

LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 5

LEFEL 5

EXAMINATION PERIOD

CYFNOD ARHOLIADAU

MODULE CODE:
M5x02064: CAE & Stress analysis

TIME: 2 HOURS

ANSWER THREE OUT OF FOUR QUESTIONS

Equipment or materials required – None

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A formula sheet is provided for reference.

Calculators are permitted.

Assume 'g' = 9.81 m/s².

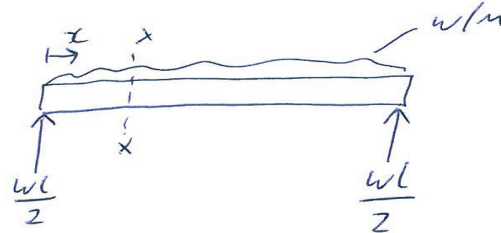
Continuation Sheet

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Question No	Question	Marks
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Q
1

1



$$M_{xx} = \frac{wlx}{2} - wx \cdot \frac{x}{2}$$

$$\therefore EI \frac{d^2 y}{dx^2} = \frac{wlx}{2} - \frac{wx^2}{2}$$

$$\therefore EI \frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} + A \quad * *$$

$$\therefore EI y = \frac{wlx^3}{12} - \frac{wx^4}{24} + Ax + B \quad *$$

WHEN $x=0, y=0 \therefore B=0$

WHEN $x=l, y=0 \therefore$

FROM *
 $0 = \frac{wl^3}{12} - \frac{wl^4}{24} + Al \therefore Al = \frac{wl^4}{24} \therefore A = \frac{wl^3}{24}$

FROM *
 $y = \frac{1}{EI} \left[\frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^3x}{24} \right]$

y_{max} WHEN $x = \frac{l}{2}$

$$\therefore y_{max} = \frac{1}{EI} \left[\frac{wl}{12} \left(\frac{l^3}{8} \right) - \frac{w}{24} \left(\frac{l^4}{16} \right) - \frac{wl^3}{24} \left(\frac{l}{2} \right) \right]$$

$$= \frac{1}{EI} \left[\frac{wl^4}{96} - \frac{wl^4}{384} - \frac{wl^4}{48} \right]$$

$$= -\frac{5wl^4}{384EI}$$

[8 MARKS]

Continuation Sheet

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Question No

Question

Marks

(ii) FROM **

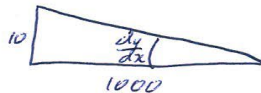
(2)

$$\frac{dy}{dx} = \frac{1}{EI} \left[\frac{wLx^2}{4} - \frac{wx^3}{6} - \frac{wL^3}{24} \right]$$

WHEN $x = 0$:

$$\left(\frac{dy}{dx} \right)_{\max} = \frac{1}{EI} \left(-\frac{wL^3}{24} \right) = -\frac{wL^3}{24EI} \quad [2 \text{ MARKS}]$$

(b) i) FIND $\left(\frac{dy}{dx} \right)_{\max}$:



$$\left(\frac{dy}{dx} \right)_{\max} = \frac{10}{1000} = \underline{0.01}$$

• FIND udl : $w = \frac{5000 \times 9.81}{5} = \underline{9810 \text{ N/m}}$

• $\left(\frac{dy}{dx} \right)_{\max} = -\frac{wL^3}{24EI}$

$$\therefore EI = \frac{wL^3}{24 \left(\frac{dy}{dx} \right)_{\max}} = \frac{9810 \times 5^3}{24 \times 0.01} = \underline{5.1 \text{ MNm}^2}$$

[5 MARKS]

(ii) IF $E = 210 \text{ GPa}$

$$I = \frac{5.1 \times 10^6}{210 \times 10^9}$$

$$= 2.4 \times 10^{-5} \text{ m}^4$$

$$= 2.4 \times 10^{-5} \times (100^4)$$

$$= \underline{2400 \text{ cm}^4}$$

FROM TABLE

$$\text{BEAM} = 254 \times 101.6$$

MASS

PER

$$\text{MASS} = 22 \text{ kg/m}$$

LENGTH

$$\therefore \text{TOTAL MASS} = 22 \times 6.5 = \underline{143 \text{ kg}}$$

[5 MARKS]

Q2

(a) Typically 5 core sequential design phases as specified in the following order:

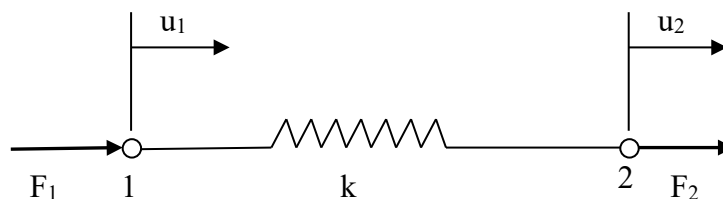
1. Define the problem; establish customer needs, develop problem statement, criteria for success.
2. Gather relevant information; search information sources, redefine problem if needed.
3. Generate multiple solutions; creative phase involving brainstorming, 'synthesis' of new ideas to solve design problem.
4. Analyse and select solution; consider functional analysis, ergonomics, product safety & liability, use decision matrix.
5. Test and implement solution; prototyping and test under real conditions, apply for Patents, release to market.

