

3.5.iv: Resources for addition, subtraction, multiplication and division

This annex contains four parts dealing with teaching mental and written methods of addition, subtraction, multiplication and division. The parts can be used as follows.

- As the starting point for a discussion within a numeracy working group or whole-school staff meeting when consideration is being given to improving consistency of mental and written methods for the four operations within the school.
- As the basis for development of the section on the four operations in a school numeracy booklet. The booklet could be made available in hard copy format or placed on the school internet or both. Decisions would need to be taken as to the most accessible and effective format for learners, teachers, teaching assistants and parents/carers. An alternative approach to a numeracy booklet would be to include age-appropriate extracts in learners' homework diaries for reference.
- To design posters. Displaying posters of consistent design and consistent content, where appropriate, throughout a school can help learners make connections between numeracy taught in different year groups or different subjects.
- As exemplar material for a series of INSET sessions for all staff. To ensure that the sessions were interactive, discussion points and practical exercises would need to be added.
- As a basis for annotating examples of learners' work, linking them to the age-related expectation statement.

Portfolios of these annotated examples of learners' work can be used to demonstrate good practice and to help ensure consistency in age-related expectations across the school.

Addition

This annex includes:

1. a brief overview of the most common mental calculation strategies for addition;
2. a step-by-step overview of a route for the development of addition methods from simple mental calculations involving pictures, number lines and practical resources to the written method of addition involving 'carrying'.

This route provides appropriate strategies for a range of addition sums from two one-digit numbers to the addition of a list of numbers with any number of digits as well as numbers with up to two decimal places.

Common mental calculation strategies

Mental recall of number bonds

$$6 + 4 = 10 \qquad \square + 3 = 10$$

$$25 + 75 = 100 \qquad 19 + \square = 20$$

Use near-doubles

$$6 + 7 = \text{double } 6 + 1 = 13$$

Add using partitioning and recombining

$$34 + 45 = (30 + 40) + (4 + 5) = 79$$

Count on or back in repeated steps of 1, 10, 100, 1000

$$86 + 57 = 143 \text{ (by counting on in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Add the nearest multiple of 10, 100 and 1000 and adjust

$$24 + 19 = 24 + 20 - 1 = 43$$

$$458 + 71 = 458 + 70 + 1 = 529$$

Use the relationship between addition and subtraction

$$36 + 19 = 55 \qquad 19 + 36 = 55$$

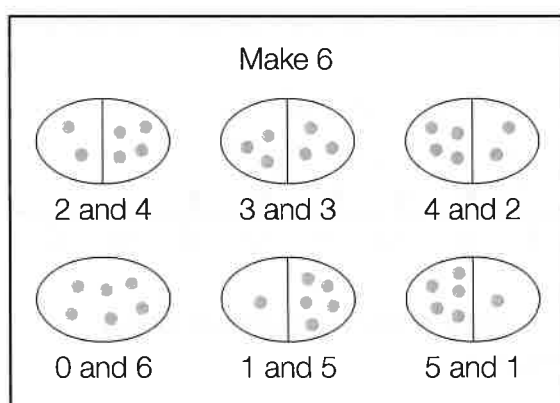
$$55 - 19 = 36 \qquad 55 - 36 = 19$$



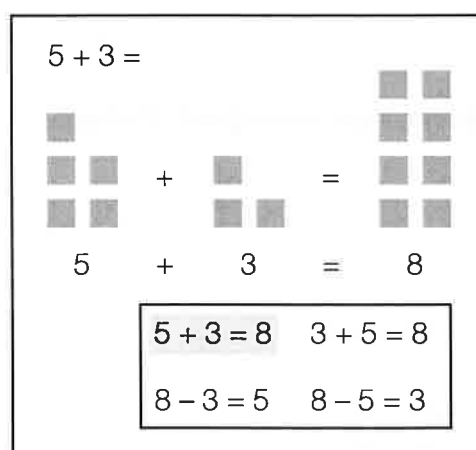
Many mental calculation strategies will continue to be used. They are not replaced by written methods.

Step 1

Learners are encouraged to develop a mental picture of the number system in their heads to use for calculation.

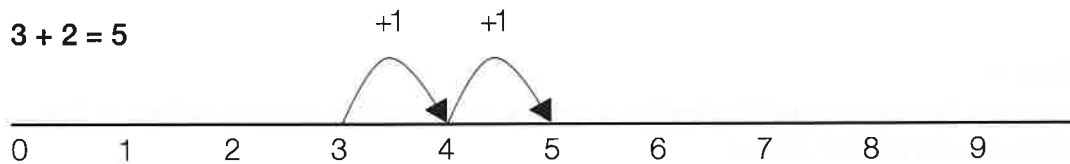


They develop ways of recording calculations using pictures, etc.



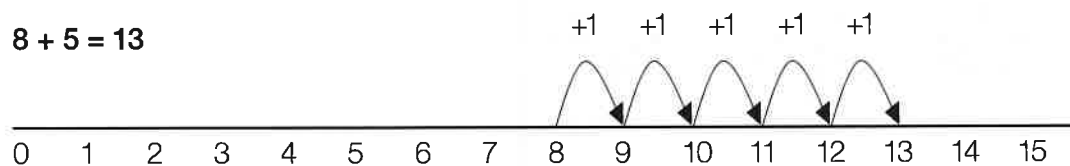
They use number lines and practical resources to support calculation and teachers **demonstrate** the use of the numberline.

$$3 + 2 = 5$$



Learners then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

$$8 + 5 = 13$$



Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

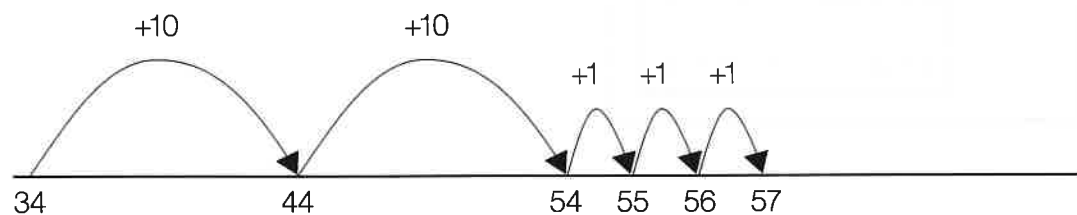


Step 2

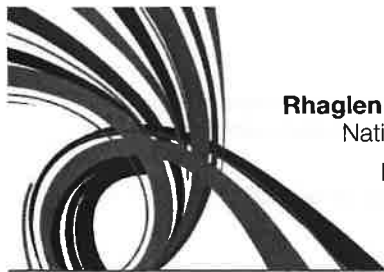
Learners will begin to use 'empty number lines' themselves, starting with the larger number and counting on.

- First counting on in tens and ones.

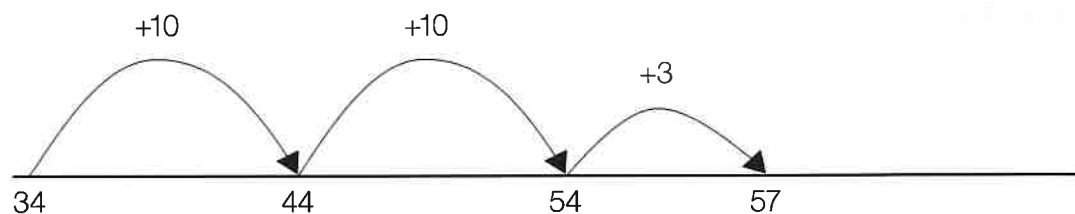
$$34 + 23 = 57$$



- Then helping learners to become more efficient by adding the units in one jump (by using the known fact $4 + 3 = 7$).

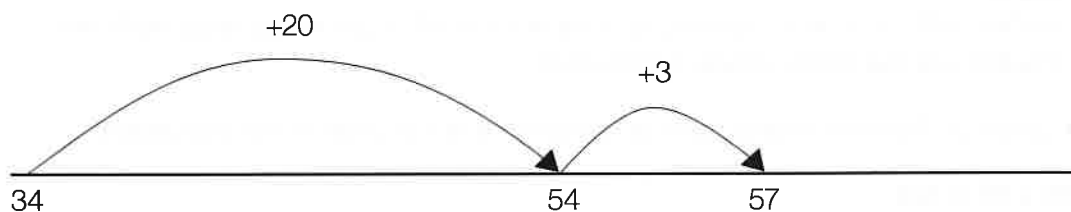


$$34 + 23 = 57$$



- Followed by adding the tens in one jump and the units in one jump.

$$34 + 23 = 57$$



4

$28 + 35 =$ 2 8 ▶ $+$ 3 5 ▶

$=$ 2 8 ▶ $+$ 3 0 ▶ $+$ 5 ▶

$+$ 3 0 ▶

$+$ 5 ▶

Number line diagram for $28 + 35 = 63$. The number line starts at 28 and has two jumps: a jump of +30 to 58, and a jump of +5 to 63.

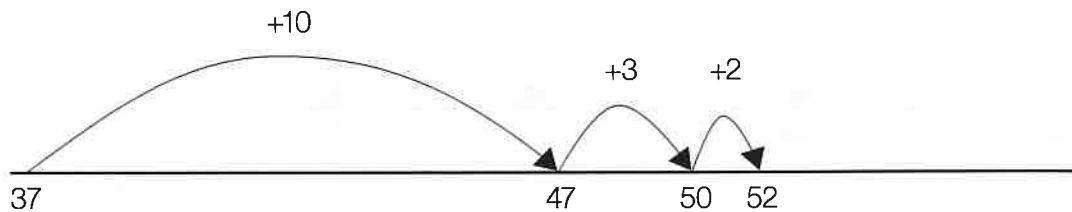
$28 + 35 = 63$

$28 + 35 = 63$
 $63 - 35 = 28$

$35 + 28 = 63$
 $63 - 28 = 35$

- Bridging through ten can help learners become more efficient.

$$37 + 15 = 52$$

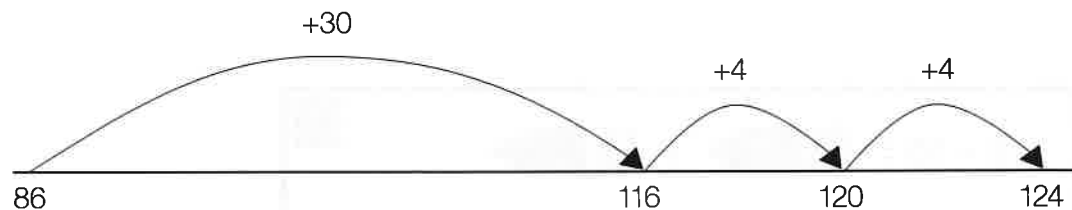


Step 3

Learners will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

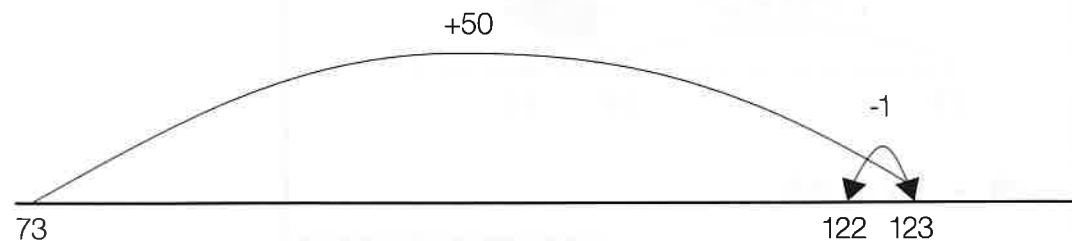
- Count on from the largest number irrespective of the order of the calculation.

$$38 + 86 = 124$$

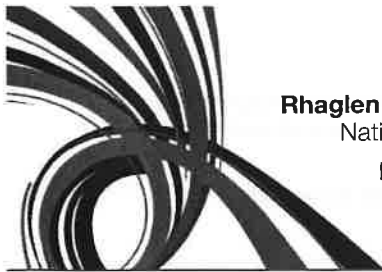


- Compensation

$$49 + 73 = 122$$



Learners will begin to use informal pencil and paper methods (jottings) to support record and explain partial mental methods building on existing mental strategies.



Phase 1: Adding most significant digits first, and then moving to adding least significant digits.

6

28 + 35

	2	8
+	3	5
	5	0
	1	3
	6	3

2

0

+

8

3

0

+

5

2

0

+

3

0

=

5

0

8

+

5

=

1

3

5

0

+

1

0

+

3

$28 + 35 = 63$	$35 + 28 = 63$
$63 - 35 = 28$	$63 - 28 = 35$

Phase 2: Moving to adding the least significant digits first in preparation for 'carrying'.

7

28 + 35

	2	8
+	3	5
	1	3
	5	0
	6	3

2

0

+

8

3

0

+

5

8

+

5

=

1

3

2

0

+

3

0

=

5

0

5

0

+

1

0

+

3

$28 + 35 = 63$	$35 + 28 = 63$
$63 - 35 = 28$	$63 - 28 = 35$

Adding the least significant digits first, without using place value cards.

	2	8	
	3	5	
+	1	3	(8 + 5)
	5	0	(20 + 30)
	6	3	

Step 4

From this, learners will begin to 'carry' below the line, using the correct mathematical vocabulary.

28 + 35

20 plus 30 is 50
50 add the
'carried' 10 is 60

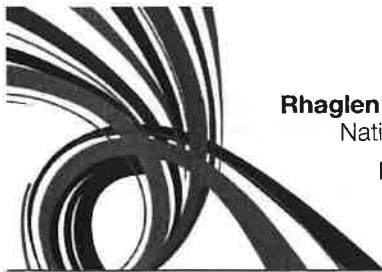
2	8
+	3
	5
	6
	3
	1

8 plus 5 is 13
(10 + 3)

3 from the 13

'carried' 10 from
the 13

$28 + 35 = 63$	$35 + 28 = 63$
$63 - 35 = 28$	$63 - 28 = 35$



239 + 53 8b

	2	3	8
+		5	3
		1	1
		8	0
	2	0	0
	2	9	1

200 + **30** + **8**

50 + **3**

8 + **3** = **11**

30 + **50** = **80**

200 = **200**

$239 + 53 = 291$ $239 + 53 = 291$
 $291 - 238 = 53$ $291 - 53 = 238$

365 + 88 8c

	3	6	5
+		8	8
		1	3
	1	4	0
	3	0	0
	4	5	3

300 + **60** + **5**

80 + **8**

5 + **8** = **13**

60 + **80** = **140**

300 = **300**

$365 + 88 = 453$ $88 + 365 = 453$
 $453 - 365 = 88$ $453 - 88 = 365$

368 + 494

	3	6	8
+	4	9	4
		1	2
	1	5	0
	7	0	0
	8	6	2

300 + 400 = 700
60 + 90 = 150
8 + 4 = 12
150 + 10 = 160
160 + 700 = 860
860 + 2 = 862

368 + 494 = 862 494 + 368 = 862
862 - 494 = 368 862 - 368 = 494

368 + 494

3a 60 plus 90 is 150
150 add the 'carried' 10 is 160

1 8 plus 4 is 12
(10 + 2)

2a 2 from the 12

3b 'carried' 100 from the 160

4 300 plus 400 is 700
700 add the 'carried' 100 is 800

2b 'carried' 10 from the 12

	3	6	8
+	4	9	4
		1	2
	1	5	0
	7	0	0
	8	6	2

368 + 494 = 862 494 + 368 = 862
862 - 494 = 368 862 - 368 = 494

Using similar methods, learners will:

- add several numbers with different numbers of digits;
- begin to add two or more three-digit sums of money, with or without adjustment



from the pence to the pounds;

- know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.

Step 5

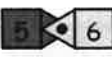
Learners should extend the 'carrying' method to numbers with at least four digits.

$$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ \hline 111 \end{array}$$

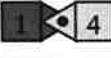
Learners should extend their methods to decimal numbers.

