

1) Calculate the directivity of an antenna that radiates a total power of 100W with a maximum radiation intensity of 80 W/sr. Express the results in both linear scale and decibels.

Soln.

$$P_{\text{rad}} = 100 \text{ W}$$

$$U_{\text{max}} = 80 \text{ W/sr.}$$

$$D = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}}$$

$$D = \frac{4\pi \times 80}{100}$$

$$D = 10.5$$

$$D_{\text{dB}} = 10 \log_{10}(D)$$
$$= 10 \log_{10}(10.5)$$

$$D_{\text{dB}} = 10.21 \text{ dB}$$

2) An antenna radiates energy at a distance of 10m from the antenna. The radiated power density in a specific direction is measured to be 0.05 W/m^2 . Calculate the radiation intensity in that direction.

Soln. $r = 10 \text{ m}$

$W_{\text{rad}} = 0.05 \text{ W/m}^2$

$$U = r^2 W_{\text{rad}} \\ = (10)^2 \times 0.05 = 5 \text{ W/sr}$$

3) An antenna radiates EM waves uniformly in all direction. At a distance of 8m, the measured power density is found to be 0.02 W/m^2 . Calculate the radiated power.

Soln. $r = 8 \text{ m}$

$W_{\text{rad}} = 0.02 \text{ W/m}^2$

$P_{\text{rad}} = 4\pi U_0$

$U = r^2 W_{\text{rad}}$

$$P_{\text{rad}} = 4\pi r^2 h_{\text{rad}}$$

$$= 4\pi (8)^2 \cdot 0.02$$

$$P_{\text{rad}} = 16.08 \text{ W}$$

4) The radiation intensity of an antenna is represented by

$$u(\theta) = \cos^2 \theta \cdot \cos^2(3\theta), \quad (0^\circ \leq \theta \leq 90^\circ, 0^\circ \leq \phi \leq 360^\circ)$$

$$(0 \leq \theta \leq \frac{\pi}{2}, 0 \leq \phi \leq 2\pi)$$

Calculate the half power beamwidth and first null beamwidth (both in radians and degree)

Soln

$$u(\theta) = \cos^2 \theta \cdot \cos^2(3\theta)$$

i) To find HPBW

$$u(\theta) = \cos^2 \theta \cos^2(3\theta) = 0.5$$

$$\cos^2 \theta \cdot \cos^2(3\theta) = 0.5$$

Taking square root on both the side.

$$\cos \theta \cdot \cos(3\theta) = \sqrt{0.5}$$

$$\cos \theta \cdot \cos 3\theta = 0.707$$

$$\cos(14.325) \cdot \cos(3 \times 14.325) = 0.707$$

$$\theta = 14.325^\circ$$

$$[0 \leq \theta \leq 90^\circ]$$

$$[\cos 10^\circ \cos(3 \times 10^\circ) = 0.85]$$

$$\cos 13^\circ \cos(3 \times 13^\circ) = 0.759$$

$$\cos 14^\circ \cos(3 \times 14^\circ) = 0.72$$

$$\cos 14.5^\circ \cos(3 \times 14.5^\circ) = 0.702$$

$$\cos(14.325) \cdot \cos(3 \times 14.325) = 0.707$$

in radians.

