

LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 4

LEFEL 4

EXAMINATION PERIOD
JANUARY 2025

CYFNOD ARHOLIADAU
IONAWR 2025

MODULE CODE: GOEN4001

MATHEMATICS TEST 1

SOLUTIONS

TIME: 2 HOURS

ANSWER ALL QUESTIONS

Equipment or materials required – calculators & formula book

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QUESTION ONE (35 marks)

a) Find the first derivative of the following functions:

i. $y = 3x^4 + 5x^3 + 4x^2 + 7x - 10$ [2]

$$\frac{dy}{dx} = 12x^3 + 15x^2 + 8x + 7$$

ii. $y = \frac{1}{x^3} + \frac{1}{\sqrt[5]{x^2}} - 2\pi x$ [3]

$$y = x^{-3} + x^{-\frac{2}{5}} - 2\pi x$$

$$\frac{dy}{dx} = -3x^{-4} - \frac{2}{5} x^{-\frac{7}{5}} - 2\pi$$

iii. $y = 2\sin(2x) + 3\cos(3x) + \frac{1}{5}\tan(5x)$ [3]

$$\frac{dy}{dx} = 4\cos(2x) - 9\sin(3x) + \csc^2 5x$$

iv. $y = 7x^3 \sin(5x)$ [4]

$$u = 7x^3 \quad \text{and} \quad \frac{du}{dx} = 21x^2$$

$$V = \sin(5x) \quad \text{and} \quad \frac{dv}{dx} = 5\cos(5x)$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{dy}{dx} = 7x^3 (5\cos(5x)) + \sin(5x) (21x^2)$$

$$\frac{dy}{dx} = 35x^3 \cos(5x) + 21x^2 \sin(5x)$$

v. $y = \frac{2 \sin(5x)}{3e^{5x}}$ [4]

$$u = 2 \sin(5x) \quad \text{and} \quad \frac{du}{dx} = 10 \cos(5x)$$

$$v = 3e^{5x} \quad \text{and} \quad \frac{dv}{dx} = 15e^{5x}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{(v)^2}$$

$$\frac{dy}{dx} = \frac{3e^{5x} (10 \cos(5x)) - 2 \sin(5x) (15e^{5x})}{(3e^{5x})^2}$$

$$\frac{dy}{dx} = \frac{30e^{5x} \cos(5x) - 30e^{5x} \sin(5x)}{9e^{10x}}$$

vi. $y = 5(2x^3 + 4)^3$ [3]

$$\frac{dy}{dx} = 15 (2x^3 + 4)^2 \times 6x^2$$

$$\frac{dy}{dx} = 90 x^2 (2x^3 + 4)^2$$

or

$$\text{let } u = 2x^3 + 4 \text{ so } y = 5 u^3$$

$$\frac{dy}{du} = 15u^2 \quad \text{also} \quad \frac{du}{dx} = 6x^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = 15u^2 \times 6x^2 = 90 x^2 u^2$$

$$\frac{dy}{dx} = 90 x^2 (2x^3 + 4)^2$$

vii. $y = \sqrt[3]{(x^4 + 5)^2}$ [3]

$$y = (x^4 + 5)^{\frac{2}{3}}$$

$$\frac{dy}{dx} = \frac{2}{3} (x^4 + 5)^{-\frac{1}{3}} (4x^3)$$

$$\frac{dy}{dx} = \frac{8}{3} x^3 (x^4 + 5)^{-1/3}$$

or

$$\text{let } u = x^4 + 5 \quad \text{so } y = u^{\frac{2}{3}}$$

$$\text{so } \frac{dy}{du} = \frac{2}{3} u^{-\frac{1}{3}}$$

$$\text{also } \frac{du}{dx} = 4x^3 + 0$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3} u^{-\frac{2}{3}} \times (4x^3)$$

$$\frac{dy}{dx} = \frac{1}{3} (x^4 + 5)^{-2/3} (4x^3)$$

$$\frac{dy}{dx} = \frac{4}{3} x^3 (x^4 + 5)^{-2/3}$$

b) The variable w is related to the variables x and y by the following:

$$w = 5x^5 + 4x^2y^4 + 3y^3 + \sin(x) - 5$$

Use Partial Differentiation techniques to find following second order Partial Derivatives,

$$(i) \frac{\partial^2 w}{\partial x^2} \quad (ii) \frac{\partial^2 w}{\partial y^2} \quad (iii) \frac{\partial^2 w}{\partial x \partial y} \quad (iv) \frac{\partial^2 w}{\partial y \partial x}$$

