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L4 Mathematics Exam

ACDA4008

Electronic Engineering Year 1

11/01/2022

1.) a.) i.)

$$y = \frac{x^2}{2x+1} \qquad u = x^2 \quad u' = 2x \qquad v = 2x+1 \quad v' = 2$$

$$\frac{dy}{dx} = \frac{(2x+1)(2x) - x^2(2)}{(2x+1)^2}$$

$$= \frac{4x^2+2x-2x^2}{(2x+1)^2}$$

$$= \frac{2x^2+2x}{(2x+1)^2}$$

$$\frac{dy}{dx} = \frac{2x(x+1)}{(2x+1)^2}$$

ii.) $y = \sqrt{x^2+1}$

$y = (x^2+1)^{\frac{1}{2}}$

let $u = x^2+1$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2}u^{-\frac{1}{2}} \times 2x$$

$$= \frac{x}{u^{\frac{1}{2}}}$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{x^2+1}}$$

1.) a.) iii.) $y = 3x^{\frac{3}{2}} + \frac{3}{x^2} - 6x = 3x^{\frac{3}{2}} + 3x^{-2} - 6x$

$\frac{dy}{dx} = \frac{3}{2} \cdot 3 \cdot x^{\frac{1}{2}} - 6x^{-3} - 6$

$= \frac{9}{2}\sqrt{x} - \frac{6}{x^3} - 6$

iv.) $y = \sec^3(4x)$

$y = \frac{1}{\cos^3(4x)}$

let $u = \cos(4x)$ $\therefore y = \frac{1}{u^3} = u^{-3}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= -3u^{-4} \cdot -\sin(4x) \cdot 4$

$= 12u^{-4}\sin(4x)$

$\frac{dy}{dx} = \frac{12\sin(4x)}{\cos^4(4x)}$

v.) $y = (x + \sin(2x))$

$\frac{dy}{dx} = 1 - \cos(2x) \cdot 2$

$\frac{dy}{dx} = 1 - 2\cos(2x)$

1.) a.) vi.) $y = \frac{1-10x}{(2x-1)^2}$

$u = 1-10x \quad u' = -10$

$v = (2x-1)^2 \quad v' = 2(2x-1)(2)$

$= 4(2x-1)$

$$\frac{dy}{dx} = \frac{(2x-1)^2(-10) - (1-10x)\cdot 4(2x-1)}{(2x-1)^4}$$

$$= \frac{-10(2x-1) - 4(1-10x)}{(2x-1)^3}$$

$$= \frac{-20x + 10 - 4 + 40x}{(2x-1)^3}$$

$$= \frac{20x+6}{(2x-1)^3}$$

$$= \frac{2(10x+3)}{(2x-1)^3}$$

b.) $y = \frac{1}{x} + \cos(2x)$

$y = \frac{1}{x} = x^{-1}$

$y' = -x^{-2}$

$y'' = 2x^{-3}$

$y''' = -6x^{-4}$

$y'''' = 24x^{-5}$

$y''''' = -120x^{-6}$

$\uparrow \sin \searrow$

$-\cos \quad \cos$

$\nwarrow -\sin \swarrow$

$y = \cos(2x)$

$y' = -\sin(2x)\cdot 2$

$y'' = -\cos(2x)\cdot 4$

$y''' = \sin(2x)\cdot 8$

$y'''' = \cos(2x)\cdot 16$

$y''''' = \cos(2x)\cdot 32$

$$\frac{d^5y}{dx^5} = -120x^{-6} + 32\cos(2x)$$

$$= 4(8\cos(2x) - 30x^{-6})$$

$$= 8(4\cos(2x) - 15x^{-6})$$

$$= 8\left[4\cos(2x) - \frac{15}{x^6}\right]$$

1) c.)

$$f(x) = \ln\left(\operatorname{cosec}(x) - \cot(x)\right)$$

$$1 = \cos^2(x) + \sin^2(x)$$

$$-\sin^2(x) = \cos^2(x) - 1$$

$$\sin^2(x) = 1 - \cos^2(x)$$

$$f'(x) = \frac{1}{\operatorname{cosec}(x) - \cot(x)}$$

$$= \frac{1}{\frac{1}{\sin(x)} - \frac{\cos(x)}{\sin(x)}}$$

$$= \frac{1}{\left(\frac{1-\cos(x)}{\sin(x)}\right)}$$

$$= \frac{\sin(x)}{1-\cos(x)} \times \frac{1+\cos(x)}{1+\cos(x)}$$

$$= \frac{\sin(x) + \sin(x)\cos(x)}{1 - \cos^2(x)}$$

$$= \frac{\sin(x) + \sin(x)\cos(x)}{\sin^2(x)}$$

$$= \frac{1+\cos(x)}{\sin(x)}$$

Q2.) a.) i.)

$$\int \frac{2x}{(4+3x^2)}\,dx$$

let $u = 4+3x^2$, $du = 6x\,dx$

$$\int \frac{2x}{u} \cdot \frac{du}{6x}$$

$$\frac{1}{3}\int \frac{1}{u}\,du$$

$$\frac{1}{3}\ln|u| + C$$

$$\frac{1}{3}\ln|4+3x^2| + C$$

ii.)

$$\int_0^4 e^{\frac{1}{2}x}\,dx$$

$$\left[\frac{e^{\frac{1}{2}x}}{\frac{1}{2}}\right]_0^4$$

$$\left[2e^{\frac{1}{2}x}\right]_0^4$$

$$2e^2 - 2$$

$$2(e^2-1)$$

2.) a.) iii.)

