



PRIFYSGOL CYMRU  
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Faculty of Applied Design  
and Engineering

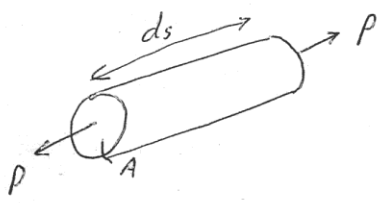
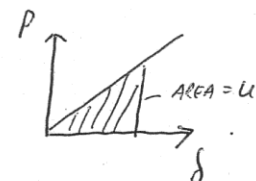
**EXAMINATION SOLUTIONS: SESSION** 2015/2016

|              |                                        |       |      |
|--------------|----------------------------------------|-------|------|
| School:      | LOGISTICS & MANUFACTURING              |       |      |
| Programme:   | Mechanical & Manufacturing Engineering | Year: | 3    |
| Module/Unit: | ADVANCED STRESS                        |       |      |
| Author:      | KELVIN LAKE                            | Page: | 1 of |

| Question No | Question | Marks |
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## Continuation Sheet

|              |                          |       |    |
|--------------|--------------------------|-------|----|
| Module/Unit: | ADVANCED STRESS ANALYSIS | Page: | of |
|--------------|--------------------------|-------|----|

| Question No | Question                                                                                                                                                                                                                                                                                                                      | Marks |
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| (1a)        |  $E = \frac{\sigma}{\epsilon}$ $= \frac{P}{A} \div \frac{\delta}{ds} = \frac{P ds}{A \delta} \quad \therefore \delta = \frac{P ds}{AE}$                                                                                                      | (2)   |
| (b)         |  <p>FOR SMALL SECTION <math>U = \frac{1}{2} P \delta</math></p> $= \frac{1}{2} P \times \frac{P ds}{AE} = \frac{P^2 ds}{2AE}$ <p><math>\therefore</math> FOR TOTAL BAR <math>U = \int_0^L \frac{P^2 ds}{2AE} = \frac{P^2 L}{2AE}</math></p> | (4)   |
| (c)         | <p>RESILIENCE = STRAIN PER UNIT VOLUME</p> $U = \frac{P^2 L}{2AE} \quad \therefore \frac{U}{AL} = \frac{\sigma^2}{2E}$ $= \frac{\sigma^2 A^2 L}{2AE}$ $= \frac{\sigma^2 AL}{2E}$ <p style="text-align: right;">= RESILIENCE</p>                                                                                               | (4)   |

# Continuation Sheet

|              |                          |       |    |
|--------------|--------------------------|-------|----|
| Module/Unit: | ADVANCED STRESS ANALYSIS | Page: | of |
|--------------|--------------------------|-------|----|

| Question No | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Marks |
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| (di)        | $\sigma = \frac{F}{A} \therefore F = \sigma \times A = 300 \times 10^6 \times (20 \times 20) \times 10^{-6}$ $= \underline{\underline{120 \text{ kN}}}$ $AREA = 20 \times 20 \times 10^{-6} = \underline{\underline{4 \times 10^{-4} \text{ m}^2}}$ $U = \frac{P^2 L}{2AE} = \frac{120,000^2 \times 4}{2 \times 4 \times 10^{-4} \times 210 \times 10^9} = \underline{\underline{342.9 \text{ J}}}$                                                                                                                                                                                                               | (3)   |
| (d)<br>ii   | <p>P.E. = mgh<br/>= mg(h + <math>\delta L</math>) ①</p> $E = \frac{\sigma}{\epsilon} = \sigma \div \frac{\delta L}{L} = \frac{\sigma L}{\delta L} \therefore \delta L = \frac{\sigma L}{E}$ <p>FROM ① P.E. = mg(h + <math>\frac{\sigma L}{E}</math>) ②</p> <p>THAT ENERGY IS ABSORBED BY SHAFT.</p> $U = \frac{P^2 L}{2AE} = \frac{\sigma^2 A^2 L}{2AE} = \frac{\sigma^2 AL}{2E} = \frac{\sigma^2 V}{2E} \text{ ③}$ <p>EQUATING ② &amp; ③</p> $\frac{\sigma^2 V}{2E} = mg(h + \frac{\sigma L}{E})$ $\frac{\sigma^2 V}{2Emg} = h + \frac{\sigma L}{E} \therefore h = \frac{\sigma^2 V}{2Emg} - \frac{\sigma L}{E}$ | (4)   |

