau_{31} \neq 0$ $\gamma_{31} = S_{55} \tau_{31}$ $\therefore S_{55} = \frac{\gamma_{31}}{\tau_{31}} = \frac{1}{G_{31}}$ FINALLY IF $\sigma_1 = \sigma_2 = \sigma_3 = \tau_{23} = \tau_{31} = 0$ & $\tau_{12} \neq 0$ $\gamma_{12} = S_{66} \tau_{12}$ $\therefore S_{66} = \frac{\gamma_{12}}{\tau_{12}} = \frac{1}{G_{12}}$ $\therefore [S] = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{13}}{E_1} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$	(1) (1) (1)

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Question No	Question	Marks
(26)	$S_{11} = \frac{1}{E_1} = \frac{1}{181 \times 10^9} = 5.525 \times 10^{-12} \text{ Pa}^{-1}$	(1/2)
	$S_{12} = \frac{-\nu_{12}}{E_1} = \frac{-0.28}{181 \times 10^9} = -1.547 \times 10^{-12} \text{ Pa}^{-1}$	(1/2)
	$S_{13} = \frac{-\nu_{13}}{E_1} = \frac{-0.27}{181 \times 10^9} = -1.492 \times 10^{-12} \text{ Pa}^{-1}$	(1/2)
	$S_{22} = \frac{1}{E_2} = \frac{1}{10.3 \times 10^9} = 9.709 \times 10^{-11} \text{ Pa}^{-1}$	(1/2)
	$S_{23} = \frac{-\nu_{23}}{E_2} = \frac{-0.60}{10.3 \times 10^9} = -5.825 \times 10^{-11} \text{ Pa}^{-1}$	(1/2)
	$S_{33} = \frac{1}{E_3} = \frac{1}{10.3 \times 10^9} = 9.709 \times 10^{-11} \text{ Pa}^{-1}$	(1/2)
	$S_{44} = \frac{1}{G_{23}} = \frac{1}{3.0 \times 10^9} = 3.333 \times 10^{-10} \text{ Pa}^{-1}$	(1/2)
	$S_{55} = \frac{1}{G_{31}} = \frac{1}{7 \times 10^9} = 1.429 \times 10^{-10} \text{ Pa}^{-1}$	(1/2)
	$S_{66} = \frac{1}{G_{12}} = \frac{1}{7.17 \times 10^9} = 1.395 \times 10^{-10} \text{ Pa}^{-1}$	(1/2)

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Question No	Question	Marks
(26) (CONT)	$[S] = \begin{bmatrix} 5.525 \times 10^{-12} & -1.547 \times 10^{-12} & -1.492 \times 10^{-12} \\ -1.547 \times 10^{-12} & 9.709 \times 10^{-11} & -5.825 \times 10^{-11} \\ -1.492 \times 10^{-12} & -5.825 \times 10^{-11} & 9.709 \times 10^{-11} \end{bmatrix}$ $\sigma_a = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $P_a = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\frac{1}{2}$ IF $\sigma_1 = 10 MPa$ $\sigma_2 = -5 MPa$ $\sigma_3 = 7 MPa$ (c) $\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & s_{23} \\ s_{13} & s_{23} & s_{33} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix}$ $\therefore \epsilon_i = s_{i1}\sigma_1 + s_{i2}\sigma_2 + s_{i3}\sigma_3$ $= 5.525 \times 10^{-12} \times 10 \times 10^6 + 1.547 \times 10^{-12} \times (-5 \times 10^6) - 1.492 \times 10^{-12} \times 7 \times 10^6$ $= 5.25 \times 10^{-5} m$	5

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Question No	Question	Marks
(3) (4)	SEVERAL LAMINA ARE USED TO FORM A LAMINATE DUE TO (1) A SINGLE LAMINA IS $\sim 0.1-0.2\text{mm}$ THICK, $\therefore$ TO GET ANYTHING OF MEANINGFUL STRENGTH SEVERAL LAMINA MUST BE COMBINED (2) A SINGLE LAMINA IS WEAK TRANSVERSE TO THE FIBRES SO SEVERAL LAMINA ARE STACKED WITH VARYING ORIENTATIONS TO OVERCOME THIS (b) PROBLEMS : - HOW MANY PLYS - WHAT ORIENTATION - WHAT STACKING SEQUENCE OPPORTUNITIES : - OPTIMISATION	(4)
(c)	(i) $[0/-45/90_2/60/0]$ $\begin{array}{r} 0 \\ -45 \\ 90 \\ 90 \\ 60 \\ 0 \end{array}$ (ii) $[0/-45/60]_s$ $\begin{array}{r} 0 \\ -45 \\ 60 \\ 60 \\ -45 \\ 0 \end{array}$ (iii) $[0/-45/60]_s$ $\begin{array}{r} 0 \\ -45 \\ 60 \\ -45 \\ 0 \end{array}$ (iv) $[0_{GR}/\pm 45_B]_s$ $\begin{array}{r} \text{GRAPHITE } 0 \\ \text{BORON } 45 \\ \text{BORON } -45 \\ \text{BORON } -45 \\ \text{BORON } 45 \\ \text{GRAPHITE } 0 \end{array}$	(4)

