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Stress analysis (ACHA5015)

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Q1)

$$L = 1.5 \text{ m}$$

$$\text{Point load} = 3.5 \text{ kN}$$

$$\text{UDL} = 1500 \text{ N/m}$$

$$EI = 2.2 \times 10^6 \text{ Nm}^2$$

a)

$$\text{Total deflection} = \frac{FL^3}{3EI} + \frac{WL^4}{8EI}$$

$$y_{\text{total}} = \frac{(-3.5 \times 10^3) \cdot (1.5)^3}{3 \cdot (2.2 \times 10^6)} + \frac{(-1500) \cdot (1.5)^4}{8 \cdot (2.2 \times 10^6)}$$

$$y_{\text{total}} = -2.21 \times 10^{-3} \text{ m} \approx -2.2 \text{ mm} \times$$

b) let $y_{\text{total}} = -5 \times 10^{-3}$

$$-5 \times 10^{-3} = \frac{FL^3}{3EI} + \frac{WL^4}{8EI}$$

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$$F \left[\frac{1}{5 \times 10^{-3}} - \frac{8EI}{WL^4} \right] = \frac{3EI}{L^3}$$

$$F = \frac{3EI}{L^3}$$

$$\frac{1}{5 \times 10^{-3}} - \frac{8EI}{WL^4}$$

$$y_t - y_{wdL} = y_{pl}$$

$$-5 \times 10^{-3} - (-4.31 \times 10^{-4}) = y_{pl}$$

$$y_{pl} = -4.56 \times 10^{-3}$$

$$-4.56 \times 10^{-3} = \frac{F(1.5)^3}{3(2.2 \times 10^6)}$$

$$F = -8934 \text{ N} \quad \text{or} \quad -8.934 \text{ kN}$$

$$8.934 - 3.5 = 5.434 \text{ kN} \quad *$$

Q2)

$$\phi_{\text{outer}} = 45 \text{ mm} \quad \phi_{\text{inner}} = 35 \text{ mm} \quad P = 115 \text{ bar}, 115 \times 10^5 \text{ Pa}$$

a)

$$\text{let } P_{\text{inner}} = 115 \times 10^5 \text{ Pa}$$

$$P_{\text{outer}} = 0$$

$$\sigma_r = a - \frac{b}{r^2} \quad \sigma_c = a + \frac{b}{r^2}$$

$$\text{let } r_i = 17.5 \text{ mm} \quad \text{or} \quad 17.5 \times 10^{-3} \text{ m}$$

$$r_o = 22.5 \text{ mm}$$

$$22.5 \times 10^{-3} \text{ m}$$

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b)

$$\sigma_r = a - \frac{b}{r^2}$$

$$-115 \times 10^5 = a - \frac{b}{(17.5 \times 10^{-3})^2}$$

$$-115 \times 10^5 (17.5 \times 10^{-3})^2 = a(17.5 \times 10^{-3})^2 - b$$

$$-3521.875 = a(17.5 \times 10^{-3})^2 - a(5.0625 \times 10^{-4})$$

$$-3521.875 = a[(17.5 \times 10^{-3})^2 - (5.0625 \times 10^{-4})]$$

$$\frac{-3521.875}{(17.5 \times 10^{-3})^2 - (5.0625 \times 10^{-4})} = a \rightarrow a = 17.6 \times 10^6 *$$

$$b = a \times 5.0625 \times 10^{-4}$$

$$b = 8914.7 * \text{ or } 8.914 \times 10^3$$

c) let $a = 17.6 \times 10^6$ $b = 8914.7$

$$\sigma_{r_{inner}} = a + \frac{b}{r^2}$$

$$\sigma_{r_{outer}} = a + \frac{b}{r^2}$$

$$\sigma_{r_{inner}} = 17.6 \times 10^6 + \frac{8914.7}{(17.5 \times 10^{-3})^2}$$

$$= 46.7 \times 10^6 \text{ Pa}$$

$$= \text{or } 46.7 \text{ MPa} *$$

$$\sigma_{r_{outer}} = 17.6 \times 10^6 + \frac{8914.7}{(22.5 \times 10^{-3})^2}$$

$$= 35.2 \times 10^6 \text{ Pa}$$

$$= \text{or } 35.2 \text{ MPa} *$$

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Q3)

