

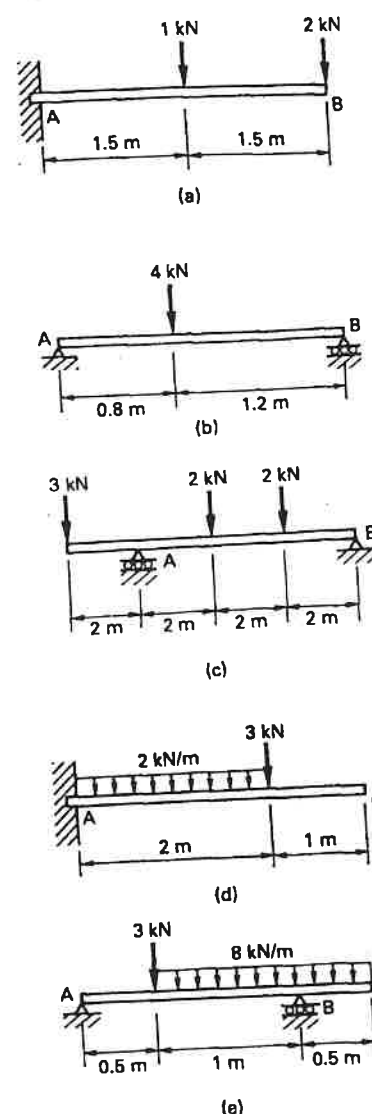


Figure 7.8

$$V\delta x = \delta M$$

As δx approaches zero this can be written as the differential:

$$V = \frac{dM}{dx} \quad [7.1]$$

In words, this means that the shear force is equal to the rate of change of the bending moment diagram. Thus when the shear force is zero the slope of the bending moment is zero, indicating either a maximum or minimum value.

EXAMPLE 7.1

For each of the beams shown in figure 7.8a-e draw the deflected shape, shear force and bending moment diagrams. Indicate the magnitude of all peak values.

Solution

Beam (a)

As usual the first step is to find the reactions. This is clearly a cantilever beam, and there is no reaction at the end B. Therefore the upward reaction at end A must, from vertical equilibrium, equal the total downward force on the beam, i.e. 3 kN. So starting at A we draw a line vertically upwards representing 3 kN (figure 7.9a). Moving rightwards there is no change in vertical load until we reach the first point load where we descend 1 kN. Continuing rightwards there is no further change until the end B where there is a further downward force of 2 kN. Thus the line returns to the baseline indicating that vertical equilibrium is satisfied, i.e. the sum of the downward forces equals the upward reaction.

As far as the bending moment is concerned, because of the point loads, we would expect it to consist of a series of straight lines. We can also see by inspection that these particular loads would cause the beam to deflect downwards at B, producing tension in the top face. We therefore plot the diagram on the top. If we consider the free-body diagram shown, it is clear that the bending moment:

$$M = 2 \times x$$

Hence it has a zero value at B (where $x = 0$) and a value of 3 kNm at the mid-point of the beam. At the support A we must take both point loads into account. Hence:

$$M_A = (2 \times 3) + (1 \times 1.5) = 7.5 \text{ kNm}$$

Remember, if we want the moment at A, then all distances must be measured from A.

Comment – This result tells us that both the wall and the beam connection at A must be capable of resisting a moment of 7.5 kNm if collapse is to be avoided.

Beam (b)

As before we must first find the unknown reactions at A and B before we can start to draw the shear force diagram.

$$\begin{aligned} \Sigma M \text{ about A) } \quad & 4 \times 0.8 = R_B \times 2 \\ & \text{clockwise} \quad \text{anticlockwise} \\ \text{Hence } R_B = & 1.6 \text{ kN } \uparrow \end{aligned}$$

We could find R_A in a similar way, by taking moments about B, but there is an easier way. Simply use vertical equilibrium.

$$\begin{aligned} \Sigma F = 0) \quad & 4 = R_A + 1.6 \\ & \text{downwards} \quad \text{upwards} \\ \text{Hence } R_A = & 2.4 \text{ kN } \uparrow \end{aligned}$$

We are now in a position to draw the shear force diagram. Again start at the left-hand end and follow the direction of the force (figure 7.9b).

