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ADVANCED VEHICLE DYNAMICS
VEHICLE TESTING AND ANALYSIS

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06-01-20

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1 Abstract

The suspension and aerodynamic setup characteristics are analysed for a simulator model of an Aston Martin DBR9 GT1 driving around Silverstone International circuit. The results of each test shall be critically analysed in terms of vehicle balance and handling and compared to logged data for validation.

2 Introduction

The way a vehicle handles is dictated by many different constraints, most prominently suspension and aerodynamic setup. As a vehicle travels around a corner, the outside tyre is loaded more than the inside. This is because of load transfer. As the tyre is loaded, a greater slip angle is required to maintain traction. If this is not met, the tyre will lose grip. If this happens at the front of the vehicle, understeer is experienced, and at the rear, oversteer is experienced. By controlling how the weight transfer is distributed between the front and rear of the vehicle, the balance can be set to as close to what the driver requires as possible (Gillespie, 1992; Bastow *et al.*, 2004).

Moreover, a large amount of available traction in most competition vehicles comes from aerodynamic devices, namely for this investigation, the splitter and wing. As air passes over each device, a pressure gradient creates a net downward force on the car, pushing the tyres harder into the road and increasing traction at the contact patch. These devices can be altered to change their behaviour. Additionally, the vehicle's flat underside means an aerodynamic advantage can be found by increasing the rake angle. This is the difference between front and rear ride height. The floor can encourage a Venturi effect, reducing pressure beneath the vehicle, and allowing the splitter to be closer to the floor, aiding ground effect (Zhang *et al.*, 2006).

For this investigation, the author shall conduct a series of tests to evaluate the effects of different suspension and aerodynamic setups on vehicle balance and handling around the Silverstone International circuit. Vehicle roll stiffness distribution will be varied through spring and anti-roll bar (ARB) rates as well as various damper settings, through changing rates of bump and rebound damping. All tests shall be compared to a base run with the vehicle's standard settings. Moreover, the effect on vehicle handling of changing the position of aerodynamic devices, as well as varying rake angle shall be analysed.

To carry out each test the author, within a group of six, shall use a Base Performance simulator running R-Factor software, the same driver and the Aston Martin DBR9 GT1. The vehicle has a double wishbone, pushrod suspension setup with Koni dampers front and rear. A front splitter and rear wing are mounted, and the vehicle has a flat underside to aid the rake investigation (Gillies, 2007).

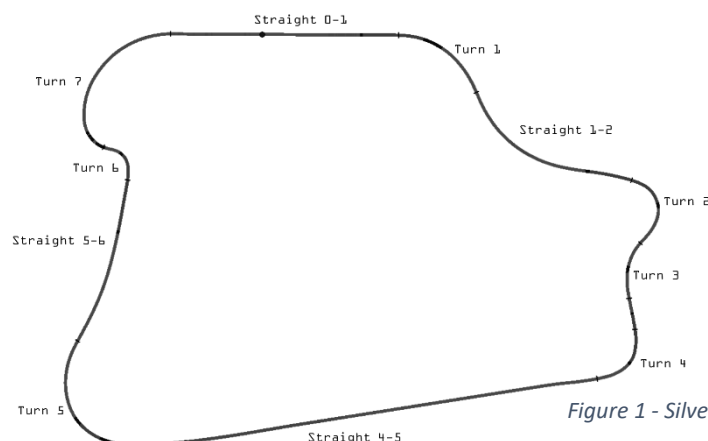


Figure 1 - Silverstone International Circuit

[illegible]

Figure 2 - Test Plan

The group created a plan prior to testing. In total, 37 runs needed to be completed within two, two hour sessions. The initial runs consisted of a base run, which was repeated for validation, two coast-down tests and a slow, steady speed run to gain steering data. The group then completed runs at all combinations of maximum and minimum aerodynamic device settings, and rake settings. These were ordered to make changing setups as quick and easy as possible in a real-life scenario. Similarly the group then completed runs at combinations of maximum and minimum ARB and spring rates before using the same method for bump and rebound damper rates. The final eight runs were further damper combinations, isolating each slow and fast damper rate to analyse how each effect vehicle handling. Figure two shows how the group pre-planned each session and assigned roles to each team member to ensure the smooth-running of testing. After each session, both supervisor Tim Tudor and the group internally gave feedback for future improvement. During the second test session, two files became corrupted. The author and another team member re-ran these tests to eliminate error.

4 Aerodynamic Setup Investigation

4.1 Calculations

☒ Aero Force Front [N]	$(\text{'Tire Load - FL' [N]} + \text{'Tire Load - FR' [N]}) - ((\text{'Static Mass FL - Motec' [kg]} \times \text{'Gravity' [m/s/s]}) + (\text{'Static Mass FR - MoTec' [kg]} \times \text{'Gravity' [m/s/s]})) + (\text{'Longitudinal WT' [N]})$
☒ Aero Force Rear [N]	$(\text{'Tire Load - RL' [N]} + \text{'Tire Load - RR' [N]}) - ((\text{'Static Mass RL - MoTec' [kg]} \times \text{'Gravity' [m/s/s]}) + (\text{'Static Mass RR - MoTec' [kg]} \times \text{'Gravity' [m/s/s]})) - (\text{'Longitudinal WT' [N]})$
☒ Total Aero Force [N]	$(\text{'Aero Force Front' [N]}) + (\text{'Aero Force Rear' [N]})$
☒ Centre of Pressure [%]	$(\text{'Aero Force Front' [N]} / \text{'Total Aero Force' [N]}) \times 100$
☒ Coefficient of Lift Front [ratio]	$(\text{'Aero Force Front' [N]}) / ((0.5 \times \text{'Density of Air' [kg/m3]} \times (\text{'Speed M/S' [m/s]}^2))$
☒ Coefficient of Lift Rear [ratio]	$(\text{'Aero Force Rear' [N]}) / ((0.5 \times \text{'Density of Air' [kg/m3]} \times (\text{'Speed M/S' [m/s]}^2))$
☒ Total Coefficient of Lift [ratio]	$(\text{'Total Aero Force' [N]}) / ((0.5 \times \text{'Density of Air' [kg/m3]} \times (\text{'Speed M/S' [m/s]}^2))$
☒ Total Drag Force [N]	$(0.5 \times (\text{'Density of Air' [kg/m3]} \times (\text{'Speed M/S' [m/s]}^2) \times (\text{'Coefficient of Drag (CD)' [ratio]})$

Figure 3 - Aerodynamics Math Channels

Figure three shows MoTeC math channels used. Equations can be clearly seen in appendix one.

As can be seen in the aerodynamics setup sheet in appendix two, a coast-down test was completed for each setup to find the resistances and drag coefficient values. These values were taken from last year's assignment calculations, but two identical coast-down tests were carried out this year to validate. Repeated data correlated well, with the largest difference only 4% out.

For all 3D graphs, known data was put into an excel table, read into MATLAB and linearly interpolated to fill missing data. The surface function was then used to plot a 3D graph.

4.2 Analysis Graphs

4.2.1 Lift and Drag Variation Over One Lap - Aero Setups

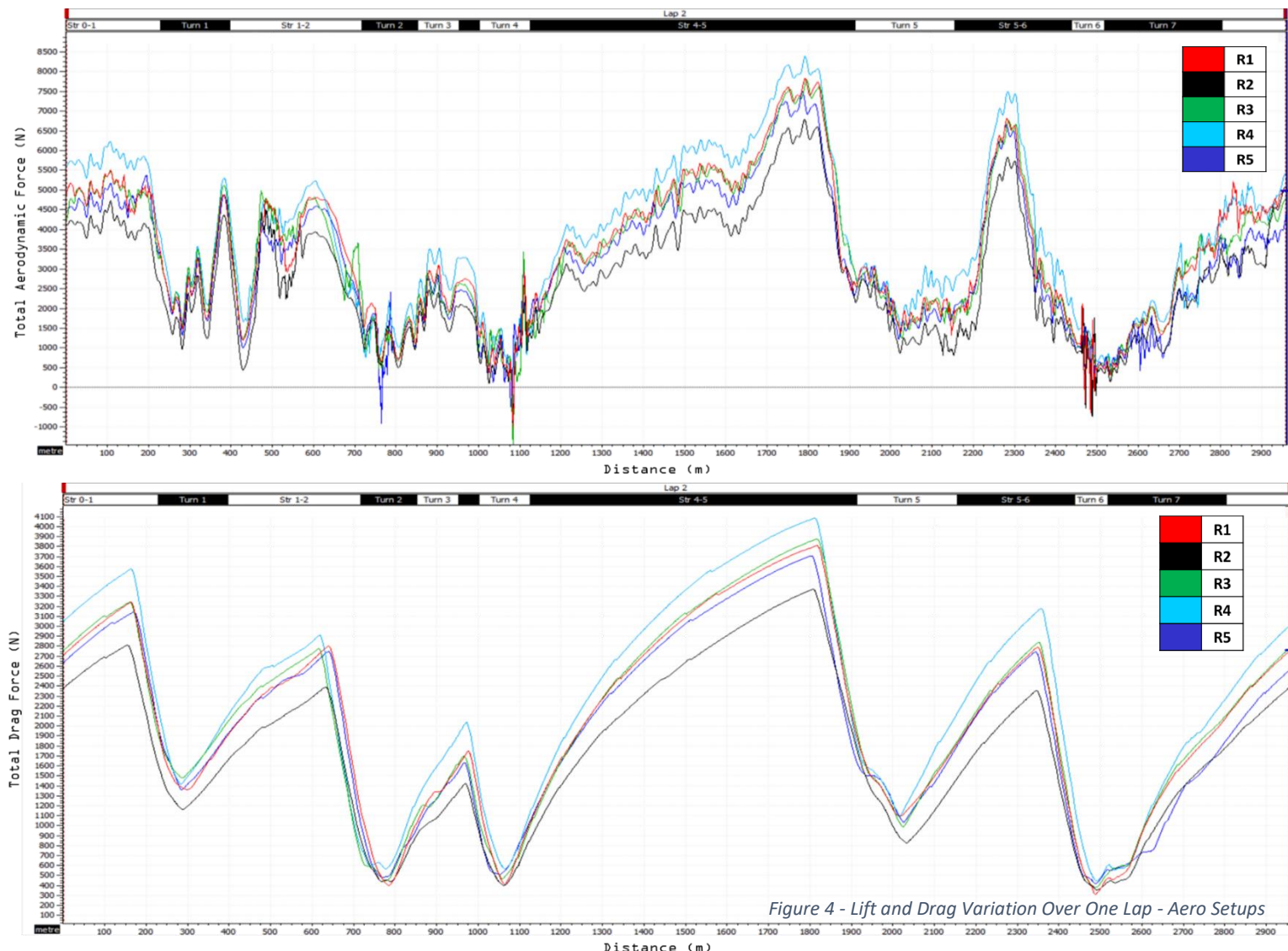


Figure 4 - Lift and Drag Variation Over One Lap - Aero Setups

- Figure four shows a positive correlation between splitter and wing position and lift force, showing the devices are functioning as expected. The highest lift comes from setup R4 with maximum splitter and wing position, minimum from R2 with minimum positions.
- The same trend is seen for drag force produced.
- It is clear that the drag graph is much smoother than the lift graph. This is because lift is calculated considering fluctuating tyre loads, and drag force considering coefficient of drag through Bernoulli's equation. Because of this, it is clear that drag is a product of speed.
- These trends can be seen further in figures six, nine and ten.

4.2.2 Lift and Drag Variation Over One Lap - Rake Setups

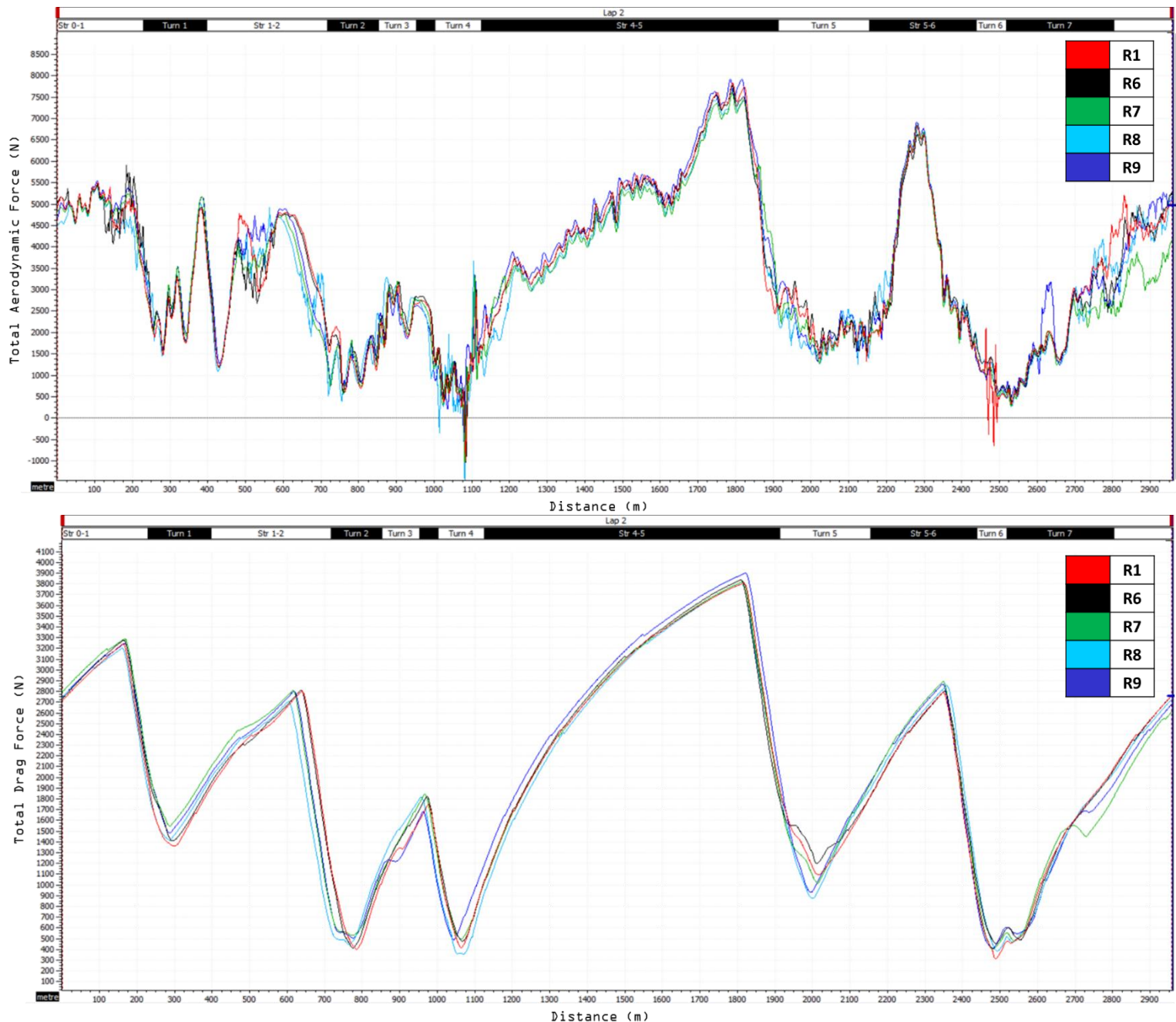


Figure 5 - Lift and Drag Variation Over One Lap - Rake Setups

- Figure five shows a considerably smaller variation in lift with varying rake angles.
- The results are not as expected. Setup R9 gives maximum lift with maximum front and minimum rear ride height. This is negative rake and should reduce lift. Overall, the lowest lift shown is from setup R7, the maximum positive rake angle. Because the effective area of the underside is increased, the diffuser can function better and the splitter gains more ground effect. However the results do not reflect this (Katz, 1995).
- The same effects are seen with drag forces produced.
- It can be concluded that the simulator does model aerodynamic device behaviour correctly, but not when at different heights relative to each other. These results would need to be repeated with real-world data to confirm and validate the findings (Barnard, 2001).

4.2.3 Lift and Drag Forces vs Speed Scatter Plot - Hangar Straight

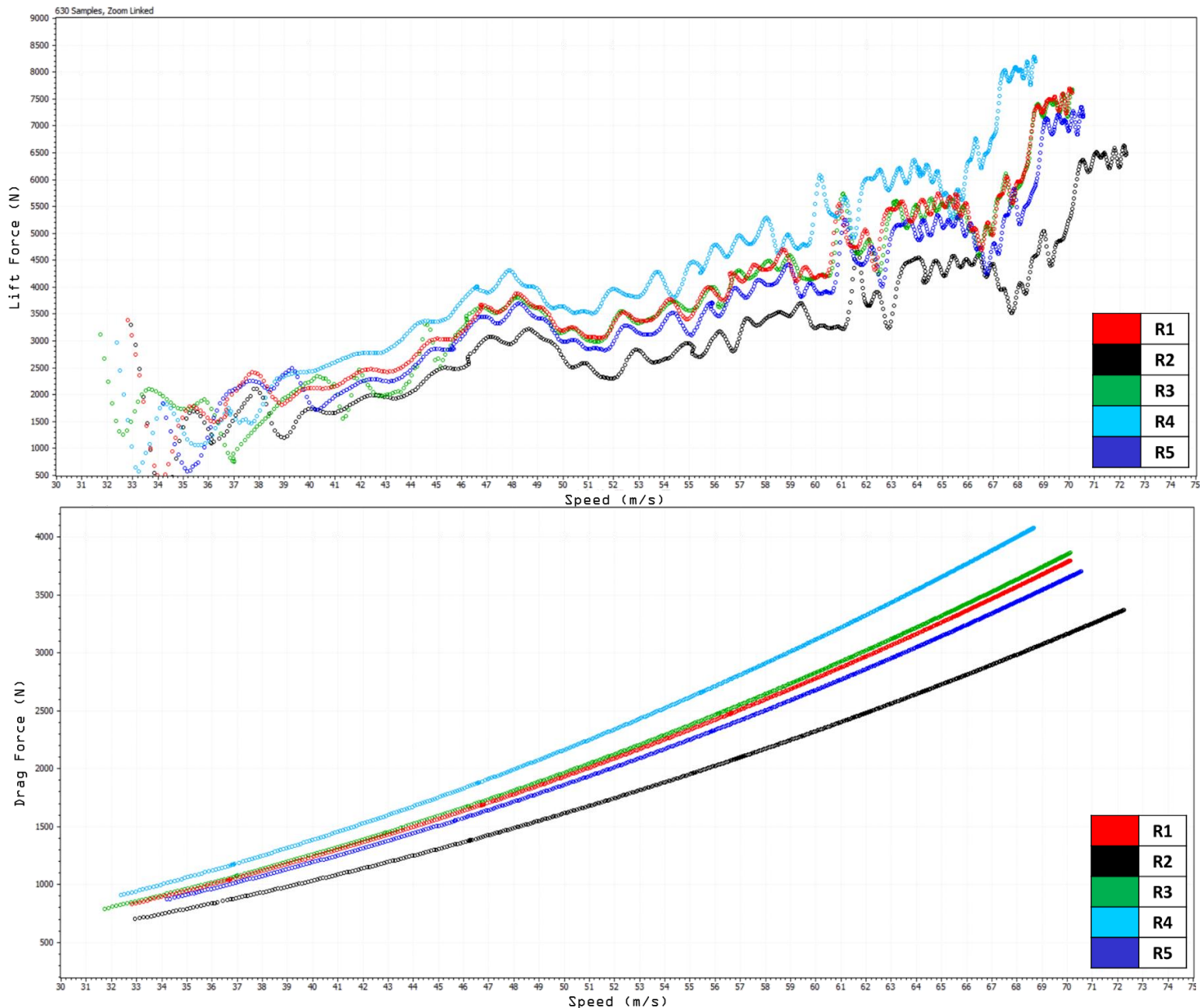


Figure 6 - Lift and Drag Forces vs Speed Scatter Plot - Hangar Straight

- Figure six shows a positive correlation between speed and lift force. Again, the lift trend is less smooth due to a changing C_l value. As speed increases, viscous effects cause the boundary layer to thin, changing the shape of the device's aerofoil. These changing flow conditions are why differing Reynolds numbers are used in aerodynamics calculations. The author could implement a constant C_l value to smooth the graph, but the trends shown confirm what is expected and clearly show the impact of each setup (Olasek and Karczewski, 2012).
- These graphs also confirm the trends seen on figure four with setup R4 showing highest lift and drag forces.