$$= \text{degree} \times \frac{\pi}{180^\circ}$$

$$= 14.325 \times \frac{\pi}{180^\circ}$$

$$= 0.25 \text{ rad.}$$

To find HPBW

$$\text{HPBW} = 2Q = 2 \times 14.325 = 28.65^\circ$$

$$\text{HPBW} = 2Q = 2 \times 0.25 = 0.5 \text{ rad}$$

(ii) FNBW

$$U(\theta) = \cos^2 \theta \cdot \cos^2(3\theta)$$

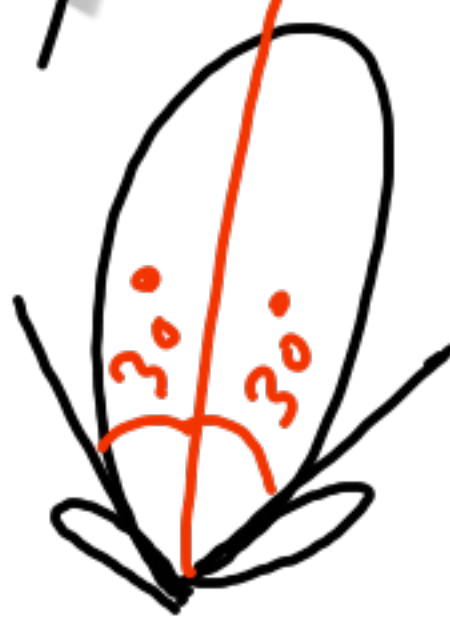
$$\cos^2 \theta \cdot \cos^2(3\theta) = 0$$

taking square root.

$$\cos \theta \cdot \cos(3\theta) = 0$$

$$\cos \theta = 0 \Rightarrow \theta = \cos^{-1}(0) = 90^\circ$$

$$\cos(3\theta) = 0 \Rightarrow \theta = \frac{1}{3} \cos^{-1}(0) = 30^\circ$$



$$\text{FNBW} = 20$$

$$= 2 \times 30^\circ = 60^\circ$$

$$\text{FNBW}(\text{rad}) = 60 \times \frac{\pi}{180}$$

$$\text{FNBW}(\text{rad}) = 1.0 \text{ rad}$$

5) An antenna has a radiation power density of $2 \sin^2 \theta$ W/m² in the far field. The radiation is over a spherical surface with a radius of 5m. Calculate the total radiated

Power.

Soln - $W_{\text{rad}} = 2 \sin^2 \theta$ W/m²

$$r = 5 \text{ m}$$

$$P_{\text{rad}} = ?$$

$$P_{\text{rad}} = \oint_S W_{\text{rad}} ds.$$

$$ds = r^2 \sin \theta d\theta d\phi$$

$$(0 \leq \theta \leq \pi), (0 \leq \phi \leq 2\pi)$$

$$P_{\text{rad}} = \oint_S \underline{W_{\text{rad}}} \cdot r^2 \sin \theta d\theta d\phi$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi} 2 \sin^2 \theta \cdot r^2 \sin \theta d\theta d\phi$$

$$\begin{aligned}
 P_{\text{rad}} &= 2r^2 \int_0^{2\pi} d\phi \int_0^\pi \sin^2\theta \sin\theta \cdot d\theta \\
 &= 2r^2 \left[\phi \right]_0^{2\pi} \int_0^\pi \sin^3\theta d\theta \\
 &= 2r^2 \left[2\pi - 0 \right] \int_0^\pi \sin^3\theta d\theta \\
 &= 4\pi r^2 \int_0^\pi \sin^3\theta d\theta
 \end{aligned}$$

$$\left[\int dx = x \right]$$

$$\left[\sin 3\theta = 3\sin\theta - 4\sin^3\theta \right]$$

$$4\sin^3\theta = 3\sin\theta - \sin 3\theta$$

$$\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$$

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$$= 4\pi r^2 \int_0^{\pi} \left[\frac{3\sin\theta - \sin 3\theta}{4} \right] d\theta$$

$$= \frac{4\pi r^2}{4} \int_0^{\pi} [3\sin\theta - \sin 3\theta] d\theta$$

$$= \pi r^2 \left[3 \int_0^{\pi} \sin\theta d\theta - \int_0^{\pi} \sin 3\theta d\theta \right]$$

$$= \pi r^2 \left[3 \left[-\cos\theta \right]_0^{\pi} - \left[-\frac{1}{3}\cos 3\theta \right]_0^{\pi} \right]$$

$$= \pi r^2 \left[3[-\cos\pi + \cos 0] + \frac{1}{3}[\cos(3\pi) - \cos 0] \right]$$