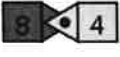
12

$5.6 + 2.8 =$  $+$ 

$=$  $+$  $+$  $+$ 

↓ ↓ ↓ ↓

$=$  $+$ 

$=$ 

$5.6 + 2.8 = 8.4$	$2.8 + 5.6 = 8.4$
$8.4 - 2.8 = 5.6$	$8.4 - 5.6 = 2.8$

5.6 + 2.8

13

	5	.	6
+	2	.	8
	1	.	4
	7	.	0
	8	.	4

5 + 2 = 7
6 + 8 = 14
14 = 10 + 4
7 + 10 = 17
17 + 0 = 17
17 + 0 = 17

5.6 + 2.8 = 8.4
2.8 + 5.6 = 8.4
8.4 - 2.8 = 5.6
8.4 - 5.6 = 2.8

5.6 + 2.8

14

5 plus 2 is 7.
7 plus the 'carried' 1 is 8

0.6 plus 0.8 is 1.4

0.4 from the 1.4

'carried' 1 from the 1.4

	5	.	6
+	2	.	8
	8	.	4
	1		

5.6 + 2.8 = 8.4
2.8 + 5.6 = 8.4
8.4 - 2.8 = 5.6
8.4 - 5.6 = 2.8

Using similar methods, learners will:

- add several numbers with different numbers of digits;
- begin to add two or more decimal fractions with up to three digits and the same number of decimal places;



- *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m – 280 cm.*

Step 6

Learners should extend the 'carrying' method to numbers with any number of digits.

$\begin{array}{r} 7648 \\ + 1486 \\ \hline 9134 \\ \hline 111 \end{array}$	$\begin{array}{r} 6584 \\ + 5848 \\ \hline 12432 \\ \hline 111 \end{array}$	$\begin{array}{r} 42 \\ 6432 \\ 786 \\ 3 \\ + 4681 \\ \hline 11944 \\ \hline 121 \end{array}$
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Using similar methods, learners will

- *add several numbers with different numbers of digits;*
- *begin to add two or more decimal fractions with up to four digits and either one or two decimal places;*
- *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 401.2 + 26.85 + 0.71.*

Learners need to have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Learners should not be made to go on to the next stage if:

1. they are not ready;
2. they are not confident.

Learners should be encouraged to approximate their answers before calculating.

Learners should be encouraged to check their answers after calculation, using an appropriate strategy.

Learners should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Subtraction

This annex includes:

1. a brief overview of the most common mental calculation strategies for subtraction;
2. a step-by-step overview of a route for the development of subtraction methods from simple mental calculations involving pictures, number lines and practical resources to a written method using decomposition.

This route provides appropriate strategies for a range of subtraction sums from two one-digit numbers to the subtraction of two numbers with any number of digits, as well as numbers with up to two decimal places.

Mental calculations

Mental recall of addition and subtraction facts

$$10 - 6 = 4 \qquad 17 - \square = 11$$

$$20 - 17 = 3 \qquad 10 - \square = 2$$

Find a small difference by counting up

$$82 - 79 = 3$$

Count on or back in repeated steps of 1, 10, 100, 1000

$$86 - 52 = 34 \text{ (by counting back in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Subtract the nearest multiple of 10, 100 and 1000 and adjust

$$24 - 19 = 24 - 20 + 1 = 5$$

$$458 - 71 = 458 - 70 - 1 = 387$$

Use the relationship between addition and subtraction

$$36 + 19 = 55 \qquad 19 + 36 = 55$$

$$55 - 19 = 36 \qquad 55 - 36 = 19$$

Many mental calculation strategies will continue to be used. They are not replaced by written methods.



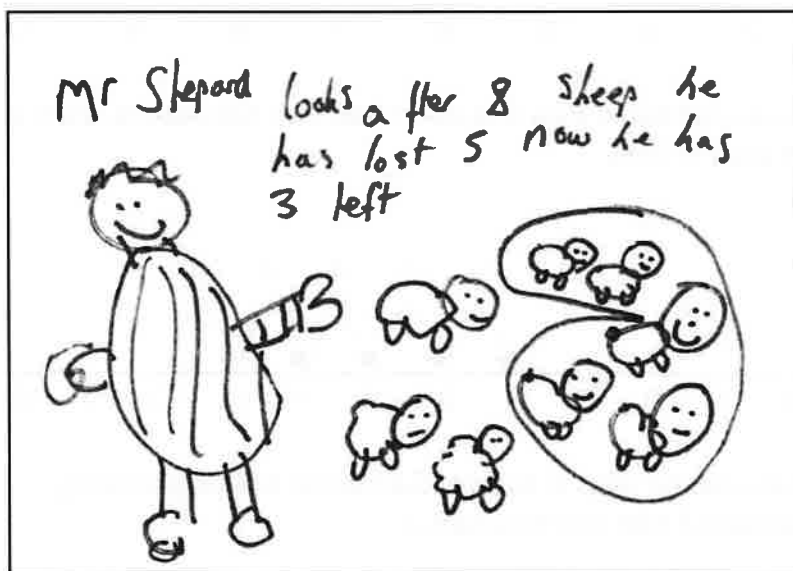
Step 1

Learners are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.

$8 - 3 =$

$8 - 3 = 5$

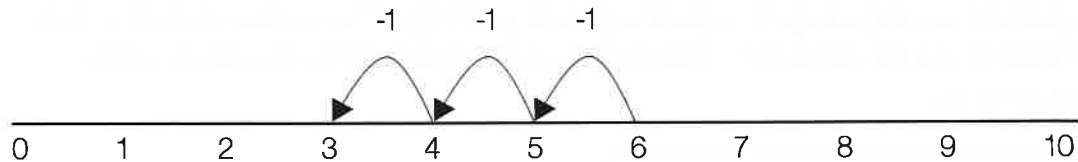
$8 - 3 = 5$	$8 - 5 = 3$
$5 + 3 = 8$	$3 + 5 = 8$



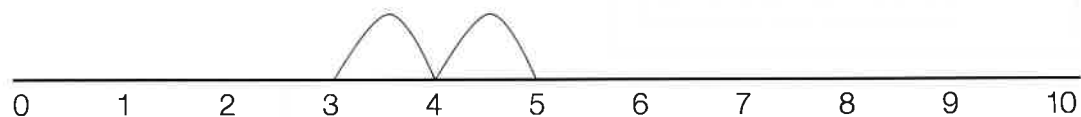
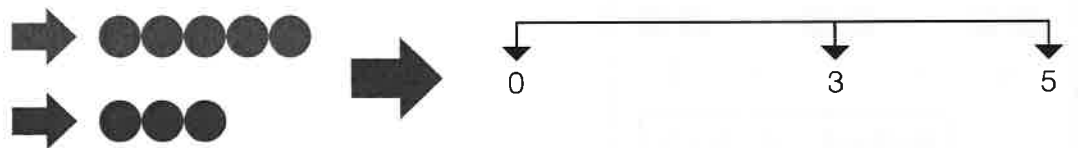
They use number lines and practical resources to support calculation.

Teachers **demonstrate** the use of the number line.

$$6 - 3 = 3$$

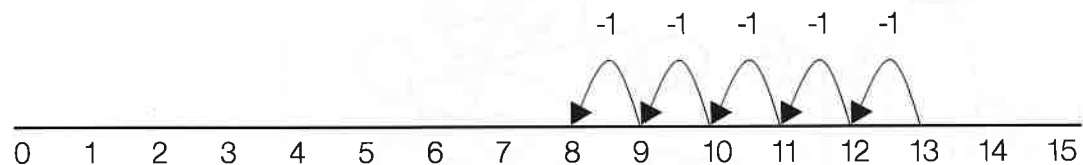


The number line should also be used to show that $5 - 3$ means the 'difference between 5 and 3' or 'the difference between 3 and 5' and how many jumps they are apart.



Learners then begin to use numbered lines to support their own calculations, using a numbered line to count back in ones.

$$13 - 5 = 8$$



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

$$13 - 5 = 8$$





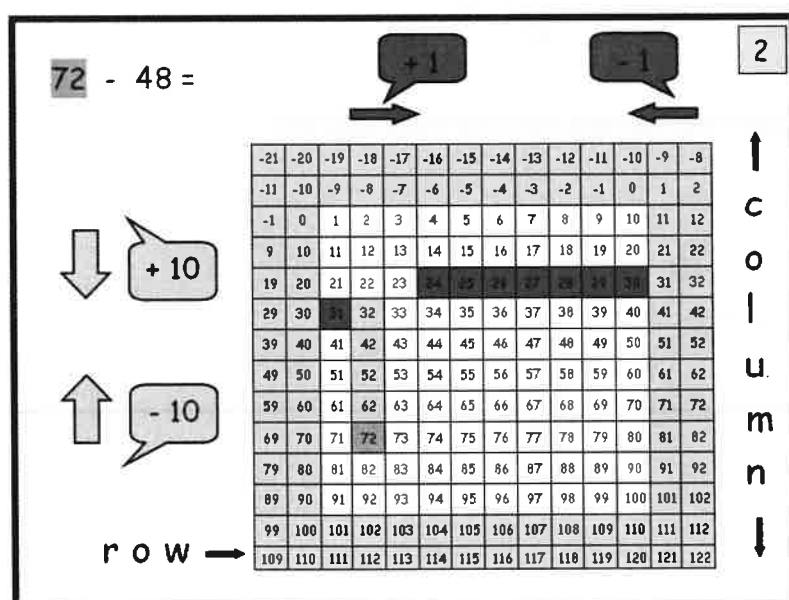
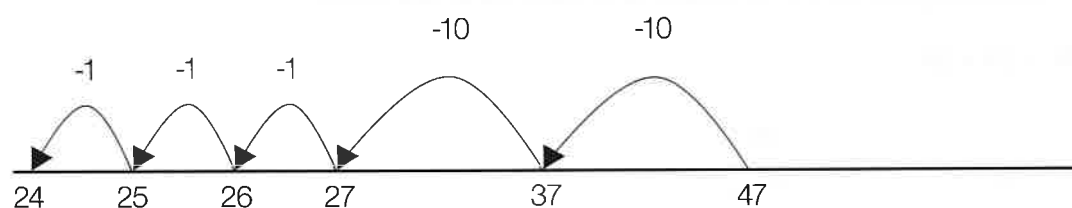
Step 2

Learners will begin to use extended hundred squares and empty number lines to support calculations.

Counting back

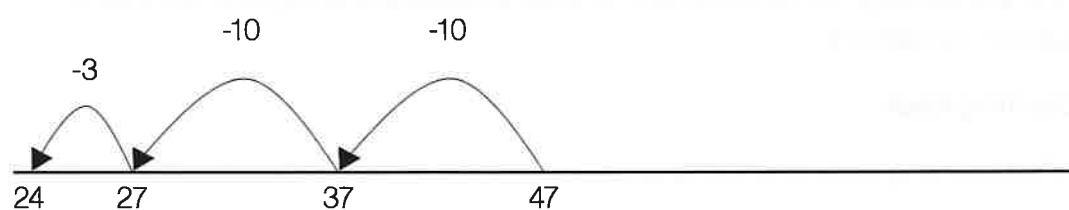
- First counting back in tens and ones.

$$47 - 23 = 24$$



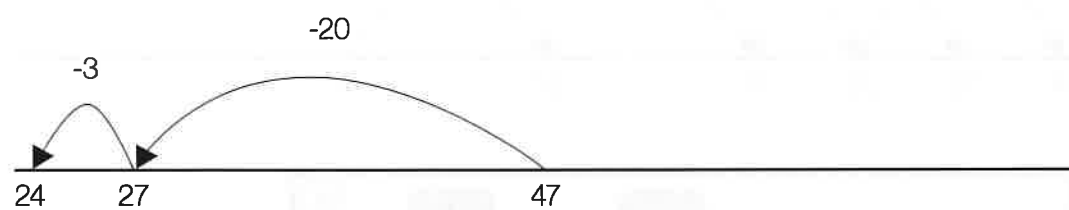
- Then helping learners to become more efficient by subtracting the units in one jump (by using the known fact $7 - 3 = 4$).

$$47 - 23 = 24$$



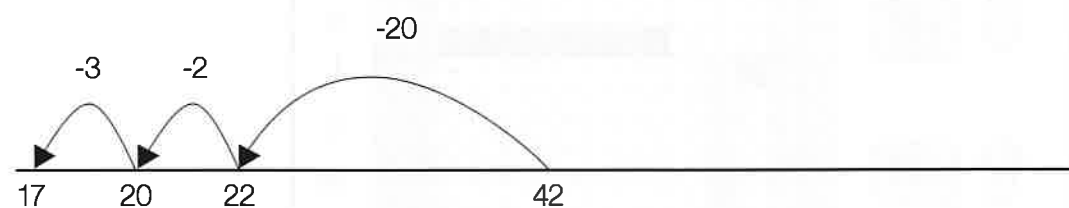
- Subtracting the tens in one jump and the units in one jump.

$$47 - 23 = 24$$



- Bridging through ten can help learners become more efficient.

$$42 - 25 = 17$$





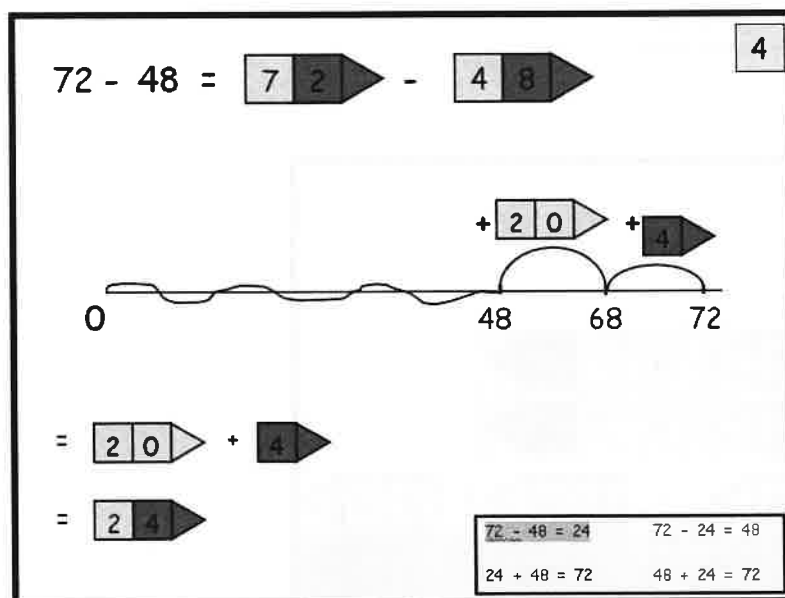
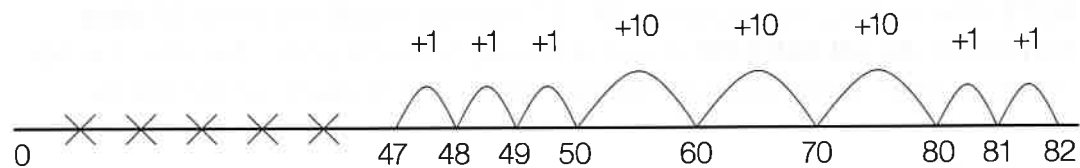
Counting on

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

The number line should still show 0 so learners can cross out the section from 0 to the smallest number. They then associate this method with 'taking away'.

82 – 47



Help learners to become more efficient with counting on by:

- subtracting the units in one jump;
- subtracting the tens in one jump and the units in one jump;
- bridging through ten.

Step 3

Learners will continue to use empty number lines with increasingly large numbers.

Learners will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods, building on existing mental strategies.

Partitioning and decomposition

This process should be demonstrated using arrow cards to show the partitioning and base 10 materials to show the decomposition of the number.

NOTE When solving the calculation $89 - 57$, learners should know that 57 **does NOT EXIST AS AN AMOUNT**, it is what you are subtracting from the other number. Therefore, when using resources, learners would need to count out only the 89.

$$\begin{array}{r} 89 \\ - 57 \\ \hline \end{array} \quad = \quad \begin{array}{r} 80 + 9 \\ 50 + 7 \\ \hline 30 + 2 = 32 \end{array}$$

89 - 57

8	9
-	5 7
3	0
	2
3	2

8 0

+

9

5 0

+

7

8 0

-

5 0

=

3 0

9

-

7

=

2

$89 - 57 = 32$
$32 + 57 = 89$

$89 - 32 = 57$
$57 + 32 = 89$

Initially, the learners will be taught using examples that do not need the learners to exchange.