4.2.4 Lateral Acceleration vs Speed Scatter Plot - Whole Lap

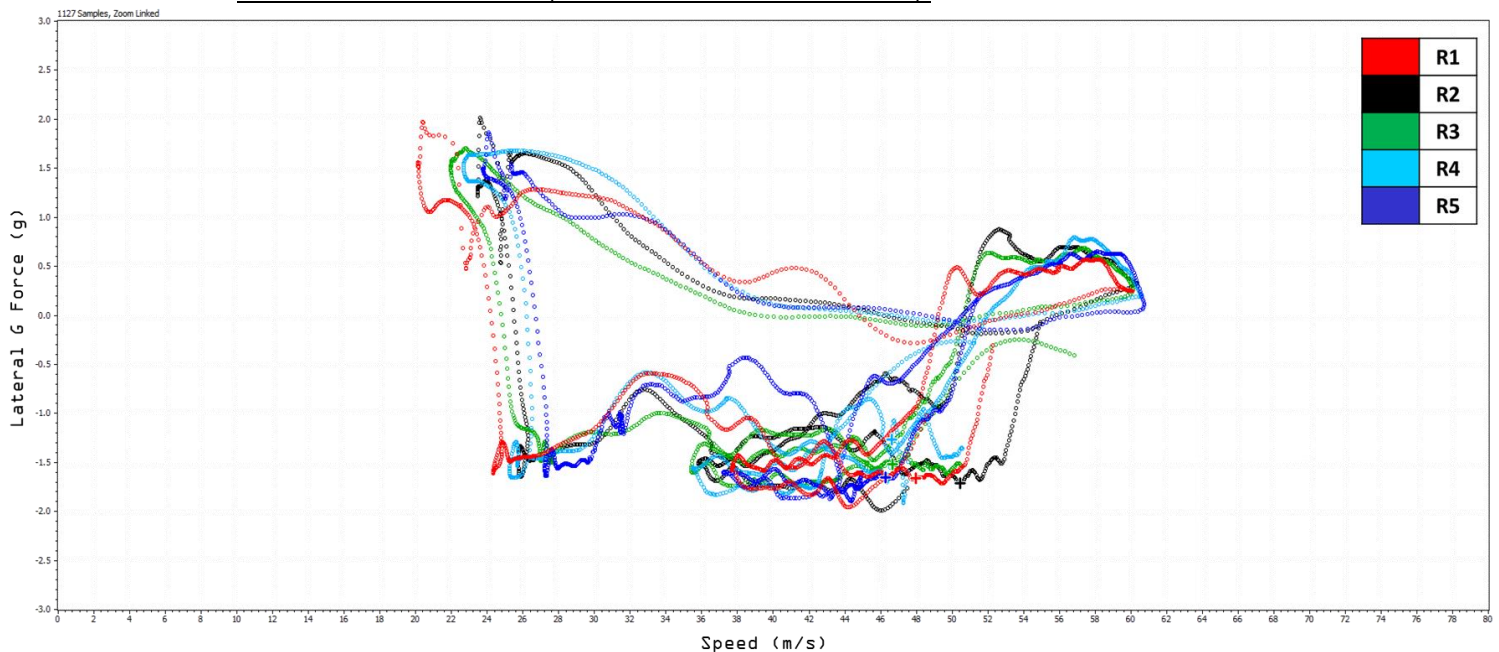


Figure 7 - Acceleration vs Speed Scatter Plot - Whole Lap

- Figure seven shows lateral acceleration vs speed between corners five and seven.
- The radius of corner has a larger impact than speed; the highest acceleration seen is at around 25 m/s. Even at high speed, if the corner radius is high, less lateral acceleration will be seen.
- Aerodynamic setup clearly impacts this. Overall, setup R4 shows the highest accelerations, showing that around both corners, a maximum aero setup will allow more speed to be carried through.

4.2.5 Corner Entry Analysis Graphs

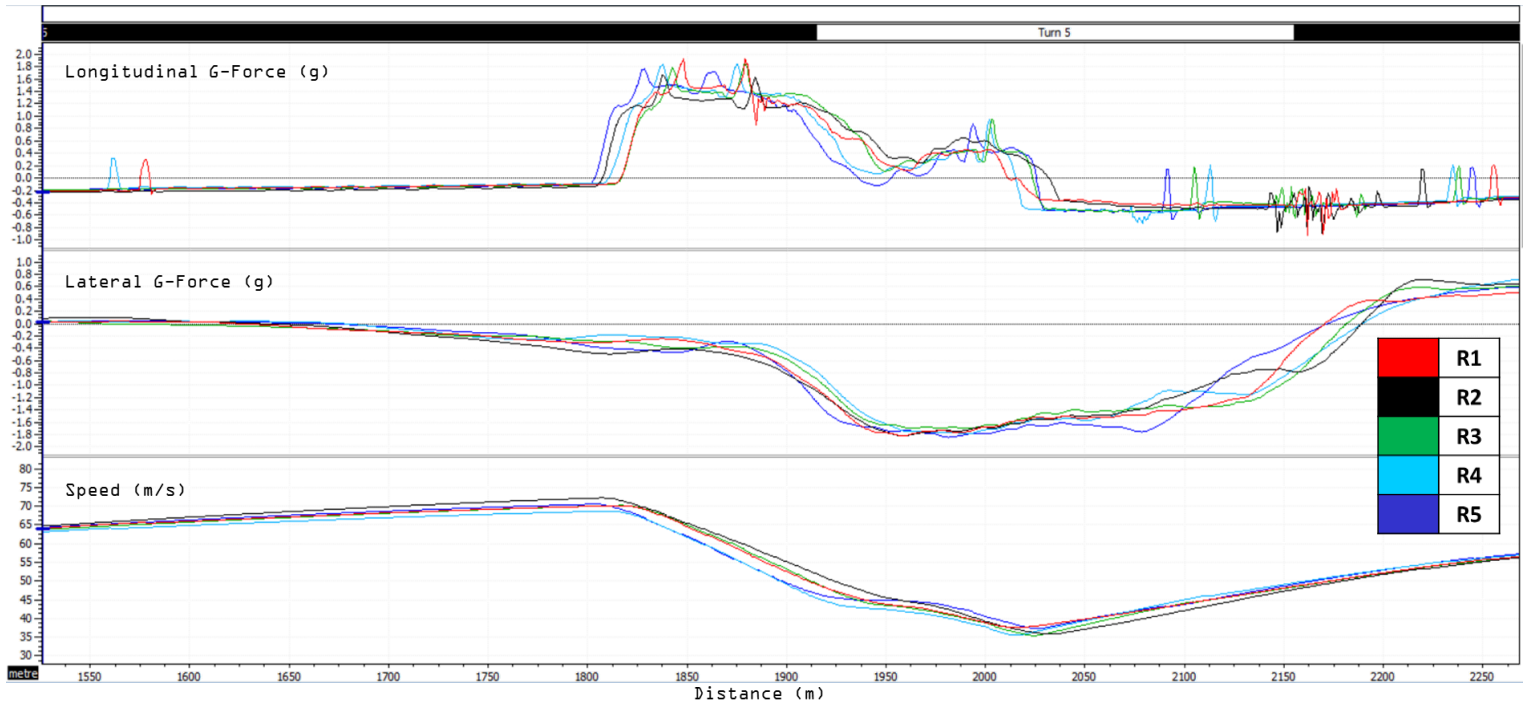


Figure 8 - Corner Entry Analysis Graphs

- Figure eight analyses the entry, apex and exit of turn five. On the straight, setup R2 at minimum aero positions produces the least drag, thus achieving the fastest speed. However, due to lack of traction, the driver is forced to brake earlier. Setup R4 shows a much later braking point with more traction available at maximum wing positions. At the apex of the corner, setup R4 also shows the highest lateral acceleration and velocity as more speed can be carried through the corner.

4.2.6 Coefficient of Lift at Varying Wing and Splitter Positions

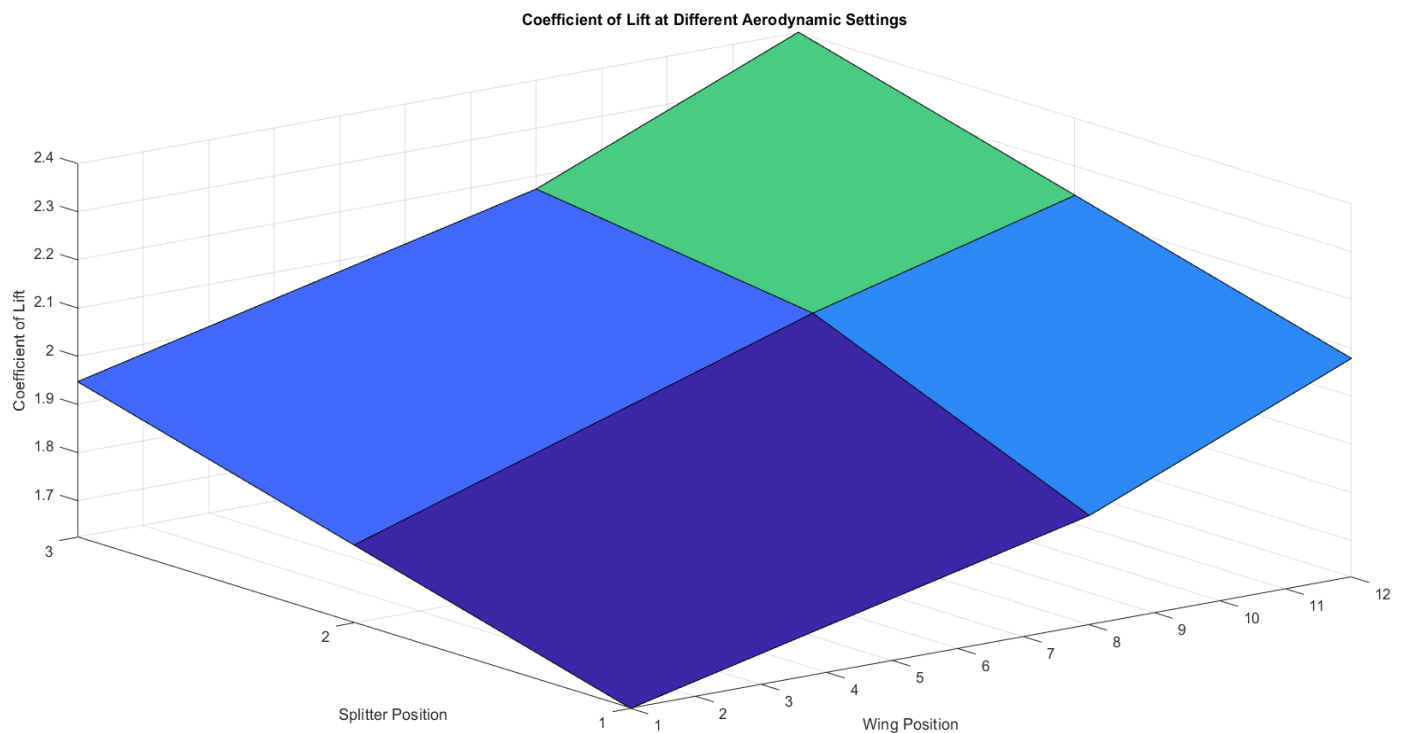


Figure 9 - Coefficient of Lift at Varying Wing and Splitter Positions

- Figure nine confirms trends shown in figure four. Throughout the setups the coefficient of lift increases by 0.78, a 32% increase; this shows the importance of a good wing and splitter setup and how effective a higher position can be.

4.2.7 Coefficient of Drag at Varying Wing and Splitter Positions

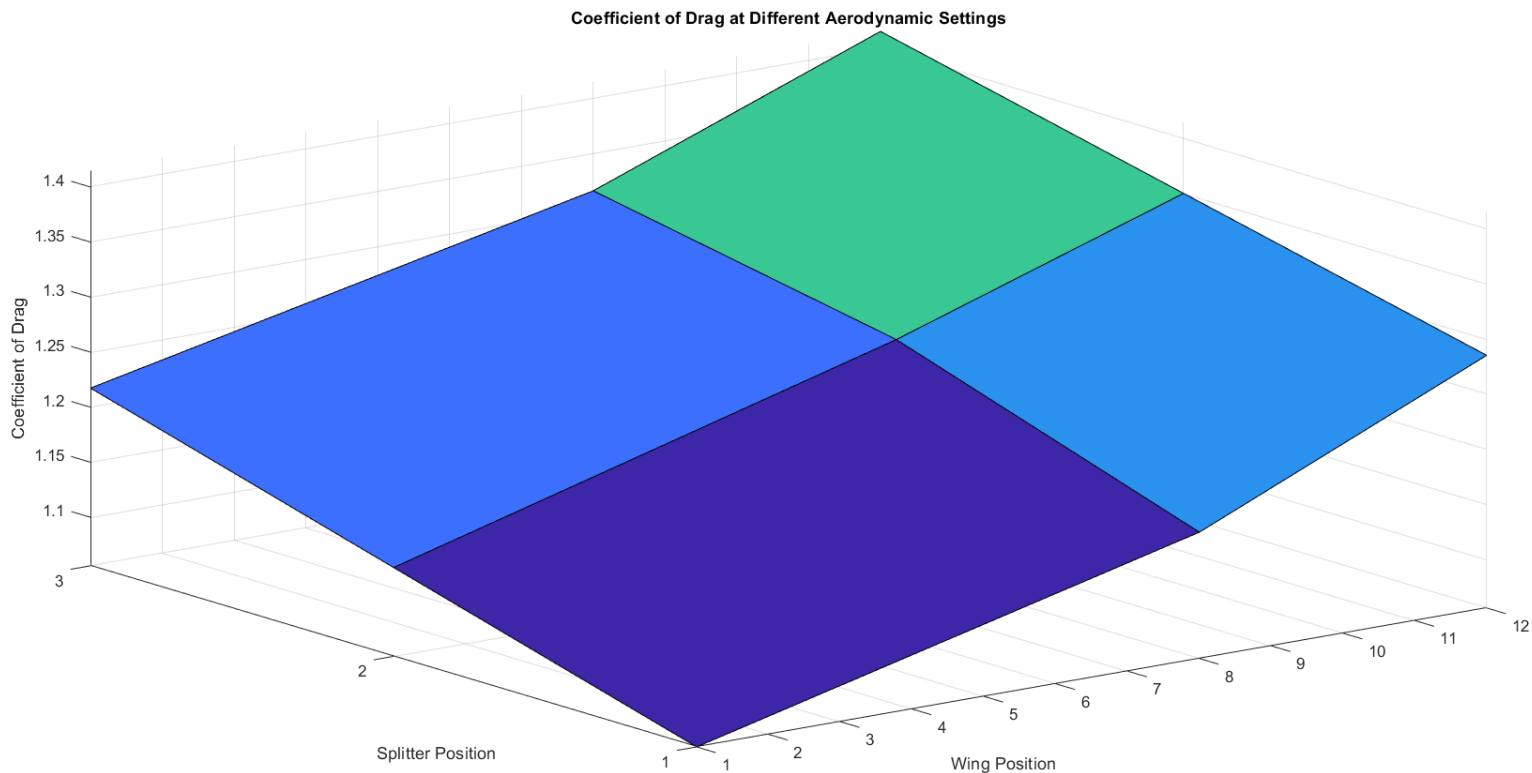


Figure 10 - Coefficient of Drag at Varying Wing and Splitter Positions

- Figure 10 confirms trends shown in figure four. Throughout the setups the coefficient of drag increases by 0.35, an increase of 25%. This is considerably less than C_L increases, which suggests that changing the device positions will not cause drag to overcome lift benefit.