$$= \pi r^2 \left[3[-(-1) + 1] + \frac{1}{3}[(-1) - 1] \right]$$

$$= \pi r^2 \left[3 \times 2 - \frac{2}{3} \right]$$

$$= \pi r^2 \left[6 - \frac{2}{3} \right]$$

$$\int \sin\theta = -\cos\theta$$

$$\int \sin 3\theta = -\frac{1}{3}\cos 3\theta$$

we know that $r = 5\text{ m}$

$$= \pi (5)^2 \left[6 - \frac{2}{3} \right]$$

$$P_{\text{rad}} = 418.87 \text{ W}$$

6) The antenna has a radiation intensity $u = 10 \cos^2 \theta$ W/Sr from $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$. Calculate the total power radiated by the antenna.

Soln. $u = 10 \cos^2 \theta$ W/Sr

$$P_{\text{rad}} = \oint_{\Omega} u \cdot d\Omega = \int_0^{2\pi} \int_0^{\pi} u \sin \theta \cdot d\theta d\phi$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi} 10 \cdot \cos^2 \theta \cdot \sin \theta \cdot d\theta \cdot d\phi$$

$$[d\Omega = \sin \theta d\theta d\phi]$$

$$\begin{aligned} P_{\text{rad}} &= 10 \int_0^{2\pi} d\phi \int_0^{\pi} \cos^2 \theta \cdot \sin \theta d\theta \\ &= 10 \left[\phi \right]_0^{2\pi} \int_0^{\pi} \cos^2 \theta \cdot \sin \theta d\theta \\ &= 10 [2\pi] \int_0^{\pi} \cos^2 \theta \cdot \sin \theta d\theta \end{aligned}$$

$$P_{\text{rad}} = 20\pi \int_0^{\pi} \cos^2 \theta \sin \theta d\theta \quad \text{--- ①.}$$

Solve using Substitution method

$$u = \cos \theta, \quad du = -\sin \theta d\theta$$

$$\text{When, } \theta = 0, \quad u = \cos 0 = 1$$

$$\theta = \pi, \quad u = \cos \pi = -1$$

$$\text{eq ①} \Rightarrow P_{\text{rad}} = 20\pi \int_1^{-1} u^2 \cdot (-du)$$

$$P_{\text{rad}} = -20\pi \int_1^{-1} u^2 \cdot du$$

$$P_{\text{rad}} = -20\pi \left[\frac{u^3}{3} \right]_1^{-1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$P_{\text{rad}} = -20\pi \left[\frac{(-1)^3}{3} - \frac{1}{3} \right]$$

$$= -20\pi \left[\frac{-1 - 1}{3} \right]$$

$$= \frac{-20\pi}{3} [-2]$$

$$P_{\text{rad}} = \frac{40\pi}{3} = 41.9 \text{ W}$$

$$(28) \quad P_{\text{rad}} = 20\pi \int_0^{\pi} \cos^2 \theta \sin \theta \, d\theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\left[\cos^2 \theta = 1 - \sin^2 \theta \right]$$

$$P_{\text{rad}} = 20\pi \int_0^{\pi} (1 - \sin^2 \theta) \sin \theta \, d\theta$$

$$P_{\text{rad}} = 20\pi \int_0^{\pi} (\sin \theta - \sin^3 \theta) \, d\theta$$

$$P_{\text{rad}} = 20\pi \left[\int_0^{\pi} \sin \theta \, d\theta - \int_0^{\pi} \sin^3 \theta \, d\theta \right]$$

$$P_{\text{rad}} = 20\pi \left[\left[-\cos \theta \right]_0^{\pi} - \int_0^{\pi} \left[\frac{3\sin \theta - \sin 3\theta}{4} \right] d\theta \right]$$

$$P_{\text{rad}} = 20\pi \left[-\cos \pi + \cos 0 \right] - \left[\frac{4}{3} \right]$$

$$P_{\text{rad}} = 20\pi \left[-(-1) + 1 \right] - \frac{4}{3}$$

$$= 20\pi \left[2 - \frac{4}{3} \right]$$

$$P_{\text{rad}} = 41.9 \, \text{W}$$

↓ Refer previous question

← Refer to questions