From this the learners will begin to exchange.

$$\begin{array}{r} 72 \\ - 48 \\ \hline \end{array}$$

(The calculation should be read as, e.g. take 8 from 2.)

Stage 1

$$\begin{array}{r} 70 + 2 \\ - 40 + 8 \\ \hline \end{array}$$

Stage 2

$$\begin{array}{r} 60 + 12 \\ - 40 + 8 \\ \hline 20 + 4 = 24 \end{array}$$

This would be recorded by the learners as:

$$\begin{array}{r} 60 \quad 1 \\ 70 + 2 \\ - 40 + 8 \\ \hline 20 + 4 = 24 \end{array}$$

72 - 48

		7	2
-		4	8

7	0
4	0

+

2
8

		6	0
-		4	8
		2	4

6	0
~~7~~	~~0~~
4	0

+

1	2
	8

72 - 48 = 24 72 - 24 = 48
24 + 48 = 72 48 + 24 = 72

Learners should know that units line up under units, tens under tens, and so on.

If the teaching staff feel that the use of addition signs within a subtraction calculation will cause confusion, then they can be replaced with arrows, as in the example below. This needs to be agreed as part of the whole-school policy and applied consistently throughout the school.

$$\begin{array}{r}
 89 = 80 \rightarrow 9 \\
 - 57 = 50 \rightarrow 7 \\
 \hline
 30 \rightarrow 2 = 32
 \end{array}$$

Where the numbers involved in the calculation are close together or near to multiples of 10, 100, etc, counting on using a number line should be used.

$$102 - 89 = 13$$



Step 4

Partitioning and decomposition

The diagram illustrates the decomposition of 754 into 700, 50, and 4, and then adjusting these components to subtract 86. The diagram shows three stages of adjustment:

- Stage 1:** 700 + 50 + 4 - 80 + 6
- Stage 2:** 600 + 140 + 14 - 80 + 6
- Stage 3:** 600 + 60 + 8 = 668

$$\begin{array}{r} 754 \\ - 86 \\ \hline \end{array}$$

Stage 1

$$\begin{array}{r} 700 + 50 + 4 \\ - 80 + 6 \\ \hline \end{array}$$

Stage 2

$$\begin{array}{r} 700 + 40 + 14 \quad (\text{adjust from } T \text{ to } U) \\ - 80 + 6 \\ \hline \end{array}$$

Stage 3

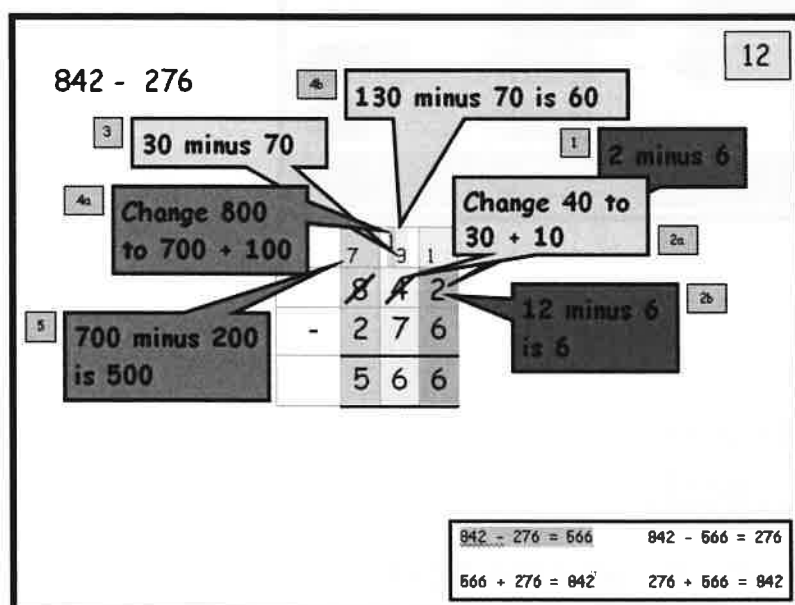
$$\begin{array}{r} 600 + 140 + 14 \quad (\text{adjust from } H \text{ to } T) \\ - 80 + 6 \\ \hline 600 + 60 + 8 = 668 \end{array}$$

This would be recorded by the learners as:

$$\begin{array}{r}
 600 \quad 140 \quad 1 \\
 700 + 50 + 4 \\
 - \quad \quad 80 + 6 \\
 \hline
 600 + 60 + 8 = 668
 \end{array}$$

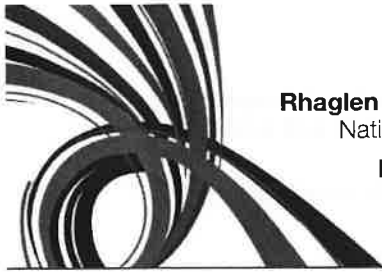
Decomposition

$$\begin{array}{r}
 7131 \\
 842 \\
 - 276 \\
 \hline
 668
 \end{array}$$



Learners should:

- be able to subtract numbers with different numbers of digits;
- using this method, also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds;
- know that decimal points should line up under each other;



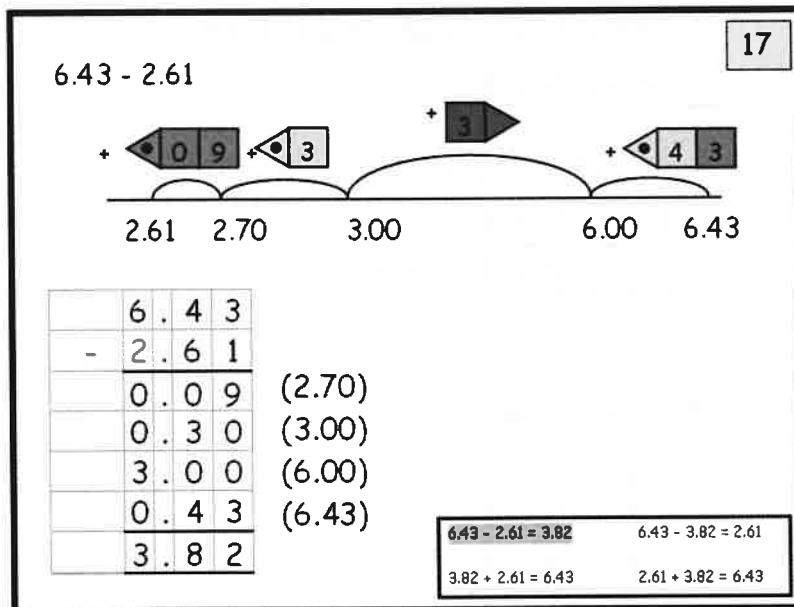
- be able to use a number line to help in making the calculation.

$$\begin{array}{r}
 \text{£}8.95 = 8 + 0.9 + 0.05 \\
 - \text{£}4.38 \quad 4 + 0.3 + 0.08 \\
 \hline
 \end{array}
 \quad \text{leading to}$$

$$\begin{array}{r}
 = 8 + 0.8 + 0.15 \quad (\text{adjust from } T \text{ to } U) \quad 8.85 \\
 4 + 0.3 + 0.08 \\
 \hline
 - 4 + 0.5 + 0.07 \\
 \hline
 = \text{£}4.57
 \end{array}$$

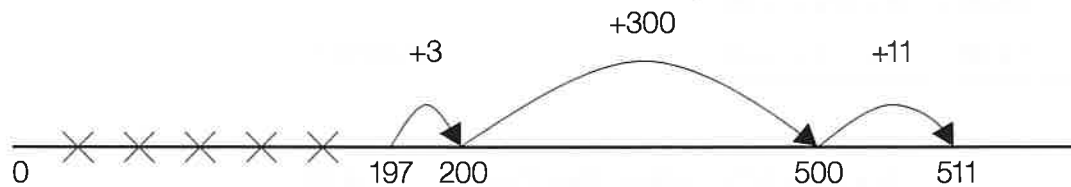
Alternatively, learners can set the amounts to whole numbers, i.e. 895 – 438 and convert to pounds after the calculation.

Using a number line to help in making the calculation:



Where the numbers involved in the calculation are close together or near to multiples of 10, 100, etc, counting on using a number line should be used.

$$511 - 197 = 314$$



Step 5

Partitioning and decomposition

$$\begin{array}{r} \text{Stage 1} \quad 754 = 700 + 50 + 4 \\ - 286 \quad - 200 + 80 + 6 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Stage 2} \quad 700 + 40 + 14 \quad (\text{adjust from } T \text{ to } U) \\ - 200 + 80 + 6 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Stage 3} \quad 600 + 140 + 14 \quad (\text{adjust from } H \text{ to } T) \\ - 200 + 80 + 6 \\ \hline 400 + 60 + 8 = 468 \end{array}$$

This would be recorded by the learners as:

$$\begin{array}{r} 600 \quad 140 \quad 1 \\ 700 + 50 + 4 \\ - 200 + 80 + 6 \\ \hline 400 + 60 + 8 = 468 \end{array}$$

Decomposition

$$\begin{array}{r} 6141 \\ 754 \\ - 286 \\ \hline 468 \end{array}$$

11

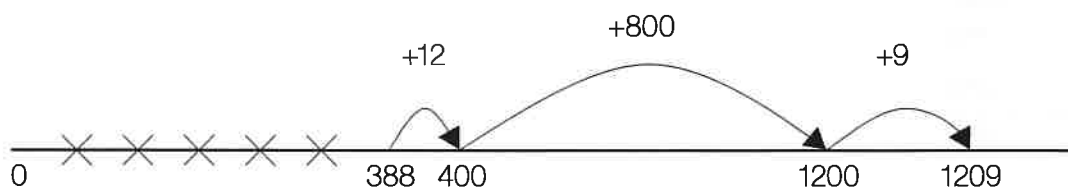
8 4 2	8 0 0	+ 4 0	+ 2
- 2 7 6	- 2 0 0	+ 7 0	+ 6
8 ³⁴ 12	8 0 0	+ 3 0	
- 2 7 6	- 2 0 0	+ 4 0	+ 1 2
7 ¹³ 4 12	7 0 0	+ 1 3 0	
- 2 7 6	- 2 0 0	+ 4 0	+ 1 2
5 6 6	5 0 0	+ 6 0	+ 6

Learners should:

- be able to subtract numbers with different numbers of digits;
- begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places;
- know that decimal points should line up under each other.

Where the numbers involved in the calculation are close together or near to multiples of 10, 100, etc, counting on using a number line should be used.

$$1209 - 388 = 821$$



Step 6 Decomposition

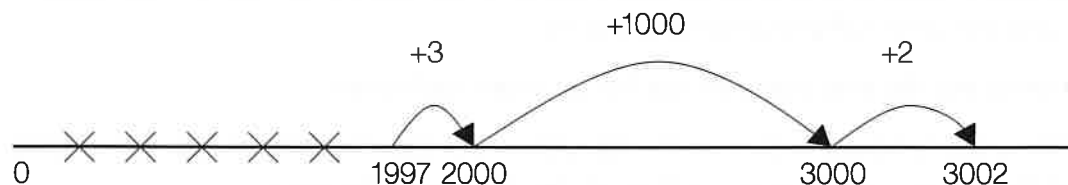
$$\begin{array}{r} 5131 \\ 6467 \\ - 2684 \\ \hline 3783 \end{array}$$

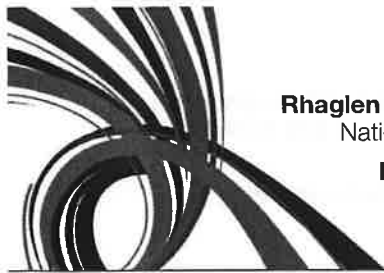
Learners should:

- be able to subtract numbers with different numbers of digits;
- be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;
- know that decimal points should line up under each other.

Where the numbers involved in the calculation are close together or near to multiples of 10, 100, etc, counting on using a number line should be used.

$$3002 - 1997 = 1005$$





Learners need to have a range of calculation methods, mental and written.
Selection will depend upon the numbers involved.

Learners should not be made to go on to the next stage if:

- 1) they are not ready;
- 2) they are not confident.

Learners should be encouraged to approximate their answers before calculating.

Learners should be encouraged to check their answers after calculation using an appropriate strategy.

Learners should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Multiplication

This annex includes:

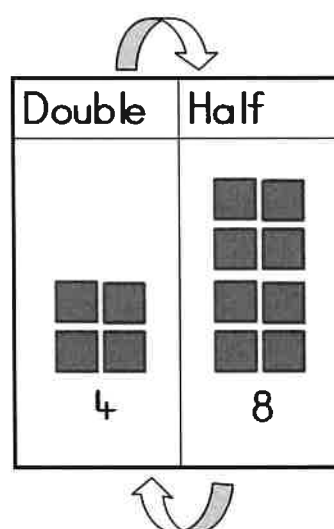
1. a brief overview of the most common mental calculation strategies for multiplication including:
 - doubling and halving;
 - understanding multiplication as repeated addition;
 - multiplication facts up to 10×10 ;
 - recognising the relationship between tables: families of tables;
 - using knowledge of tables to derive other facts;
 - multiplying by 10 and 100;
 - partitioning;
 - using factors.
2. a step-by-step overview of one route by which multiplication can be learnt from counting equal groups of objects through to using the Grid Method (also known as the Area Method) for short and long division of whole numbers and decimals.

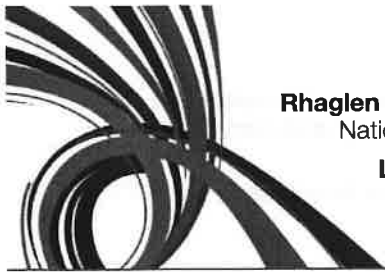
Mental calculations

Step 1

Doubling and halving

Apply the knowledge of doubles and halves to known facts, e.g. 4×4 is double 2×4 .