4.2.8 Aerodynamic Efficiency at Varying Wing and Splitter Positions

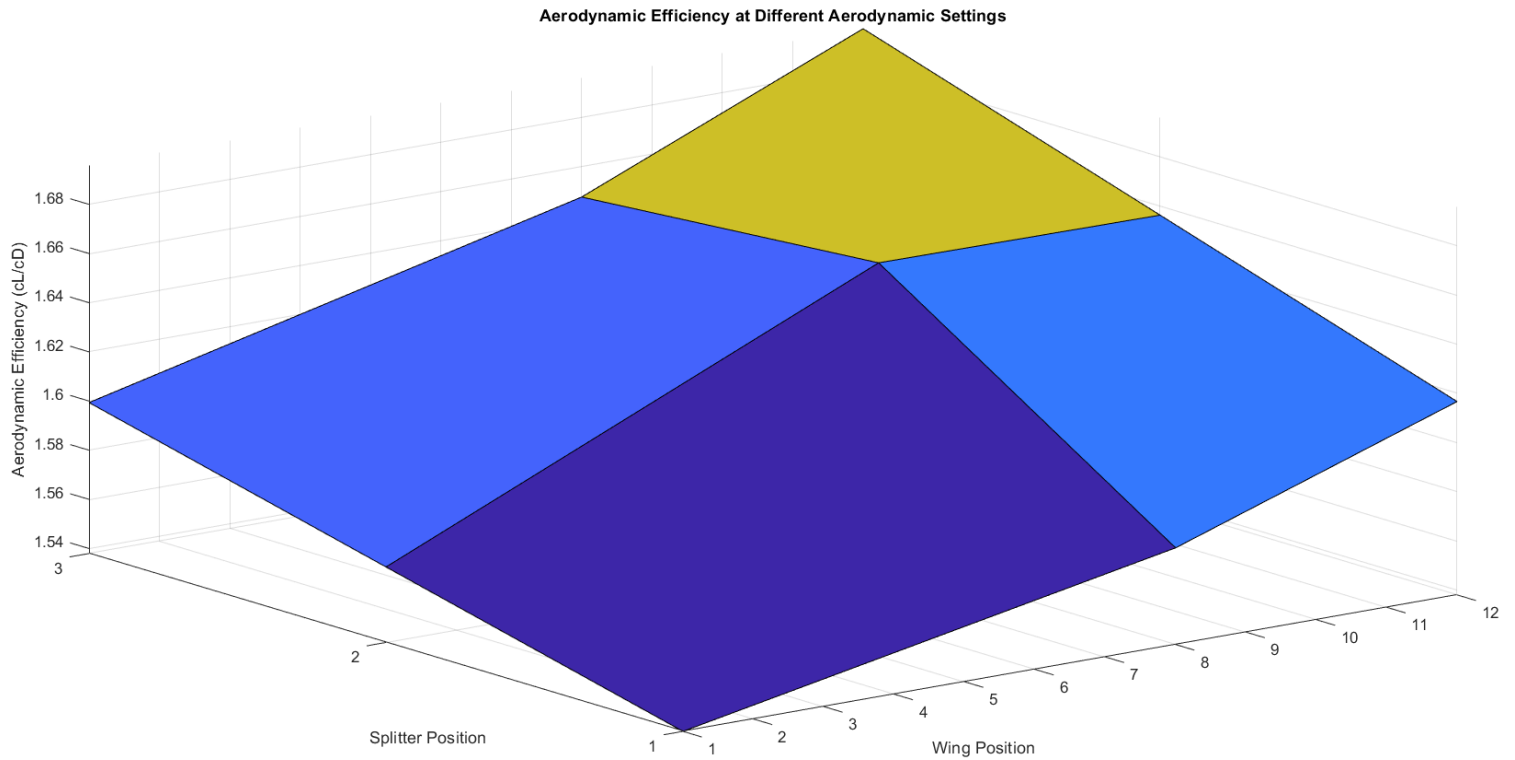


Figure 11 - Aerodynamic Efficiency at Varying Wing and Splitter Positions

- Figure 11 validates the findings of figures nine and ten. Setup R4 shows the greatest aerodynamic efficiency, by 10%. This confirms that the increase in drag with increased device position does not eliminate the increase in lift. However the setup with the highest efficiency is not necessarily the best, as figure twelve shows.

4.2.9 Absolute Centre of Pressure at Varying Wing and Splitter Positions

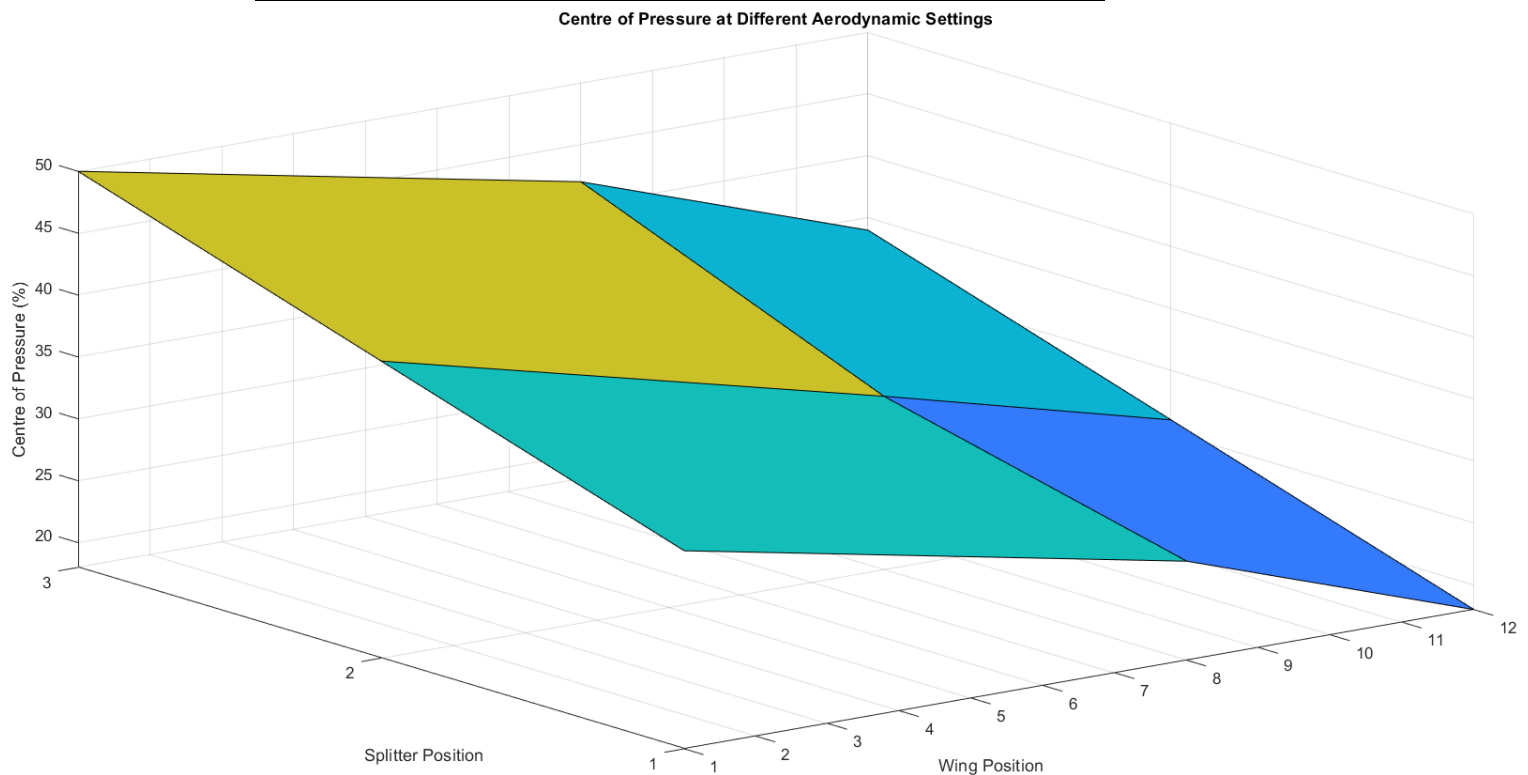


Figure 12 - Absolute Centre of Pressure at Varying Wing and Splitter Positions

- Setup R4 has a centre of pressure of 34%. This places aerodynamic load far back on the vehicle and will disrupt vehicle balance, encouraging understeer (Quintarelli, 2018).
- Overall, as a device's position increases, the centre of pressure moves towards it.
- Each setup shows a centre of pressure towards the rear of the vehicle. This is preferable as the vehicle is rear wheel drive and will increase traction at the drive wheels; this shows that the wing is a much more powerful source of lift than the splitter. Even with maximum splitter and minimum wing, the centre of pressure is 50%. Never biased to the front.
- Centre of pressure can be varied with relation to centre of gravity to influence handling characteristics. When behind the centre of gravity, front grip will reduce, making the vehicle susceptible to understeer. Conversely, when in front, the vehicle may be inclined to oversteer. From the results, it seems that a low wing position and middle splitter position may give an ideal centre of gravity, around 40-45%, ensuring a balance of handling and adequate drive wheel traction (McBeath, 1998; Quintarelli, 2018).

4.2.10 Variation of Centre of Pressure During a Lap

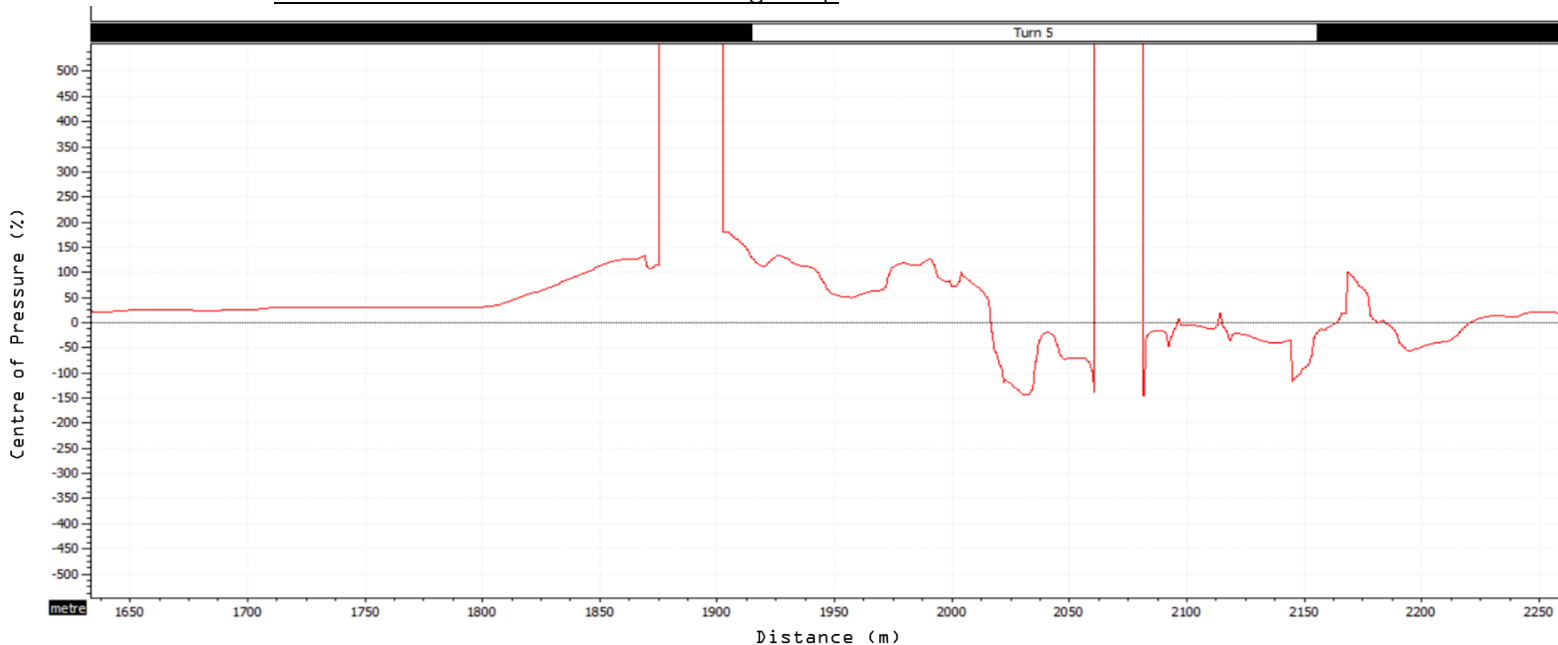


Figure 5b - Centre of Pressure During a Lap

- As a vehicle travels around a track, longitudinal acceleration constantly changes the pitch. This changes the angle of attack of the aerodynamic devices and thus changes the centre of pressure.
- Figure 5b shows the variation of centre of pressure at the standard setup through turn five. As the vehicle brakes at the end of the straight, the front dives bringing the splitter closer to the ground. This creates more ground effect, making the splitter more functional and improving front grip. This brings the centre of pressure forwards (Katz, 1995) .
- This demonstrates the inaccuracy of the simulator. The lift produced by the splitter increases infinitely until an erroneous spike as the splitter hits the floor. In reality there would be a choke point and the lift force would tail off.

4.3 Data Validation

The coefficient of drag for an Aston Martin DB9 road car is given as 0.32. The frontal area is given as 2.03m^2 (SCCA Racing, 2009). For this assignment the author took the frontal area as 1m^2 . Therefore, to validate findings:

$$\text{Calculated } Cd \text{ at standard setup, } 1\text{m}^2 \text{ frontal area} = 1.263$$

$$\therefore Cd \cdot Area = (1.263) \cdot (1) = 1.263$$

$$\therefore Cd \cdot Area \text{ for frontal area of } 2.03\text{m}^2 = \frac{1.263}{2.03} = 0.622$$

$$\therefore Cd = \frac{0.622}{2.03} = 0.306$$

It can be seen that the calculated Cd is 4.3% away from the given value for the road car. This gives a good idea for validation, although the DBR9 GT1 will have a higher frontal area and Cd than the road car, so the error will likely be smaller. These show the results to be trustworthy.

4.4 Limitations and Inaccuracies

The main source of inaccuracy within the aerodynamics data will be from the simulator. There is no indication of a specific splitter length or wing angle, just a generic position. The author could not find real-life information about the standard setup of the vehicle, if this information was available it could have been used to estimate values for the range of positions used.

The simulator does not model flow separation at high device positions. The coefficient of lift will increase seemingly infinitely. At the maximum positions, in the real world less lift would be produced as the large pressure gradient would slow the airflow, causing stall and separation. This would also increase drag, which again appeared to not be modelled (Katz, 1995; Elsenaar, 2000).

For improved accuracy, the group could have collected data from a real-world logging system or validated against one. For real-world data, correction algorithms can be applied to correct for ambient conditions through partial differential error analysis. This analysis can create a uniform, standard environment in which data comes from, reducing the possibility of erroneous results (Emtage, 1983). See Appendix three. It is unknown how the simulator takes such corrections into account. It is likely the simulator sets perfect ambient conditions initially, however how well it does this is again unknown.

Overall, numerical validation shows the results are always within an acceptable tolerance to what is expected. The author believes the tests were carried out correctly and professionally, with enough validation and repetition to be considered trustworthy. Data collected was clearly and securely stored, group members were given specific, clear instructions and the setups differed a sufficient amount. These tests were run in accordance with ISO 10521-1:2006 (ISO, 2011).

5 Suspension Setup Investigation

5.1 Calculations

☒ Front Tyre Load [kg]	$(\text{'Tire Load - FL' [N]} + \text{'Tire Load - FR' [N]}) / \text{'Gravity' [m/s/s]}$
☒ Rear Tyre Load [kg]	$(\text{'Tire Load - RL' [N]} + \text{'Tire Load - RR' [N]}) / 9.81$
☒ Load Transfer Front [N]	$((\text{'Total Mass Front - MoTec' [kg]} * \text{'G Force Lat' [G]} * \text{'Gravity' [m/s/s]}) * \text{'Roll Centre Height Front' [m]}) / \text{'Track Front' [m]}$
☒ Load Transfer Rear [N]	$((\text{'Total Mass Rear - MoTec' [kg]} * \text{'G Force Lat' [G]} * \text{'Gravity' [m/s/s]}) * \text{'Roll Centre Height Rear' [m]}) / \text{'Track Rear' [m]}$
☒ Overturning Moment [Nm]	$\text{'Total Sprung Mass' [kg]} * \text{'G Force Lat' [G]} * \text{'Gravity' [m/s/s]} * \text{'COG Height Above Roll Centre (H)' [m]}$
☒ Roll Angle [rad]	$(\text{'Overturning Moment' [Nm]} / \text{'Total Roll Stiffness' [Nm/rad]})$
☒ Sprung Elastic WTF [N]	$(\text{'Wheel Rate Front' [N/m]} * \text{'Track Front' [m]} * \text{'Roll Angle' [rad]}) / 2$
☒ Sprung Elastic WTR [N]	$(\text{'Wheel Rate Rear' [N/m]} * \text{'Track Rear' [m]} * \text{'Roll Angle' [rad]}) / 2$
☒ Sprung Geometric WTF [N]	$(\text{'Sprung Mass Front' [kg]} * \text{'G Force Lat' [G]} * \text{'Gravity' [m/s/s]} * \text{'Roll Centre Height Front' [m]}) / \text{'Track Front' [m]}$
☒ Sprung Geometric WTR [N]	$((\text{'Sprung Mass Rear' [kg]} * \text{'G Force Lat' [G]} * \text{'Gravity' [m/s/s]}) * \text{'Roll Centre Height Rear' [m]}) / \text{'Track Rear' [m]}$
☒ Unsprung Geometric WTF [N]	$(\text{'Unsprung Mass Front' [kg]} * \text{'G Force Lat' [G]} * \text{'Gravity' [m/s/s]} * \text{'Wheel Centre Height Front' [m]}) / \text{'Track Front' [m]}$
☒ Unsprung Geometric WTR [N]	$((\text{'Unsprung Mass Rear' [kg]} * \text{'G Force Lat' [G]} * \text{'Gravity' [m/s/s]}) * \text{'Wheel Centre Height Rear' [m]}) / \text{'Track Rear' [m]}$
☒ Longitudinal WT [N]	$((\text{'Total Mass - MoTec' [kg]} * \text{'G Force Long' [G]} * \text{'Gravity' [m/s/s]}) * (\text{'Centre of Gravity Height (H+h)' [m]}) / \text{'Wheelbase' [m]}$
☒ WTF - Calculated [N]	$\text{'Sprung Elastic WTF' [N]} + \text{'Sprung Geometric WTF' [N]} + \text{'Unsprung Geometric WTF' [N]}$
☒ WTF - Calculated PLUS DAMPERS [N]	$\text{'Sprung Elastic WTF' [N]} + \text{'Sprung Geometric WTF' [N]} + \text{'Unsprung Geometric WTF' [N]} + \text{'Damper WTF' [N]}$
☒ WTF - MoTeC [N]	$(\text{'Tire Load - FR' [N]} - \text{'Tire Load - FL' [N]}) / 2$
☒ WTR - Calculated [N]	$\text{'Sprung Elastic WTR' [N]} + \text{'Sprung Geometric WTR' [N]} + \text{'Unsprung Geometric WTR' [N]}$
☒ WTR - Calculated PLUS DAMPERS [N]	$\text{'Sprung Elastic WTR' [N]} + \text{'Sprung Geometric WTR' [N]} + \text{'Unsprung Geometric WTR' [N]} + \text{'Damper WTR' [N]}$
☒ WTR - MoTeC [N]	$(\text{'Tire Load - RR' [N]} - \text{'Tire Load - RL' [N]}) / 2$
☒ Damper WTF [N]	$(\text{'Damper Force FR' [N]} - \text{'Damper Force FL' [N]}) / 2$
☒ Damper WTR [N]	$(\text{'Damper Force RR' [N]} - \text{'Damper Force RL' [N]}) / 2$
☒ Overall WT - Calculated [N]	$((\text{'Total Mass - MoTec' [kg]} * \text{'G Force Lat' [G]} * \text{'Gravity' [m/s/s]}) * \text{'COG Height Above Roll Centre (H)' [m]}) / \text{'Track Front' [m]}$
☒ Overall WT - MoTeC [N]	$(\text{'WTF - MoTeC' [N]} + \text{'WTR - MoTeC' [N]})$
☒ Weight Transfer Distribution - Calculated	$((\text{'WTF - Calculated' [N]}) / (\text{'Overall WT - Calculated' [N]})) * 100$
☒ Weight Transfer Distribution - MoTeC	$((\text{'WTF - MoTeC' [N]}) / (\text{'Overall WT - MoTeC' [N]})) * 100$
☒ Damper Weight Transfer Distribution	$(\text{'Damper WTF' [N]}) / (\text{'Damper WTF' [N]} + \text{'Damper WTR' [N]}) * 100$
☒ Contact Patch Load FL [N]	$(\text{'Static Mass FL - MoTec' [kg]} * 9.81) - (\text{'WTF - Calculated' [N]}) - (\text{'Longitudinal WT' [N]}) + (\text{'Aero Force Front' [N]}) / 2$