[6]

$$w = 5x^5 + 4x^3y^4 + 3y^3 + \sin(3x) - 5$$

$$\frac{\partial w}{\partial x} = 25x^4 + 12x^2y^4 + 3\cos(3x)$$

[1]

$$\frac{\partial^2 w}{\partial x^2} = 100x^3 + 24xy^4 - 9\sin(3x)$$

[1]

$$\frac{\partial^2 w}{\partial y \partial x} = 48x^2y^3$$

[1]

$$w = 5x^5 + 4x^3y^4 + 3y^3 + \sin(3x) - 5$$

$$\frac{\partial w}{\partial y} = 16x^3y^3 + 9y^2$$

[1]

$$\frac{\partial^2 w}{\partial y^2} = 48x^3y^2 + 18y$$

[1]

$$\frac{\partial^2 w}{\partial x \partial y} = 48x^2y^3$$

[1]

c) If $y = \cos(2x) + x^2$ show that

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 4x^2 - 8x + 8\sin(2x) + 2$$

[4]

$$LHS = \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y$$

$$y = \cos(2x) + x^2$$

$$\frac{dy}{dx} = -2\sin(2x) + 2x$$

[1]

$$\frac{d^2y}{dx^2} = -4 \cos(2x) + 2$$

[1]

$$LHS = \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 4y$$

$$LHS = -4 \cos(2x) + 2 - 4(-2 \sin(2x) + 2x) + 4(\cos(2x) + x^2)$$

$$LHS = -4 \cos(2x) + 2 + 8 \sin(2x) - 8x + 4 \cos(2x) + 4x^2$$

$$LHS = 4x^2 - 8x + 8 \sin(2x) + 2$$

$$LHS = RHS$$

[2]

d) Find the 3th derivative of:

$$y = xe^{2x}$$

[3]

$$\frac{dy}{dx} = 2xe^{2x} + e^{2x}$$

$$\frac{d^2y}{dx^2} = 4xe^{2x} + 2e^{2x} + 2e^{2x}$$

$$\frac{d^3y}{dx^3} = 8xe^{2x} + 4e^{2x} + 4e^{2x} + 4e^{2x}$$

$$\frac{d^3y}{dx^3} = 8xe^{2x} + 12e^{2x}$$

QUESTION TWO (33 marks)

a) Express, $\frac{3x-4}{(x+2)(x-5)}$ in partial fractions

[10]

$$\frac{3x-4}{(x+2)(x-5)} = \frac{A}{(x+2)} + \frac{B}{(x-5)}$$

$$\frac{(3x-4)(x+2)(x-5)}{(x+2)(x-5)} = \frac{A(x+2)(x-5)}{(x+2)} + \frac{B(x+2)(x-5)}{(x-5)}$$

$$3x-5 = A(x-5) + B(x+2)$$

[2]

let $x = 5$

$$3x-4 = A(x-5) + B(x+2)$$

$$15-4 = A(5-5) + B(5+2)$$

$$11 = A(0) + B(7)$$

$$11 = 7B$$

$$B = \frac{11}{7}, \quad B = 1.57$$

[4]

let $x = -2$

$$3x-4 = A(x-5) + B(x+2)$$

$$-6-4 = A(-2-5) + B(-2+2)$$

$$-10 = A(-7) + B(0)$$

$$-10 = -7B$$

$$A = \frac{-10}{-7}, \quad A = 1.43$$

[2]

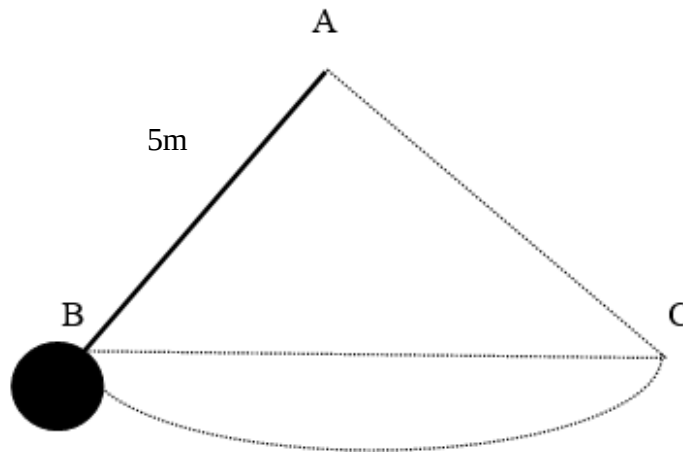
$$\frac{x-5}{(x+3)(x-1)} = \frac{A}{(x+3)} + \frac{B}{(x-1)}$$

$$\frac{x-5}{(x+3)(x-1)} = \frac{1.43}{(x+3)} - \frac{1.57}{(x-1)}$$

[2]

- b) A pendulum of length 5m swings from point B to point C, creating an angle at point A of 100° . Calculate the horizontal distance between point B and point C.