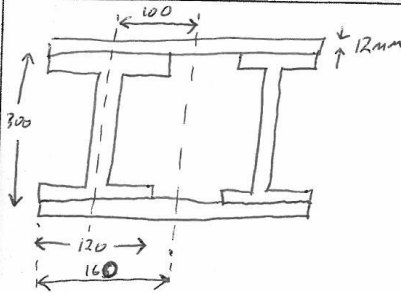
# Continuation Sheet

|              |                          |       |        |
|--------------|--------------------------|-------|--------|
| Module/Unit: | ADVANCED STRESS ANALYSIS | Page: | 7 of 9 |
|--------------|--------------------------|-------|--------|

| Question No | Question                                                                                                                                                                                                                                                                                                                                                                                                      | Marks |
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| d.<br>iii   | $h = \frac{\sigma^2 V}{2Emg} - \frac{\sigma L}{E}$ $= \frac{(75 \times 10^6)^2 \times (4 \times 10^{-4} \times 4)}{2 \times 210 \times 10^9 \times 150 \times 9.81} - \frac{75 \times 10^6 \times 4}{210 \times 10^9}$ $= \frac{9 \times 10^{12}}{6.18 \times 10^{14}} - \frac{300 \times 10^6}{210 \times 10^9}$ $= 14.56 \times 10^{-3} - 1.43 \times 10^{-3}$ $= \underline{\underline{13.13 \text{ mm}}}$ | (3)   |

## Continuation Sheet

|              |                          |       |  |
|--------------|--------------------------|-------|--|
| Module/Unit: | ADVANCED STRESS ANALYSIS | Page: |  |
|--------------|--------------------------|-------|--|

| Question No | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | Marks |
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| 2           |  <p>FOR EACH JOIST</p> $I_{xx} = 96 \times 10^{-6} \text{ m}^4$ $I_{yy} = 4.8 \times 10^{-6} \text{ m}^4$ $A = 6 \times 10^{-3} \text{ m}^2$ <p>(a) XX: JOISTS <math>I_{xx} = 2 \times 96 \times 10^{-6} = \underline{192 \times 10^{-6} \text{ m}^4}</math></p> <p>PLATES: <math>I_{xx} = \square - \square</math></p> $= \left[ \frac{320 \times 324^3}{12} - \frac{320 \times 300^3}{12} \right] \times 10^{-12}$ $= \underline{187 \times 10^{-6} \text{ m}^4}$ <p><math>I_{\text{TOTAL}} = 187 \times 10^{-6} + 192 \times 10^{-6}</math></p> $= \underline{379 \times 10^{-6} \text{ m}^4}$ |       |

## Continuation Sheet

| Module/Unit:  | ADVANCED STRESS ANALYSIS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | Page: |  |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|--|
| Question No   | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | Marks |  |
| 2 (a)<br>CONT | <p>YY NOW <math>I_G = I_{NA} + Ah^2</math></p> <p>JOISTS: <math>I_{YY} = 2 (4.2 \times 10^{-6} + 6 \times 10^{-3} \times 0.1^2)</math><br/> <math>= 128.4 \times 10^{-6} \text{ m}^4</math></p> <p>PLATES <math>I_{YY} = \frac{BD^3}{12}</math><br/> <math>= 2 \left( \frac{12 \times 320^3}{12} \right) \times 10^{-12}</math><br/> <math>= 6.55 \times 10^{-5} \text{ m}^4</math></p> <p><math>\therefore I_{YY \text{ total}} = 128.4 \times 10^{-6} + 6.55 \times 10^{-5}</math><br/> <math>= 193.9 \times 10^{-6} \text{ m}^4</math></p> <p>SO, <del>IT WILL BUCKLE ABOUT XX</del></p> <p><math>I_{XX} = 379 \times 10^{-6} &gt; I_{YY} = 193.9 \times 10^{-6}</math></p> <p><math>\therefore</math> IT WILL BUCKLE ABOUT YY.</p> | (5)   |  |