$$\int_0^4 \frac{5x+13}{(2x+1)(x+4)}\,dx$$

$$\int_0^4 \frac{5x+13}{2x^2+9x+4}\,dx$$

let $u = 5x+13$ $\quad du = 5\,dx$

$dv = (2x^2+9x+4)^{-1}dx$ $\quad v = 2x^2+9x+4$

$$= \left[(5x+13)(2x^2+9x+4)\right]_0^4 - 5\int_0^4 2x^2+9x+4\,dx$$

$$= \left[10x^3+45x^2+20x+6x^2+27x+12\right]_0^4 - 5\left[\frac{2x^3}{3}+\frac{9x^2}{2}+4x\right]_0^4$$

$$= \left[10x^3+51x^2+47x+12\right]_0^4 - 5\left[\frac{2x^3}{3}+\frac{9x^2}{2}+4x\right]_0^4$$

$$= \left[640+51(16)+188+12-12\right] - 5\left[\frac{2}{3}(64)+\frac{9}{2}(16)+16\right]$$

$$= 1644 - 5\left[\frac{392}{3}\right]$$

$$= \frac{2972}{3}$$

2b.) $\int \frac{4}{x(1+4\ln x)^2}\,dx$

$4\int \left[x(1+4\ln x)^2\right]^{-1}\,dx$

$4\left[\ln\left[x(1+4\ln x)^2\right]\right]+C$

$\cancel{4\ln(x+4x\ln}$

$4\ln\left(x(1+4\ln x)^2\right)+C$

$4\left[\ln(x)+\ln\left((1+4\ln x)^2\right)\right]+C$

$4\left[\ln(x)+2\ln(1+4\ln x)\right]+C$

$\cancel{4[\ln(x)+2\ln}$

$4\ln(x)+8\ln(1+4\ln(x))+C$

2.) c.) i.) $\displaystyle\int_{\ln(2)}^{\ln(5)} \frac{3e^{2x}}{(e^x-1)^{\frac{1}{2}}}\,dx$

$u = 3e^{2x} \qquad du = 6e^{2x}\,dx$

$dv = (e^x-1)^{-\frac{1}{2}}dx \qquad v = \cancel{\ln\left((e^x-1)^{\frac{1}{2}}\right) = \frac{1}{2}\ln(e^x-1)}\ \dfrac{2(e^x-1)^{\frac{1}{2}}}{e^x}$

$= uv - \int v\,du$

$\cancel{= \left[3e^{2x}\cdot\frac{1}{2}\ln(e^x-1)\right]_{\ln(2)}^{\ln(5)} - \frac{1}{2}\int_{\ln(2)}^{\ln(5)} \ln(e^x-1)\cdot 6e^{2x}\,dx}$

$= 3e^{2x}\left(\dfrac{2(e^x-1)^{\frac{1}{2}}}{e^x}\right) - \displaystyle\int_{\ln(2)}^{\ln(5)} \frac{2(e^x-1)^{\frac{1}{2}}}{e^x}\,6e^{2x}\,dx$

$= \left[6e^x(e^x-1)^{\frac{1}{2}}\right]_{\ln(2)}^{\ln(5)} - 12\displaystyle\int_{\ln(2)}^{\ln(5)} e^x(e^x-1)^{\frac{1}{2}}\,dx$

2.) c.) ii.)

$$\int_1^e \ln(x)\,dx$$

$$\left[x\ln(x) - x\right]_1^e$$

$$\left[e\ln(e) - e\right] - \left[\ln(1) - 1\right]$$

$$\cancel{0 - [(0) - 1]}$$

$$\underline{\underline{1}}$$

$$\left[e(1) - e\right] - \left[(0) - 1\right]$$

$$0 + 1$$

$$\underline{\underline{1}}$$

3.) a.) i.) $\left(\frac{1+i}{1-i}\right)^3 = \left(\frac{(1+i)(1+i)}{(1-i)(1+i)}\right)^3 = \left(\frac{1+2i-1}{1+1}\right)^3 = (i)^3 = \underline{\underline{0-i}}$

$= \left(\frac{2i}{2}\right)^3$

ii.) $\frac{3}{(1+i)(2+i)} = \frac{3}{2+3i-1} = \frac{3}{1+3i} \times \frac{1-3i}{1-3i} = \frac{3-9i}{1+9} = \underline{\underline{\frac{3}{10} - \frac{9}{10}i}}$

iii.) $\frac{1+i}{1-i} - (1+2i)(2+2i) + \frac{3-i}{1+i}$

$$= \frac{(1+i)^2}{2} - (2+6i-4) + \frac{3-i}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{1+2i-1}{2} + 2 - 6i + \frac{3-3i-i-1}{1+1}$$

$$= i + 2 - 6i + \frac{2-4i}{2}$$

$$= 2 - 5i + 1 - 2i$$

$$= \underline{\underline{3-7i}}$$

3.) a.) iv.) $2i(i-1) + (\sqrt{3}-i) + (1+i)(\overline{1+i})$

$= (-2-2i) + (\sqrt{3}-i) + (1+i)(1-i)$

$= (-2+\sqrt{3} - 3i) + (1+1)$

$= -4+2\sqrt{3} - 6i$

b.) $z = (4+7i)(3-2i)$

$= 12 - 8i + 21i + 14$

$z = 26 + 13i$

$|z| = \sqrt{(26)^2+(13)^2}$

$= \sqrt{845}$

$|z| = 13\sqrt{5}$

$\arg(z) = \operatorname{atan}\left(\frac{13}{26}\right)$

$= 1.11^c$ (3sf)

$= 13\sqrt{5}\left(\cos(1.11) + i\sin(1.11)\right)$ (3sf)

c.) $= \frac{1+3i}{1+i} = \frac{(1+3i)(1-i)}{(1+i)(1-i)} = \frac{1-i+3i+3}{2} = \frac{4+2i}{2} = 2+i$