Figure 13 - Weight Transfer Math Channels

☒ Speed M/S [m/s]	$\text{'Corr Speed' [km/h]} * 0.277778$
☒ Corner Radius [m]	$(\text{'Speed M/S' [m/s]}^2) / (\text{'G Force Lat' [G]} * \text{'Gravity' [m/s/s]})$
☒ Theoretical Wheel Steer Angle [rad]	$(\text{'Wheelbase' [m]} / \text{'Corner Radius' [m]})$
☒ Theoretical Steer % [ratio]	$(\text{'Theoretical Wheel Steer Angle' [deg]} * \text{'Steering X Constant' [-]})$
☒ Understeer	$\text{abs}(\text{'Steering Wheel Pct' [\%]} - \text{abs}(\text{'Theoretical Steer \%' [ratio]}))$

Figure 14 - Steering Math Channels

```
(((('Damper Velocity FL' [m/s] > 0) & ('Damper Velocity FL' [m/s] <= 0.09)) * 'Front Slow Bump Damping Coefficient' [N/m/s]) +
((('Damper Velocity FL' [m/s] > 0) & ('Damper Velocity FL' [m/s] > 0.09)) * 'Front Fast Bump Damping Coefficient' [N/m/s]) +
(((('Damper Velocity FL' [m/s] < 0) & ('Damper Velocity FL' [m/s] >= -0.09)) * 'Front Slow Rebound Damping Coefficient' [N/m/s]) +
(((('Damper Velocity FL' [m/s] < 0) & ('Damper Velocity FL' [m/s] < -0.09)) * 'Front Fast Rebound Damping Coefficient' [N/m/s])
```

Figure 15 - Damper Rates if Statements

☒ Damper Force FL [N]	$(\text{'Damper Velocity FL' [m/s]}) * (\text{'Damper Rate FL' [N/m/s]})$
☒ Damper Force FR [N]	$(\text{'Damper Velocity FR' [m/s]}) * (\text{'Damper Rate FR' [N/m/s]})$
☒ Damper Force RL [N]	$(\text{'Damper Velocity RL' [m/s]}) * (\text{'Damper Rate RL' [N/m/s]})$
☒ Damper Force RR [N]	$(\text{'Damper Velocity RR' [m/s]}) * (\text{'Damper Rate RR' [N/m/s]})$

Figure 16 - Damper Math Channels

Figures 13-16 show the MoTeC math channels used for the suspension calculations. The equations can be found more clearly in appendices four-six.

For constant values for each setup, a MoTeC setup sheet was used. This can be found in appendix seven.

5.2 Analysis Graphs

Weight transfer has been analysed through turn three as it is a fast chicane where weight transfer will be apparent and trends will be clear.

5.2.1 Front and Rear Load Transfer for Varying ARB Rates

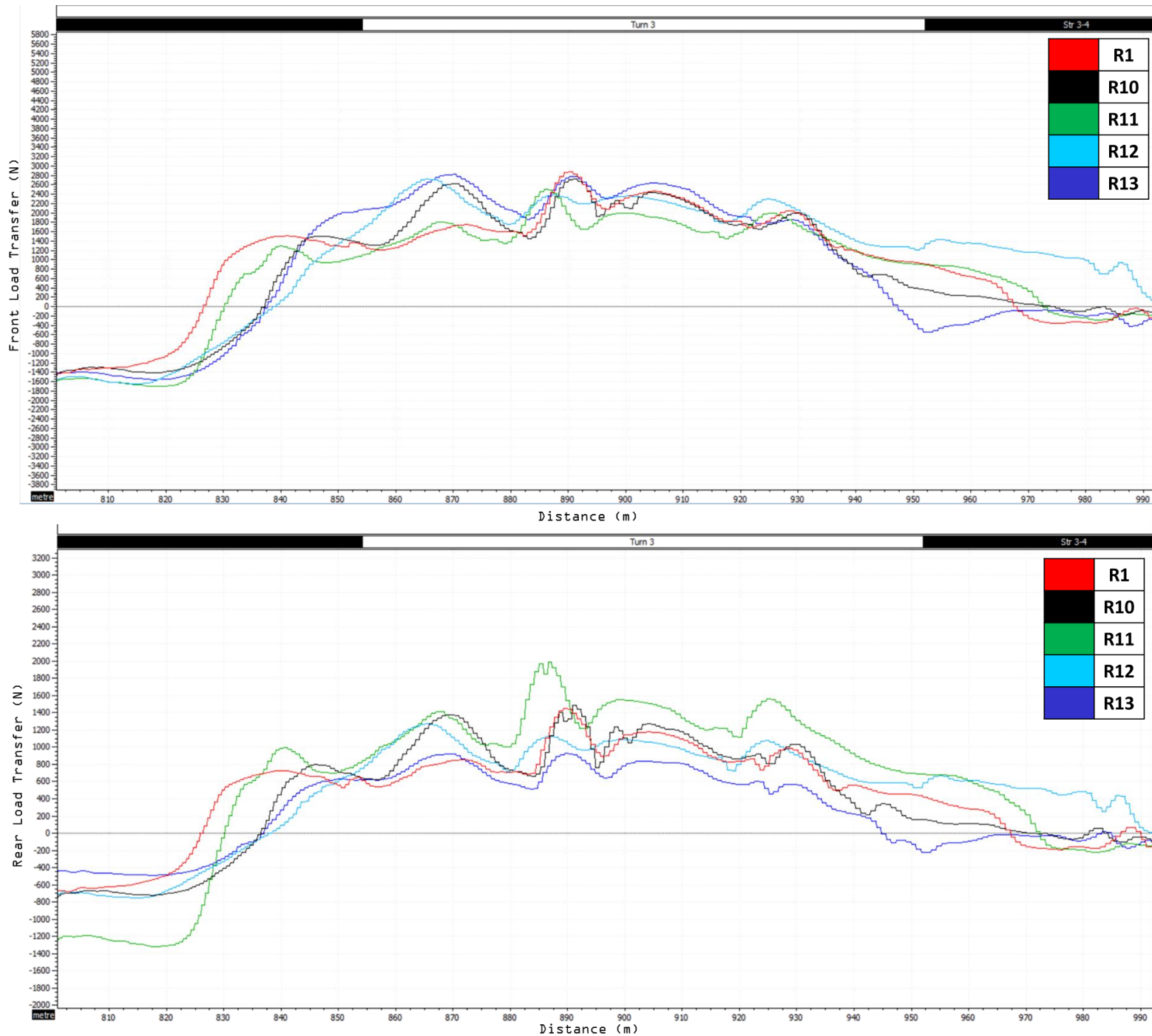


Figure 17 - Front and Rear Load Transfer for Varying ARB Rates

- Figure 17 shows that varying ARB rates has a large effect on weight transfer.
- R13 shows the highest front weight transfer, with maximum front stiffness and minimum rear.
- R11 shows the highest rear weight transfer, with maximum rear stiffness and minimum front.
- This confirms the accepted theory that the highest proportion of weight transfer is seen in the area of the car with the highest roll stiffness (Bastow *et al.*, 2004).

5.2.2 Front and Rear Load Transfer for Varying Spring Rates

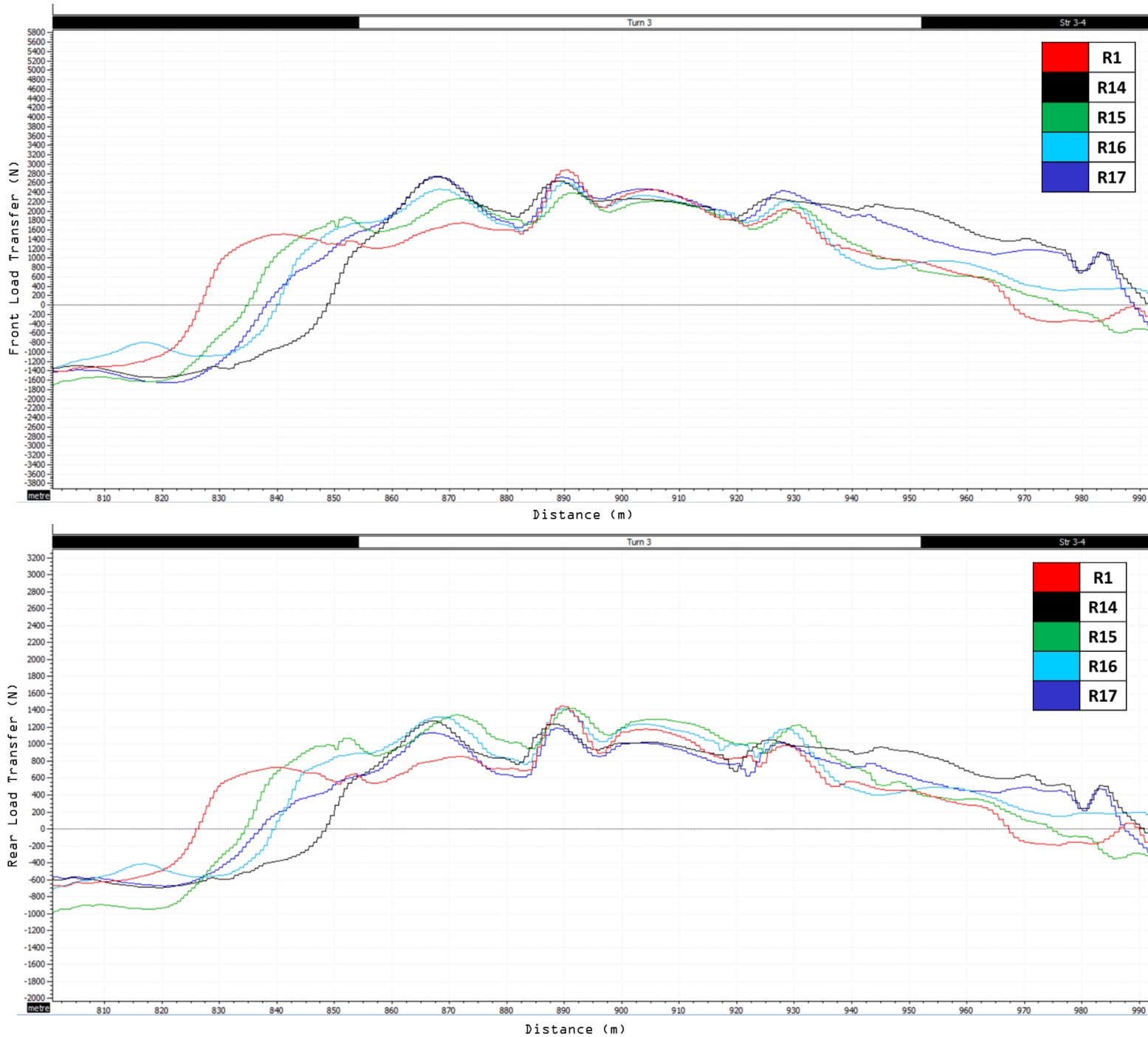


Figure 18 - Front and Rear Load Transfer for Varying Spring Rates

- The same trends are seen as figure 17 with the end of the vehicle with highest roll stiffness distribution experiencing the most weight transfer.
- It is clear that the datasets are much closer together here, showing that varying spring rates effect roll stiffness distribution less than ARB's (Agrawal and Gustafsson, 2017).
- If these results were added, total weight transfer would be the same for each setup. Varying roll stiffness only changes the distribution. Because total weight transfer is quantified as: $\frac{mass \cdot lateral\ accel. \cdot COG\ height}{track}$, varying roll stiffness makes no difference .

5.2.3 Front and Rear Load Transfer for Varying Damping Rates

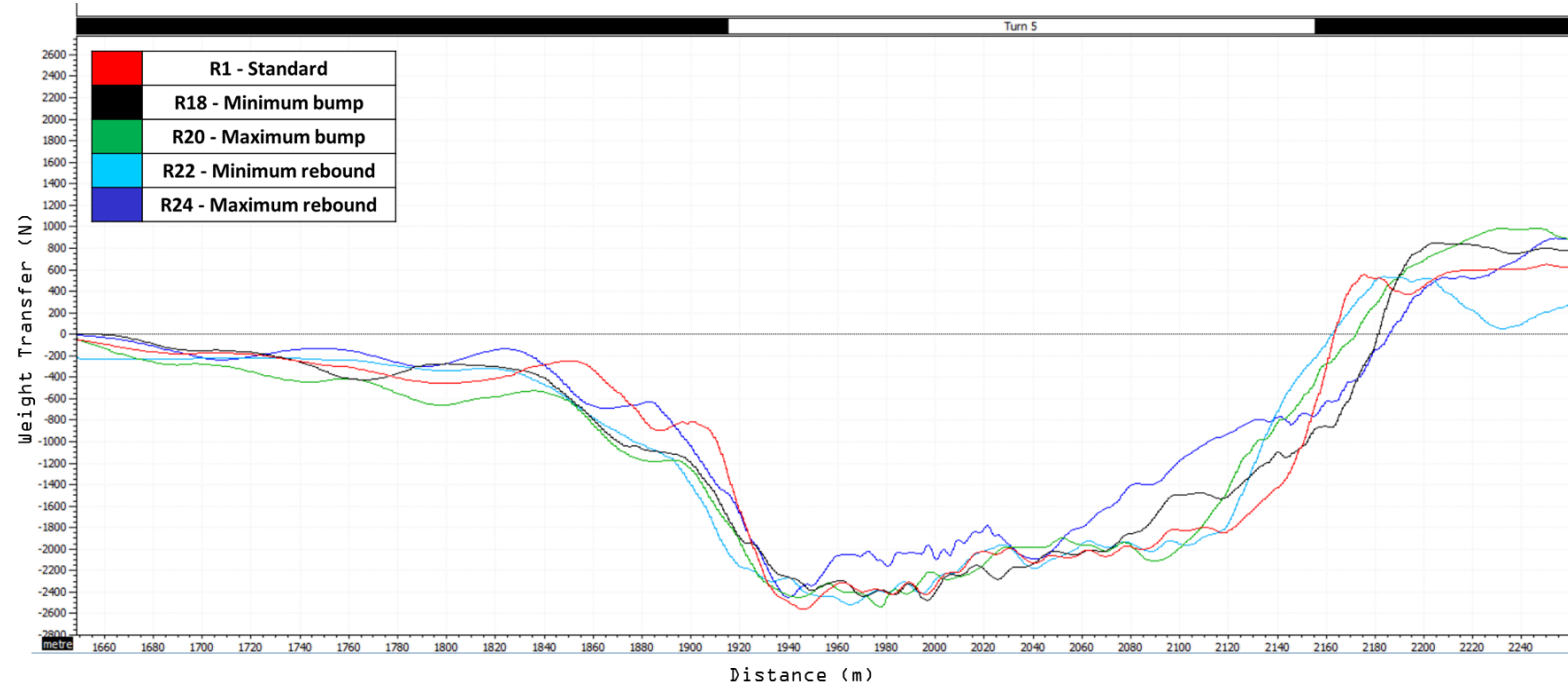


Figure 19 - Front Load Transfer for Varying Damping Rates

- Figure 19 shows how dampers do not alter the amount of weight transfer, but change when the transfer happens.
- Setup R20 starts transferring load onto the outside tyre first at the corner entrance and reaches maximum load first. Setup R18 is considerably slower.
- Setup R20 starts transferring load back to the inside tyre first at the corner exit. Setup R22 is considerably slower.
- These trends are generally apparent but fluctuate somewhat. This is due to both driver inconsistency and how the data is not perfectly synchronised, only against distance on the x axis which is somewhat inaccurate for comparison.
- Rebound controls the unsprung mass, thus controlling the load leaving the tyre. Bump controls the sprung mass, thus controlling the loading of the tyre. However as the outside tyre is loaded, the inside is unloaded so both are always affecting corner entry and exit.
- Higher damping rates speed up weight transfer, softer damping rates slow down weight transfer. This is why softer damping rates are preferable in wet conditions. The car reacts slower and is less inclined to spin (Kasprzak, 2007; Alexander, 2015).
- The technical report from Jim Kasprzak suggests that bump solely controls corner entry, as this is when the outside tyre is loaded, and rebound controls corner exit, when the load is released. The webpage 'Chassis Tuning with Dampers' by Don Alexander disagrees with this, as dampers work oppositely. On entry, rebound controls tyre unloading on the inside whilst bump controls tyre loading on the outside. Information from both sources were cross-referenced and validated before further research.
- On corner entry, the vehicle is usually braking and steering. When steering, the outside dampers compress and inside extend; when braking the front compress and the rear extend. This all happens at once; the opposite is true on corner exit. This explains why dampers are influential in weight transfer.
- Dampers are not often used for major setup changes, but for fine-tuning vehicle balance. If understeering on entry, more traction is needed on the front and less on the rear; a stiffer rear rebound rate will help this. If a stiffer rear bump rate is chosen, the car is likely to oversteer more on exit (Sekulic, 2011).