$\epsilon_x = -10 \times 10^{-6}$, $\epsilon_y = 13 \times 10^{-6}$

$\nu = 0.20$

$E = 165 \times 10^9$

a)

$$\sigma_x = \frac{E}{1-\nu^2} [\epsilon_x + \nu \epsilon_y]$$

$$= \frac{165 \times 10^9}{1-(0.2)^2} [-10 \times 10^{-6} + (0.2)(13 \times 10^{-6})]$$

$$= -1.27 \times 10^6 \text{ N/m}^2 \quad \text{Compression} *$$

$$\sigma_y = \frac{E}{1-\nu^2} [\epsilon_y + \nu \epsilon_x]$$

$$= +1.89 \times 10^6 \quad \text{tensile} *$$

b) let $\epsilon_x = 20 \times 10^{-6}$, $\epsilon_y = -15$ $\sigma_x = 20 \times 10^6$, $\sigma_y = -15 \times 10^6$
Find ϵ_x, ϵ_y

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y)$$

$$\epsilon_x = \frac{1}{165 \times 10^9} [20 \times 10^6 - (0.2)(-15 \times 10^6)]$$

$$\epsilon_x = 139.4 \times 10^{-6} *$$

$$\epsilon_y = \frac{1}{165 \times 10^9} [-15 \times 10^6 - (0.2 \times 20 \times 10^6)]$$

$$\epsilon_y = -115.2 \times 10^{-6} *$$

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Q4) Thin walled:
 $\phi_{outer} = 200\text{mm}$
 $L = 390\text{mm}$
 $T = 1.5\text{mm}$
 $\mu_L = 0.9$, $\mu_c = 0.6$
 Find P when $\sigma = 100\text{MPa}$

$$\sigma_c = \frac{PD}{2t\mu_L}$$

$$\sigma_c = \frac{PD}{4t\mu_c}$$

$$P = \frac{2t\mu_L\sigma_c}{D}$$

$$P = \frac{4t\mu_c\sigma_c}{D}$$

$$P = \frac{2(1.5 \times 10^{-3}) \times 0.9 \times 100 \times 10^6}{200 \times 10^{-3}}$$

$$P = \frac{4(1.5 \times 10^{-3}) \times 0.6 \times 100 \times 10^6}{200 \times 10^{-3}}$$

$$P = 1.35 \times 10^6 \text{ N/m}^2$$

or 13.5 Bar

$$P = 1.8 \times 10^6 \text{ N/m}^2$$

or $P = 18 \text{ bar}$

$$P_{max} = 13.5 \text{ bar} *$$

Max pressure is irrespective of its length

Q5) Isothermic, length = 350mm, $P = 50 \times 10^6 \text{ Pa}$, $\nu = 0.3$,
 $E = 165 \times 10^9$

a) $K = \frac{E}{3(1-2\nu)} = \frac{165 \times 10^9}{3(1-2(0.3))} = 1.375 \times 10^{11} \text{ N/m}^2 *$

$$K = \frac{-P}{\epsilon_v} \quad \therefore \quad \epsilon_v = \frac{-P}{K} \quad \therefore \quad \epsilon_v = \frac{-50 \times 10^6}{1.375 \times 10^{11}}$$

$$\epsilon_v = -3.64 \times 10^{-4} *$$

$$\epsilon_v = \frac{\Delta V}{V} \quad \therefore \quad \epsilon_v \times V = \Delta V \quad \therefore \quad \Delta V = -3.64 \times 10^{-4} \times (350 \times 10^{-3})^3$$

$$\Delta V = -1.56 \times 10^{-5} \text{ m}^3$$

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- 6) An orthotropic material is one which shows the different ~~same~~ mechanical and Thermal properties in each of its axial directions. Unlike anisotropic materials which may have the same thermal properties in 2/3 directions. An example of an orthotropic material is WOOD.