[4]



$$AB = AC = 5m$$

Applying the Cosine rule:

$$BC^2 = AB^2 + AC^2 - 2(AB)(AC) \cos A$$

$$BC^2 = 5^2 + 5^2 - 2(5)(5) \cos 100^\circ$$

$$BC^2 = 25 + 25 - 50 \cos 100^\circ$$

$$BC^2 = 25 + 25 - 50(-0.174)$$

$$BC = \sqrt{58.7}$$

$$BC = 7.66 \text{ m}$$

[4]

- c) Factorise $x^2 - 6x + 8$, hence solve the following trigonometric equation for $0^\circ \leq x \leq 360^\circ$:

$$\tan^2 x - 6 \tan x + 8 = 0$$

[6]

$$x^2 - 6x + 8$$

$$(x - 4)(x - 2)$$

[2]

$$\tan^2 x - 6 \tan x + 8 = 0$$

$$(\tan x - 4)(\tan x - 2) = 0$$

$$\tan x - 4 = 0$$

$$\tan x = 4$$

$$x = \tan^{-1} 4$$

$$x = 75.96^\circ \quad \text{or} \quad x = 255.96^\circ$$

[2]

$$\tan x - 2 = 0$$

$$\tan x = 2$$

$$x = \tan^{-1}(2)$$

$$x = 63.43^\circ \quad \text{or} \quad x = 243.43^\circ$$

[2]

d) A curve $y = x^2$ and the line $y = x + 20$ intersect at the points

M and N. By solving these equations simultaneously, find the co-ordinates of M and N.

[6]

$$y = x^2$$

$$y = x + 20$$

$$x^2 = x + 20$$

[1]

$$x^2 - x - 20 = 0$$

[1]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - (4)(1)(-20)}}{2 \times 1}$$

$$= \frac{1 \pm \sqrt{1 + 80}}{2}$$

$$x = 5$$

$$x = -4$$

[2]

$$y = x + 20$$

$$\text{if } x = 5 \quad y = 5 + 20$$

$$\mathbf{y = 25}$$

$$\text{if } x = -4 \quad \text{and } y = x + 20$$

$$y = -4 + 20$$

$$\mathbf{y = 16}$$

[2]

Solution: Coords are

$$\mathbf{(5, 25) \quad \text{and} \quad (-4, 16)}$$

e) Solve the following trigonometric equations for $0^\circ \leq x \leq 360^\circ$:

i. $\sin x = 0.6$

[2]

$$\sin x = 0.6$$

$$x = \sin^{-1} 0.6$$

$$x = 36.87^\circ$$

$$x = 180^\circ - 36.87^\circ$$

$$x = 143.13^\circ$$

$$\mathbf{x = 36.87^\circ \quad , \quad x = 143.13^\circ}$$

ii. $7 \cos x = 6$

[3]

$$7 \cos x = 6 \quad \text{so} \quad \cos x = \frac{6}{7}$$

$$\cos x = 0.857$$

$$x = \cos^{-1} 0.857$$

$$x = 31^\circ$$

$$x = 360^\circ - 31^\circ$$

$$x = 329^\circ$$

$$\mathbf{x = 31^\circ \quad , \quad x = 329^\circ}$$

iii. $-3 \tan x = 2$

[2]

$$\tan x = -\frac{2}{3}$$

$$x = \tan^{-1}\left(\frac{2}{3}\right)$$

$$x = 33.69^\circ \text{ not a solution}$$

$$x = 180^\circ - 33.69^\circ$$

$$x = \mathbf{146.31^\circ}$$

$$x = 360^\circ - 33.69^\circ$$

$$x = \mathbf{326.31^\circ}$$

$$x = \mathbf{146.31^\circ, 326.31^\circ}$$

QUESTION THREE (32 marks)

a) Solve the following equations, using the appropriate method, for x :

i. $\ln(x^2 + 2x) = \ln(3) + \ln(x + 1)$

[4]

$$\ln(x^2 + 2x) = \ln(3) + \ln(x + 1)$$

$$\cancel{\ln}(x^2 + 2x) = \cancel{\ln}(3)(x + 1)$$

$$(x^2 + 2x) = 3(x + 1)$$

[1]

$$x^2 + 2x = 3x + 3$$

$$x^2 - x - 3 = 0$$

[1]