Roll angle has been analysed through turn one as it is a high speed corner so roll will be apparent. Turn two, a slower corner, is also analysed for comparison. Coming out of turn one, the track is very bumpy hence noise within the data, but trends are still apparent.

5.2.4 Vehicle Roll Angle for Varying ARB Rates

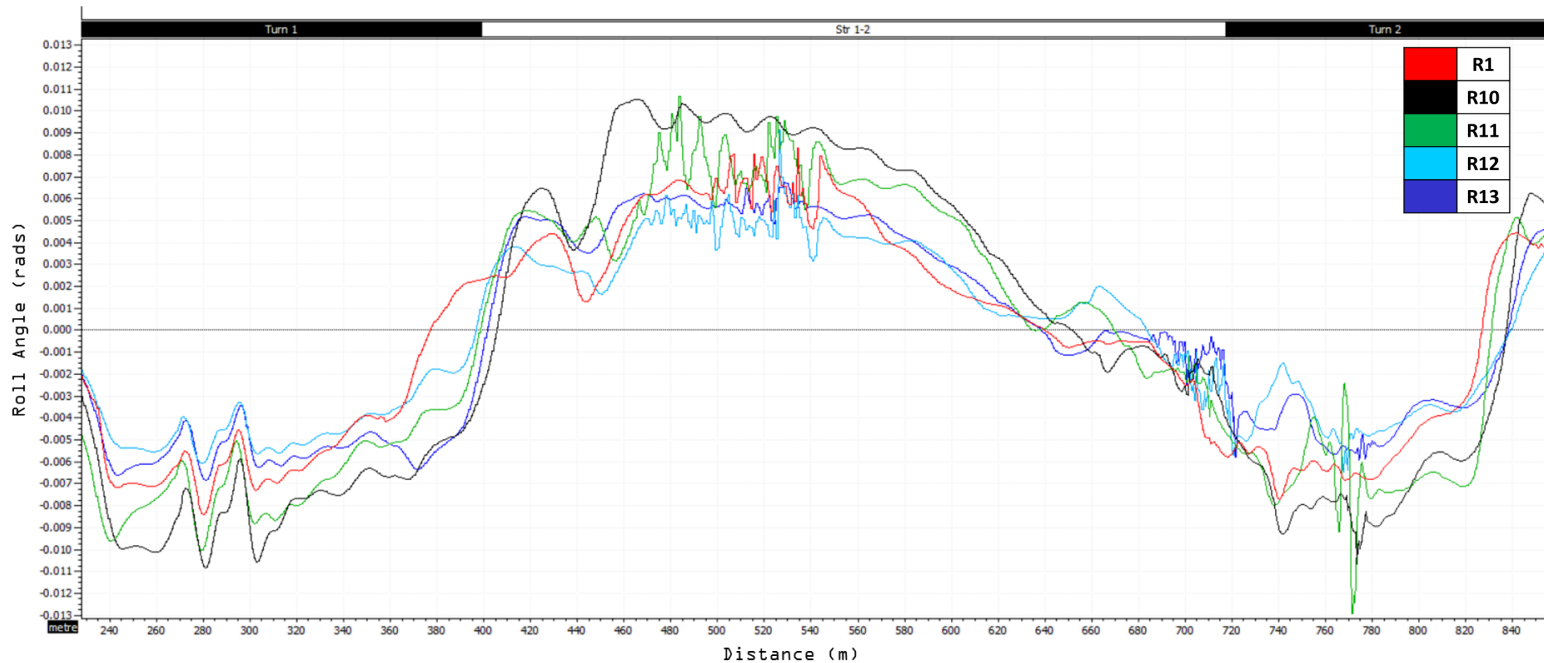


Figure 20 - Vehicle Roll Angle for Varying ARB Rates

- As expected, ARB rates have a large effect on vehicle roll.
- R10 has minimum ARB rates front and rear and experiences the most roll. With no input from ARB's, the wishbones can move independently through the full range of travel, encouraging body roll (Simmonds, 2013).
- Conversely, setup R12 shows minimum body roll. This is maximum ARB rates front and rear. This is not necessarily a good setup as maximum stiffness front and rear will make the vehicle incredibly twitchy and prone to snap oversteer.

5.2.5 Vehicle Roll Angle for Varying Spring Rates

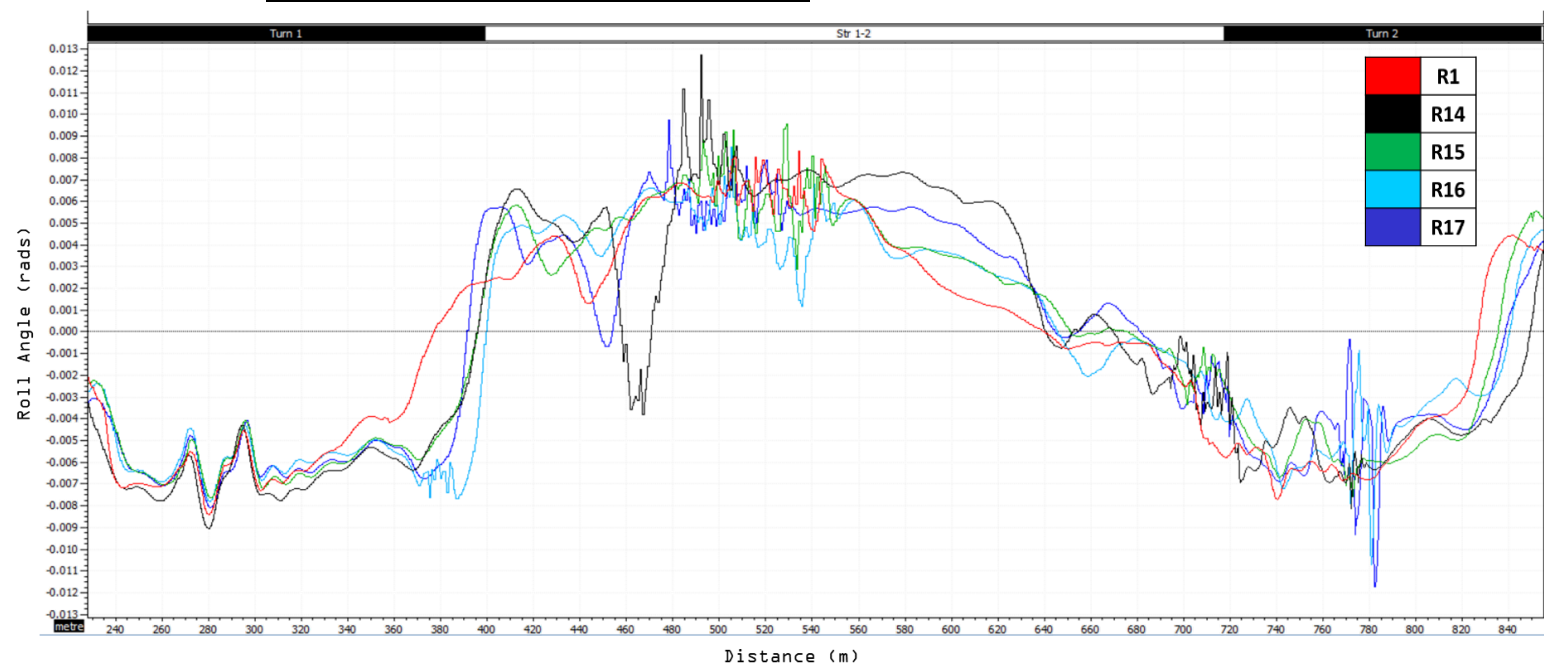


Figure 21 - Vehicle Roll Angle for Varying Spring Rates

- Figure 22 shows that spring rates effect roll angle less than ARB rates. As expected, the most roll angle is seen at R14 with minimum spring rates.

5.2.6 Vehicle Roll Angle for Varying Damping Rates

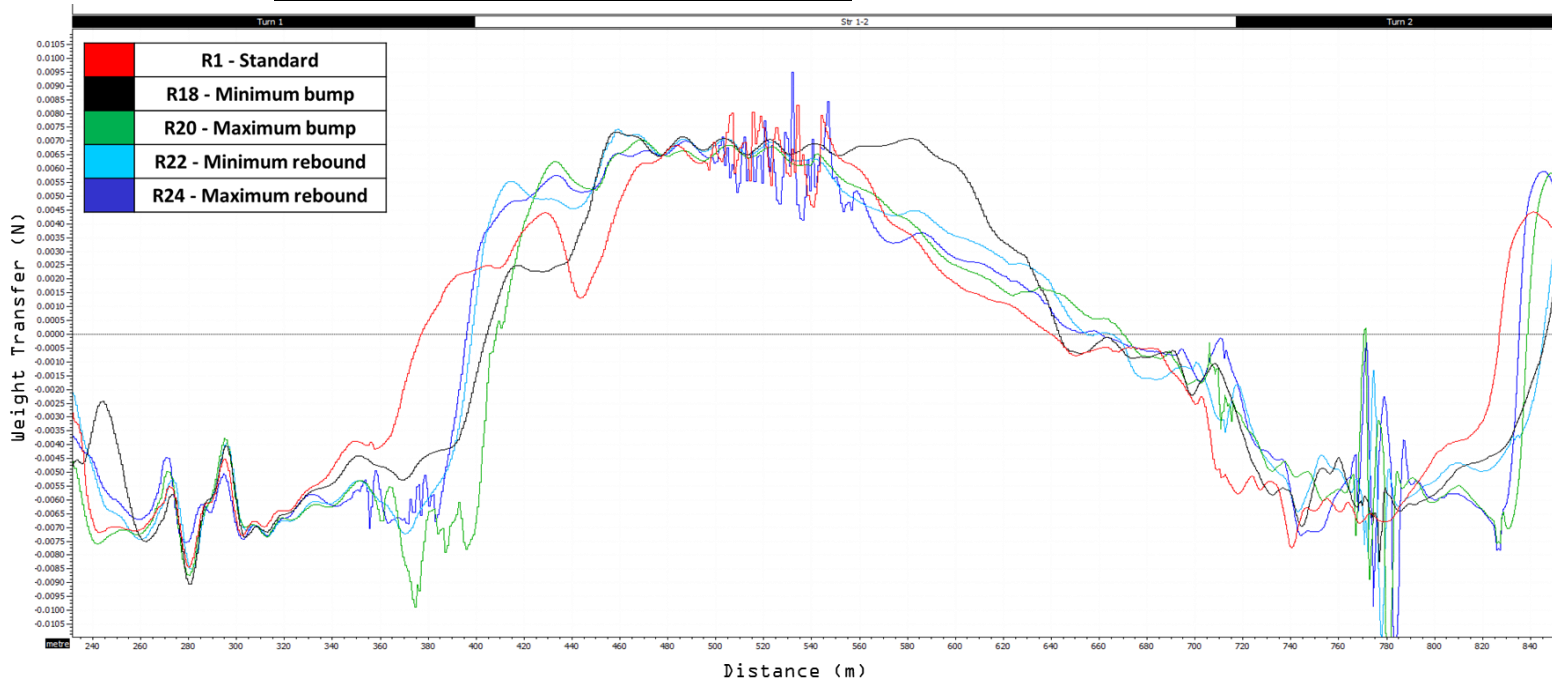


Figure 22 - Vehicle Roll Angle for Varying Damping Rates

- Figure 23 shows that roll angle is not influenced by damper rates. Overall, all setups see a very similar maximum roll angle.
- Dampers usually effect body roll more in the high speed region. Most race cars benefit from a stiff suspension setup, however if the setup is too stiff and the vehicle hits a bump such as a kerb, the wheel can lift off of the track altogether, losing traction.
- Slow bump rates can be high to ensure a sufficiently rigid setup, whilst high bump rates can be a lot softer to allow the damper to absorb the bump, and thus allow the vehicle to cope with the jolt better (Albinsson and Routledge, 2013) .

Understeer has been analysed at turns five and seven as this is a fast section, therefore any understeer will be apparent. Turn six is slower, and good for comparison.

5.2.7 Understeer Experienced Compared to Roll Stiffness Distribution

Graph to Show Amount of Understeer Experienced vs Roll Stiffness Distribution

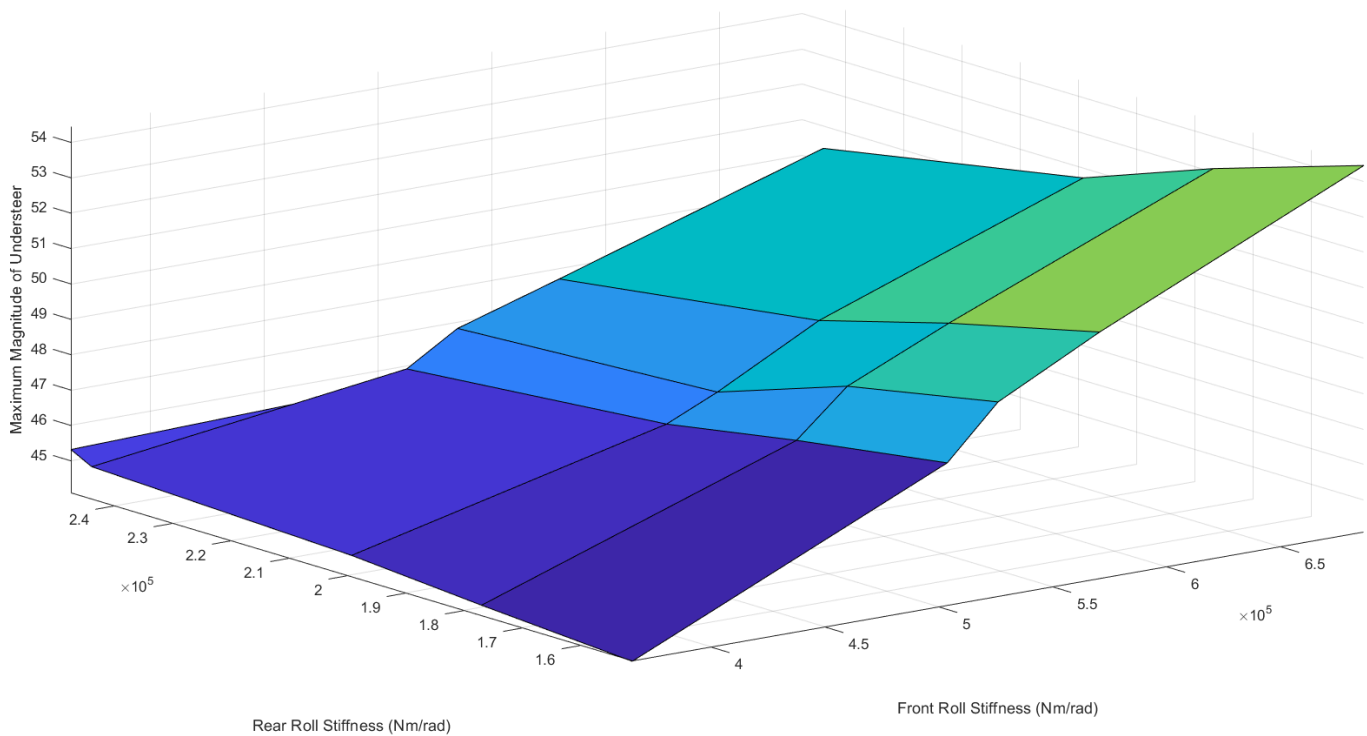
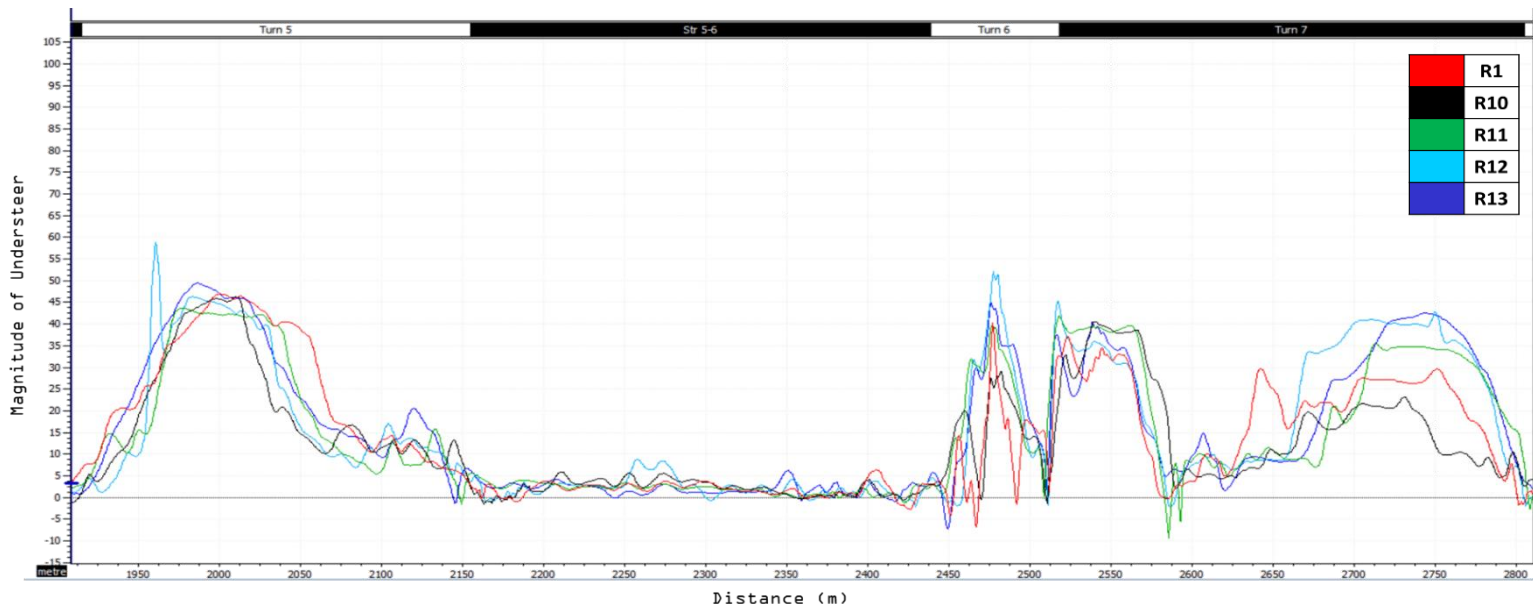


Figure 23 - Understeer Experienced Compared to Roll Stiffness Distribution

- The Z axis contains the maximum understeer value seen through corners five - seven for each corresponding setup.
- It is clear that the most understeer is experienced when the roll stiffness distribution is furthest forwards. This matches expectations; a stiffer front setup means more frontal elastic weight transfer, meaning less lateral force can be taken by the suspension, thus larger slip angle at the front wheels and a loss of traction, causing understeer (Staniforth, 1994).
- Even with maximum stiffness front and rear, thus a relatively central distribution, understeer is still high as the front stiffness is high.

