

Q1.a) Simplify: $G(s) = \frac{K}{(s+1)(s+6)(s+3)}$ (1mark)

Then with unity feedback and gain of 63, the open loop TF becomes:

$$GH(s) = \frac{63}{(s+1)(s+6)(s+3)}$$

Let s approach $j\omega$:

$$GH(s) = \frac{63}{(j\omega+1)(j\omega+6)(j\omega+3)} \quad (1mark)$$

$$\therefore |GH(j\omega)| = \frac{63}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+6^2})(\sqrt{\omega^2+3^2})} \quad (1mark)$$

$$|GH(j\omega)| = \frac{63}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+36})(\sqrt{\omega^2+9})} \quad (1mark)$$

$$\therefore |GH(j\omega)|_{dB} = 20 \log_{10} \left(\frac{63}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+36})(\sqrt{\omega^2+9})} \right) \quad (1mark)$$

$$\angle GH(j\omega) = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{6}\right) - \tan^{-1}\left(\frac{\omega}{3}\right) \quad (1mark)$$

$$\therefore |GH(j\omega)|_{dB} = 20 \log_{10} \left(\frac{63}{(\sqrt{(0.32)^2+1})(\sqrt{(0.32)^2+36})(\sqrt{(0.32)^2+9})} \right) \\ = 10.4 \text{ dB} \quad (1mark)$$

$$\angle GH(j\omega) = -\tan^{-1}(0.32) - \tan^{-1}\left(\frac{0.32}{6}\right) - \tan^{-1}\left(\frac{0.32}{3}\right) = -26.62^\circ \quad (1mark)$$

1(b) From the figure DC gain = 12dB

$$G(\text{dB}) = 20 \log k \quad (1 \text{ mark})$$

$$12 = 20 \log k \quad (1 \text{ mark})$$

$$\frac{12}{20} = \log k \quad (1 \text{ mark})$$

$$k = 10^{12/20} = 3.98 \approx 4 \quad (1 \text{ mark})$$

Therefore in standard form :

$$G(j\omega) = \frac{4 \left(1 + \frac{j\omega}{2}\right)}{\left(1 + \frac{j\omega}{15}\right) \left(1 + \frac{j\omega}{20}\right)} \quad (2 \text{ marks})$$

$$= \frac{4 \times \left(\frac{15 \times 20}{2}\right) \times (j\omega + 2)}{(j\omega + 15)(j\omega + 20)} \quad (2 \text{ marks})$$

$$= \frac{600 (j\omega + 2)}{(j\omega + 15)(j\omega + 20)} \quad (1 \text{ mark})$$

$$= \frac{600 (s + 2)}{(s + 15)(s + 20)} \quad (1 \text{ mark})$$

c) Annotate figure and show phase never reaches -180 Therefore Gain Margin method cannot be used (2 marks)

Phase Margin method shows that at 0dB phase is approx -85° (Annotations to show this) (2 marks)

$$\text{PM} = -85 - (-180) = 95^\circ \quad (2 \text{ marks})$$

Since this is greater than 45° , system is stable (1 mark)

Q2 a) i) $s^3 + 5s^2 + 3s + 2$

Since all coefficients are present and none are negative, this system can be stable.

ii) $s^4 + 4s^3 + s^2 + 2$

Since one coefficient is missing and none are negative, this system is at best critically stable.

iii) $s^5 + 3s^4 + 5s^3 + s^2 - 2s + 4$

All coefficients are present but one is negative, this system is unstable.