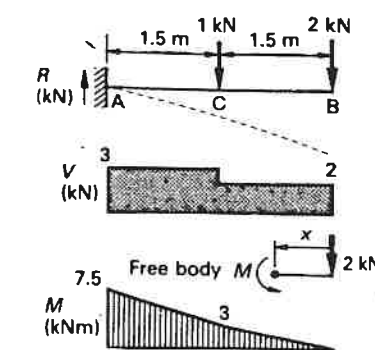
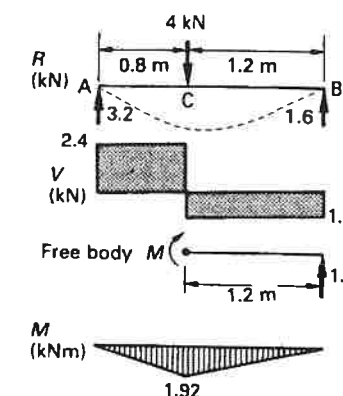
The peak bending moment will occur at the position of the point load, C, and from the free body, is given by:

$$M_C = 1.2 \times R_B = 1.2 \times 1.6 = 1.92 \text{ kNm}$$

Clearly we would get the same answer if, as an alternative, we had considered the equilibrium of the left-hand end:

$$M_C = 0.8 \times R_A = 0.8 \times 2.4 = 1.92 \text{ kNm}$$

This time the bottom of the beam is in tension.


Figure 7.9a
Solution to beam a

Figure 7.9b
Solution to beam b

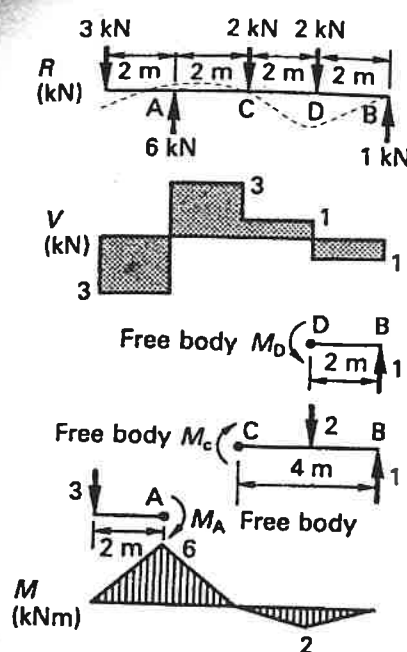


Figure 7.9c
Solution to beam c

Beam (c)

First find the reactions.

$$(\Sigma M \text{ about } A) \quad (2 \times 2) + (2 \times 4) = (3 \times 2) + (R_B \times 6)$$

clockwise anticlockwise

$$R_B = 1 \text{ kN } \uparrow$$

$$(\Sigma V = 0)$$

$$3 + 2 + 2 = R_A + 1$$

downwards upwards

$$R_A = 6 \text{ kN } \uparrow$$

Comment – Examination of the above calculations shows that if the 3 kN load on the cantilever was doubled, the reaction at B would reduce to zero as the moments became balanced about A. A further increase in the load would result in R_B becoming negative, which implies that the beam must be held down at B in order to maintain equilibrium. Thus when calculating reactions, if a negative sign results, it simply means that the direction of the reaction was originally guessed wrongly.

Hence the shear force diagram can be drawn (figure 7.9c).

We can now determine the bending moment at the significant points by breaking the beam, and considering the equilibrium of the resulting free body by summing the moments to one side of the break. We choose either the left-hand or right-hand side depending upon which is the simpler calculation.

$$M_D = -1 \times 2 = -2$$

$$M_C = (-1 \times 4) + (2 \times 2) = 0$$

$$M_A = -3 \times 2 = -6$$

If these values are plotted and joined up by straight lines we have the bending moment diagram.

Comment – In the preceding calculations clockwise moments have been considered positive and anticlockwise moments negative. However, inspection of the bending moment diagram reveals no apparent connection between these signs and the side of the beam on which the bending moment is plotted. This is because we changed from considering the right-hand end to considering the left-hand end. Remember that our convention is always to plot the diagram on the tension side.