5.2.8 Magnitude of Understeer for Varying ARB Rates



- Figure 26 shows R13, maximum front and detached rear ARB's generally experiences the most understeer
- Turn five is a generally slow, tight turn. Understeer is not apparent.
- Turn seven is less tight, but the vehicle picks up speed on exit onto the straight. This higher speed increases the amount of understeer seen. It is clear that R12 and R13, both with maximum front ARB rates produce a lot of understeer here.
- This shows that with adequate spring rates, a poor ARB setup will only drastically impact handling on high speed corners (Gillespie, 1992).

5.2.9 Magnitude of Understeer for Varying Spring Rates

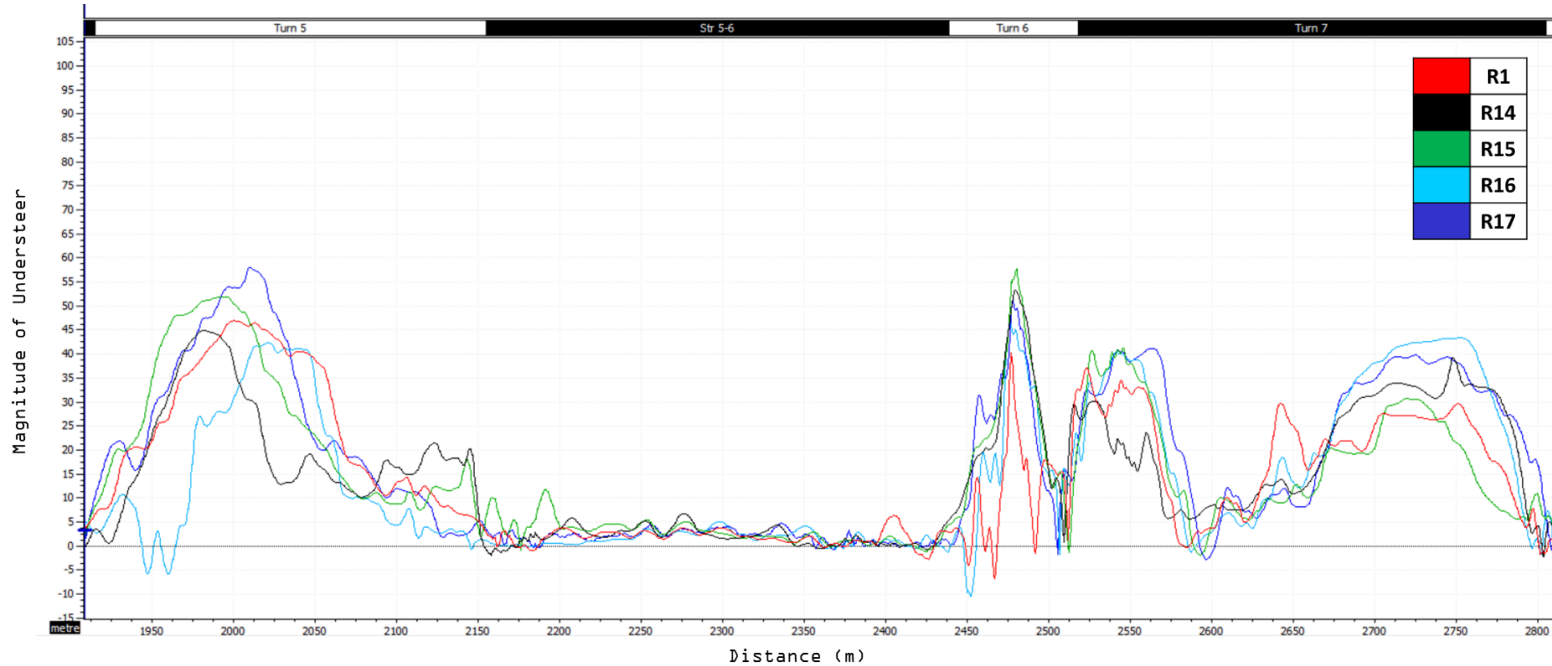


Figure 25 - Magnitude of Understeer for Varying Spring Rates

- Figure 27 shows a large variation in understeer through all corners. This shows that varying spring rate is more correlated to understeer (Smith, 1978).
- Even on slow speed sections, setup R17 with maximum front and minimum rear rates sees a considerably higher amount of understeer. This is closely followed by R16 with maximum rate all round.

5.2.10 Magnitude of Understeer for Varying Damping Rates

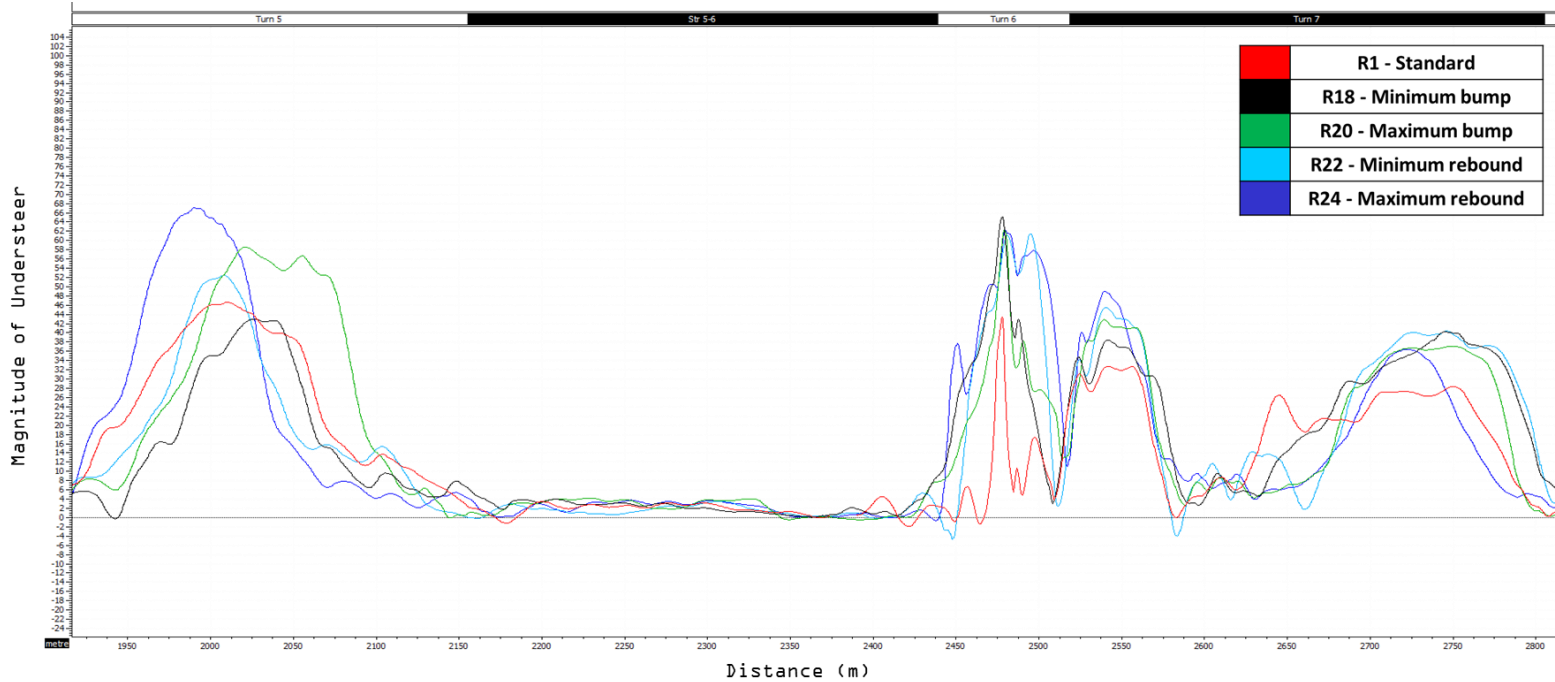


Figure 26 - Magnitude of Understeer for Varying Damping Rates

- As stated in figure 19, a higher rebound rate means load leaves the tyre much quicker, high bump rate means tyres are loaded quicker. This is apparent as setups R20 and R24, with maximum rates, show the most amount of understeer.
- A higher rate means a higher proportion of overall weight transfer across the front axle at a quicker rate, encouraging more understeer
- Setup R22 with minimum rebound rates is also higher than standard. This includes a comparatively high bump rate, showing that the outside tyre is loaded faster, hence high amounts of understeer (F1technical, 2012).

5.2.11 Weight Transfer Distribution Compared to Roll Stiffness Distribution

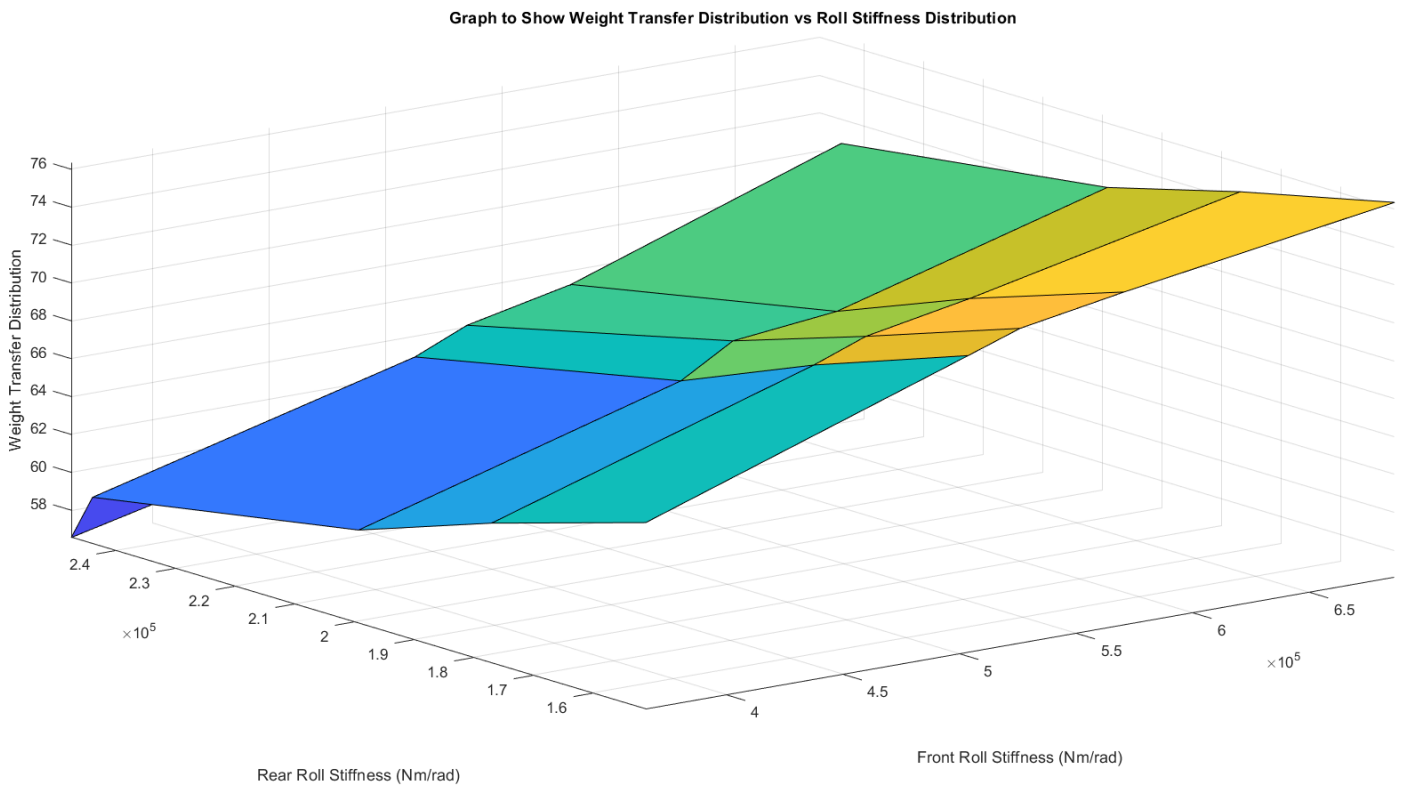


Figure 27 - Weight Transfer Distribution Compared to Roll Stiffness Distribution

- Roll stiffness and weight transfer distributions have a positive correlation. As the front of the car becomes stiffer, the centre of weight transfer moves towards the rear.
- Weight transfer distribution, sometimes referred to as the 'magic number' is a measure of the balance of weight transfer throughout the vehicle.
- This relationship shows that as the centre of weight transfer moves rearwards, the vehicle is more likely to understeer (Gillespie, 1992).

5.2.12 Damper Histograms

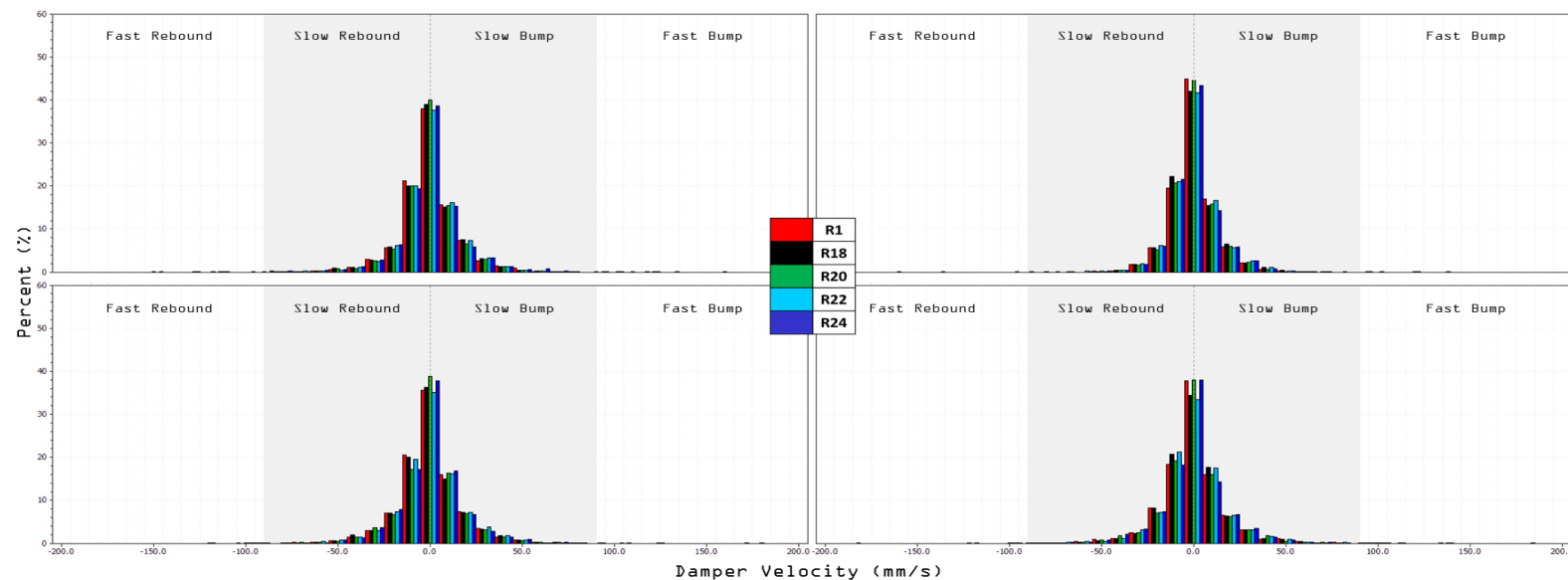


Figure 28 - Damper Histograms

Table 1 - Damper Histogram Data

Average Proportion of Time Spent in Each Damper Rate				
Setup	Fast Rebound	Slow Rebound	Fast Bump	Slow Bump
Standard (R1)	0.4%	52.3%	0.5%	47.5%
Minimum Bump (R18)	0.4%	52.6%	0.5%	46.5%
Maximum Bump (R20)	0.8%	51.4%	0.6%	47.4%
Minimum Rebound (R22)	0.4%	51.8%	0.5%	47.25%
Maximum Rebound (R24)	0.5%	53.6%	0.8%	45.2%

- A damper histogram should be a symmetrical bell curve in an ideal setup. For example, it is desirable to spend an equal amount of time in fast bump as fast rebound (Kasprzak, 2007).
- Table one (data from figure 31) shows good, but not perfect symmetry.
- It is expected that increasing the damping rate of a particular zone will reduce the time spent in that zone. For example, a higher slow rebound rate will decrease percentage of time in the slow bump zone as suspension action within this region is being damped out (DataMC, 2019).
- When at maximum bump, less time is spent in the slow bump region compared to standard, however when at maximum rebound this trend is not followed. All setups show more time spent in rebound than bump.
- It is clear that the vehicle hardly ever reaches the fast damping rate regions. The knee point for this vehicle is 90mm/s, which the velocity usually stays below.

5.2.13 Calculated vs Actual Front Left Contact Patch Loads



Figure 29 - Calculated vs Actual Contact Patch Loads

- Figure 32 analyses how dampers effect contact patch loading on the front left tyre across the whole lap, on standard damper setup.

- The ‘calculated’ contact patch load is found from:

$$(FL\ Corner\ Mass \cdot g) - (Front\ WT) - (Longitudinal\ WT) + \left(\frac{Front\ Aero\ Force}{2} \right)$$

- This is a mathematical model of contact patch load, considering only weight transfer and aerodynamic loading. This value does not include damper data.
- The ‘actual’ contact patch load is from MoTeC tyre load channels.
- It is clear that dampers work by exerting a force onto the suspension system, therefore adding to the contact patch load as along the straight the loading is the same. It is only at entry and exit, the transient points of a corner, where there is inconsistency (Alexander, 2015).