## Continuation Sheet

|              |                   |       |  |
|--------------|-------------------|-------|--|
| Module/Unit: | ADVANCED STRESSES | Page: |  |
|--------------|-------------------|-------|--|

| Question No | Question                                                                                                                                                                                                                                                                                                                          | Marks |
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| (b)         | $K = \sqrt{\frac{I}{A}}$ $\text{TOTAL AREA} = 2 \times \text{TUBES} + 2 \times \text{PLATES}$ $= 2 \times 6 \times 10^{-3} + 2 (320 \times 12) \times 10^{-6}$ $= \underline{\underline{0.01968 \text{ m}^2}}$ $K = \sqrt{\frac{I}{A}} = \sqrt{\frac{193.9 \times 10^{-6}}{0.01968}}$ $= \underline{\underline{0.1 \text{ m}^2}}$ | (2)   |
| (c)         | $\text{SLENDERNESS RATIO} = \frac{L}{K}$ $= \frac{7}{0.1}$ $= \underline{\underline{70}}$                                                                                                                                                                                                                                         | (2)   |

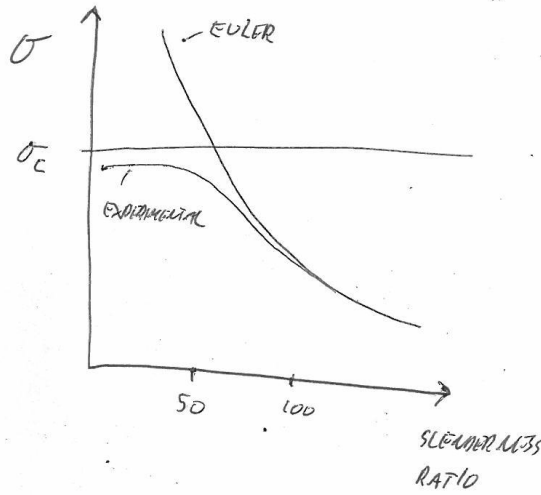
## Continuation Sheet

|              |                 |       |  |
|--------------|-----------------|-------|--|
| Module/Unit: | ADVANCED STRESS | Page: |  |
|--------------|-----------------|-------|--|

| Question No | Question                                                                                                                                                                                                         | Marks             |
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| d           | $P_e = \frac{\pi^2 EI}{L^2}$ $= \frac{\pi^2 \times 200 \times 10^9 \times 193.9 \times 10^{-6}}{7^2}$ $= \cancel{7.8 \times 10^6} = \underline{\underline{7.8 \times 10^6 \text{ N}}}$                           |                   |
| e           | $P_R = \frac{\sigma_y A}{1 + a \left( \frac{L}{r} \right)^2}$ $= \frac{320 \times 10^6 \times 0.01968}{1 + \frac{1}{7500} \left( \frac{7}{0.1} \right)^2}$ $= \underline{\underline{3.8 \times 10^6 \text{ N}}}$ | <p>3</p> <p>3</p> |

## Continuation Sheet

|              |                 |       |  |
|--------------|-----------------|-------|--|
| Module/Unit: | ADVANCED STRIPS | Page: |  |
|--------------|-----------------|-------|--|

