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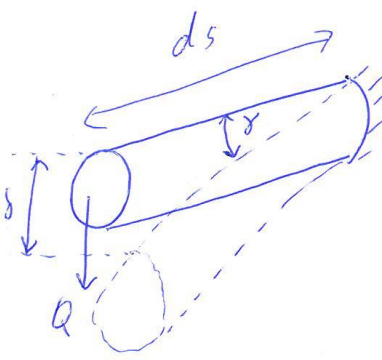
EXAMINATION SOLUTIONS: SESSION 2015/2016

School:	LOGISTICS & MANUFACTURING		
Programme:	Mechanical & Manufacturing Engineering	Year:	3
Module/Unit:	ADVANCED STRESS		
Author:	KELVIN LAKE	Page:	1 of

Question No	Question	Marks
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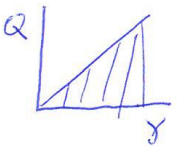
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Question No	Question	Marks
1 (a) i	 SHEAR MODULUS $G = \frac{\text{SHEAR STRESS}}{\text{SHEAR STRAIN}}$ $= \frac{\tau}{\gamma}$ $= \frac{Q}{\gamma A} \quad \left(\text{SINCE } \tau = \frac{Q}{A} \right)$ $\therefore \gamma = \frac{Q}{AG}$	(2)

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Question No	Question	Marks
1 (a) ii	 STRAIN ENERGY = $\frac{1}{2} Q y$ IN SMALL ELEMENT $= \frac{1}{2} Q \frac{Q}{AL}$ $= \frac{1}{2} \frac{Q^2}{AL}$ $\therefore$ TOTAL STRAIN ENERGY = $\int_0^L \frac{Q^2}{2AL} ds$ $\therefore U = \frac{Q^2 L}{2AL}$	① ② ①

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Question No	Question	Marks
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2a(iii)	RESILANCE = STRAIN PER UNIT VOLUME $U = \frac{P^2 L}{2AE}$ $= \frac{\sigma^2 A^2 L}{2AE}$ $= \frac{\sigma^2 AL}{2E}$	$\therefore \frac{U}{AL} = \frac{\sigma^2}{2E}$ = RESILANCE (4)
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Question No	Question	Marks
2b(i)	$\sigma = \frac{F}{A} \quad \therefore F = \sigma \times A = 300 \times 10^6 \times (15 \times 15) \times 10^{-6}$ $= \underline{\underline{67.5 \text{ kN}}}$ $A_{\text{CEA}} = 15 \times 15 \times 10^{-6} = \underline{\underline{2.25 \times 10^{-4} \text{ m}^2}}$ $U = \frac{P^2 L}{2AE} = \frac{67500^2 \times 1}{2 \times 2.25 \times 10^{-4} \times 200 \times 10^9} = \underline{\underline{50.6 \text{ J}}}$	3
2b(ii)	$P.E. = mgh$ $= mg(h + \delta L) \quad (1)$ $E = \frac{\sigma}{\epsilon} = \sigma \div \frac{\delta L}{L} = \frac{\sigma L}{\delta L} \quad \therefore \delta L = \frac{\sigma L}{E}$ $\text{from (1)} \quad P.E. = mg\left(h + \frac{\sigma L}{E}\right) \quad (2)$ TAKE ENERGY IS ASSOCIATED BY SHAFT. $U = \frac{P^2 L}{2AE} = \frac{\sigma^2 A^2 L}{2AE} = \frac{\sigma^2 AL}{2E} = \frac{\sigma^2 V}{2E} \quad (3)$ Equate (2) & (3) $\frac{\sigma^2 V}{2E} = mg\left(h + \frac{\sigma L}{E}\right)$ $\frac{\sigma^2 V}{2Emg} = h + \frac{\sigma L}{E} \quad \therefore h = \frac{\sigma^2 V}{2Emg} - \frac{\sigma L}{E}$	4