Step 2

1 times table

1 times table

2 times table

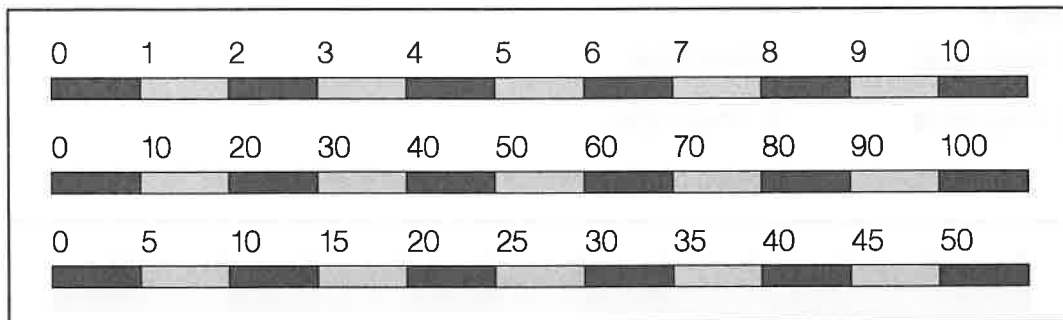
10 times table

5 times table



	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

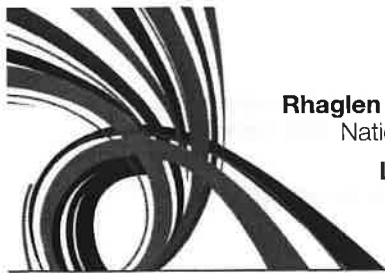
	0	x	2	=		0
	1	x	2	=		2
	2	x	2	=		4
	3	x	2	=		6
	4	x	2	=		8
	5	x	2	=	1	0
	6	x	2	=	1	2
	7	x	2	=	1	4
	8	x	2	=	1	6
	9	x	2	=	1	8
1	0	x	2	=	2	0



	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	1	0	=			0
	1	x	1	0	=		1	0
	2	x	1	0	=		2	0
	3	x	1	0	=		3	0
	4	x	1	0	=		4	0
	5	x	1	0	=		5	0
	6	x	1	0	=		6	0
	7	x	1	0	=		7	0
	8	x	1	0	=		8	0
	9	x	1	0	=		9	0
1	0	x	1	0	=	1	0	0

	0	x	5	=		0
	1	x	5	=		5
	2	x	5	=	1	0
	3	x	5	=	1	5
	4	x	5	=	2	0
	5	x	5	=	2	5
	6	x	5	=	3	0
	7	x	5	=	3	5
	8	x	5	=	4	0
	9	x	5	=	4	5
1	0	x	5	=	5	0



Step 3

1 times table

1 times table

1 times table

2 times table

10 times table

3 times table

4 times table

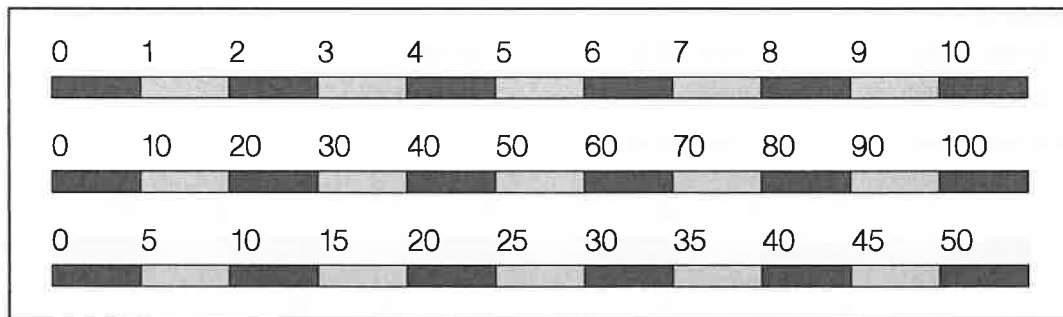
5 times table

0	1	2	3	4	5	6	7	8	9	10
0	2	4	6	8	10	12	14	16	18	20
0	4	8	12	16	20	24	28	32	36	40

	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	2	=		0
	1	x	2	=		2
	2	x	2	=		4
	3	x	2	=		6
	4	x	2	=		8
	5	x	2	=	1	0
	6	x	2	=	1	2
	7	x	2	=	1	4
	8	x	2	=	1	6
	9	x	2	=	1	8
1	0	x	2	=	2	0

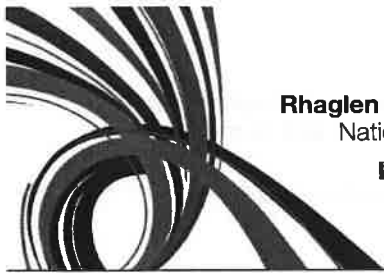
	0	x	4	=		0
	1	x	4	=		4
	2	x	4	=		8
	3	x	4	=	1	2
	4	x	4	=	1	6
	5	x	4	=	2	0
	6	x	4	=	2	4
	7	x	4	=	2	8
	8	x	4	=	3	2
	9	x	4	=	3	6
1	0	x	4	=		0



	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	1	0	=			0
	1	x	1	0	=		1	0
	2	x	1	0	=		2	0
	3	x	1	0	=		3	0
	4	x	1	0	=		4	0
	5	x	1	0	=		5	0
	6	x	1	0	=		6	0
	7	x	1	0	=		7	0
	8	x	1	0	=		8	0
	9	x	1	0	=		9	0
1	0	x	1	0	=	1	0	0

	0	x	5	=		0
	1	x	5	=		5
	2	x	5	=	1	0
	3	x	5	=	1	5
	4	x	5	=	2	0
	5	x	5	=	2	5
	6	x	5	=	3	0
	7	x	5	=	3	5
	8	x	5	=	4	0
	9	x	5	=	4	5
1	0	x	5	=	5	0



0	1	2	3	4	5	6	7	8	9	10
0	3	6	9	12	15	18	21	24	27	30

	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	3	=		0
	1	x	3	=		3
	2	x	3	=		6
	3	x	3	=		9
	4	x	3	=	1	2
	5	x	3	=	1	5
	6	x	3	=	1	8
	7	x	3	=	2	1
	8	x	3	=	2	4
	9	x	3	=	2	7
1	0	x	3	=	3	0

Step 4

1 times table

1 times table

1 times table

2 times table

10 times table

3 times table

4 times table

5 times table

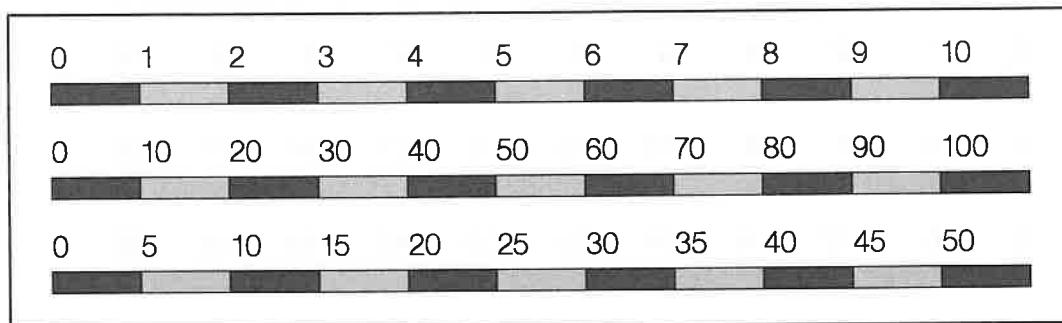
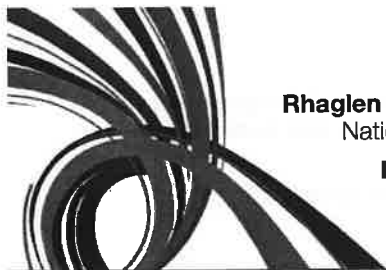
6 times table

0	1	2	3	4	5	6	7	8	9	10
0	2	4	6	8	10	12	14	16	18	20
0	4	8	12	16	20	24	28	32	36	40

0	x	1	=	0
1	x	1	=	1
2	x	1	=	2
3	x	1	=	3
4	x	1	=	4
5	x	1	=	5
6	x	1	=	6
7	x	1	=	7
8	x	1	=	8
9	x	1	=	9
10	x	1	=	10

0	x	2	=	0
1	x	2	=	2
2	x	2	=	4
3	x	2	=	6
4	x	2	=	8
5	x	2	=	10
6	x	2	=	12
7	x	2	=	14
8	x	2	=	16
9	x	2	=	18
10	x	2	=	20

0	x	4	=	0
1	x	4	=	4
2	x	4	=	8
3	x	4	=	12
4	x	4	=	16
5	x	4	=	20
6	x	4	=	24
7	x	4	=	28
8	x	4	=	32
9	x	4	=	36
10	x	4	=	40



	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	1	0	=			0
	1	x	1	0	=		1	0
	2	x	1	0	=		2	0
	3	x	1	0	=		3	0
	4	x	1	0	=		4	0
	5	x	1	0	=		5	0
	6	x	1	0	=		6	0
	7	x	1	0	=		7	0
	8	x	1	0	=		8	0
	9	x	1	0	=		9	0
1	0	x	1	0	=	1	0	0

	0	x	5	=		0
	1	x	5	=		5
	2	x	5	=	1	0
	3	x	5	=	1	5
	4	x	5	=	2	0
	5	x	5	=	2	5
	6	x	5	=	3	0
	7	x	5	=	3	5
	8	x	5	=	4	0
	9	x	5	=	4	5
1	0	x	5	=	5	0

0	1	2	3	4	5	6	7	8	9	10
0	3	6	9	12	15	18	21	24	27	30
0	6	12	18	24	30	36	42	48	54	60

	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	3	=		0
	1	x	3	=		3
	2	x	3	=		6
	3	x	3	=		9
	4	x	3	=	1	2
	5	x	3	=	1	5
	6	x	3	=	1	8
	7	x	3	=	2	1
	8	x	3	=	2	4
	9	x	3	=	2	7
1	0	x	3	=	3	0

	0	x	6	=		0
	1	x	6	=		6
	2	x	6	=	1	2
	3	x	6	=	1	8
	4	x	6	=	2	4
	5	x	6	=	3	0
	6	x	6	=	3	6
	7	x	6	=	4	2
	8	x	6	=	4	8
	9	x	6	=	5	4
1	0	x	6	=	6	0



Step 5

Derive and recall quickly all multiplication facts up to 10×10 .

1 times table	1 times table	1 times table	1 times table
2 times table	10 times table	3 times table	7 times table
4 times table	5 times table	6 times table	
8 times table		9 times table	

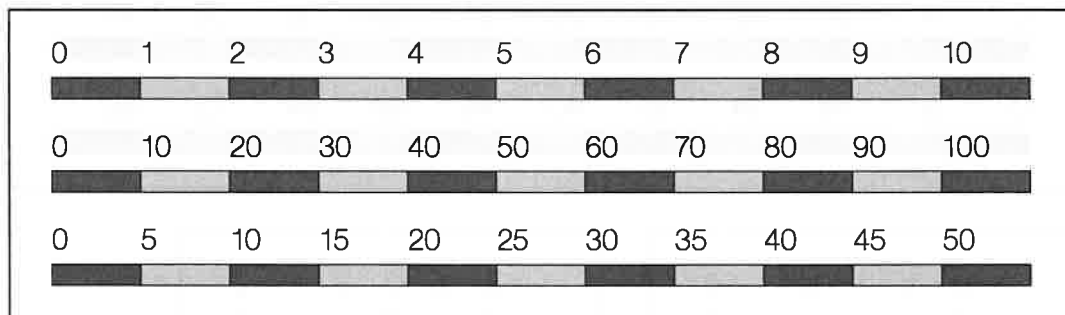
0	1	2	3	4	5	6	7	8	9	10
0	2	4	6	8	10	12	14	16	18	20
0	4	8	12	16	20	24	28	32	36	40
0	8	16	24	32	40	48	56	64	72	80

	0	x	1	=	0
	1	x	1	=	1
	2	x	1	=	2
	3	x	1	=	3
	4	x	1	=	4
	5	x	1	=	5
	6	x	1	=	6
	7	x	1	=	7
	8	x	1	=	8
	9	x	1	=	9
1	0	x	1	=	1 0

	0	x	2	=	0
	1	x	2	=	2
	2	x	2	=	4
	3	x	2	=	6
	4	x	2	=	8
	5	x	2	=	1 0
	6	x	2	=	1 2
	7	x	2	=	1 4
	8	x	2	=	1 6
	9	x	2	=	1 8
1	0	x	2	=	2 0

	0	x	4	=		0
	1	x	4	=		4
	2	x	4	=		8
	3	x	4	=	1	2
	4	x	4	=	1	6
	5	x	4	=	2	0
	6	x	4	=	2	4
	7	x	4	=	2	8
	8	x	4	=	3	2
	9	x	4	=	3	6
1	0	x	4	=	4	0

	0	x	8	=		0
	1	x	8	=		8
	2	x	8	=	1	6
	3	x	8	=	2	4
	4	x	8	=	3	2
	5	x	8	=	4	0
	6	x	8	=	4	8
	7	x	8	=	5	6
	8	x	8	=	6	4
	9	x	8	=	7	2
1	0	x	8	=	8	0



	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	1	0	=			0
	1	x	1	0	=		1	0
	2	x	1	0	=		2	0
	3	x	1	0	=		3	0
	4	x	1	0	=		4	0
	5	x	1	0	=		5	0
	6	x	1	0	=		6	0
	7	x	1	0	=		7	0
	8	x	1	0	=		8	0
	9	x	1	0	=		9	0
1	0	x	1	0	=	1	0	0

0	x	5	=	0
1	x	5	=	5
2	x	5	=	10
3	x	5	=	15
4	x	5	=	20
5	x	5	=	25
6	x	5	=	30
7	x	5	=	35
8	x	5	=	40
9	x	5	=	45
10	x	5	=	50

0	1	2	3	4	5	6	7	8	9	10
0	3	6	9	12	15	18	21	24	27	30
0	6	12	18	24	30	36	42	48	54	60
0	9	18	27	36	45	54	63	72	81	90

0	x	1	=	0
1	x	1	=	1
2	x	1	=	2
3	x	1	=	3
4	x	1	=	4
5	x	1	=	5
6	x	1	=	6
7	x	1	=	7
8	x	1	=	8
9	x	1	=	9
10	x	1	=	10

0	x	3	=	0
1	x	3	=	3
2	x	3	=	6
3	x	3	=	9
4	x	3	=	12
5	x	3	=	15
6	x	3	=	18
7	x	3	=	21
8	x	3	=	24
9	x	3	=	27
10	x	3	=	30

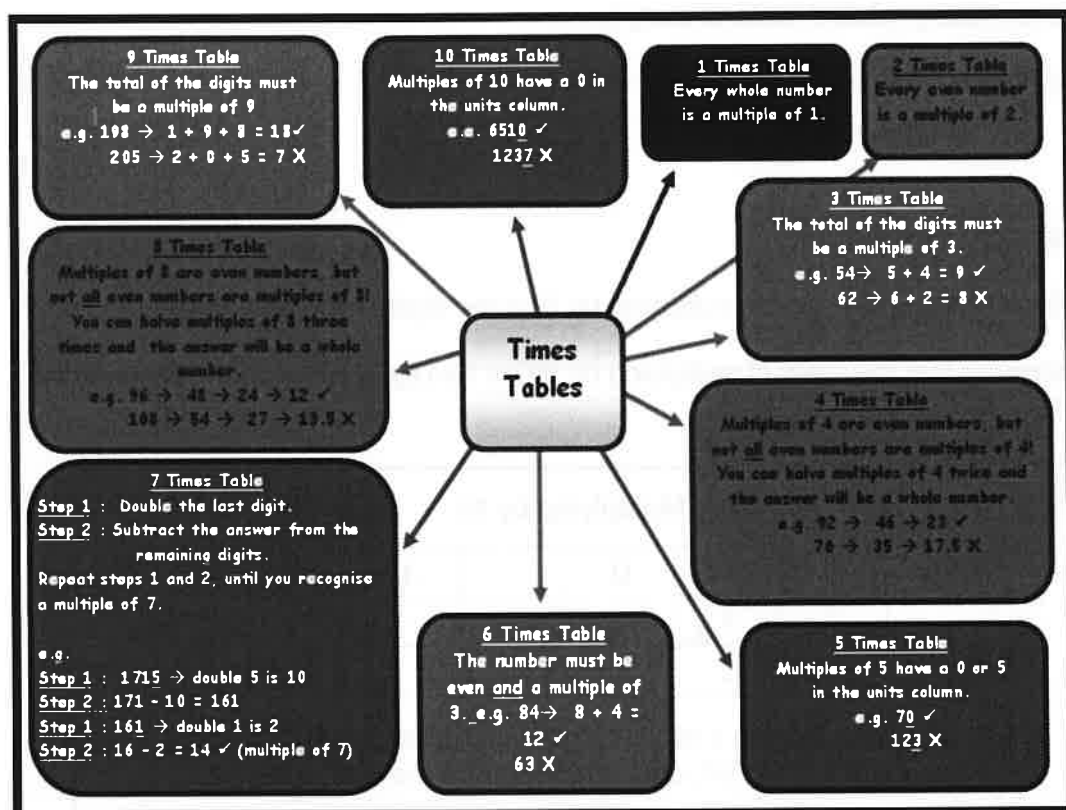
	0	x	6	=		0
	1	x	6	=		6
	2	x	6	=	1	2
	3	x	6	=	1	8
	4	x	6	=	2	4
	5	x	6	=	3	0
	6	x	6	=	3	6
	7	x	6	=	4	2
	8	x	6	=	4	8
	9	x	6	=	5	4
1	0	x	6	=	6	0

	0	x	9	=		0
	1	x	9	=		9
	2	x	9	=	1	8
	3	x	9	=	2	7
	4	x	9	=	3	6
	5	x	9	=	4	5
	6	x	9	=	5	4
	7	x	9	=	6	3
	8	x	9	=	7	2
	9	x	9	=	8	1
1	0	x	9	=	9	0

0	1	2	3	4	5	6	7	8	9	10
0	7	14	21	28	35	42	49	56	63	70

	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	7	=		0
	1	x	7	=		7
	2	x	7	=	1	4
	3	x	7	=	2	1
	4	x	7	=	2	8
	5	x	7	=	3	5
	6	x	7	=	4	2
	7	x	7	=	4	9
	8	x	7	=	5	6
	9	x	7	=	6	3
1	0	x	7	=	7	0



Step 6

Learners should be able to use their knowledge of tables to derive other facts; e.g. If I know $3 \times 7 = 21$, what else do I know?

$$30 \times 7 = 3 \times 7 \times 10 = 210$$

$$300 \times 7 = 3 \times 7 \times 10 \times 10 = 3 \times 7 \times 100 = 2100$$

$$3000 \times 7 = 3 \times 7 \times 10 \times 10 \times 10 = 3 \times 7 \times 10 \times 100 = 3 \times 7 \times 1000 = 21\,000$$

$$0.3 \times 7 = (3 \times 7) \div 10 = 2.1$$

Use closely related facts already known

$$\begin{aligned} 13 \times 11 &= (13 \times 10) + (13 \times 1) \\ &= 130 + 13 \\ &= 143 \end{aligned}$$

Multiply by 10 or 100

Knowing that the effect of multiplying by 10 is the digit(s) moving one place to the left.

