

Simple Guide to Number Conversion Techniques

Md Shantanu Islam

February 13, 2024

1 Introduction

This guide is a simple tutorial on number system conversion techniques, especially for students starting in computer science. We will explore three methods: Multiplication, Division, and Substitution, and apply them in various scenarios like converting numbers to and from binary, octal, and hexadecimal systems.

2 Conversion Methods Overview

2.1 Table of Methods

The table below outlines which method is suited for which kind of conversion.

Conversion Type	Method Used	Base Used
Non-decimal to Decimal	Multiplication Method	Depends on the original number system (e.g., 2 for binary)
Decimal to Non-decimal	Division Method	Depends on the target number system (e.g., 2 for binary)
Non-decimal to Non-decimal (excluding decimal)	Substitution Method	-

3 Multiplication Method

This method is used when converting a number from a non-decimal system (like binary, octal, or hexadecimal) to decimal. Each digit is multiplied by the base of its original system raised to the power of its position, counted from right to left starting from 0.

3.1 Examples

3.1.1 Binary to Decimal

Example: Convert $(1101)_2$ (binary) to decimal.

Each digit's position determines the power of 2 it's multiplied with:

$$(1 \times 2^3) \quad (1 \times 2^2) \quad (0 \times 2^1) \quad (1 \times 2^0)$$

Calculating the above gives us: $8 + 4 + 0 + 1 = (13)_{10}$.

3.1.2 Octal to Decimal

Example: Convert $(17)_8$ (octal) to decimal.

Here, each digit is multiplied by 8 raised to the power of its position:

$$(1 \times 8^1) \quad (7 \times 8^0)$$

Calculating the above gives us: $8 + 7 = (15)_{10}$.

3.1.3 Hexadecimal to Decimal

Example: Convert $(A1)_{16}$ (hexadecimal) to decimal.

Each digit (keeping in mind that A = 10) is multiplied by 16 raised to the power of its position:

$$(10 \times 16^1) \quad (1 \times 16^0)$$

Calculating the above gives us: $160 + 1 = (161)_{10}$.

4 Division Method

This technique is applied when converting a decimal number to a non-decimal system (like binary, octal, or hexadecimal). The number is continuously divided by the base of the target system, and the remainders form the number in the new base when read in reverse order.

4.1 Examples

Each example below involves dividing the decimal number by the target base and taking note of the remainder until the quotient is zero.

4.1.1 Decimal to Binary

Example: Convert $(13)_{10}$ to binary (2 is the base for binary).

Quotient and remainder:

$$\begin{array}{ll} 13/2 = 6 & \text{remainder } 1 \\ 6/2 = 3 & \text{remainder } 0 \\ 3/2 = 1 & \text{remainder } 1 \\ 1/2 = 0 & \text{remainder } 1 \end{array}$$

Reading the remainders from bottom to top, we get $(1101)_2$.

4.1.2 Decimal to Octal

Example: Convert $(29)_{10}$ to octal (8 is the base for octal).

Quotient and remainder:

$$\begin{array}{rcl} 29/8 & = & 3 \text{ remainder } 5 \\ 3/8 & = & 0 \text{ remainder } 3 \end{array}$$

Reading the remainders from bottom to top, we get $(35)_8$.

4.1.3 Decimal to Hexadecimal

Example: Convert $(254)_{10}$ to hexadecimal (16 is the base for hexadecimal).

Quotient and remainder:

$$\begin{array}{rcl} 254/16 & = & 15 \text{ remainder } 14(E) \\ 15/16 & = & 0 \text{ remainder } 15(F) \end{array}$$

Reading the remainders from bottom to top, we get $(FE)_{16}$.

5 Substitution Method

This method is handy when converting numbers between non-decimal systems (like binary to octal or binary to hexadecimal and vice versa). It involves substituting each group of digits with its equivalent in the target base, usually done using a reference table.

5.1 Binary Equivalents for Substitution

Below are tables showcasing binary equivalents that are used for substitution.

5.1.1 Binary to Octal

Octal Digit	Binary Equivalent
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

5.1.2 Binary to Hexadecimal

Hexadecimal Digit	Binary Equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

5.2 Examples

The substitution method uses the tables above to convert numbers from binary to octal or hexadecimal and vice versa.

5.2.1 Binary to Octal

Example: Convert $(101110)_2$ to octal.

First, group the binary digits into groups of three (starting from the right), then substitute with the octal equivalent:

$$(101)_2 \quad (110)_2$$

Looking at our table, we find:

$$(101)_2 = (5)_8 \quad (110)_2 = (6)_8$$

So, $(101110)_2 = (56)_8$.

5.2.2 Binary to Hexadecimal

Example: Convert $(10101100)_2$ to hexadecimal.

Group the binary digits into groups of four (starting from the right), then substitute with the hexadecimal equivalent:

$$(1010)_2 \quad (1100)_2$$

From our table:

$$(1010)_2 = (A)_{16} \quad (1100)_2 = (C)_{16}$$

So, $(10101100)_2 = (AC)_{16}$.

5.2.3 Octal to Binary

Example: Convert $(71)_8$ to binary.

Using the octal to binary table, substitute each digit:

$$(7)_8 = (111)_2 \quad (1)_8 = (001)_2$$

So, $(71)_8 = (111001)_2$.

5.2.4 Hexadecimal to Binary

Example: Convert $(3A)_{16}$ to binary.

Using the hexadecimal to binary table, substitute each digit:

$$(3)_{16} = (0011)_2 \quad (A)_{16} = (1010)_2$$

So, $(3A)_{16} = (00111010)_2$.

6 Conclusion

This guide simplifies the process of converting numbers between different systems, a crucial skill in computer science and related fields. By understanding the Multiplication, Division, and Substitution methods, and practicing with various examples, students can confidently tackle problems related to number conversions. Remember, practice is key to mastering these techniques!