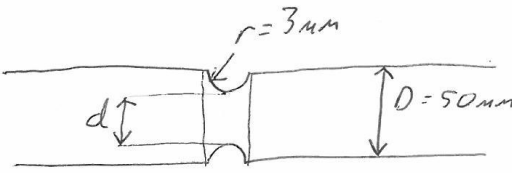
| Question No | Question                                                                                                                                                                                                                                                 | Marks |
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| (F)         | <p>FOR THIS STRUT, WHICH IS APPROACHING THE VALIDITY LIMIT FOR EULER'S THEORY, THERE IS AN APPRECIABLE DIFFERENCE BETWEEN THE RESULTS GIVEN BY THE TWO THEORIES.</p>  |       |

## Continuation Sheet

|              |                          |       |  |
|--------------|--------------------------|-------|--|
| Module/Unit: | ADVANCED STRESS ANALYSIS | Page: |  |
|--------------|--------------------------|-------|--|

| Question No | Question                                                                                                                                                                                                                                                       | Marks |
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| (3) (a)     | <p>ITEMS SUCH AS</p> <ul style="list-style-type: none"> <li>- GROOVES</li> <li>- FILLETS</li> <li>- THREADS</li> <li>- HOLES</li> <li>- KEYWAYS</li> <li>- CONCENTRATED LOADING</li> <li>- MATERIAL PLAYS</li> <li>- ETC</li> </ul> <p>SHOULD BE DISCUSSED</p> | (5)   |
| (b)         | <p>IF <math>\frac{A}{B} = 1</math> THEN <math>K_t = 3</math> i.e.</p> $K_t = 1 + 2\left(\frac{A}{B}\right)$                                                                                                                                                    | (6)   |

## Continuation Sheet

| Module/Unit:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | ADVANCED STRESS ANALYSIS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Page:                                                                                                                                                                   |  |
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| Question No                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Marks                                                                                                                                                                   |  |
| <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center; margin: 10px;"> <div style="border: 1px solid black; border-radius: 50%; width: 15px; height: 15px; display: flex; align-items: center; justify-content: center;">C</div> <div style="border: 1px solid black; border-radius: 50%; width: 15px; height: 15px; display: flex; align-items: center; justify-content: center;">1</div> </div> | <div style="text-align: center; margin-bottom: 20px;">  </div> <p> <math>D = 50 \text{ mm}</math><br/> <math>d = 50 - 6 = 44 \text{ mm}</math> </p> $\frac{D}{d} = \frac{50}{44} = \underline{\underline{1.14}}$ $\frac{r}{d} = \frac{3}{44} = \underline{\underline{0.07}}$ $\sigma_{nom} = \frac{4P}{\pi d^2} = \frac{4 \times 50 \times 10^3}{\pi (44 \times 10^{-3})^2} = \underline{\underline{32.9 \text{ MPa}}}$ <p>FROM DIAGRAM</p> $K_t = 2.51$ $\sigma_{max} = 2.51 \times 32.9$ $= \underline{\underline{82.6 \text{ MPa}}}$ | <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center; margin: 10px;">3</div> |  |

## Continuation Sheet

|              |                          |       |  |
|--------------|--------------------------|-------|--|
| Module/Unit: | ADVANCED STRESS ANALYSIS | Page: |  |
|--------------|--------------------------|-------|--|

| Question No | Question                                                                                                                                                                                                                                                                                     | Marks |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| (C)<br>(11) | <p>BENDING</p> $\sigma_{nom} = \frac{32M}{\pi d^3} = \frac{32 \times 150}{\pi \times (4 \times 10^{-3})^3}$ $= \underline{\underline{17.9 \text{ MPa}}}$ <p>FROM DIAGRAM <math>K_t = 2.24</math></p> $\therefore \sigma_{max} = 2.24 \times 17.9 = \underline{\underline{39.9 \text{ MPa}}}$ | (3)   |
| (111)       | <p>TORSION</p> $\tau_{nom} = \frac{16T}{\pi d^3} = \frac{16 \times 320}{\pi \times (4 \times 10^{-3})^3} = \underline{\underline{19.1 \text{ MPa}}}$ <p>FROM DIAGRAM <math>K_t = 1.65</math></p> $\therefore \sigma_{max} = 1.65 \times 19.1$ $= \underline{\underline{31.5 \text{ MPa}}}$   | (3)   |