6 Data Validation

6.1 Damper Rates Validation

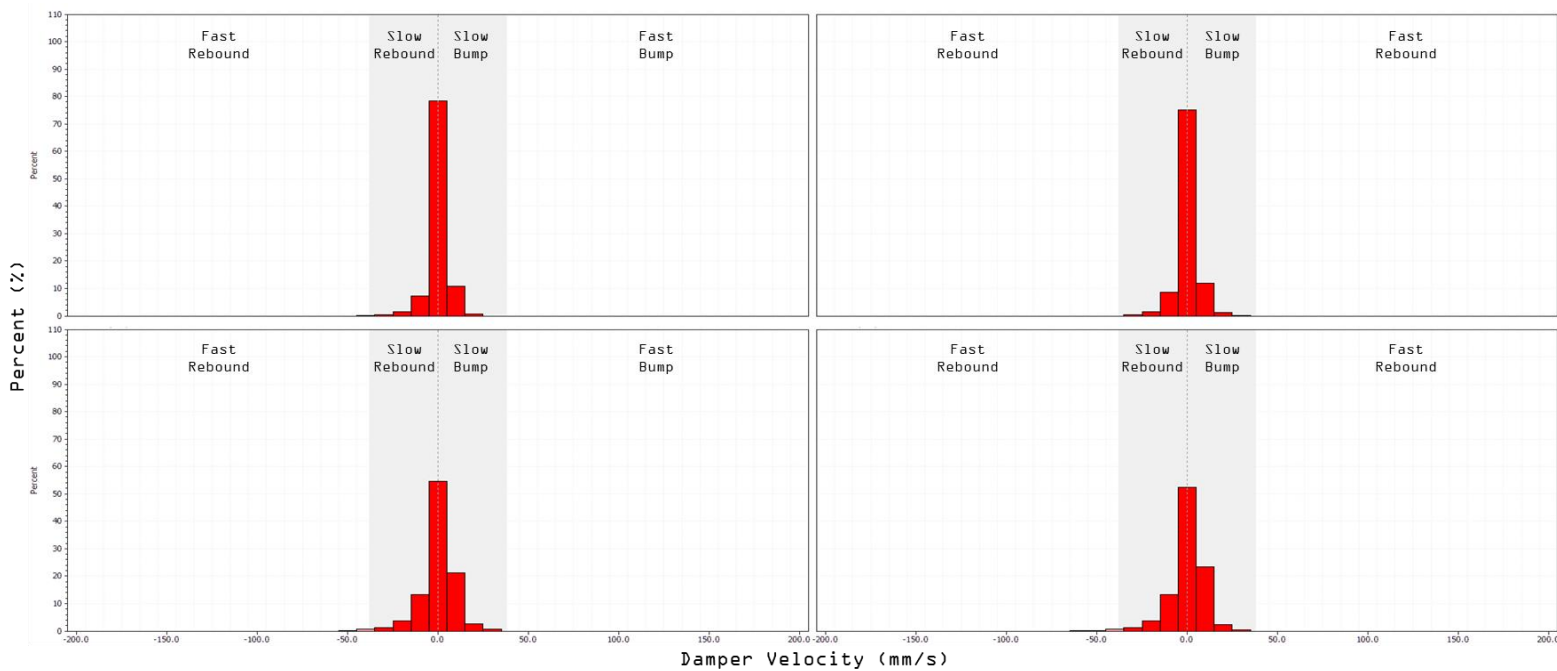


Figure 30 - MCR Damper Histogram

Table 2 - MCR Damper Histogram Data

Average Proportion of Time Spent in Each Damper Rate - MCR Sports 2000				
Setup	Fast Rebound	Slow Rebound	Fast Bump	Slow Bump
Standard dry	0.6%	44.9%	0.3%	54.4%

- The author extracted data from real-world testing of the MCR Sports 2000 around the same circuit to compare damper data.
- Figure 30 shows a clear correlation to the results in figure 28. A very small amount of time is spent in high speed ranges and the distribution is close to, but not perfectly symmetrical.
- These are different cars with different dampers, but as the trends match, this data can be validated and assumed correct.
- Both datasets agree with section 5.2.6, high speed damping is only used very occasionally when the vehicle hits a bump or kerb.

6.2 Roll Angle Validation

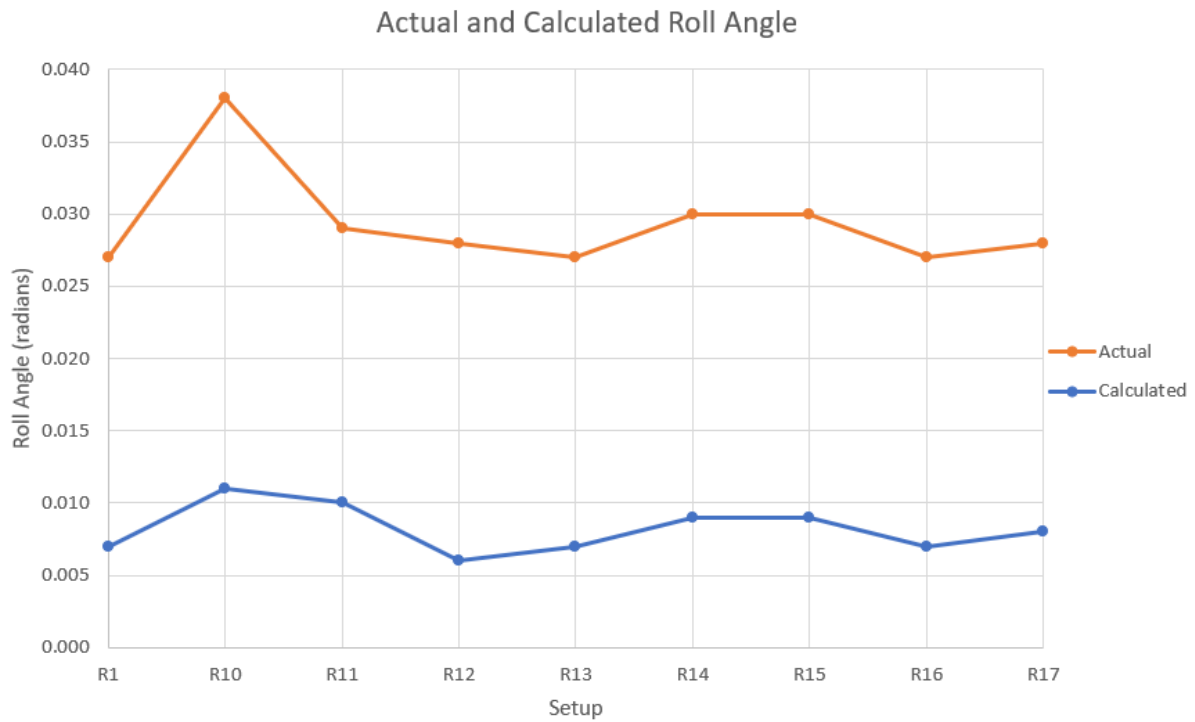


Figure 31 - Roll Angle Validation

- The calculated roll angle is consistently just less than half the actual roll angle. Despite this, the trends match well so this data can be trusted.
- The simulator does not allow for tyre deflection, a large contributor to body roll, this could explain the error (Parczewski, 2017).

6.3 Weight Transfer Validation

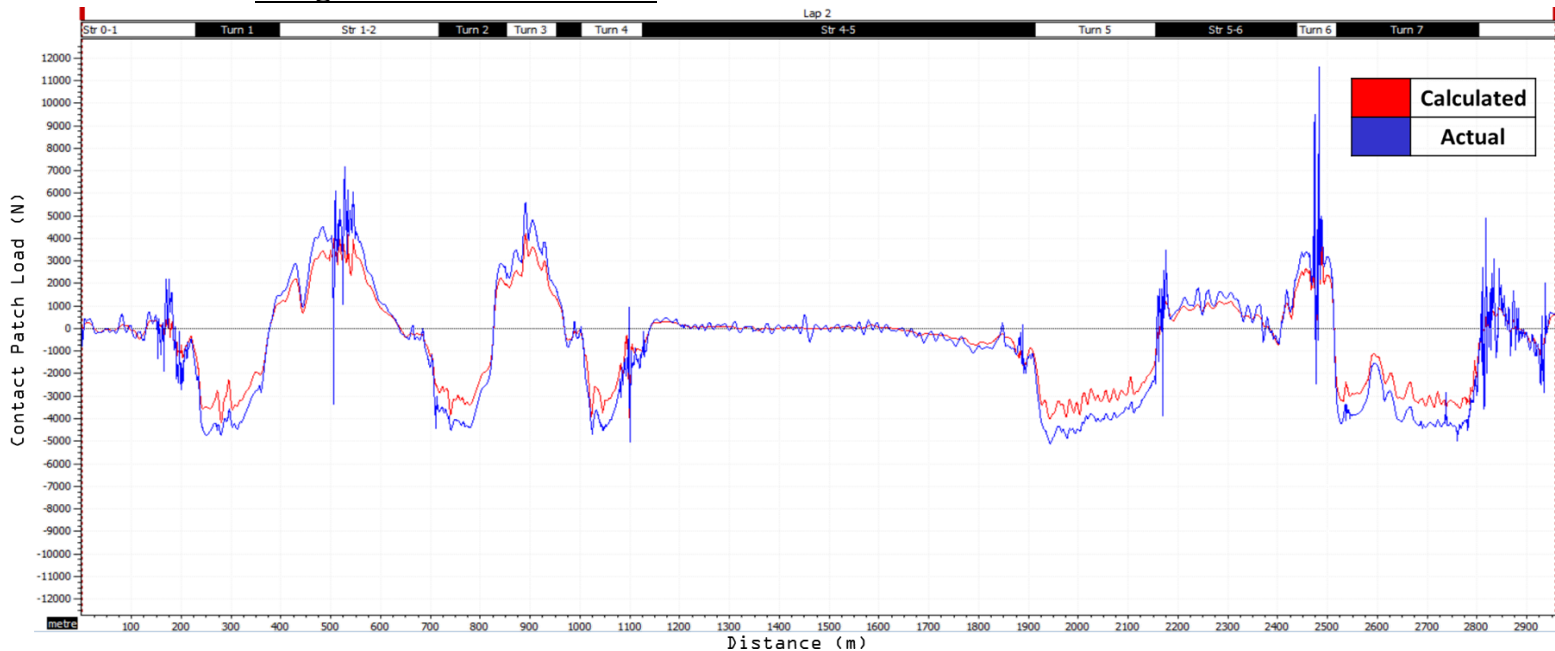


Figure 32 - Weight Transfer Validation

- The roll angle error is carried forward into the weight transfer calculations, hence similar inaccuracies are seen.
- On average the actual weight transfer is 20% higher than the calculated value, however the trends are the same, therefore the data is trusted.

- The author expected a closer correlation between lines on this graph, as the calculated value includes damper weight transfer. However the inaccuracy is still evident. This could suggest there is a problem in the way the simulator models damper behaviour, or the damping rates are not suitable for the vehicle. Further testing would have to be carried out to confirm.

6.4 Weight Transfer Distribution Validation

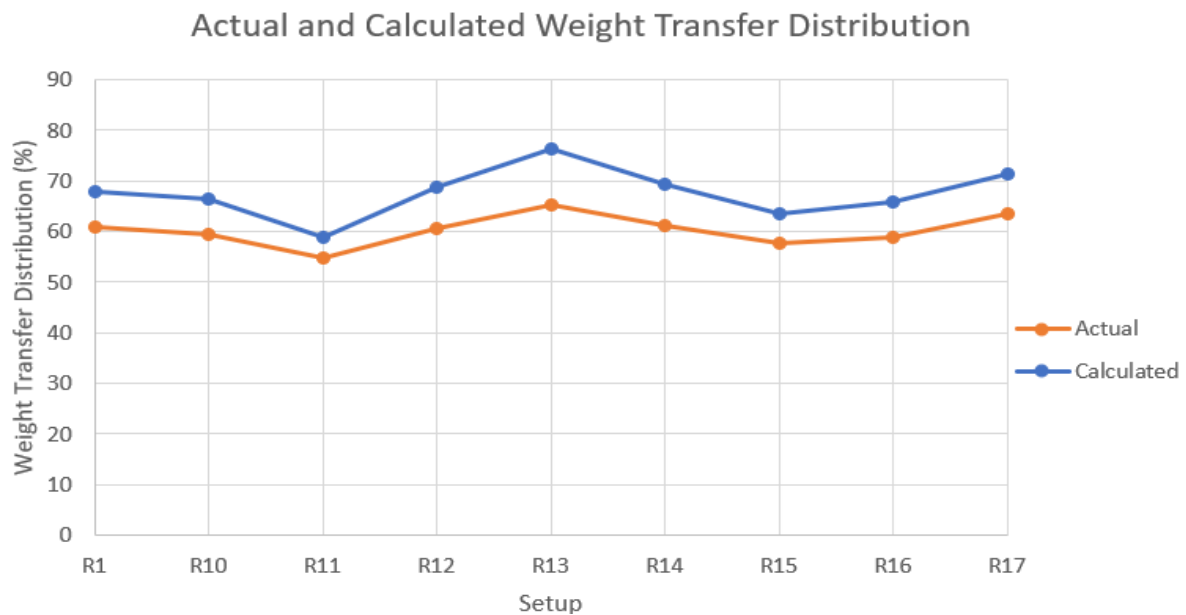


Figure 33 - Weight Transfer Distribution Validation

- On average, the actual weight transfer distribution is 10.7% higher than the calculated value, however the trends match so this data is trustworthy.
- To calculate weight transfer distribution, the inverse of weight transfer is used. This correlates to seeing the opposite error in the weight transfer graphs with actual value as 20% larger (figure 32).

7 Limitations and Inaccuracies

To compare data, the author mainly used graphs directly from MoTeC with distance on the x axis. This proved useful, with clear trends shown, however the distance travelled may not be identical at a certain point on different laps, making comparing when events happened graphically challenging and less accurate.

Assumptions have been made where more accurate data could not be found. For example wheel centre is assumed to be rolling radius. This may bring inaccuracy because of wheel weighing and brake components. Various data were provided to the author with no formal validation. Values such as motion ratio, which is presumed to be one, linear, value and roll axis heights, which are assumed to stay constant and are not dynamic, could lead to inaccuracy. Without access to the vehicle for measurement, these are difficult to rectify (Rowley, 2003). Static mass values were provided; however these did not include fuel or driver mass. To improve accuracy the author took static wheel loads from MoTeC, however error may still be apparent here, and the centre of gravity may be incorrect. Moreover, the simulator will not model fuel slosh, which will add to weight transfer.

Upon loading a car onto a track, the simulator aligns the vehicle and track coordinate systems. This could cause suspension movement and disrupt static mass readings. To counter this, the group moved the car forwards, stopped and depressed all pedals for a more accurate reading.

Despite this, without real-world validation, error may still be apparent (RFactor, 2005; Purdy, 2006).

The simulator may also bring inaccuracy by completing the tests in perfect ambient conditions and not modelling contact patch deformation or slip angle change (Purdy, 2006).

Finally, the accuracy to which the simulator models dampers is unknown. The damper forces produced appear to be quite low, with very low damper weight transfer values.

8 Evaluation and Analysis

As the angle of attack of an aerodynamic device is increased, the pressure difference increases, producing more lift. Moreover, Newton's third law dictates that the harder the air hits the underside of the aerodynamic device, the greater the equal and opposing downwards force (Barnard, 2001; Lajos, 2002). This is ideal so long as drag this causes does not outweigh the lift produced, maintaining a good aerodynamic efficiency.

This, as discussed, is only true within a certain range. The simulator is unhelpful at higher positions, as results are inaccurate, because in reality flow separation occurs, resulting in a loss of lift force. The wing and splitter should be run at the highest positions that do not create flow separation. This can only be achieved and validated through real world testing. It is also important to consider centre of pressure. If this is incorrect, the vehicles balance will be severely compromised.

No clear conclusion can be drawn from the rake data as the simulator models this incorrectly and the results do not follow clear trends.

A good aerodynamic setup will consist of as higher wing and splitter position possible, whilst flow is still attached and the centre of pressure is far enough rearwards to provide traction to the drive wheels, but not so far back that the front becomes light and the vehicle understeers. This must of course be balanced with the vehicles longitudinal mass distribution, as this distribution will have significant influence on the tyre loads. More negative lift force produced means later braking points, more entry speed and more corner speed; however, less straight line speed depending on the drag produced.

Weight transfer distribution has the most impact on vehicle balance and therefore handling and is most impacted by roll stiffness distribution. Thus, the best way to drastically change vehicle handling characteristics is by making either the front or rear of the vehicle more stiff. By making the front stiffer, moving the roll stiffness distribution forwards, the vehicle experiences a higher proportion of overall weight transfer at the front, and is more inclined to understeer. By moving the distribution rearwards, the car will oversteer. Total weight transfer will not change.

From this, dampers can be used to fine-tune vehicle balance. If a car is well set up but is slightly unbalanced in certain areas, dampers can be adjusted to control the rate of weight transfer and balance the vehicle as desired. Bump and rebound settings can be altered as means of controlling dive under braking, squat under acceleration and front-rear stabilisation during dynamic transition to fine tune handling balance, with a higher damping rate speeding up the weight transfer. Equally, a softer rate will slow down weight transfer and make the car less twitchy, good for wet conditions.

An optimal roll stiffness distribution should ensure overall stiffness is adequate. If too low, the body will be inclined to roll, if too high, the vehicle will tend to understeer and snap.

An optimal setup for any vehicle will depend on driver preference and track layout. This report has detailed ways in which an engineer should change a setup in order to achieve the best vehicle balance for any situation.