(3 marks)

b)

$$G_o = \frac{4}{(s+1)} \times \frac{1}{s(s+2)}$$

$$= \frac{4}{s(s+1)(s+2)}$$

$$= \frac{4}{s(s^2 + 3s + 2)}$$

$$= \frac{4}{s^3 + 3s^2 + 2s}$$

(2 marks)

$$G(s) = \frac{G_o(s)}{1 + G_o(s)H(s)}$$

$$H(s) = K$$

$$= \frac{\frac{4}{s^3 + 3s^2 + 2s}}{1 + \frac{4}{s^3 + 3s^2 + 2s} \times K}$$

$$= \frac{4}{s^3 + 3s^2 + 2s + 4K}$$

(2 marks)

From denominator

$$a_n = 1 \quad a_{n-1} = 3 \quad a_{n-2} = 2 \quad a_{n-3} = 4k$$

(1 mark)

$$\begin{array}{l|ll} s^3 & 1 & 2 \\ s^2 & 3 & 4k \\ s^1 & 2 - \frac{4}{3}k & 0 \\ s^0 & 4k & 0 \end{array}$$

$$b_1 = \frac{3 \times 2 - 1 \times 4k}{3}$$

$$= \frac{6 - 4k}{3} = 2 - \frac{4}{3}k$$

$$C_1 = \frac{(2 - \frac{4}{3}k)4k - 3 \times 0}{2 - \frac{4}{3}k}$$

(8 marks)

$$= 4k$$

For a stable system there must be no sign changes in column 1

$\therefore 2 - \frac{4}{3}k$ must be positive

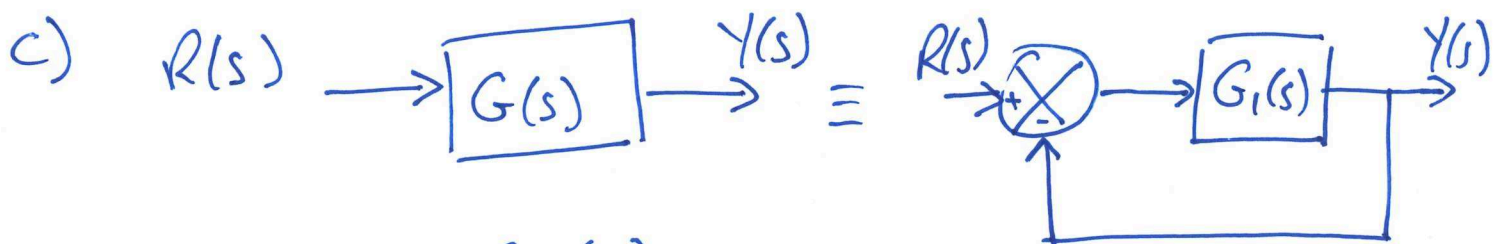
$$2 - \frac{4}{3}k > 0$$

$$2 > \frac{4}{3}k$$

$$\frac{3 \times 2}{4} > k$$

$$\therefore k < 1.5 \text{ and } k > 0$$

(3 marks).



$$\therefore G(s) = \frac{G_1(s)}{1 + G_1(s)}$$

(2 marks)

$$G(s)[1 + G_1(s)] = G_1(s)$$

$$G(s) + G(s)G_1(s) = G_1(s)$$

$$G(s) = G_1(s) - G(s)G_1(s)$$

$$G(s) = G_1(s)[1 - G(s)]$$

$$G_1(s) = \frac{G(s)}{1 - G(s)}$$

(2marks)

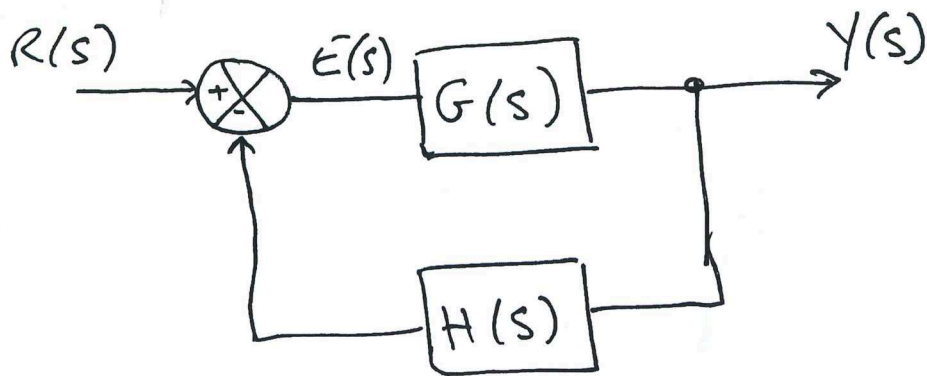
$$\begin{aligned} \therefore G_1(s) &= \frac{4}{s^3 + 3s^2 + 2s + 4k} \\ &= \frac{1}{1 - \frac{4}{s^3 + 3s^2 + 2s + 4k}} \\ &= \frac{4}{s^3 + 3s^2 + 2s + 4k - 4} \\ &= \frac{4}{s^3 + 3s^2 + 2s + 4(k-1)} \end{aligned}$$