## Continuation Sheet

|              |                 |       |  |
|--------------|-----------------|-------|--|
| Module/Unit: | ADVANCED STRESS | Page: |  |
|--------------|-----------------|-------|--|

| Question No                                                                                                                                                                                                                                                                                                                                   | Question                                                                                                                                                                                                                                                                                                                                                         | Marks                                                                                                                                                                     |
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| <div style="border: 1px solid black; border-radius: 50%; width: 20px; height: 20px; display: flex; align-items: center; justify-content: center; margin: 5px;">4</div> <div style="border: 1px solid black; border-radius: 50%; width: 20px; height: 20px; display: flex; align-items: center; justify-content: center; margin: 5px;">a</div> | $\frac{\sigma}{y} = \frac{M}{I}$ $\therefore \sigma = \frac{My}{I}$ <p>now <math>I_o = \frac{\pi d^4}{64}</math></p> <del><math display="block">\sigma = \frac{My}{I_o}</math></del> $\sigma = \frac{64My}{\pi d^4}$ $= \frac{64 \times 1.9 \times 10^3 \times 25 \times 10^{-3}}{\pi \times (50 \times 10^{-3})^4}$ $= \underline{\underline{155 \text{ MPa}}}$ | <div style="border: 1px solid black; border-radius: 50%; width: 20px; height: 20px; display: flex; align-items: center; justify-content: center; margin: 0 auto;">3</div> |

## Continuation Sheet

|              |                          |       |  |
|--------------|--------------------------|-------|--|
| Module/Unit: | ADVANCED STRESS ANALYSIS | Page: |  |
|--------------|--------------------------|-------|--|

| Question No | Question                                                                                                                                                                                                                                                                                 | Marks |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| (b)         | $\frac{I}{J} = \frac{\tau}{R}$ $\therefore \tau = \frac{TR}{J}$ $\text{NOW } J_0 = \frac{\pi d^4}{32}$ $\therefore \tau = \frac{32TR}{\pi d^4}$ $= \frac{32 \times T \times 25 \times 10^{-3}}{\pi \times (50 \times 10^{-3})^4}$ $= \underline{\underline{40.7 \times 10^3 \text{ T}}}$ | (3)   |

## Continuation Sheet

|              |                          |       |  |
|--------------|--------------------------|-------|--|
| Module/Unit: | ADVANCED STRESS ANALYSIS | Page: |  |
|--------------|--------------------------|-------|--|

| Question No | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Marks |
|-------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| (C)<br>(1)  | <p>MAX SHEAR STRESS THEORY</p> $\sigma_1 - \sigma_3 = \sigma_y$ <p>FROM COMPLEX STRESS</p> $\sigma_1 = \frac{1}{2} [ (\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} ]$ <p>let <math>a = (\sigma_x + \sigma_y)</math><br/> <math>b = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}</math></p> $\therefore \sigma_1 = \frac{1}{2}(a + b)$ $= \underline{\underline{\frac{1}{2}a + \frac{1}{2}b}}$<br>$\sigma_3 = \frac{1}{2} [ (\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} ]$ $= \frac{1}{2}(a - b)$ $= \frac{1}{2}a - \frac{1}{2}b$ <p>now <math>\sigma_1 - \sigma_3 = \frac{1}{2}a + \frac{1}{2}b - (\frac{1}{2}a - \frac{1}{2}b)</math></p> $= b$ $= \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$ |       |