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10 Appendices

Appendix One - Aerodynamics Equations

$$\text{Aero Force Front [N]} = (\text{Tyre Load FL} + \text{Tyre Load FR}) - (\text{Static Weight FL} + \text{Static Weight FR}) \cdot g + \text{Longitudinal WT}$$

$$\text{Aero Force Rear [N]} = (\text{Tyre Load RL} + \text{Tyre Load RR}) - (\text{Static Weight RL} + \text{Static Weight RR}) \cdot g - \text{Longitudinal WT}$$

$$\text{Total Aero Force} = \text{Aero Force Rear} + \text{Aero Force Front}$$

$$\text{Centre of Pressure [\%]} = \frac{\text{Aero Force Front}}{\text{Total Aero Force}} \cdot 100$$

$$\text{Total Resistive Forces [N]} = \frac{V_2 - V_1}{T_2 - T_1} \cdot \text{Total Mass}$$

$$\text{Rolling Resistive Forces [N]} = \frac{(V_4 - V_3)}{T_4 - T_3} \cdot \text{Total Mass}$$

$$\text{Aerodynamic Resistive Forces [N]} = \text{Total Resistance} - \text{Rolling Resistance}$$

$$\text{Coefficient of Lift} = \frac{\text{Total Aero Force}}{\frac{1}{2} \cdot \text{Density of Air} \cdot \text{Velocity}^2}$$

$$\text{Coefficient of Drag} = \frac{2 \cdot \text{Aerodynamic Resistance}}{\text{Density of Air} \cdot \text{Average Velocity}^2}$$

$$\text{Aerodynamic Efficiency} = \frac{CL}{CD}$$

$$\text{Total Drag Force [N]} = \frac{1}{2} \cdot \text{Density of Air} \cdot \text{Speed}^2 \cdot \text{Coefficient of Drag}$$

Appendix Two - Aerodynamics Setup Sheet

	A	B	C	BK	BL	BM	BN	BO	BP	BQ	BR	BS	BT	BU	BV	BW	BX	BY	BZ	CA	CB	CC	CD	CE	CF	CG	
	MoTeC			Aerodynamics Data Wing Position Splitter Position V1 V2 V3 V4 T1 T2 T3 T4 Aerodynamic Resistance Rolling Resistance Total Resistance Coefficient of Lift (CL) Coefficient of Drag (CD) Aerodynamic Efficiency (CL/CD) Centre of Pressure Other Data Initial Fuel Mass Steering X Constant Density of Air Gravity																							
1																											
2	Hide Values	Date	Time		-	-	m/s	m/s	m/s	m/s	s	s	s	s	N	N	N	ratio	ratio	ratio	ratio		kg	-	kg/m3	m/s/s	
3	Set																										
4	Set																										
5	Difference																										
6	CD1 *(R0)* [understeer coeff.]	28-Oct-19	15:02:20		8	2	-	-	-	-	-	-	-	-	-	-	-	2.096	1.276	-	-		50.000	-5.590	1.223	9.810	
7																											
8	R0 *(CD1)*	28-Oct-19	15:04:09		8	2	55.094	52.108	2.450	2.033	29.302	30.721	103.115	104.298	2242.311	451.192	2693.502	2.096	1.276	1.642	33.000		50.000	-5.590	1.223	9.810	
9	Aero Runs:																										
10	R1 - Standard	28-Oct-19	15:06:40		8	2	55.042	52.928	2.436	2.081	44.628	45.628	118.216	119.216	2251.520	454.400	2705.920	2.094	1.263	1.657	32.000		50.000	-5.590	1.223	9.810	
11	R2	28-Oct-19	15:12:00		1	1	55.384	53.530	2.625	2.267	15.500	16.500	93.867	94.867	1914.880	458.240	2373.120	1.624	1.056	1.538	34.000		50.000	-5.590	1.223	9.810	
12	R3	28-Oct-19	15:15:40		12	1	54.958	52.811	2.439	2.075	16.417	17.417	89.931	90.931	2282.240	465.920	2748.160	2.078	1.285	1.617	18.000		50.000	-5.590	1.223	9.810	
13	R4	28-Oct-19	15:20:50		12	3	54.472	52.194	2.408	2.053	15.606	16.606	85.980	86.980	2461.440	454.400	2915.840	2.400	1.415	1.696	34.000		50.000	-5.590	1.223	9.810	
14	R5	28-Oct-19	15:23:30		1	3	55.138	53.089	2.486	2.140	15.400	16.400	90.417	91.417	2179.840	442.880	2622.720	1.947	1.217	1.599	50.000		50.000	-5.590	1.223	9.810	
15	Ride Height Runs:																										
16	R6	28-Oct-19	15:30:20		8	2	55.008	52.869	2.453	2.097	16.157	17.157	89.975	90.975	2282.240	455.680	2737.920	2.107	1.283	1.642	33.000		50.000	-5.590	1.223	9.810	
17	R7	28-Oct-19	15:33:00		8	2	54.900	52.756	2.431	2.075	15.905	16.905	88.840	89.840	2288.640	455.680	2744.320	2.019	1.292	1.563	31.000		50.000	-5.590	1.223	9.810	
18	R8	28-Oct-19	15:35:50		8	2	55.031	52.914	2.478	2.119	16.292	17.292	89.781	90.781	2250.240	459.520	2709.760	2.015	1.263	1.595	32.000		50.000	-5.590	1.223	9.810	
19	R9	28-Oct-19	15:38:40		8	2	55.036	52.900	2.603	2.247	15.291	16.291	88.877	89.877	2278.400	455.680	2734.080	2.105	1.279	1.645	32.000		50.000	-5.590	1.223	9.810	
20																											
21	CD2 - (R4 Setup)	28-Oct-19	15:43:10		3	12	55.894	52.417	2.356	1.003	28.577	30.063	100.313	103.804	2498.906	496.087	2994.993	2.094	1.393	1.503	34.000		50.000	-5.590	1.223	9.810	

Appendix Three - Coast down test ambient correction algorithms.

$$f(x_s) = f(x_s+h) - h \times f'(x_s) - \frac{h^2}{2} \times f''(x_s) - \text{etc} \quad (3.1)$$

where,

$$f(x_s) = t_s$$

and,

$$f(x_s+h) = t_o$$

and,

x_s = datum (or standard) value of parameter

and,

$x_s + h = x_o$ = observed value of parameter

and,

h = the aberration in the ambient parameter

In this particular case time is a function of six variables (T, P, v_{hw} , K_Y , i , m), and so the corrected time is given by:

$$t_s = t_o - \Delta \quad (3.2)$$

where,

$$\begin{aligned} \Delta = & \left\{ \frac{\partial t}{\partial T} \times \delta T + \frac{\partial t}{\partial P} \times \delta P + \frac{\partial t}{\partial v_{hw}} \times \delta v_{hw} + \frac{\partial t}{\partial K_Y} \times \delta K_Y + \frac{\partial t}{\partial i} \times \delta i + \frac{\partial t}{\partial m} \times \delta m \right\} \\ & + \frac{1}{2} \left\{ \frac{\partial^2 t}{\partial T^2} \times \delta T^2 + \frac{\partial^2 t}{\partial P^2} \times \delta P^2 + \frac{\partial^2 t}{\partial v_{hw}^2} \times \delta v_{hw}^2 + \frac{\partial^2 t}{\partial K_Y^2} \times \delta K_Y^2 + \frac{\partial^2 t}{\partial i^2} \times \delta i^2 + \frac{\partial^2 t}{\partial m^2} \times \delta m^2 \right\} \\ & + \left\{ \frac{\partial^2 t}{\partial T \cdot \partial P} \times \delta T \cdot \delta P + \frac{\partial^2 t}{\partial T \cdot \partial v_{hw}} \times \delta T \cdot \delta v_{hw} + \frac{\partial^2 t}{\partial T \cdot \partial K_Y} \times \delta T \cdot \delta K_Y \right. \\ & + \frac{\partial^2 t}{\partial T \cdot \partial i} \times \delta T \cdot \delta i + \frac{\partial^2 t}{\partial T \cdot \partial m} \times \delta T \cdot \delta m + \frac{\partial^2 t}{\partial P \cdot \partial v_{hw}} \times \delta P \cdot \delta v_{hw} \\ & + \frac{\partial^2 t}{\partial P \cdot \partial K_Y} \times \delta P \cdot \delta K_Y + \frac{\partial^2 t}{\partial P \cdot \partial i} \times \delta P \cdot \delta i + \frac{\partial^2 t}{\partial P \cdot \partial m} \times \delta P \cdot \delta m \\ & + \frac{\partial^2 t}{\partial v_{hw} \cdot \partial K_Y} \times \delta v_{hw} \cdot \delta K_Y + \frac{\partial^2 t}{\partial v_{hw} \cdot \partial i} \times \delta v_{hw} \cdot \delta i + \frac{\partial^2 t}{\partial v_{hw} \cdot \partial m} \times \delta v_{hw} \cdot \delta m \\ & \left. + \frac{\partial^2 t}{\partial K_Y \cdot \partial i} \times \delta K_Y \cdot \delta i + \frac{\partial^2 t}{\partial K_Y \cdot \partial m} \times \delta K_Y \cdot \delta m + \frac{\partial^2 t}{\partial i \cdot \partial m} \times \delta i \cdot \delta m \right\} \quad (3.3) \end{aligned}$$

(Emtage, 1983)

Appendix Four - Weight Transfer Equations

$$ACTUAL\ Static\ Mass\ [kg] = \frac{Tyre\ Load\ FR + Tyre\ Load\ FL + Tyre\ Load\ RR + Tyre\ Load\ RL}{g}$$

$$COG\ Dist.\ from\ Front\ (Weight\ Distribution)\ [m] = \left(\frac{Static\ Mass\ Rear}{Total\ Static\ Mass} \right) \cdot Wheelbase$$

$$Roll\ Axis\ Height\ at\ COG\ [m] = \left(\frac{Roll\ Axis\ Height\ R - Roll\ Axis\ Height\ F \cdot Weight\ Distribution}{Wheelbase} \right) + Roll\ Axis\ Height\ F$$

$$Wheel\ Rate\ [N/m] = (Spring\ Rate \cdot Motion\ Ratio^2) + AntiRoll\ Bar\ Rate$$

$$Overturning\ moment\ [Nm] = Total\ Sprung\ Mass \cdot Lateral\ Acceleration \cdot COG\ Height\ above\ Roll\ Centre$$

$$Roll\ Angle\ [rads] = \frac{Overturning\ Moment}{Total\ Roll\ Stiffness}$$

$$Sprung\ Elastic\ WT\ [N] = \frac{Wheel\ Rate \cdot Track \cdot Roll\ Angle}{2}$$

$$Sprung\ Geometric\ WT\ [N] = \frac{Sprung\ Mass \cdot Lateral\ Acceleration \cdot Roll\ Centre\ Height}{Track}$$

$$Unsprung\ Geometric\ WT\ [N] = \frac{Unsprung\ Mass \cdot Lateral\ Acceleration \cdot Wheel\ Centre\ Height}{Track}$$

$$Longitudinal\ WT\ [N] = \frac{Total\ Mass \cdot Longitudinal\ Acceleration \cdot Centre\ of\ Gravity\ Height}{Wheelbase}$$

$$CALCULATED\ Weight\ Transfer\ Front\ and\ Rear\ [N] = Sprung\ Elastic\ WT + Sprung\ Geometric\ WT + Unsprung\ Geometric\ WT$$

$$ACTUAL\ Weight\ Transfer\ Front\ and\ Rear\ [N] = \frac{Tyre\ Load\ Right - Tyre\ Load\ Left}{2}$$

$$CALCULATED\ Total\ Weight\ Transfer\ [N] = \frac{Total\ Mass \cdot Lateral\ Acceleration \cdot COG\ Height\ above\ roll\ centre}{Track}$$

$$ACTUAL\ Total\ Weight\ Transfer\ [N] = ACTUAL\ Weight\ Transfer\ Front + ACTUAL\ Weight\ Transfer\ Rear$$

$$Centre\ of\ Weight\ Transfer\ [\%] = \left(\frac{Front\ Weight\ Transfer}{Total\ Weight\ Transfer} \right) \cdot 100$$

$$Roll\ Stiffness\ Distribution\ [\%] = \left(\frac{Front\ Roll\ Stiffness}{Total\ Roll\ Stiffness} \right) \cdot 100$$

Appendix Five - Steering Equations

$$\text{Corner Radius [m]} = \frac{\text{Velocity}^2}{\text{Lateral Acceleration}}$$

$$\text{Theoretical Wheel Steer Angle [rads]} = \frac{\text{Wheelbase}}{\text{Corner Radius}}$$

$$\text{Steering 'X' Constant} = \frac{\text{Theoretical Steer \%}}{\text{Theoretical Steer Angle}} \quad (\text{At low speeds when car is within limit of grip})$$

$$\text{Theoretical Steer \%} = \text{Theoretical Angle} \cdot \text{Steering 'X' Constant}$$

$$\text{Understeer Magnitude} = \text{abs(Steering Wheel \%)} - \text{abs(Theoretical Steer \%)}$$

Appendix Six - Dampers Equations

$$\text{Damper Rate} \left[\frac{\text{N}}{\text{m/s}} \right]:$$

if $0 < \text{Damper velocity} \leq 0.09 = \text{Slow Bump Damping Rate}$

if $\text{Damper velocity} > 0.09 = \text{Fast Bump Damping Rate}$

if $0 > \text{Damper velocity} \geq -0.09 = \text{Slow Rebound Damping Rate}$

if $\text{Damper velocity} < -0.09 = \text{Fast Rebound Damping Rate}$

$$\text{Damper Force [N]} = \text{Damper Velocity} \cdot \text{Damper Rate}$$

$$\text{Damper Weight Transfer} = \frac{\text{Damper Force R} - \text{Damper Force L}}{2}$$

$$\text{Centre of Damper Weight Transfer} = \left(\frac{\text{Front Damper Weight Transfer}}{\text{Total Damper Weight Transfer}} \right) \cdot 100$$

Appendix Seven - Suspension Setup Sheet

	A	B	C	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	AQ	AR	AS	AT	AU	AV	AW	AX	AY	AZ	BA	BB	BC	BD	BE	BF	BG	BH	BI	BJ		
				Left Front	Left Centre Height Rear	Spring Rate Front	Spring Rate Rear	Roll Bar Rate Front	Roll Bar Rate Rear	Wheel Rate Front	Wheel Rate Rear	Roll Stiffness Front	Roll Stiffness Rear	Total Roll Stiffness	Centre of Gravity Height (mm)	Roll axle height at COG (N)	COG Height Above Roll Centre (N)	Roll Centre Height Front	Roll Centre Height Rear	Meach Ratio Front	Meach Ratio Rear	Front Ride Height	Rear Ride Height	Roll Angle	Front Slow Bump Damping Coefficient	Front Fast Bump Damping Coefficient	Rear Slow Bump Damping Coefficient	Rear Fast Bump Damping Coefficient	Front Slow Rebound Damping Coefficient	Front Fast Rebound Damping Coefficient	Rear Slow Rebound Damping Coefficient	Rear Fast Rebound Damping Coefficient	Front Bump Knead Point	Rear Bump Knead Point			
1	Hide Values	Date	Time		N/m	N/m	N/m	N/m	N/m	N/m	N/m	N/m	N/m	N/m	m	m	m	m	ratio	ratio	m	m	degree	N/m/s	N/m/s	N/m/s	N/m/s	N/m/s	N/m/s	N/m/s	N/m/s	N/m/s	N/m/s	mm/s	mm/s	mm/s	mm/s
2	Set																																				
3	Set																																				
4	Difference																																				
22	ARB Run:																																				
23	R10	28-Oct-19	15:46:00	237000.000	237000.000	160000.000	0.000	272635.700	112835.700	365121.830	151002.150	516123.980	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
24	R11	28-Oct-19	15:48:30	237000.000	237000.000	160000.000	72000.000	272635.700	184835.700	365121.830	247356.006	612477.836	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
25	R12	28-Oct-19	15:51:00	237000.000	237000.000	400000.000	72000.000	512635.700	184835.700	686301.350	247356.006	933657.356	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
26	R13	28-Oct-19	15:53:30	237000.000	237000.000	400000.000	0.000	512635.700	112835.700	686301.350	151002.150	837303.500	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
27	Spring Run:																																				
28	R14	28-Oct-19	15:56:20	202000.000	202000.000	280000.000	36000.000	376172.200	132172.200	503411.694	176873.182	680290.877	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
29	R15	28-Oct-19	15:58:50	202000.000	307000.000	280000.000	36000.000	376172.200	182162.700	503411.694	243778.869	747190.563	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
30	R16	11-Nov-19	16:02:00	307000.000	307000.000	280000.000	36000.000	426162.700	182162.700	570311.381	243778.869	840490.250	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
31	R17	11-Nov-19	16:04:40	307000.000	202000.000	280000.000	36000.000	426162.700	132172.200	570311.381	176873.182	747190.563	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
32	Dampers Bump 1																																				
33	R18	11-Nov-19	16:08:35	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6000.000	2800.000	6000.000	2800.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
34	R19	11-Nov-19	16:12:30	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6000.000	4000.000	6000.000	4000.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
35	R20	11-Nov-19	16:19:50	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	8400.000	4000.000	8400.000	4000.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
36	R21	11-Nov-19	16:22:30	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	8400.000	2800.000	8400.000	2800.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
37	Dampers Rebound 1																																				
38	R22	11-Nov-19	16:25:40	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	1000.000	4600.000	1000.000	4600.000	90.000	90.000	-90.000	-90.000			
39	R23	11-Nov-19	16:28:23	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	1000.000	7000.000	1000.000	7000.000	90.000	90.000	-90.000	-90.000			
40	R24	11-Nov-19	16:31:15	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	14800.000	7000.000	14800.000	7000.000	90.000	90.000	-90.000	-90.000			
41	R25	11-Nov-19	16:34:00	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	14800.000	4600.000	14800.000	4600.000	90.000	90.000	-90.000	-90.000			
42	Dampers Bump 2																																				
43	R26	11-Nov-19	16:37:00	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6000.000	3100.000	8400.000	3100.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
44	R27	11-Nov-19	16:39:40	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	8400.000	3100.000	6000.000	3100.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
45	R28	11-Nov-19	16:42:20	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	2800.000	6600.000	4000.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
46	R29	11-Nov-19	16:44:55	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	4000.000	6600.000	2800.000	1800.000	5500.000	1800.000	5500.000	90.000	90.000	-90.000	-90.000			
47	Dampers Rebound 2																																				
48	R30	11-Nov-19	16:45:35	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	1000.000	5500.000	14800.000	5500.000	90.000	90.000	-90.000	-90.000			
49	R31	11-Nov-19	16:47:35	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	14800.000	5500.000	1000.000	5500.000	90.000	90.000	-90.000	-90.000			
50	R32	11-Nov-19	16:52:45	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3100.000	6600.000	3100.000	1800.000	4600.000	1800.000	7000.000	90.000	90.000	-90.000	-90.000			
51	R33	11-Nov-19	16:55:00	237000.000	237000.000	280000.000	36000.000	392835.700	148835.700	525711.590	199173.078	724890.668	0.280	0.02005	0.260	0.008	0.031	0.690	0.690	0.046	0.062	0.333	6600.000	3													