Since s cannot be factored out, this system is a type / class 0.

(2marks)

Q3

a)



$$Y(s) = G(s) E(s) \quad \text{--- (1)}$$

$$E(s) = R(s) - H(s) Y(s) \quad \text{--- (2)}$$

(1 mark)

Subs (2) into (1)

$$Y(s) = G(s) [R(s) - H(s) Y(s)]$$

(1 mark)

$$Y(s) = G(s) R(s) - G(s) H(s) Y(s)$$

(1 mark)

$$Y(s) + G(s) H(s) Y(s) = G(s) R(s)$$

(1 mark)

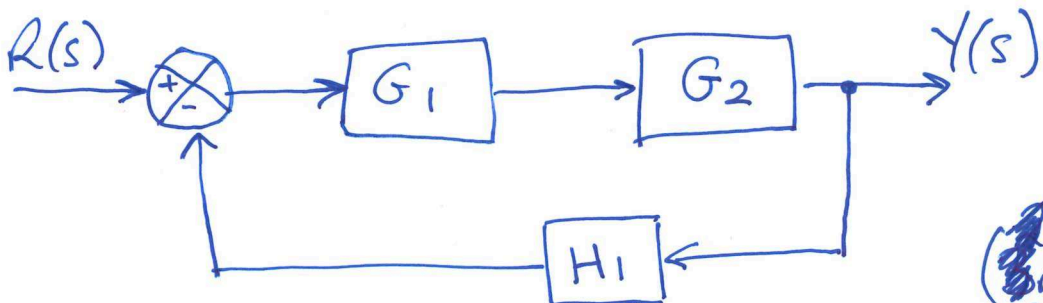
$$Y(s) [1 + G(s) H(s)] = G(s) R(s)$$

(1 mark)

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s) H(s)}$$

(1 mark)

b) i)



(3 marks)

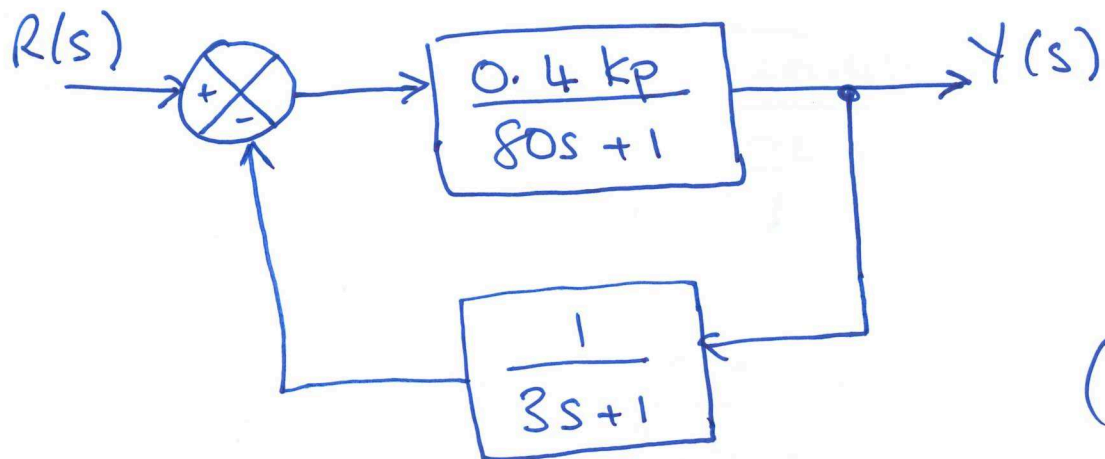
where $G_2 = \frac{0.4}{80s + 1}$

and $H_1 = \frac{1}{3s + 1}$

(2 marks)

$$G_1 = K_p$$

ii) Block diagram becomes



(1 mark)