Knowing that the effect of multiplying by 100 is the digit(s) moving two places to the left.

Multiplying by 10

Th	H	T	U		.1	.01	.001
	3	4	0	.			
		3	4				

When multiplying a whole number by 10 the digits move one column to the left, and a 0 is placed in the units column

Multiplying by 100

Th	H	T	U		.1	.01	.001
3	4	0	0	.			
		3	4				

When multiplying a whole number by 100 the digits move two columns to the left, and a 0 is placed in the tens and units columns

Partitioning

$$\begin{aligned} 38 \times 7 &= (30 \times 7) + (8 \times 7) \\ &= 210 + 56 \\ &= 266 \end{aligned}$$

2

38 × 7

×	30	8
7	210	56

7 × 3 × 10

7 × 8

2	1	0
+	5	6
2	6	6

38 × 7 = 266	7 × 38 = 266
266 ÷ 38 = 7	266 ÷ 7 = 38

Use of factors

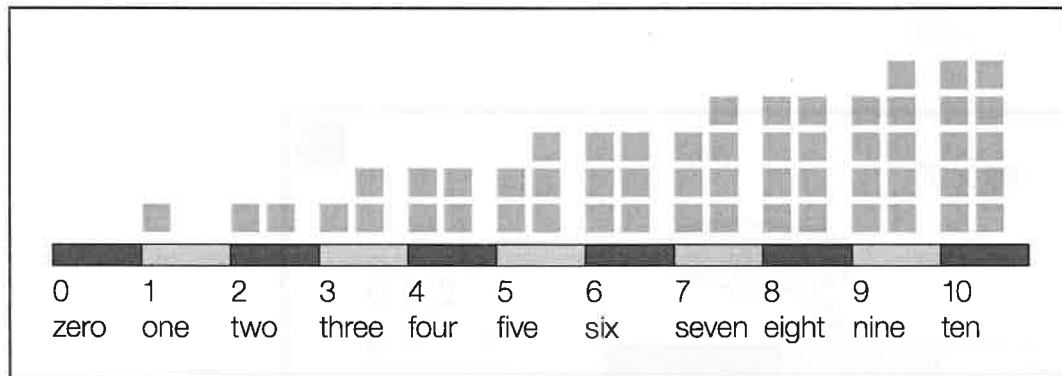
$$\begin{aligned} 21 \times 18 &= 21 \times 9 \times 2 \\ 21 \times 9 &= 189 \\ 189 \times 2 &= 378 \end{aligned}$$

$$\begin{aligned} 11 \times 50 &= 11 \times 5 \times 10 \\ 11 \times 5 &= 55 \\ 55 \times 10 &= 550 \end{aligned}$$

Many mental calculation strategies will continue to be used. They are not replaced by written methods.

Step 1

Learners will experience equal groups of objects and will count in 1s, 2s and 10s and begin to count in 5s. They will work on practical problem-solving activities involving equal sets or groups.



Step 2

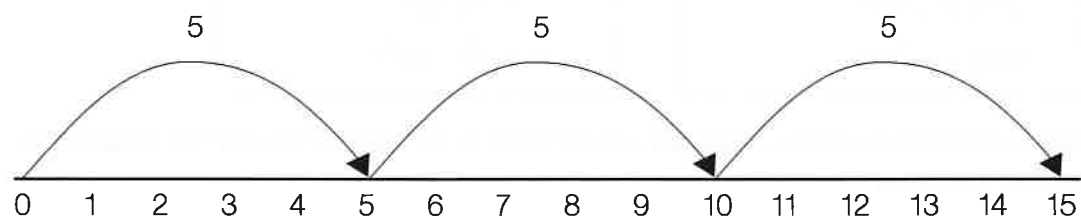
Learners will develop their understanding of multiplication and use jottings to support calculation:

Repeated addition

3 times 5 is $5 + 5 + 5 = 15$ or 3 lots of 5 or 5×3

Repeated addition can be shown easily on a number line:

$$5 \times 3 = 5 + 5 + 5$$



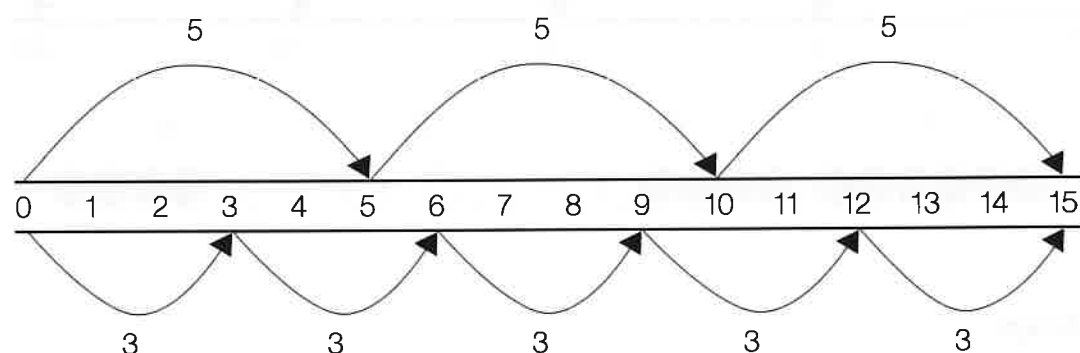
and on a bead bar:

$$5 \times 3 = 5 + 5 + 5$$



Commutative

Learners should know that 3×5 has the same answer as 5×3 . This can also be shown on the number line.



Arrays

Learners should be able to model a multiplication calculation using an array. This knowledge will support the development of the grid method.



$$3 \times 5 = 15$$

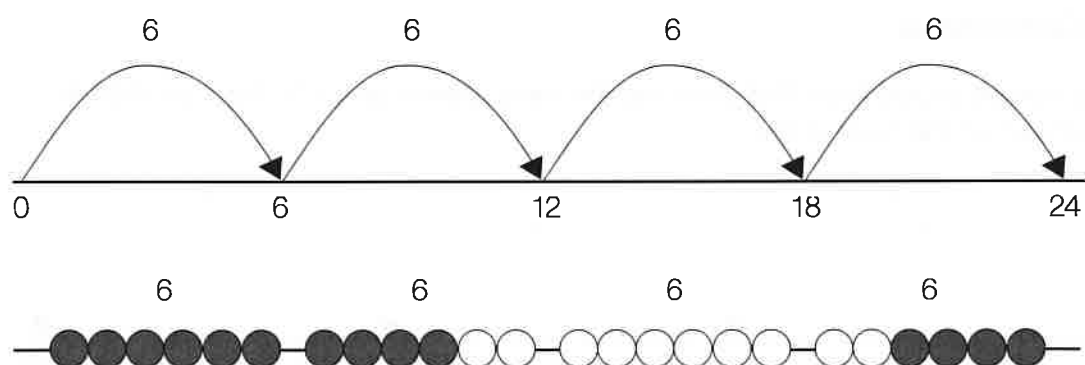
Step 3

Learners will continue to use:

Repeated addition

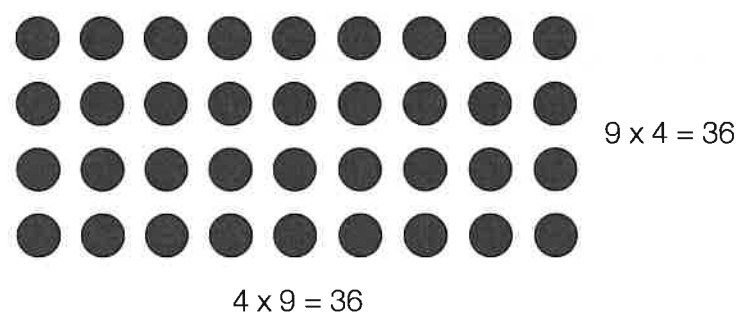
4 times 6 is $6 + 6 + 6 + 6 = 24$ or 4 lots of 6 or 6×4

Learners should use number lines or bead bars to support their understanding.



Arrays

Learners should be able to model a multiplication calculation using an array. This knowledge will support the development of the grid method.



Learners will also develop an understanding of:

Scaling

For example: find a ribbon that is 4 times as long as the purple ribbon.



- Using symbols to stand for unknown numbers to complete equations using inverse operations

$$\square \times 5 = 20$$

$$3 \times \triangle = 18$$

$$\square \times \bigcirc = 32$$

Partitioning

$$\begin{aligned} 38 \times 5 &= (30 \times 5) + (8 \times 5) \\ &= 150 + 40 \\ &= 190 \end{aligned}$$

38×5

	1	5	0
+		4	0
	1	9	0

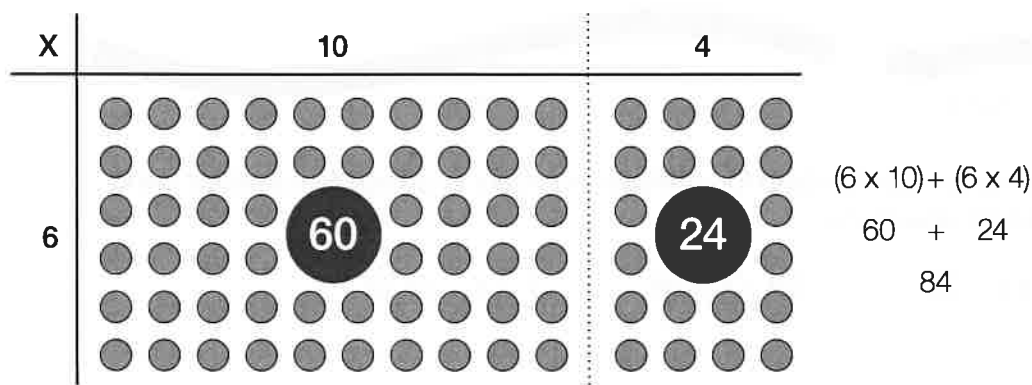
$38 \times 5 = 190$
 $190 \div 38 = 5$

$5 \times 38 = 190$
 $190 \div 5 = 38$

[NNS Section 5, page 47]

Step 4

Learners will continue to use arrays where appropriate leading into the grid method of multiplication.



Grid method

TU x U

(Short multiplication – multiplication by a single digit)

23 x 8

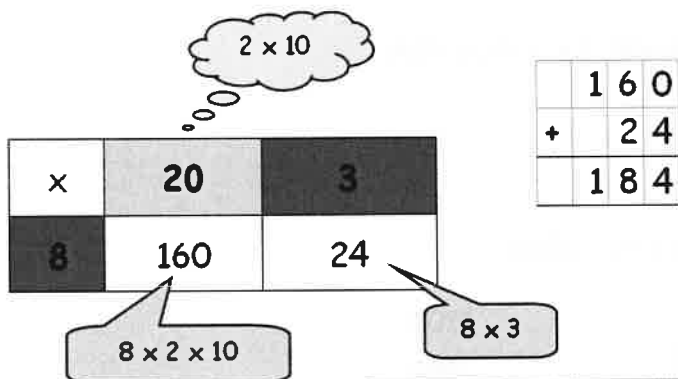
Learners will approximate first:

23 x 8 is approximately 25 x 8 = 200

x	20	3
8	160	24

$$\begin{array}{r} 160 \\ + 24 \\ \hline 184 \end{array}$$

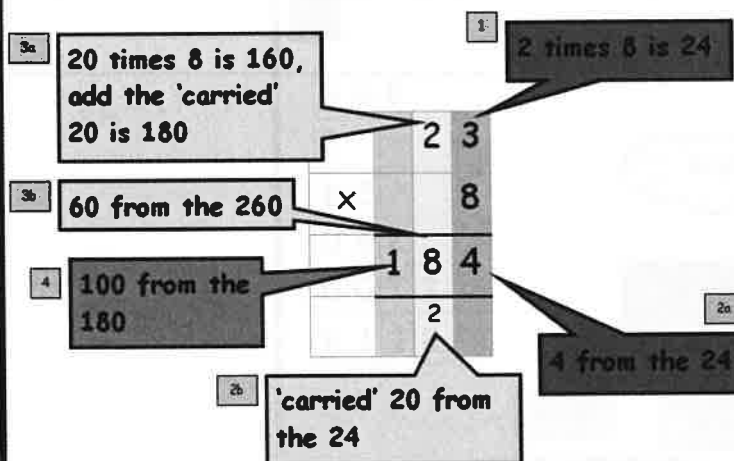
$$23 \times 8$$



$$23 \times 8 = 184 \quad 8 \times 23 = 184$$

$$184 \div 23 = 8 \quad 184 \div 8 = 23$$

$$23 \times 8$$



Step 5 Grid method

HTU x U

(Short multiplication – multiplication by a single digit)

346 x 9

Learners will approximate first:

346 x 9 is approximately $350 \times 10 = 3500$

x	300	40	6
9	2700	360	54

$$\begin{array}{r}
 2700 \\
 + 360 \\
 + 54 \\
 \hline
 3114 \\
 \hline
 11
 \end{array}$$

4

346 x 9

3×100
or
 $3 \times 10 \times 10$

4×10

x	300	40	6
9	2700	360	54

$9 \times 3 \times 100$
or
 $9 \times 3 \times 10 \times 10$

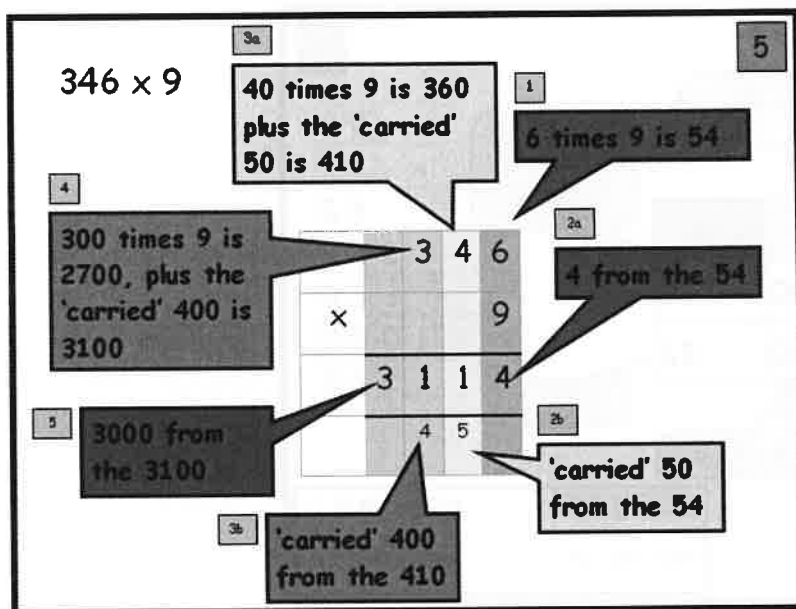
$9 \times 4 \times 10$

9×6

$346 \times 9 = 3114$
 $3114 \div 346 = 9$

$9 \times 346 = 3114$
 $3114 \div 9 = 346$

	3	4	6
x			9
		5	4
	3	6	0
	2	7	0
	3	1	1
	4		



TU x TU

(Long multiplication – multiplication by more than a single digit)

56 x 27

Learners will approximate first:

56 x 27 is approximately 60 x 30 = 1800

x	20	7	
50	1000	350	1000
6	120	42	+ 350
			+ 120
			+ 42

			1512

6

56×27

5 × 10

×	20	7
50	1000	350
6	120	42

2 × 10

5 × 10 × 7
or
5 × 7 × 10

5 × 10 × 2 × 10
or
5 × 2 × 10 × 10

6 × 2 × 10

6 × 7

		56
×		27
		42
	350	
	120	
	1000	
	1512	

$56 \times 27 = 1512$
 $1512 \div 56 = 27$

$27 \times 56 = 1512$
 $1512 \div 27 = 56$

7

A
B

$56 \times 27 = (56 \times 20) + (56 \times 7)$

A $56 \times 20 = (56 \times 10) + (56 \times 10)$
 $= 560 + 560 = 1120$

B $56 \times 7 = (50 \times 7) + (6 \times 7)$
 $= (5 \times 10 \times 7) + (6 \times 7)$
 $= (5 \times 7 \times 10) + (6 \times 7)$
 $= 350 + 42 = 392$

$\rightarrow 56 \times 27 = 1120 + 392 = 1512$

56 x 27

3a 50 times 7 is 350, plus the 'carried' 40 is 390

4a 6 times 20 is 120

5a 50 times 20 is 1000, plus the 'carried' 100 is 1100

6a 6 times 7 is 42

7a 2 from the 42

8a 'carried' 40 from the 42

9a 'carried' 100 from the 120

10a 300 from the 390

11a 1000 from the 1100

12a 1 1 2 0

13a 1 5 1 2

14a 1 5 1 2

Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other.

e.g. 4.9 x 3

Learners will approximate first:

4.9 x 3 is approximately 5 x 3 = 15

x	4	0.9
3	12	2.7

12
+ 2.7
14.7

11

4.9 × 3

9 ÷ 10

3 × 4

3 × 9 ÷ 10

x	4	0.9
3	12	2.7

	4	.	9
x			3
	2	.	7
1	2	.	0
1	4	.	7

4.9 × 3 = 14.7

3 × 4.9 = 14.7

14.7 ÷ 4.9 = 3

14.7 ÷ 3 = 4.9

12

4.9 × 3

3a 4 times 3 is 12, plus the 'carried' 2 is 14

3b 4 from the 14

3c 10 from the 14

2b 'carried' 2 from the 2.7

1 0.9 times 3 is 2.7

2a 0.7 from the 2.7

	4	.	9	
x			3	
	1	4	.	7
		2	.	