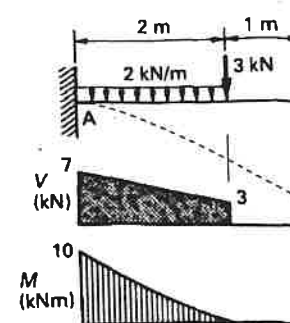


Figure 7.9d
Solution to beam d

Beam (d)

This is a cantilever, similar to beam (a), but with the added complication of a UDL.

$$\text{Reaction, } R_A = 3 + (2 \times 2) = 7 \text{ kN down}$$

The gradually applied load (at the rate of 2 kN/m, i.e. over 2 m, represents a downward force of 4 kN. We then have a 3 kN vertical drop. The last metre is unloaded and hence remains at zero on the shear force diagram (figure 7.9d).

Because of the UDL, the bending moment will be curved.

$$M_A = (3 \times 2) + (2 \times 2 \times 1)$$

$$= 10 \text{ kNm}$$

In the above term for the UDL we multiply the magnitude (2 kN/m) by the length over which it acts (2 m) by the distance from A to the centre of the UDL load (1 m). Figure 7.9d shows the resulting bending moment diagram.

Comment – It is an easy matter to plot the actual shape of the curve by evaluating the bending moment at several intermediate points. Simply cut the structure at any point, and determine the value of M to put the resulting free body into equilibrium.

Beam (e)

Firstly evaluate the reactions:

$$(\Sigma M \text{ about } A) \quad (3 \times 0.5) + (8 \times 1.5 \times 1.25) = R_B \times 1.5$$

$$R_B = 11 \text{ kN } \uparrow$$

$$(\Sigma V = 0) \quad 3 + (8 \times 1.5) = R_A + 11$$

$$R_A = 4 \text{ kN } \uparrow$$

The shear force diagram is shown in figure 7.9e. Look at it carefully and make sure that you understand the shape. A significant point is the fact that the diagram crosses the zero axis at two points, and we have already shown how this indicates maximum or minimum values of bending moment. We can calculate the distance of the zero shear point from B as follows:

$$\text{Distance, } x = 7/8 = 0.875 \text{ m}$$

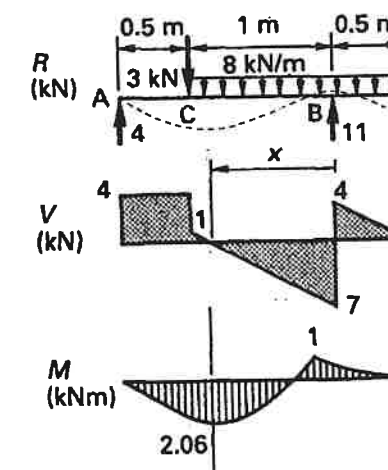


Figure 7.9e
Solution to beam e

The above is simply the value of the shear force at B divided by the slope of the line (which is equal to the UDL).

For the bending moment, firstly calculate M at B by considering the cantilever end to the right:

$$\begin{aligned} M_B &= 8 \times 0.5 \times 0.25 \\ &= 1 \text{ kNm} \end{aligned}$$

Now evaluate the moment at C by considering the left-hand end:

$$M_C = 4 \times 0.5 = 2 \text{ kNm}$$

Lastly evaluate the maximum value of moment which occurs 0.875 m from B:

$$\begin{aligned} M_{\max} &= (8 \times 1.375 \times 1.375/2) - (11 \times 0.875) \\ &= -2.06 \text{ kNm} \end{aligned}$$

The final bending moment diagram is shown in figure 7.9e.

Bending moment diagrams can become quite complicated, however, you should not lose sight of what they mean in physical terms. Quite simply, the shear force and bending moment at any point in a structure are those values necessary to satisfy equilibrium if a free body is considered to one side of the point.

Beams therefore have to resist shear forces which try to deform them as shown in figure 7.10a, and bending moments which try to deform them as shown in figure 7.10b. We shall consider bending moments first, and the next stage is to understand the relationship between the bending moment, M , on a beam and the bending stresses, σ , that this produces in the beam.

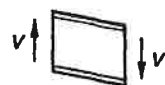


Figure 7.10a
The effect of shear



Figure 7.10b
The effect of bending

7.4 Bending stresses

7.4.1 Approximate bending strength (ignoring the web)

An 'I' beam resists bending by one flange 'pushing' and the other flange 'pulling'. The web also contributes to the bending strength, but it is less effective than the flanges. A crude, but safe, approximation to 'I' beam behaviour is to ignore the contribution of the web to bending strength and hence assume that it is the flanges that do all the work. In practice this would not be an economic