Using the feedback equation

$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

(1 mark)

$$= \frac{0.4K_p}{80s + 1} \frac{1}{1 + \frac{0.4K_p}{(80s + 1)} \times \frac{1}{(3s + 1)}}$$

(1 mark)

$$= \frac{0.4kp(3s + 1)}{(80s + 1)(3s + 1) + 0.4kp}$$

(1 mark)

$$= \frac{1.2kp(s + 1/3)}{240s^2 + 83s + 1 + 0.4kp}$$

$$= \frac{1.2kp(s + 1/3)}{240(s^2 + 0.346s + (1 + 0.4kp)/240)}$$

$$= \frac{1.2kp(s + 1/3)}{240(s^2 + 0.346s + (1 + 0.4kp)/240)}$$

$$= \frac{K_p}{200} \frac{(s + 1/3)}{s^2 + 0.346s + \left(\frac{1 + 0.4K_p}{240} \right)} \quad (4 \text{ marks})$$

when $K_p = 0.2$

$$T(s) = \frac{0.2}{200} \frac{(s + 1/3)}{s^2 + 0.346s + 4.5 \times 10^{-3}} \quad (1 \text{ mark})$$

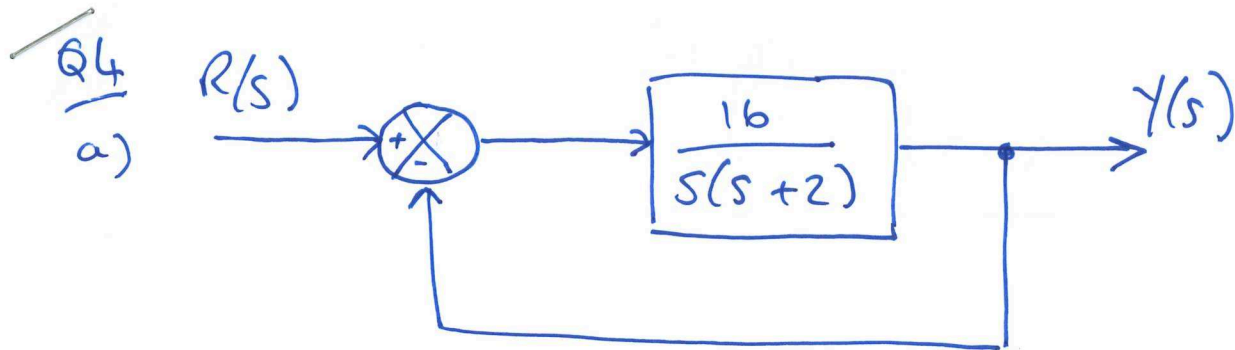
$$\text{Overall gain} = \frac{0.2}{200} = 0.001 \quad (1 \text{ mark})$$

$$y_{ss} = \lim_{s \rightarrow 0} \{ s Y(s) \} = \lim_{s \rightarrow 0} \left\{ s \times \frac{1}{s} \times T(s) \right\} \quad (1 \text{ mark})$$

$$= 0.001 \times \frac{1/3}{4.5 \times 10^{-3}}$$

$$= 0.0741 \quad (1 \text{ mark})$$

$$e_{ss} = 1 - 0.0741 = 0.926 = 92.6\% \quad (2 \text{ marks})$$



$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \quad \text{And since } H(s) = 1 \quad (1 \text{ mark})$$