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Question No	Question	Marks
2b(iii)	$h = \frac{\sigma^2 V}{2E\pi y} - \frac{\sigma^2 L}{E}$ $= \frac{(75 \times 10^6)^2 \times (225 \times 10^{-4} \times 1)}{2 \times 200 \times 10^9 \times 200 \times 9.81} - \frac{75 \times 10^6 \times 1}{200 \times 10^9}$ $= 1.61 \times 10^{-3} - 3.75 \times 10^{-4}$ $= 1.2 \times 10^{-3}$ $= \underline{\underline{1.2 \text{ mm}}}$	(3)

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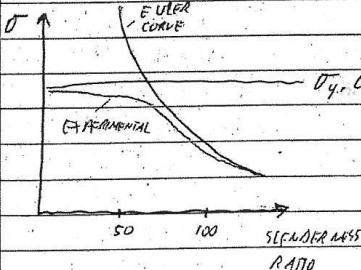
Module/Unit:	STRUCTURAL & MATERIALS	Page:	2 of
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Question No	Question	Marks
(1)(a)	COMPLETE SECTION:	
	$I_{xx} = 2 \times I_{xx} \text{ CHANNEL} + 2 \times I_{xx} \text{ PLATE}$	
	$= 2 \times 26.2 \times 10^6 + 2 \times (I_c + Ah^2)$	
	$= 2 \times 26.2 \times 10^6 + 2 \times \left(\frac{bd^3}{12} + Ah^2 \right)$	
	$= 2 \times 26.2 \times 10^6 + 2 \times \left(\frac{300 \times 10^3}{12} + 300 \times 10 \times 105^2 \right)$	
	$= 52.4 \times 10^6 + 66.2 \times 10^6$	
	$= 118.6 \times 10^6 \text{ mm}^4$	2
	$I_{zz} = 2I_{zz} \text{ CHANNEL} + 2 \times I_{zz} \text{ PLATE}$	
	$= 2(I_{yy} + Ah^2) + 2 \times I_{zz} \text{ PLATE}$	
	$= 2 \times (1.57 \times 10^6 + 4400 \times 76^2) + 2 \times \frac{10 \times 300^3}{12}$	
	$= 53.97 \times 10^6 + 45 \times 10^6$	
	$= 98.97 \times 10^6 \text{ mm}^4$	2
	IT WILL BUCKLE ABOUT ZZ	1

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Question No.	Question	Marks
(1b)	RADIUS OF GYRATION $K = \sqrt{\frac{I_{zz}}{A}}$	
	AREA = $2(4400 + 300 \times 10)$	
	$= 14800 \text{ mm}^2$	
	$K = \sqrt{\frac{98.97 \times 10^6}{14800}} = 81.8 \text{ mm}$	2
(c)	SLENDerness RATIO = $\frac{L}{K}$	
	$= \frac{8}{81.8 \times 10^{-3}}$	
	$= 97.9$	2
(d)	EULER, PINNED ENDS : $P_e = \frac{\pi^2 EI}{L^2}$	
	$= \frac{\pi^2 \times 200 \times 10^9 \times 98.97 \times 10^{-6}}{8^2}$	
	$= 3.05 \times 10^6 \text{ N}$	3

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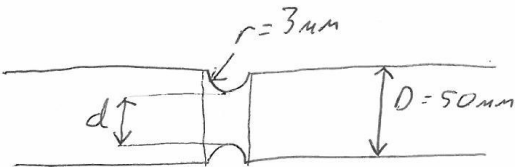
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Question No.	Question	Marks	
(1e)	RANKINE LOADS : $P_R = \frac{\sigma_y A}{1 + a \left(\frac{L}{\pi} \right)^2}$ $= \frac{320 \times 10^6 \times 0.0148}{1 + \frac{1}{7500} \times 97.9^2}$ $= \frac{4736 \times 10^3}{2.2779}$ $= 2.08 \times 10^6 \text{ N}$	3	
(f)	FOR THIS STRUT, WHICH IS APPROACHING THE VALIDITY LIMIT FOR EULER'S THEORY, THERE IS AN APPRECIABLE DIFFERENCE BETWEEN THE RESULTS GIVEN BY THE TWO THEORIES.	2	
		3	
			20