Step 6

ThHTU x U

(Short multiplication – multiplication by a single digit)

$$4346 \times 8$$

Learners will approximate first:

4346×8 is approximately $4346 \times 10 = 43460$

x	4000	300	40	6	
8	32000	2400	320	48	

32000
+ 2400
+ 320
+ 48
34768

HTU x TU

(Long multiplication – multiplication by more than a single digit)

$$235 \times 24$$

Learners will approximate first:

235×24 is approximately $240 \times 20 = 4800$

x	200	30	5	
20	4000	600	100	
4	800	120	20	

4000
+ 600
+ 100
+ 800
120
20
5640
1

235 × 24

2 × 10
2 × 100
or
2 × 10 × 10
3 × 10

×	200	30	5
20	4000	600	100
4	800	120	20

2 × 10 × 2 × 10 × 10
or
2 × 2 × 10 × 10 × 10

4 × 2 × 100
or
4 × 2 × 10 × 10

2 × 10 × 3 × 10
or
2 × 3 × 10 × 10

4 × 5
4 × 3 × 10

2 × 10 × 5
or
2 × 5 × 10

235 × 24 = 5640 24 × 235 = 5640
5640 ÷ 235 = 24 5640 ÷ 24 = 235

	2	3	5
×	2	4	
	2	0	
	1	2	0
	8	0	0
	1	0	0
	6	0	0
	4	0	0
	5	6	4
	0		

235 × 24

3a

1

2a

2b

3a

3b

4

5a

6

7

30 times 4 is 120, plus the 'carried' 20 is 140

5 times 4 is 20

0 from the 20

20 'carried' from the 20

100 carried from the 140

100 carried from the 100

200 times 4 is 800, plus the 'carried' 100 is 900

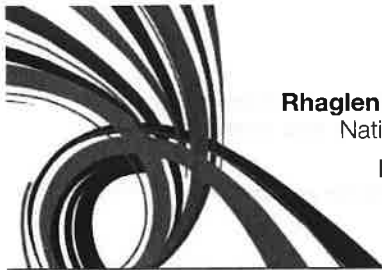
5 times 20 is 100

30 times 20 is 600, plus the 'carried' 100 is 700

200 times 20 is 4000

	2	3	5
×	2	4	
	9	4	0
	1	2	
	4	7	0
	1		
	5	6	4
	0		

Using similar methods, they will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.



For example:

$$5.82 \times 3$$

Learners will approximate first:

5.82×3 is approximately $6 \times 3 = 18$

x	5	0.8	0.02
3	15	2.4	0.06

$$\begin{array}{r} 15 \\ + 2.4 \\ + 0.06 \\ \hline 17.46 \end{array}$$

5.82 × 3

8 ÷ 10

×	5	0.8	0.02
3	15	2.4	0.06

3 × 5

(3 × 8) ÷ 10

(3 × 2) ÷ 100
or
(3 × 2) ÷ 10 ÷ 10

2 ÷ 100
or
2 ÷ 10 ÷ 10

5	.	8	2
×			3
	0	.	0 6
	2	.	4 0
1	5	.	0 0
1	7	.	4 6

5.82 × 3 = 17.46

17.46 ÷ 5.82 = 3

3 × 5.82 = 17.46

17.46 ÷ 3 = 5.82

13

Learners need to have a range of calculation methods, mental and written.
Selection will depend upon the numbers involved.

Learners should not be made to go on to the next stage if:

- 1) they are not ready;
- 2) they are not confident.

Learners should be encouraged to approximate their answers before calculating.

Learners should be encouraged to check their answers after calculation using an appropriate strategy.

Learners should be encouraged to consider if a mental calculation would be appropriate before using written methods.



Division

This annex includes:

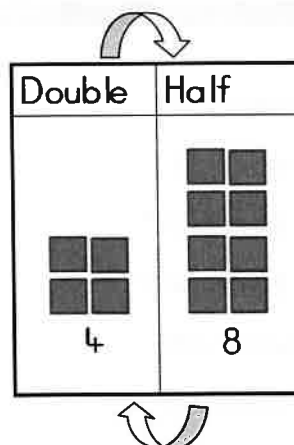
1. a brief overview of the most common mental calculation strategies for division including:
 - doubling and halving;
 - understanding division as repeated subtraction;
 - multiplication facts up to 10×10 ;
 - recognising the relationship between division and fractions;
 - deriving and recalling division facts;
 - using and applying division facts;
 - dividing by 10 and 100;
 - using factors;
 - using related facts.
2. a step by step overview of one route by which division can be learnt from sharing equal groups of objects through to using the written vertical method for short and long division of numbers and decimals. The Grid method of division is also introduced.

Mental calculations

Step 1

Doubling and halving

Knowing that halving is dividing by 2



Step 2

Deriving and recalling division facts

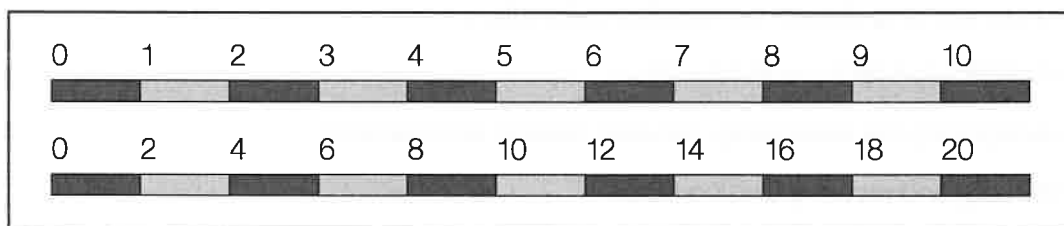
1 times table

1 times table

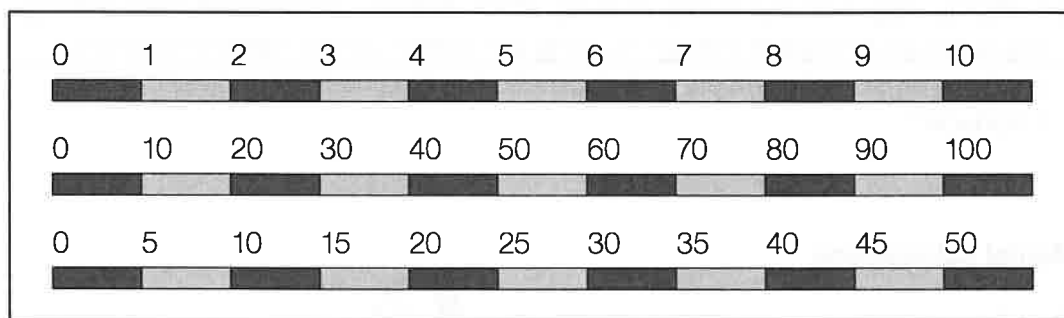
2 times table

10 times table

5 times table



- How many 1s are there in 8?
- How many 1s are there in 3?
- How many 2s are there in 14?
- How many 2s are there in 8?



- How many 1s are there in 9?
- How many 1s are there in 5?
- How many 10s are there in 70?
- How many 10s are there in 40?
- How many 5s are there in 25?
- How many 5s are there in 50?



Step 3

1 times table

1 times table

1 times table

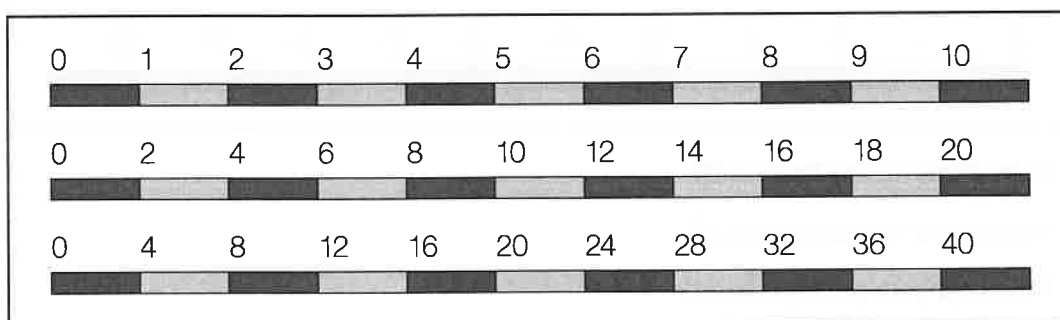
2 times table

10 times table

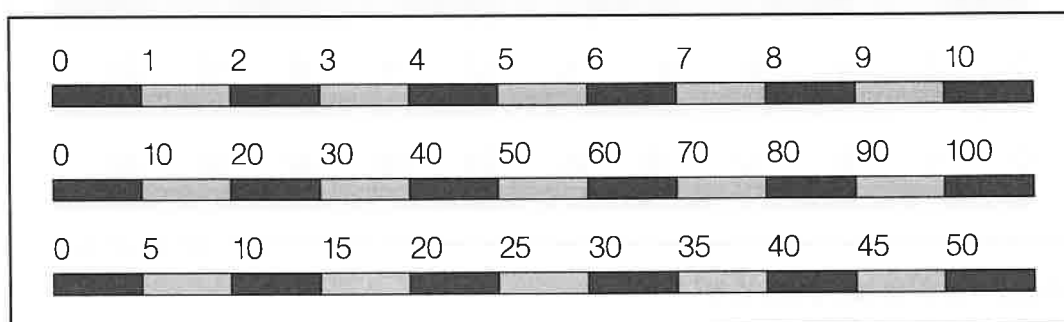
3 times table

4 times table

5 times table

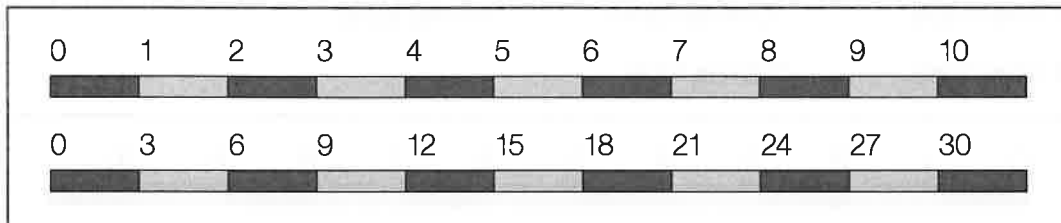


- How many 1s are there in 6?
- How many 1s are there in 8?
- How many 2s are there in 12?
- How many 2s are there in 20?
- How many 4s are there in 16?
- How many 4s are there in 32?



- How many 1s are there in 9?
- How many 1s are there in 5?
- How many 10s are there in 70?
- How many 10s are there in 40?

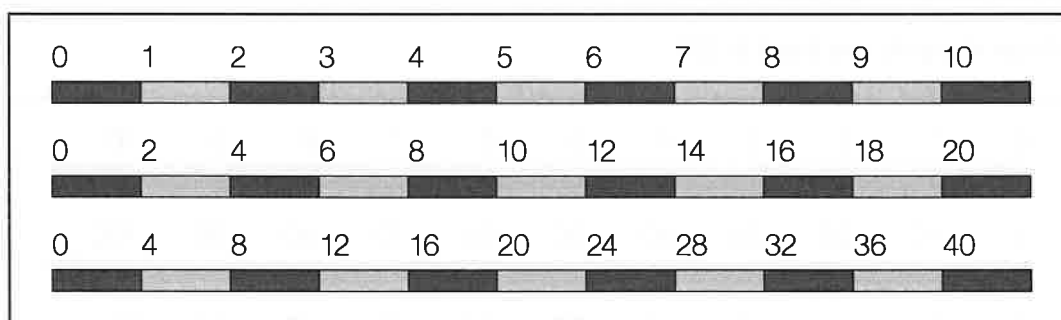
- How many 5s are there in 25?
- How many 5s are there in 50?



- How many 1s are there in 8?
- How many 1s are there in 2?
- How many 3s are there in 21?
- How many 3s are there in 9?

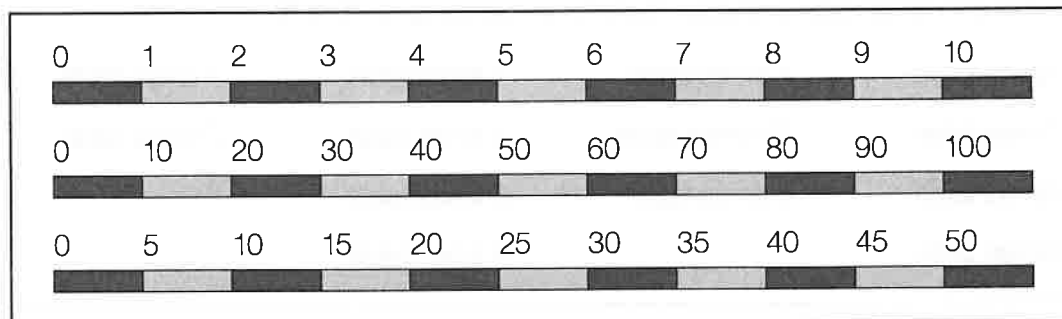
Step 4

1 times table	1 times table	1 times table
2 times table	10 times table	3 times table
4 times table	5 times table	6 times table

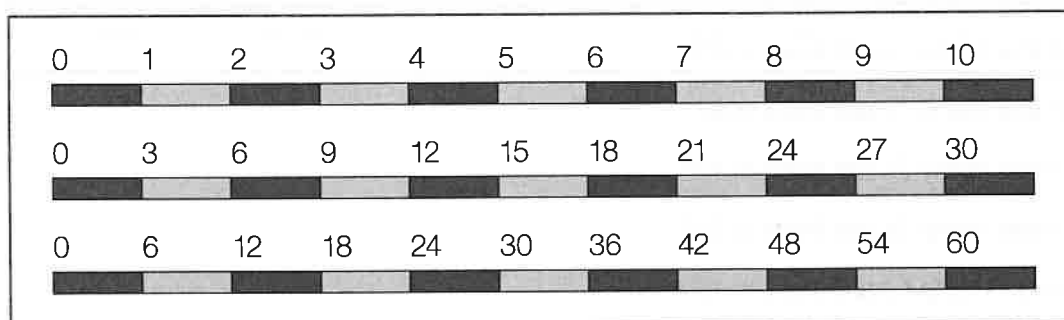


- How many 1s are there in 6?
- How many 1s are there in 8?
- How many 2s are there in 12?
- How many 2s are there in 20?
- How many 4s are there in 16?