$$= \frac{\frac{16}{s(s+2)}}{1 + \frac{16}{s(s+2)}} \quad (1 \text{ mark})$$

$$= \frac{16}{s(s+2) + 16} \quad (1 \text{ mark})$$

$$= \frac{16}{s^2 + 2s + 16} \quad (1 \text{ mark})$$

b) compare with $\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (1 \text{ mark})$

i) $\omega_n = \sqrt{16} = 4 \text{ rad/s} \quad (1 \text{ mark})$

ii) $2\zeta\omega_n = 2 \quad \therefore \zeta = \frac{2}{2 \times 4} = 0.25 \quad (2 \text{ marks})$

iii) $\omega = \omega_n \sqrt{1 - \zeta^2} = 4 \sqrt{1 - 0.25^2} = 3.873 \text{ rad/s} \quad (2 \text{ marks})$

iv) $t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1 - \zeta^2}}{\zeta}\right)}{\omega_n \sqrt{1 - \zeta^2}}$

$$= \frac{\pi - \tan^{-1} \left(\frac{\sqrt{1 - 0.25^2}}{0.25} \right)}{3.873}$$

$$= \frac{\pi - \tan^{-1} 3.873}{3.873} = 0.471 \text{ sec} \quad (2 \text{ marks})$$

$$v) t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} = \frac{\pi}{3.873} = 0.811 \text{ sec} \quad (2 \text{ marks})$$

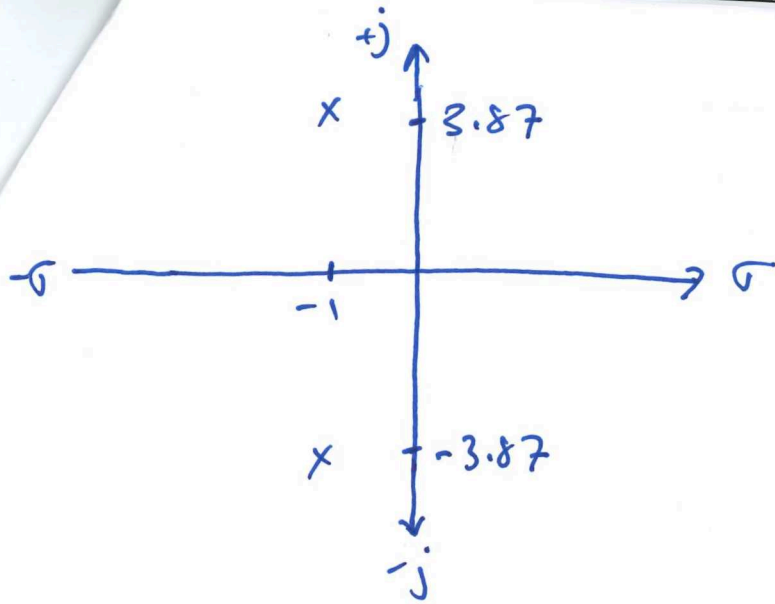
$$\begin{aligned} \text{vi) } \% OS &= 100 e^{\frac{-\zeta \pi}{\sqrt{1 - \zeta^2}}} \\ &= 100 e^{\frac{-0.25\pi}{0.968}} \\ &= 100 e^{-0.811} = 44.4\% \quad (2 \text{ marks}) \end{aligned}$$

$$\text{vii) } t_s \approx \frac{4}{\zeta \omega_n} = \frac{4}{0.25 \times 4} = 4 \text{ sec} \quad (2 \text{ marks})$$

$$\begin{aligned} \text{viii) } n &= \frac{2}{\pi} \sqrt{\frac{1}{\zeta^2} - 1} = \frac{2}{\pi} \sqrt{\frac{1}{0.25^2} - 1} \\ &= 2.466 \quad (2 \text{ marks}) \end{aligned}$$

$$\text{c) Poles found by: } s^2 + 2s + 16 = 0 \quad (1 \text{ mark})$$

$$\begin{aligned} s &= \frac{-2 \pm \sqrt{2^2 - 4(1)(16)}}{2(1)} = -1 \pm j \frac{\sqrt{60}}{2} \\ &= -1 \pm j 3.87 \quad (2 \text{ marks}) \end{aligned}$$