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Question No	Question	Marks
(3) (a)	ITEMS SUCH AS - GROOVES - FILLETS - THREADS - HOLES - KEYWAYS - CONCENTRATED LOADING - MATERIAL PLAYS - ETC SHOULD BE DISCUSSED	(5)
(b)	IF $\frac{A}{B} = 1$ THEN $K_t = 3$ i.e. $K_t = 1 + 2\left(\frac{A}{B}\right)$	(6)

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Question No	Question	Marks	
C 1	 $D = 50 \text{ mm}$ $d = 50 - 6 = 44 \text{ mm}$ $\frac{D}{d} = \frac{50}{44} = \underline{\underline{1.14}}$ $\frac{r}{d} = \frac{3}{44} = \underline{\underline{0.07}}$ $\sigma_{nom} = \frac{4P}{\pi d^2} = \frac{4 \times 50 \times 10^3}{\pi (44 \times 10^{-3})^2} = \underline{\underline{32.9 \text{ MPa}}}$ FROM DIAGRAM $K_t = 2.51$ $\sigma_{max} = 2.51 \times 32.9$ $= \underline{\underline{82.6 \text{ MPa}}}$	3	

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Question No	Question	Marks
(C) (11)	BENDING $\sigma_{nom} = \frac{32M}{\pi d^3} = \frac{32 \times 150}{\pi \times (4 \times 10^{-3})^3}$ $= \underline{\underline{17.9 \text{ MPa}}}$ FROM DIAGRAM $K_t = 2.24$ $\therefore \sigma_{max} = 2.24 \times 17.9 = \underline{\underline{39.9 \text{ MPa}}}$	(3)
(11)	TORSION $\tau_{nom} = \frac{16T}{\pi d^3} = \frac{16 \times 320}{\pi \times (4 \times 10^{-3})^3} = \underline{\underline{19.1 \text{ MPa}}}$ FROM DIAGRAM $K_t = 1.65$ $\therefore \sigma_{max} = 1.65 \times 19.1$ $= \underline{\underline{31.5 \text{ MPa}}}$	(3)

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Question No	Question	Marks
4 a	$\frac{\sigma}{y} = \frac{M}{I}$ $\therefore \sigma = \frac{My}{I}$ now $I_o = \frac{\pi d^4}{64}$ ~~$\sigma = \frac{My}{I_o}$~~ $\sigma = \frac{64My}{\pi d^4}$ $= \frac{64 \times 1.9 \times 10^3 \times 25 \times 10^{-3}}{\pi \times (50 \times 10^{-3})^4}$ $= \underline{\underline{155 \text{ MPa}}}$	3

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Question No	Question	Marks
(b)	$\frac{I}{J} = \frac{\tau}{R}$ $\therefore \tau = \frac{TR}{J}$ $\text{NOW } J_0 = \frac{\pi d^4}{32}$ $\therefore \tau = \frac{32TR}{\pi d^4}$ $= \frac{32 \times T \times 25 \times 10^{-3}}{\pi \times (50 \times 10^{-3})^4}$ $= \underline{\underline{40.7 \times 10^3 \text{ T}}}$	(3)

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Question No	Question	Marks
(C) (1)	MAX SHEAR STRESS THEORY $\sigma_1 - \sigma_3 = \sigma_y$ FROM COMPLEX STRESS $\sigma_1 = \frac{1}{2} [(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}]$ let $a = (\sigma_x + \sigma_y)$ $b = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$ $\therefore \sigma_1 = \frac{1}{2}(a + b)$ $= \underline{\underline{\frac{1}{2}a + \frac{1}{2}b}}$ $\sigma_3 = \frac{1}{2} [(\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}]$ $= \frac{1}{2}(a - b)$ $= \frac{1}{2}a - \frac{1}{2}b$ NOW $\sigma_1 - \sigma_3 = \frac{1}{2}a + \frac{1}{2}b - (\frac{1}{2}a - \frac{1}{2}b)$ $= b$ $= \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$	