## Continuation Sheet

|              |                          |       |  |
|--------------|--------------------------|-------|--|
| Module/Unit: | ADVANCED STRESS ANALYSIS | Page: |  |
|--------------|--------------------------|-------|--|

| Question No        | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Marks |
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| (C)<br>(i)<br>CONT | $\therefore \sigma_{y_{\text{IELD}}} = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$ <p>SINCE <math>\sigma_y = 0</math></p> $\sigma_{y_{\text{IELD}}} = \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$ $\therefore \tau_{xy} = \sqrt{\frac{\sigma_{y_{\text{IELD}}}^2 - \sigma_x^2}{4}}$ $= \sqrt{\frac{(200 \times 10^6)^2 - (155 \times 10^6)^2}{4}}$ $= \underline{\underline{63.2 \times 10^6 \text{ Pa}}}$ <p>SINCE <math>\tau = 40.7 \times 10^3 \text{ T}</math></p> $63.2 \times 10^6 = 40.7 \times 10^3 \text{ T}$ $\therefore \underline{\underline{T = 1.55 \text{ kNm}}}$ | (7)   |

## Continuation Sheet

|              |                          |       |  |
|--------------|--------------------------|-------|--|
| Module/Unit: | ADVANCED STRESS ANALYSIS | Page: |  |
|--------------|--------------------------|-------|--|

| Question No | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Marks |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| (C) ii      | $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2$ $\sigma_1 = \frac{1}{2}a + \frac{1}{2}b$ $\sigma_2 = 0$ $\sigma_3 = \frac{1}{2}a - \frac{1}{2}b$ $(\frac{1}{2}a + \frac{1}{2}b)^2 + (-\frac{1}{2}a + \frac{1}{2}b)^2 + (\frac{1}{2}a - \frac{1}{2}b - \frac{1}{2}a + \frac{1}{2}b)^2 = 2\sigma_y^2$ $(\frac{1}{2}a + \frac{1}{2}b)(\frac{1}{2}a + \frac{1}{2}b) + (-\frac{1}{2}a + \frac{1}{2}b)(-\frac{1}{2}a + \frac{1}{2}b) + b^2 = 2\sigma_y^2$ $\frac{1}{4}a^2 + \frac{1}{4}ab + \frac{1}{4}ab + \frac{1}{4}b^2 + \frac{1}{4}a^2 - \frac{1}{4}ab - \frac{1}{4}ab + \frac{1}{4}b^2 + b^2 = 2\sigma_y^2$ $\frac{1}{2}a^2 + \frac{3}{2}b^2 = 2\sigma_y^2$ <p>now</p> $a = \sigma_x + \sigma_y$ $= \sigma_x \text{ (since } \sigma_y = 0)$ $b = \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$ |       |

## Continuation Sheet

|              |                          |       |  |
|--------------|--------------------------|-------|--|
| Module/Unit: | ADVANCED STRESS ANALYSIS | Page: |  |
|--------------|--------------------------|-------|--|

| Question No       | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Marks |
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| (C) (ii)<br>CONT. | <p>now <math>\frac{1}{2}a^2 + \frac{3}{2}b^2 = 2\sigma_y^2</math></p> $\frac{1}{2}\sigma_x^2 + \frac{3}{2}(\sigma_x^2 + 4\tau_{xy}^2) = 2\sigma_y^2$ $\frac{1}{2}\sigma_x^2 + \frac{3}{2}\sigma_x^2 + 6\tau_{xy}^2 = 2\sigma_y^2$ $2\sigma_x^2 + 6\tau_{xy}^2 = 2\sigma_y^2$ $\sigma_x^2 + 3\tau_{xy}^2 = \sigma_y^2$<br>$\sigma_x = 155 \text{ MPa}$ $\tau_{xy} = 40.7 \times 10^3 \text{ Pa}$ $= 40.7 \times 10^{-3} \text{ T MPa}$<br>$\therefore 155^2 + 3(40.7 \times 10^{-3})^2 = 200^2$ $155^2 + 3(40.7 \times 10^{-3})^2 T^2 = 200^2$ $(40.7 \times 10^{-3})^2 T^2 = \frac{200^2 - 155^2}{3}$<br>$\therefore T^2 = \frac{5325}{(40.7 \times 10^{-3})^2} \quad \therefore T = \underline{\underline{1.79 \text{ kNm}}}$ |       |