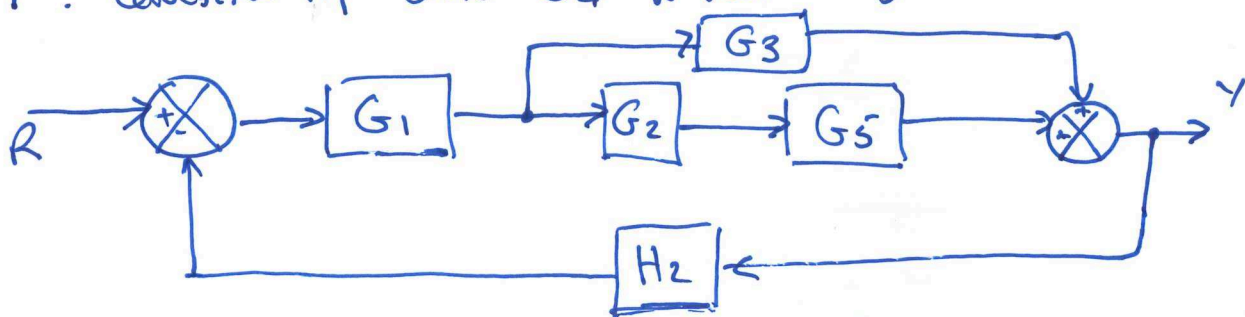


poles are in LHP $\therefore$ stable.

(2 marks)

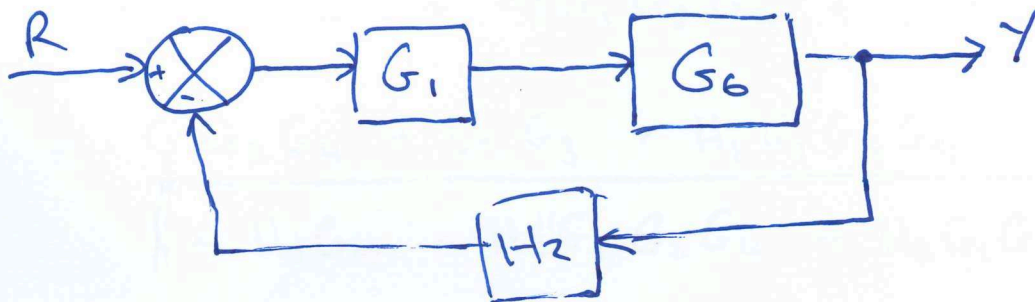
~~Q5~~

Step 1 : Combine H_1 with G_4 to form G_5



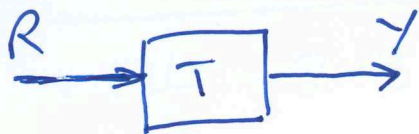
where $G_5 = \frac{G_4}{1 + H_1 G_4}$ (2 marks)

Step 2. Combine G_2 and G_5 in parallel with G_3 to form G_6



where $G_6 = G_2 G_5 + G_3$ (2 marks)

Step 3 Combine G_1 with G_6 with feedback via H_2



where $T = \frac{G_1 G_6}{1 + H_2 G_1 G_6}$ (2 marks)

Subs for G_6

$$\therefore T = \frac{G_1 [G_2 G_5 + G_3]}{1 + H_2 G_1 [G_2 G_5 + G_3]}$$

$$= \frac{G_1 G_2 G_5 + G_1 G_3}{1 + H_2 G_1 G_2 G_5 + H_2 G_1 G_3}$$

(2 marks)

ubs for G_5

$$T = \frac{G_1 G_2 \left[\frac{G_4}{1 + H_1 G_4} \right] + G_1 G_3}{1 + H_2 G_1 G_2 \left[\frac{G_4}{1 + G_4 H_1} \right] + H_2 G_1 G_3}$$