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Question No	Question	Marks
(C) (i) CONT	$\therefore \sigma_{y_{\text{IELD}}} = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$ SINCE $\sigma_y = 0$ $\sigma_{y_{\text{IELD}}} = \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$ $\therefore \tau_{xy} = \sqrt{\frac{\sigma_{y_{\text{IELD}}}^2 - \sigma_x^2}{4}}$ $= \sqrt{\frac{(200 \times 10^6)^2 - (155 \times 10^6)^2}{4}}$ $= \underline{\underline{63.2 \times 10^6 \text{ Pa}}}$ SINCE $\tau = 40.7 \times 10^3 \text{ T}$ $63.2 \times 10^6 = 40.7 \times 10^3 \text{ T}$ $\therefore \underline{\underline{T = 1.55 \text{ kNm}}}$	(7)

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Question No	Question	Marks
(C) ii	$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2$ $\sigma_1 = \frac{1}{2}a + \frac{1}{2}b$ $\sigma_2 = 0$ $\sigma_3 = \frac{1}{2}a - \frac{1}{2}b$ $(\frac{1}{2}a + \frac{1}{2}b)^2 + (-\frac{1}{2}a + \frac{1}{2}b)^2 + (\frac{1}{2}a - \frac{1}{2}b - \frac{1}{2}a + \frac{1}{2}b)^2 = 2\sigma_y^2$ $(\frac{1}{2}a + \frac{1}{2}b)(\frac{1}{2}a + \frac{1}{2}b) + (-\frac{1}{2}a + \frac{1}{2}b)(-\frac{1}{2}a + \frac{1}{2}b) + b^2 = 2\sigma_y^2$ $\frac{1}{4}a^2 + \frac{1}{4}ab + \frac{1}{4}ab + \frac{1}{4}b^2 + \frac{1}{4}a^2 - \frac{1}{4}ab - \frac{1}{4}ab + \frac{1}{4}b^2 + b^2 = 2\sigma_y^2$ $\frac{1}{2}a^2 + \frac{3}{2}b^2 = 2\sigma_y^2$ now $a = \sigma_x + \sigma_y$ $= \sigma_x \text{ (since } \sigma_y = 0)$ $b = \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$	

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Question No	Question	Marks
(C) (ii) CONT.	now $\frac{1}{2}a^2 + \frac{3}{2}b^2 = 2\sigma_y^2$ $\frac{1}{2}\sigma_x^2 + \frac{3}{2}(\sigma_x^2 + 4\tau_{xy}^2) = 2\sigma_y^2$ $\frac{1}{2}\sigma_x^2 + \frac{3}{2}\sigma_x^2 + 6\tau_{xy}^2 = 2\sigma_y^2$ $2\sigma_x^2 + 6\tau_{xy}^2 = 2\sigma_y^2$ $\sigma_x^2 + 3\tau_{xy}^2 = \sigma_y^2$ $\sigma_x = 155 \text{ MPa}$ $\tau_{xy} = 40.7 \times 10^3 \text{ Pa}$ $= 40.7 \times 10^{-3} \text{ T MPa}$ $\therefore 155^2 + 3(40.7 \times 10^{-3})^2 = 200^2$ $155^2 + 3(40.7 \times 10^{-3})^2 T^2 = 200^2$ $(40.7 \times 10^{-3})^2 T^2 = \frac{200^2 - 155^2}{3}$ $\therefore T^2 = \frac{5325}{(40.7 \times 10^{-3})^2} \quad \therefore T = \underline{\underline{1.79 \text{ kNm}}}$	