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Question No	Question	Marks
(a) (i)	A CROSS PLY ONLY HAS 0/90 PLYS	(4)
(ii)	AN ANGLE PLY ARE ONLY ORIENTATED $\pm$ THETA	
(iii)	SYMMETRICAL ABOUT THE MIDPLANE EXCEPT FOR SIGN	
(iv)	PLYs ARE IN $\pm$ PAIRS	
(c)	HYLTROTHERMAL STRESS & STRAINS ARE CAUSED BY THE DIFFERENT THERMAL EXPANSION COEFFICIENTS OF THE REINFORCEMENT PHASE & THE MATRIX WHEN THE COMPOSITE COOLS DOWN AFTER PROCESSING AT A MICROMECHANICAL LEVEL. IN ADDITION MOISTURE ABSORBED BY THE 2 PHASES RESULTING IN DIFFERENT AMOUNTS OF SWELLING CAN ALSO CAUSE ISSUES. THESE CAN BE RELATED IF THE LAMINATE IS SYMMETRICAL	(4)

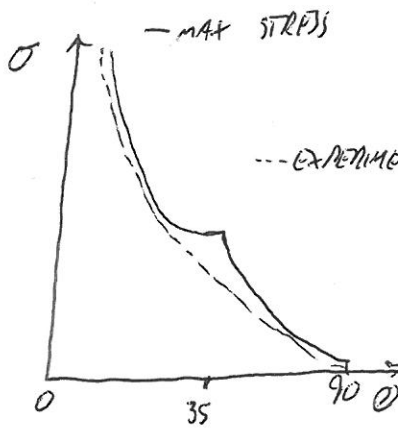
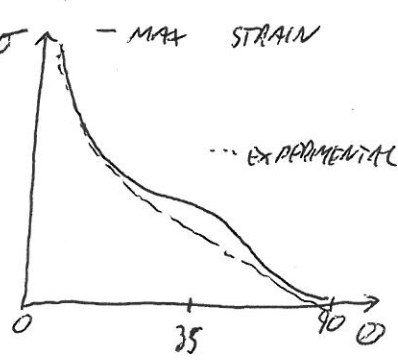
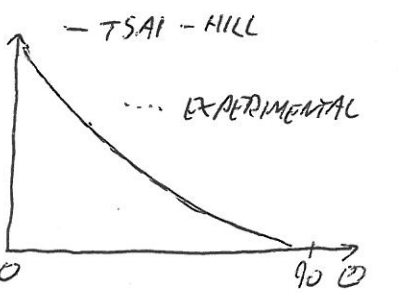
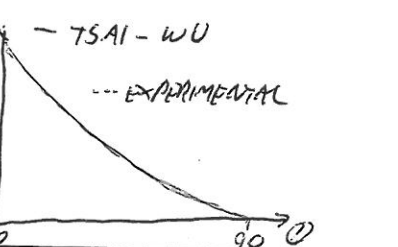
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Question No	Question	Marks
(4) (6)	INSTEAD OF THE PRINCIPAL STRESSES, THE FOLLOWING <u>ULTIMATES</u> ARE CONSIDERED WHEN PREDICTING FAILURE OF A LAMINATE IN THE 12 PLANE: • COMPRESSIVE & TENSILE STRESS - LONGITUDINAL • " " " " - LATERAL • " " " " STRAIN - LONGITUDINAL • " " " " " - LATERAL • IN-PLANE SHEAR STRESS • IN-PLANE SHEAR STRAIN	(7)
(6)	FAILURE THEORIES: ① <u>MAX STRESS FAILURE THEORY</u> - FAILURE PREDICTED IF NORMAL OR SHEAR STRESSES IN LOCAL AXIS EXCEED THOSE OF THE UNIDIRECTIONAL LAMINATE. ② <u>MAX STRAIN FAILURE THEORY</u> - AS ABOVE BUT WITH STRAINS ③ <u>TSAI-HILL FAILURE THEORY</u> - FAILURE WILL OCCUR WHEN THE DISTORTION ENERGY IS GREATER THAN THE FAILURE DISTORTION ENERGY OF THE MATERIAL: BASED ON VON-MISES FOR ISO-TROPIC MATERIALS	

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Question No	Question	Marks
(4b)	(4) <u>TSAI-WU FAILURE THEORY</u> - AS TSAI-HILL BUT ACCOUNTS FOR COMPRESSIVE STRESSES ALSO	(8)
(c)	(1)  - MAX STRESS - MAX STRESS & MAX STRAIN HAVE A CUSP AT ABOUT 35°. MAX STRESS IS MORE PRONOUNCED - TSAI-HILL & TSAI-WU ARE IN GOOD AGREEMENT & CLOSE TO THE EXPERIMENTAL RESULTS (2)  - MAX STRAIN (3)  - TSAI-HILL (4)  - TSAI-WU	(5)