(6 marks)

(b) CAE tools can document changes in design and analyse mechanical, electromagnetic, heat transfer aspects of design before prototype even manufactured. Ergonomic and material properties also assessed from computer drawings of design, and use of rapid prototyping to build mock-up of solution(s). Design time reduced and Design for Manufacturing techniques considered to reduce costs of design solution. Utilise concurrent engineering principles so designs also consider safety, marketability and regulatory compliance.

(4 marks)

(c)



Spring supports axial load only and displacement is directly proportional to applied axial load. Constant of proportionality is spring stiffness k . Assuming both nodal displacements are zero when spring undeformed, net spring deformation given by;

$$x = u_2 - u_1 \text{ and resultant axial force is, } f = k \cdot x = k \cdot (u_2 - u_1)$$

For equilibrium, $F_1 = -F_2$ which can be expressed in matrix form as;

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad (5 \text{ marks})$$

(d) FEA obtains approximate solutions of boundary value problems, also called field problems, where field is domain of interest. Model body (field) divided into equivalent system of finite elements which are connected to adjacent elements via nodes; known as discretisation process. Field variables are dependent variables of interest governed by differential equations (known as governing equations) that describes & defines their reaction to applied loading conditions. Boundary conditions (loads and constraints) are specified values of field variables on boundaries of the field.

Students may also mention Preprocessing / Solution / Postprocessing; acceptable if some of the above is also described.

(5 marks)

Q3

- (a) Equate all forces normal to AB so that: $AB\sigma_\theta = \sigma_x AC \cos\theta + \sigma_y BC \sin\theta$

Where,

$$AC = AB \cos\theta \quad \text{and,} \quad BC = AB \sin\theta$$

Therefore, $\sigma_\theta = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta$

From double-angle formula;

$$\sigma_\theta = \frac{\sigma_x}{2} + \frac{\sigma_x}{2} \cos 2\theta + \frac{\sigma_y}{2} - \frac{\sigma_y}{2} \cos 2\theta$$

Hence, $\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta$

(7 marks)

- (b) Given that no shear couple present, then equation above used, where;

$$\sigma_x = -30 \text{ MN/m}^2 \text{ and, } \sigma_y = 55 \text{ MN/m}^2$$

Therefore, $\sigma_\theta = 19.88 \text{ MN/m}^2$ acting as tensile stress on plane AB

And, $\tau_\theta = \frac{1}{2}(-30 - 55) \sin 100^\circ = -41.85 \text{ MN/m}^2$ acting upwards from B to A.

(4 marks)

- (c) Using equation for $\sigma_{1,2}$ from formula sheet, where $\sigma_x = 30 \text{ MN/m}^2$ and $\tau_{xy} = 45 \text{ MN/m}^2$;

$$\sigma_1 = \frac{1}{2} \left[(30 + 55) + \sqrt{(30 - 55)^2 + 4(45)^2} \right] = 89.2 \text{ MN/m}^2 \text{ (tensile)}$$

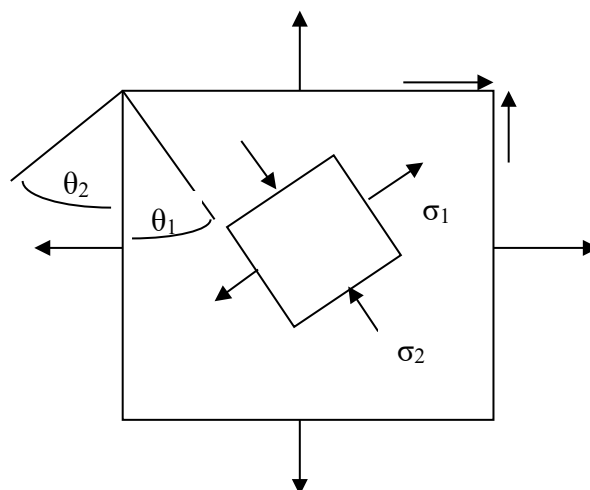
And, $\sigma_2 = -4.2 \text{ MN/m}^2$ (compressive)

Principal planes found from equation in formula sheet, where σ_p are principal stresses;

$$\theta_1 = 52.76^\circ \quad \text{and, } \theta_2 = -37.24^\circ \text{ or } 142.76^\circ$$

(6 marks)

- (d)



(3 marks)

Q4

- (a) 3D sketch showing half cylinder of length L, including internal pressure p, wall thickness t, diameter d, and hoop stress acting on both diametrically opposing walls. Also, possibly includes 'projected area'.

Total *force* on half cylinder due to pressure = p x projected area = pdL

Resisting *force* owing to hoop stress in walls = $2\sigma_H \times Lt$ (i.e. stress x area)

Therefore, $2\sigma_H \times Lt = pdL$

Thus circumferential stress is defined as; $\sigma_H = \frac{pd}{2t}$

(6 marks)

- (b) Using appropriate equation from formula sheet, where all data given in SI units;

Change in diameter, $\Delta d = 1.245 \text{ mm}$

Change in length, $\Delta L = 0.469 \text{ mm}$

(4 marks)

- (c) (i) First determine internal pressure from change in volume equation;
 $p = 5003.61 \text{ N/m}^2$

Therefore, $\sigma_H = 0.782 \text{ MN/m}^2$ and, $\sigma_L = 0.391 \text{ MN/m}^2$

(6 marks)

- (d) Hoop stress will become 1.043 MN/m^2 and longitudinal stress 0.978 MN/m^2

(4 marks)