(2 marks)

* top & btm by $(1 + G_4 H_1)$

$$\begin{aligned} T &= \frac{G_1 G_2 G_4 + G_1 G_3 (1 + H_1 G_4)}{(1 + H_1 G_4) + H_2 G_1 G_2 G_4 + (1 + H_1 G_4) H_2 G_1 G_3} \\ &= \frac{G_1 G_2 G_4 + G_1 G_3 + H_1 G_1 G_3 G_4}{1 + H_1 G_4 + H_2 G_1 G_2 G_4 + H_2 G_1 G_3 + H_1 H_2 G_1 G_3 G_4} \end{aligned}$$

(2 marks)

b)

$$\begin{aligned} T &= \frac{3 \times \frac{2}{(s+1)} \times \frac{1}{s} + 3 \times 2 + 1 \times 3 \times 2 \times \frac{1}{s}}{1 + 1 \times \frac{1}{s} + 2 \times 3 \times \frac{2}{(s+1)} \times \frac{1}{s} + 2 \times 3 \times 2 + 1 \times 2 \times 3 \times 2 \times \frac{1}{s}} \\ &= \frac{\frac{6}{s(s+1)} + 6 + \frac{6}{s}}{1 + \frac{1}{s} + \frac{12}{s(s+1)} + 12 + \frac{12}{s}} \end{aligned}$$

(1 mark)

(1 mark)

* top & btm by $s(s+1)$

$$T = \frac{6 + 6s(s+1) + 6(s+1)}{s(s+1) + (s+1) + 12 + 12s(s+1) + 12(s+1)} \quad (2 \text{ marks})$$

$$T = \frac{6 + 6s^2 + 6s + 6s + 6}{s^2 + s + s + 1 + 12 + 12s^2 + 12s + 12s + 12}$$

$$= \frac{6s^2 + 12s + 12}{13s^2 + 26s + 25} \quad (2 \text{ marks})$$

c) Zeros occur where

$$6s^2 + 12s + 12 = 0$$

$$s = \frac{-12 \pm \sqrt{12^2 - 4(6)(12)}}{2(6)}$$

$$= \frac{-12 \pm \sqrt{-144}}{12}$$

$$= -1 \pm j1$$

(2 marks)

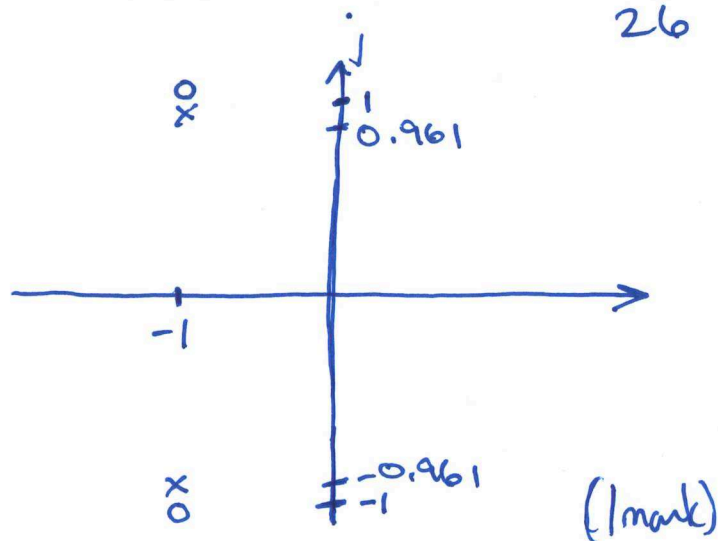
Poles occur where

$$13s^2 + 26s + 25 = 0$$

$$s = \frac{-26 \pm \sqrt{26^2 - 4(13)(25)}}{2(13)}$$

$$= \frac{-26 \pm \sqrt{-624}}{26} = -1 \pm j0.961$$

(2 marks)



Since the poles are in the L.H. plane, the system is stable and since poles have imaginary parts, oscillations will occur before system settles.

(2 marks).