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - (4)(1)(-3)}}{2 \times 1} \\ &= \frac{1 \pm \sqrt{1 + 12}}{2} \end{aligned}$$

$$x = 2.302$$

$$x = -1.302 \quad \text{No a Valid solution}$$

[2]

ii. $\log(x + 3) + \log(2x + 1) = 2\log(3x - 1)$

[4]

$$\log(x + 3) + \log(2x + 1) = 2\log(3x - 1)$$

$$\cancel{\log}(x + 3) (\cancel{2x + 1}) = \cancel{\log}(3x - 1)^2$$

$$(x + 3)(2x + 1) = (3x - 1)(3x - 1)$$

[1]

$$2x^2 + x + 6x + 3 = 9x^2 - 3x - 3x + 1$$

$$2x^2 + 7x + 3 = 9x^2 - 6x + 1$$

$$7x^2 - 13x - 2 = 0$$

[1]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{13 \pm \sqrt{(-13)^2 - 4(7)(-2)}}{2(7)} = x = \frac{13 \pm \sqrt{169 + 56}}{14}$$

$$x = \frac{13 + \sqrt{225}}{14} = 2$$

$$x = \frac{13 - \sqrt{225}}{14} = -\frac{1}{7}$$

$$x = -\frac{1}{7} \quad \text{not a valid solution} \quad x = 2 \quad \text{is the solution}$$

[2]

$$\text{iii.} \quad 5^{(x+2)} \times 2^{(2-x)} = 3^{(x+3)} \quad [4]$$

$$\log 5^{(x+2)} \times 2^{(2-x)} = \log 3^{(x+3)} \quad [1]$$

$$\log 5^{(x+2)} + \log 2^{(2-x)} = \log 3^{(x+3)} \quad [1]$$

$$(x+2) \log 5 + (2-x) \log 2 = (x+3) \log 3 \quad [1]$$

$$(x+2) 0.7 + (2-x) 0.3 = (x+3)(0.5)$$

$$0.7x + 1.4 + 0.6 - 0.3x = 0.5x + 1.5$$

$$0.7x - 0.3x - 0.5x = 1.5 - 1.4 - 0.6$$

$$-0.1x = -0.5$$

$$x = \frac{0.5}{0.1}$$

$$x = 5$$

[1]

$$\text{iv.} \quad \frac{5x+2}{2x-3} = \frac{3}{4} \quad [3]$$

$$4(5x+2) = 3(2x-3) \quad [1]$$

$$20x + 8 = 6x - 9$$

$$20x - 6x = -9 - 8$$

$$14x = -17$$

$$x = \frac{-17}{14} = -1.21$$

[2]

$$\text{v. } \frac{(3x-2)}{4} + \frac{(x+5)}{2} = \frac{7}{3}$$

[3]

$$12 \left(\frac{(3x-2)}{4} + \frac{(x+5)}{2} = \frac{7}{3} \right)$$

[1]

$$\frac{12(3x-2)}{4} + \frac{12(x+5)}{2} = \frac{12(7)}{3}$$

$$3(3x-2) + 6(x+5) = 4(7)$$

[1]

$$9x - 6 + 6x + 30 = 28$$

$$15x = 4$$

$$x = 4/15$$

[1]

b) Factorise completely, the following algebraic expressions:

$$\text{i. } z^2 - 12z + 35$$

[2]

$$z^2 - 12z + 35$$

$$(z-7)(z-5)$$

$$\text{ii. } m^2 - 121$$

[2]

$$m^2 - 121$$

$$(m+11)(m-11)$$

$$\text{iii. } kl + 2km + 5nl + 10nm \quad [2]$$

$$= k(l + 2m) + 5n(l + 2m) \\ = (l + 2m)(k + 5n)$$

c) Solve the following quadratic equations **by using the factorisation method**:

$$\text{i. } 4p + 11p + 6 = 0 \quad [2]$$

$$(4p + 3)(p + 2) = 0$$

$$4p + 3 = 0 \quad \text{or} \quad p + 2 = 0$$

$$4p + 3 = 0 \quad 4p = -3 \quad p = -\frac{3}{4} \quad p = -0.75$$

$$p + 2 = 0 \quad p = -2$$

[2]

$$\text{ii. } s^2 + 16s + 63 = 0 \quad [2]$$

$$s^2 + 16s + 63 = 0$$

$$(s + 9)(s + 7) = 0$$

$$s + 9 = 0 \quad \therefore s = -9$$

$$s + 7 = 0 \quad \therefore s = -7$$

[2]

d) Solve the following quadratic equation by using the quadratic formula:

$$3x^2 + 4x - 10 = 0$$

[4]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(-4)^2 - 4(3)(-10)}}{2(3)}$$

[1]

$$x = \frac{-4 \pm \sqrt{16 + 120}}{6} = \frac{-4 \pm \sqrt{136}}{6}$$

[1]

$$x = \frac{-4 \pm 11.662}{6}$$

$$x = \frac{-4 + 11.662}{6} = \mathbf{1.277}$$

$$x = \frac{-4 - 11.662}{6} = \mathbf{-2.610}$$

[2]

End