- How many 4s are there in 32?



- How many 1s are there in 9?
- How many 1s are there in 5?
- How many 10s are there in 70?
- How many 10s are there in 40?
- How many 5s are there in 25?
- How many 5s are there in 50?



- How many 1s are there in 8?
- How many 1s are there in 2?
- How many 3s are there in 21?
- How many 3s are there in 9?
- How many 6s are there in 36?
- How many 6s are there in 54?

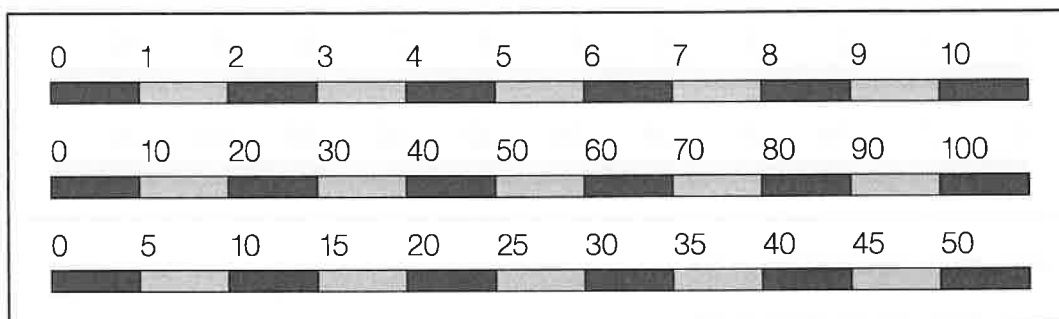
Step 5

Derive and recall quickly division facts for all tables up to 10 x 10

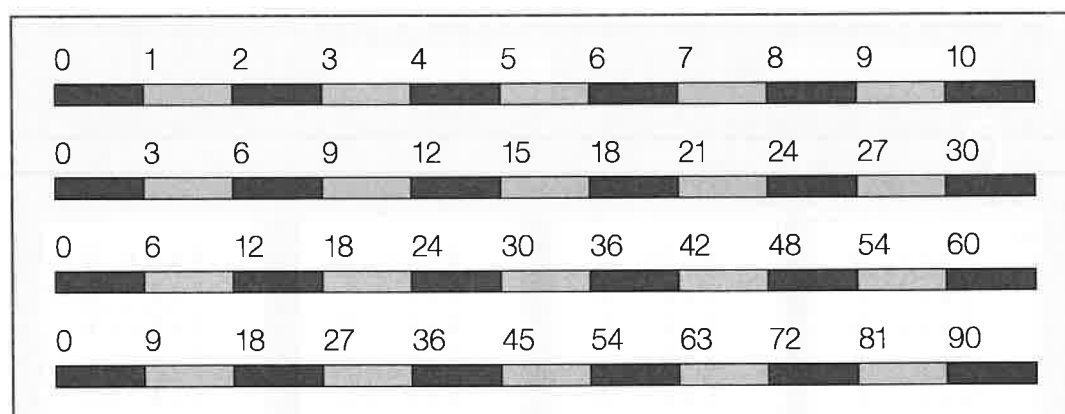
1 times table	1 times table	1 times table	1 times table
2 times table	10 times table	3 times table	7 times table
4 times table	5 times table	6 times table	
8 times table		9 times table	

0	1	2	3	4	5	6	7	8	9	10
0	2	4	6	8	10	12	14	16	18	20
0	4	8	12	16	20	24	28	32	36	40
0	8	16	24	32	40	48	56	64	72	80

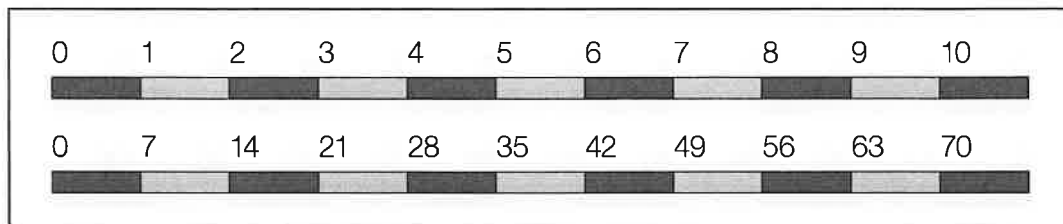
- How many 1s are there in 6?
- How many 1s are there in 8?
- How many 2s are there in 12?
- How many 2s are there in 20?
- How many 4s are there in 16?
- How many 4s are there in 32?
- How many 8s are there in 48?
- How many 8s are there in 72?



- How many 1s are there in 9?
- How many 1s are there in 5?
- How many 10s are there in 70?
- How many 10s are there in 40?
- How many 5s are there in 25?
- How many 5s are there in 50?

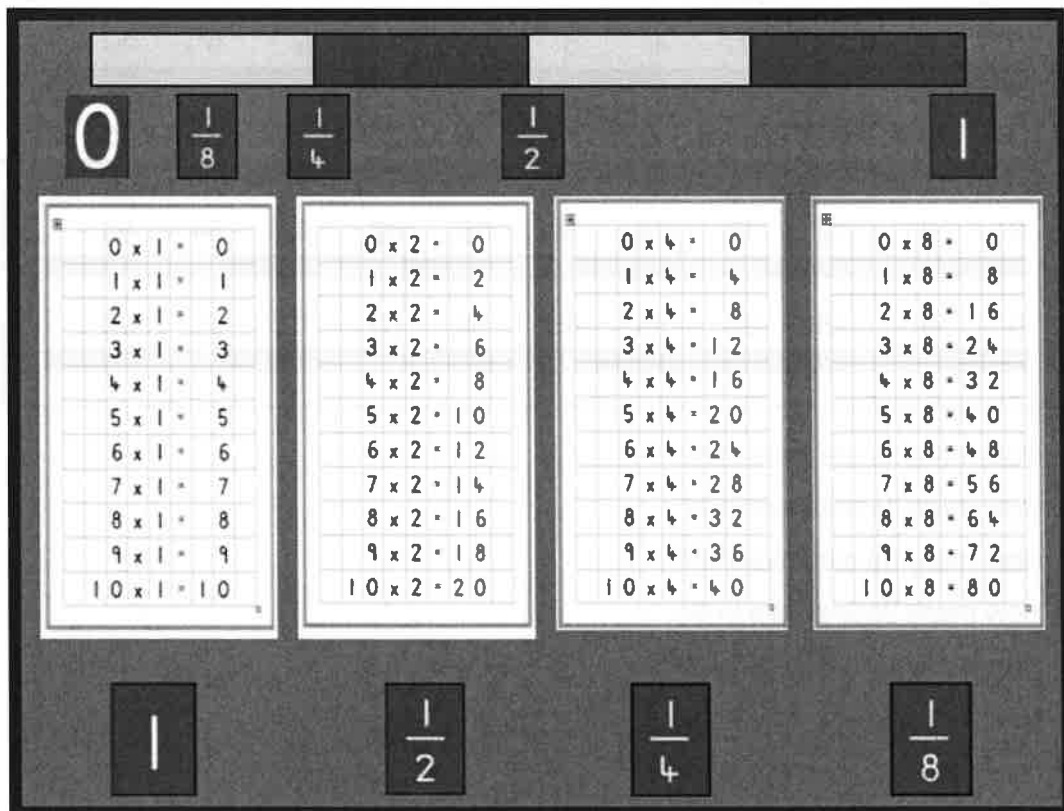


- How many 1s are there in 8?
- How many 1s are there in 2?
- How many 3s are there in 21?
- How many 3s are there in 9?
- How many 6s are there in 36?
- How many 6s are there in 54?
- How many 9s are there in 45?
- How many 9s are there in 90?



- How many 1s are there in 8?
- How many 1s are there in 2?
- How many 7s are there in 21?
- How many 7s are there in 49?

N.B. Learners should understand the close relationship between multiplication, division and fractions.





The board features a grid of colored squares at the top: grey, black, grey, black, grey, black, grey, black, grey, black. Below this grid are three multiplication tables and several fraction tiles.

Top Left Fraction Tiles:

- 0
- $\frac{1}{10}$
- $\frac{1}{5}$

Top Right Fraction Tile:

- 1

Multiplication Tables:

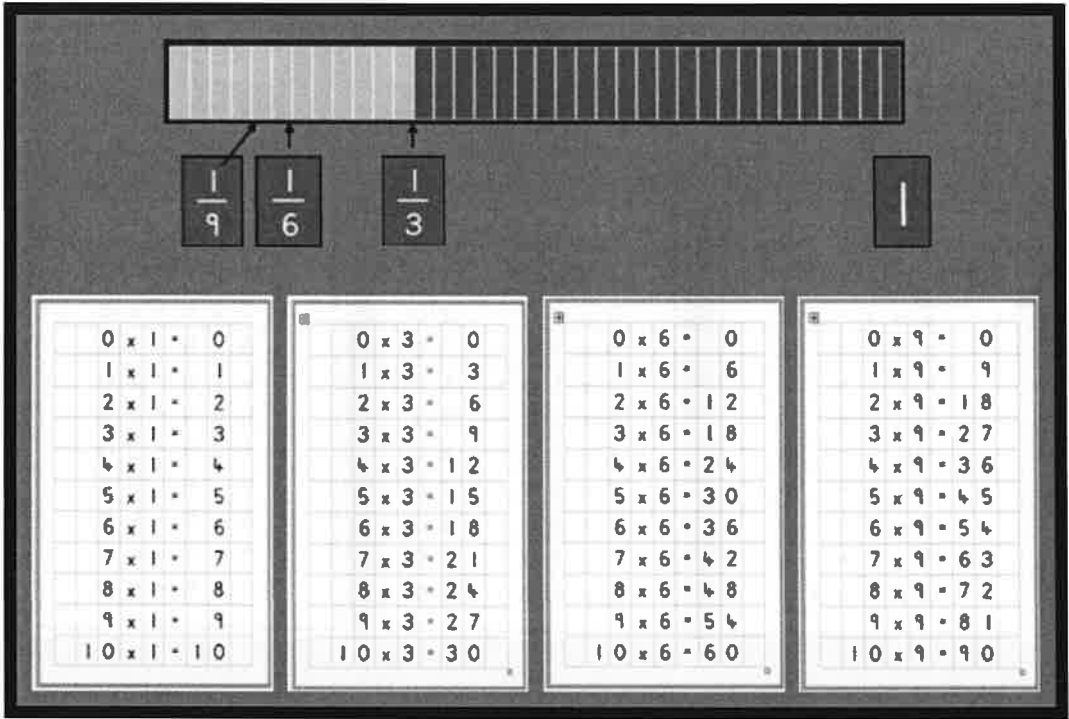
0	x	1	=	0
1	x	1	=	1
2	x	1	=	2
3	x	1	=	3
4	x	1	=	4
5	x	1	=	5
6	x	1	=	6
7	x	1	=	7
8	x	1	=	8
9	x	1	=	9
10	x	1	=	10

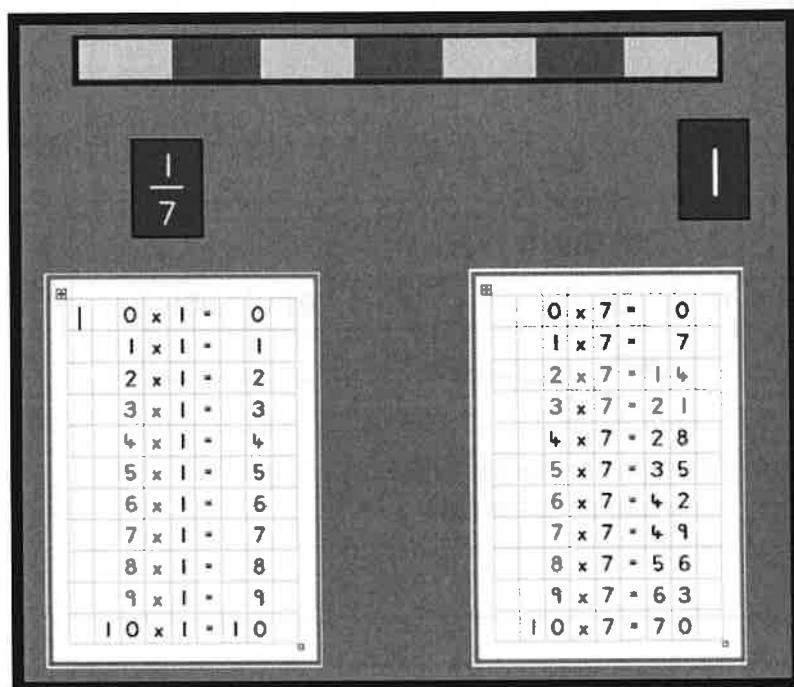
0	x	10	=	0
1	x	10	=	10
2	x	10	=	20
3	x	10	=	30
4	x	10	=	40
5	x	10	=	50
6	x	10	=	60
7	x	10	=	70
8	x	10	=	80
9	x	10	=	90
10	x	10	=	100

0	x	5	=	0
1	x	5	=	5
2	x	5	=	10
3	x	5	=	15
4	x	5	=	20
5	x	5	=	25
6	x	5	=	30
7	x	5	=	35
8	x	5	=	40
9	x	5	=	45
10	x	5	=	50

Bottom Fraction Tiles:

- 1
- $\frac{1}{10}$
- $\frac{1}{5}$





Step 6

Using and applying division facts

Learners should be able to use their knowledge of tables to derive other facts; e.g. If I know $3 \times 7 = 21$, what else do I know?

$$30 \times 7 = 3 \times 7 \times 10 = 210$$

$$300 \times 7 = 3 \times 7 \times 10 \times 10 = 3 \times 7 \times 100 = 2100$$

$$3000 \times 7 = 3 \times 7 \times 10 \times 10 \times 10 = 3 \times 7 \times 10 \times 100 = 3 \times 7 \times 1000 = 21\,000$$

$$0.3 \times 7 = (3 \times 7) \div 10 = 2.1$$

Dividing by 10 or 100

Knowing that the effect of dividing by 10 is moving the digits one place to the right.

Knowing that the effect of dividing by 100 is moving the digits two places to the right.

Dividing by 10

Th	H	T	U		.1	.01	.001
		4	3	.	4		
		3	4				

When dividing a whole number by 10 the digits move one column to the right

Dividing by 100

Th	H	T	U		.1	.01	.001
			0	.	3	4	
		3	4				

When dividing a whole number by 100 the digits move two columns to the right

Use of factors

$$\begin{aligned}
 550 \div 50 &= \\
 550 \div 10 &= 55 \\
 55 \div 5 &= 11 \\
 550 \div 50 &= 11
 \end{aligned}$$

$$\begin{aligned}
 378 \div 21 &= \\
 378 \div 3 &= 126 \\
 126 \div 7 &= 18 \\
 378 \div 21 &= 18
 \end{aligned}$$



Use of factors

Given that $124 \times 8 = 992$

What is $992 \div 8$?

or

$992 \div 124$?

Given that $1.4 \times 1.1 = 1.54$

What is $1.54 \div 1.4$?

or

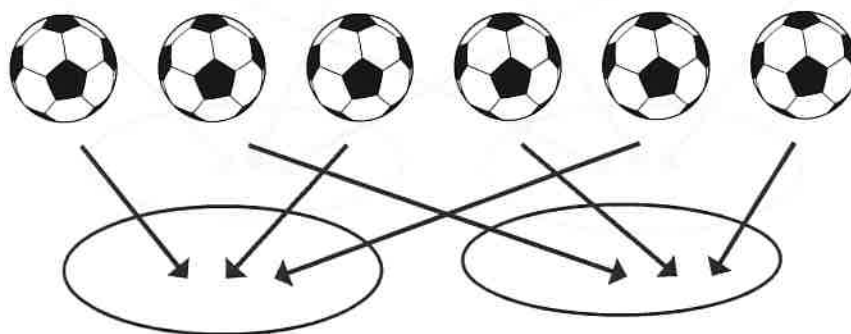
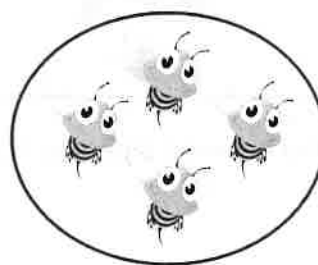
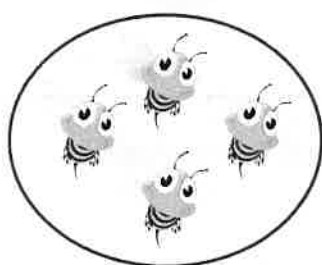
$1.54 \div 1.1$?

Many mental calculation strategies will continue to be used. They are not replaced by written methods.

Step 1

Learners will understand **equal groups**, and **share items** out in play and problem solving.

They will count in 2s and 10s and later in 5s.

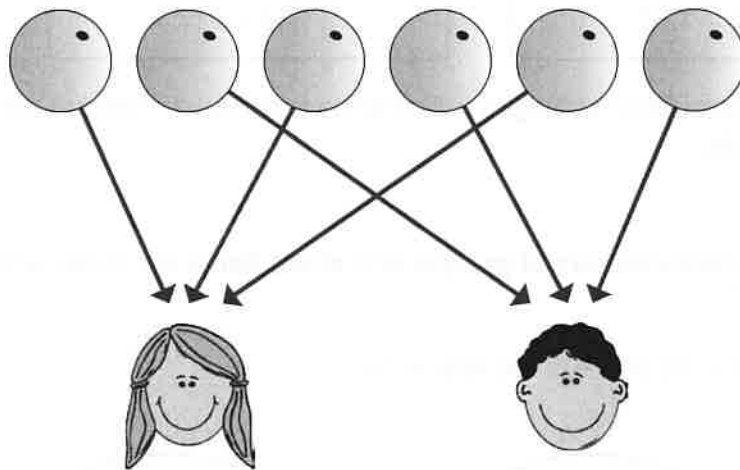


Step 2

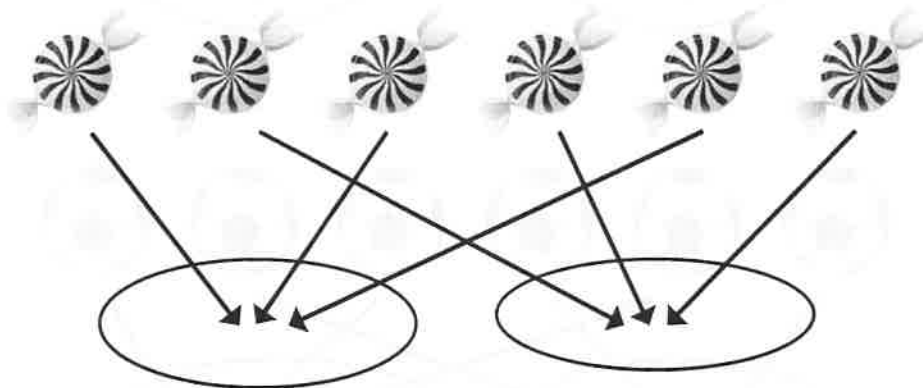
Learners will develop their understanding of division and use jottings to support calculation.

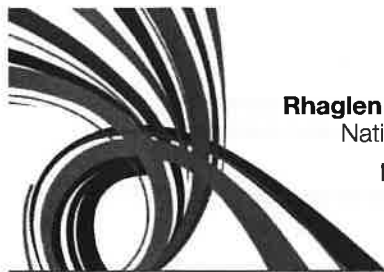
Sharing equally

6 oranges shared between 2 people, how many do they each get?

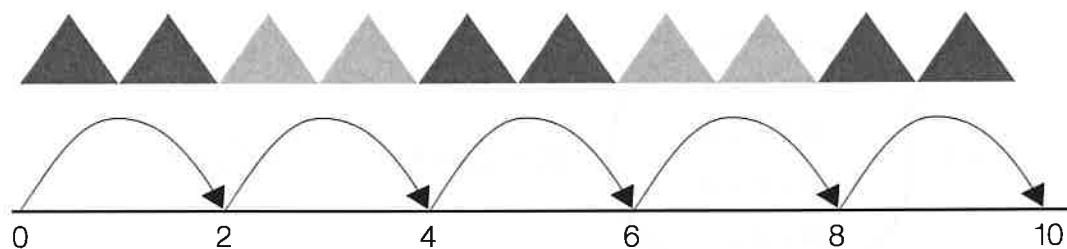


6 sweets shared between 2 people, how many do they each get?





Grouping or repeated subtraction

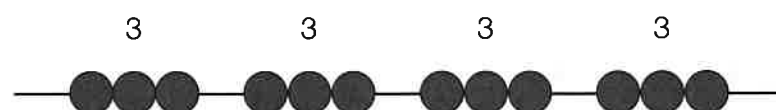
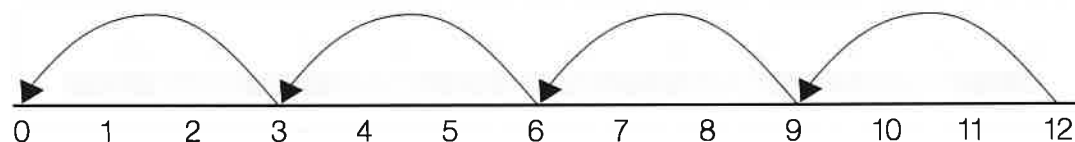


There are 6 bars of chocolate, how many people can have 2 chocolate bars each?



Repeated subtraction using a number line or bead bar

$$12 \div 3 = 4$$



A bead bar will help learners with interpreting division calculations such as $10 \div 5$ as 'how many 5s make 10?'

Using symbols to stand for unknown numbers to complete equations using inverse operations

$\square \div 2 = 4$

$4 \times 2 = 8$
 $2 \times 4 = 8$
 $8 \div 2 = 4$
 $8 \div 4 = 2$

$20 \div \triangle = 4$ $\square \div \triangle = 4$

Step 3

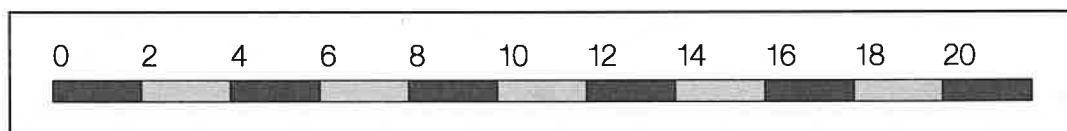
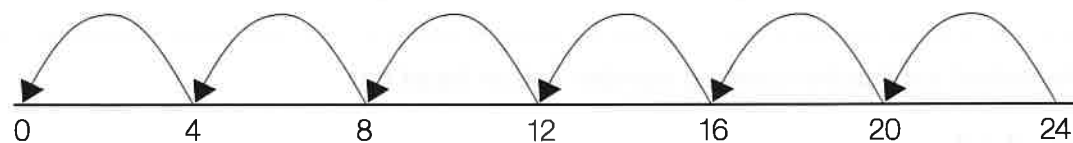
Ensure that the emphasis in Year 3 is on grouping rather than sharing.

Learners will continue to use:

Repeated subtraction using a number line

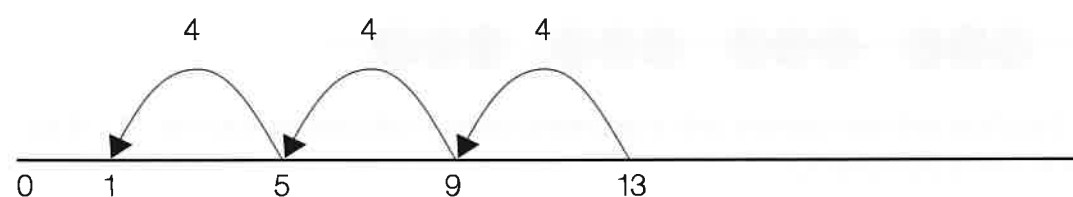
Learners will use an empty number line to support their calculation.

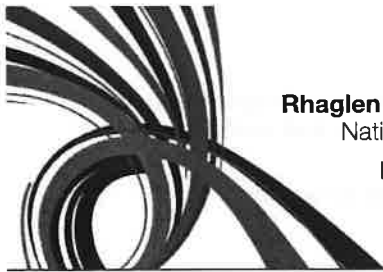
$24 \div 4 = 6$



Learners should also move onto calculations involving remainders.

$13 \div 4 = 3 \text{ r } 1$





Using symbols to stand for unknown numbers to complete equations using inverse operations

$$26 \div 2 = \square$$

$$24 \div \triangle = 8$$

$$8 \times 3 = 24$$

$$3 \times 8 = 24$$

$$24 \div 3 = 8$$

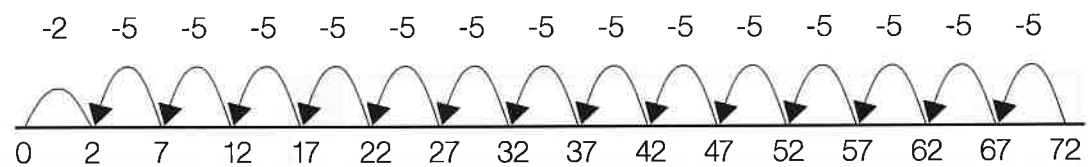
$$24 \div 8 = 3$$

$$\square \div 10 = 8$$

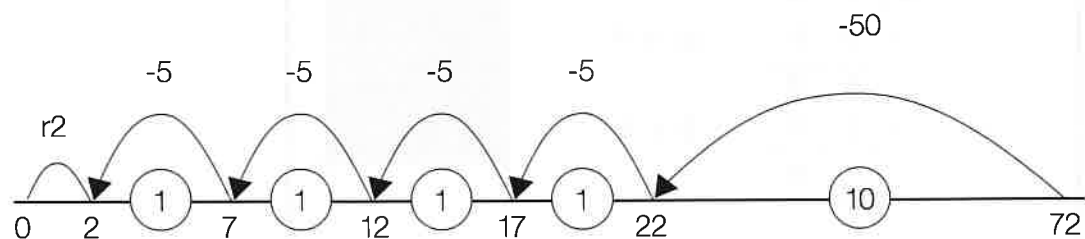
Step 4

Learners will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s – numbers with which the learners are more familiar.

$72 \div 5$



Moving onto:



Then onto the vertical method:

Short division TU ÷ U

72 ÷ 3

3	72	10x 10x 2x 2x ↓ 24
	- 30	
	42	
	- 30	
	12	
	- 6	
	6	
	- 6	
	0	

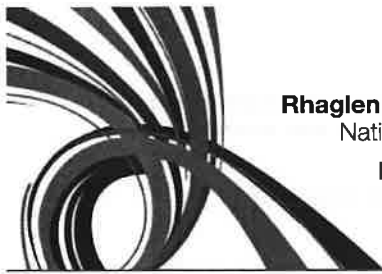
Answer:

75 ÷ 5
3

	1	5				
5	7	5				
-	5	0		10	x	5
	2	5				
-	2	5		5	x	5
		0				

+						
1	x	5	=	5		
2	x	5	=	10		
5	x	5	=	25		
10	x	5	=	50		

75 ÷ 5 = 15	75 ÷ 15 = 5
15 x 5 = 75	5 x 15 = 75

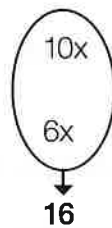


Leading to subtraction of other multiples:

$$96 \div 6$$

$$\begin{array}{r} 16 \\ 6 \overline{) 96} \\ \underline{- 60} \\ 36 \\ \underline{- 36} \\ 0 \end{array}$$

Answer:



Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

69 ÷ 4

	1	7	r. 1
4	6	9	
-	4	0	10 × 4
	2	9	
-	2	0	5 × 4
		9	
-		8	2 × 4
		1	

1 × 4 = 4
2 × 4 = 8
3 × 4 = 12
4 × 4 = 16
5 × 4 = 20
6 × 4 = 24
7 × 4 = 28
8 × 4 = 32
9 × 4 = 36
10 × 4 = 40

$69 \div 4 = 17r.1$

$69 \div 17 = 4r.1$

$17 \times 4 + 1 = 69$

$4 \times 17 + 1 = 69$

Learners need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example $62 \div 8$ is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

For example: I have 62p. Sweets are 8p each. How many can I buy?

Answer: 7 (the remaining 6p is not enough to buy another sweet).

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?

Answer: 8 (the remaining 6 apples still need to be placed into a box).

Step 5

Learners will continue to use written methods to solve short division $TU \div U$.

Learners can start to subtract larger multiples of the divisor, e.g. $30x$

Short division $HTU \div U$

$$196 \div 6$$

$$\begin{array}{r} 32 \text{ r } 4 \\ 6 \overline{) 196} \\ \underline{- 180} \\ 16 \\ \underline{- 12} \\ 4 \end{array}$$

Answer:

$$\begin{array}{c} 30x \\ 2x \end{array}$$

↓
32 remainder 4
or 32 r 4

755 ÷ 6

	1	2	5	r.5
6	7	5	5	
-	6	0	0	<u>100</u> × 6
	1	5	5	
-	1	2	0	<u>20</u> × 6
		3	5	
-		3	0	<u>5</u> × 6
			5	

6

+

1	×	6	=	6
2	×	6	=	12
5	×	6	=	30
10	×	6	=	60

10	×	6	=	60
20	×	6	=	120
50	×	6	=	300
100	×	6	=	600

755 ÷ 6 = 125r.5

125 × 6 + 5 = 755

755 + 125 = 6r.5

6 × 125 + 5 = 755

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Learners need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example, $240 \div 52$ is 4 remainder 32, but whether the answer should be rounded up to 5 or rounded down to 4 depends on the context.

Division Grid

Stage 1

755 ÷ 6			
	100		
6	700	50	5
	600		
	100		

Stage 2

	100	20	
6	700	150	5
	600	120	
		30	

Stage 3

	100	20	5
6	700	150	35
	600	120	30
			5

Stage 4

	100	20	5	r5
6	700	150	35	
	600	120	30	

Stage 5

	100	20	5	5/6
6	700	150	35	
	600	120	30	

Stage 6

	100	20	5	.833
6	700	150	35	
	600	120	30	

Step 6

Learners will continue to use written methods to solve short division $TU \div U$ and $HTU \div U$.

Long division $HTU \div TU$

$$972 \div 36$$

$$\begin{array}{r} 27 \\ 36 \overline{) 972} \\ \underline{- 720} \\ 252 \\ \underline{- 252} \\ 0 \end{array}$$

Answer: 27

Diagram illustrating the steps: $20 \times$ and $7 \times$ are shown in a circle, with an arrow pointing down to the final answer 27.

972 ÷ 36 7

			2	7	
3	6	9	7	2	
	-	7	2	0	<u>20</u> × 36
		2	5	2	
	-	1	8	0	<u>5</u> × 36
			7	2	
	-		7	2	<u>2</u> × 36
				0	

1	×	36	=	36
2	×	36	=	72
5	×	36	=	180
10	×	36	=	360

10	×	36	=	360
20	×	36	=	720
50	×	36	=	1800
100	×	36	=	3600

$972 - 36 = 27$	$972 \div 36 = 27$
$27 \times 36 = 972$	$36 \times 27 = 972$

Any remainders should be shown as fractions, i.e. if the learners were dividing 32 by 10, the answer should be shown as $3 \frac{2}{10}$ which could then be written as $3 \frac{1}{5}$ in its lowest terms.

$87.5 \div 7$

Answer:

12.5

9



Learners need to have a range of calculation methods, mental and written.
Selection will depend upon the numbers involved.

Learners should not be made to go on to the next stage if:

- 1) they are not ready;
- 2) they are not confident.

Learners should be encouraged to approximate their answers before calculating.

Learners should be encouraged to check their answers after calculation using an appropriate strategy.

Learners should be encouraged to consider if a mental calculation would be appropriate before using written methods.

