

Maths Matrices & MATLAB

Lecture 6

Basics

- A matrix is an $m \times n$ array of scalars. The individual values in the matrix are called entries:

- $$A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

- The size of the array is—written as $m \times n$, where m is number rows and n is number of columns

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{mn} \end{bmatrix}$$

↖
← rows
↙

↖ ↙ ↗
columns

A := uppercase denotes a matrix

a := lower case denotes an entry of a matrix $a \in F$.

- **Square matrices:**

- When the number of rows is the same as the number of columns, i.e. $m = n$, the matrix is said to be square and of order n (or m).
- A square matrix where all the non-zero elements are along the leading diagonal is called a diagonal matrix, often denoted by D .

$$D = \begin{bmatrix} d_{11} & 0 & 0 & \dots & 0 \\ 0 & d_{22} & 0 & \dots & 0 \\ 0 & 0 & \dots & \dots & 0 \\ 0 & 0 & 0 & \dots & d_{nn} \end{bmatrix} \quad d_{ij} = 0 \quad \text{when } i \neq j$$

- A square matrix in which all the elements above the leading diagonal are zero is called a lowertriangular matrix, often denoted by L .

$$L = \begin{bmatrix} l_{11} & 0 & 0 & \dots & 0 \\ l_{21} & l_{22} & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \dots & 0 \\ l_{n1} & l_{n2} & \vdots & \dots & l_{nn} \end{bmatrix} \quad l_{ij} = 0 \quad \text{when } i < j$$

- A square matrix in which all the elements below the leading diagonal are zero is called an upper triangular matrix, often denoted by U.

$$U = \begin{bmatrix} u_{11} & u_{12} & \dots & \dots & u_{1n} \\ 0 & u_{22} & \dots & \dots & u_{2n} \\ 0 & 0 & \dots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & u_{nn} \end{bmatrix} \quad u_{ij} = 0 \quad \text{when } i > j$$

Examples of matrices :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & 0 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -2 & 0 \\ 3 & -5 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Equality of matrices

- We say two matrices A, B are equal to each other only if A and B have the same number of rows and the same number of columns and if each element of A is equal to the corresponding element of B.

$$A = \begin{bmatrix} 1 & \alpha \\ -1 & -\beta \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ -1 & -4 \end{bmatrix}$$

A=B if $\alpha = 2$ and $\beta = 4$.

- The unit matrix

- The unit matrix or the identity matrix, denoted by I_n is the diagonal matrix of order n in which all diagonal elements are 1.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- The zero matrix

- The zero matrix or null matrix is the matrix all of whose elements are zero

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- The transpose of a matrix

- The transpose of a matrix A is a matrix where the rows of A become the columns of the new matrix and the columns of A become its rows.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \text{ becomes } \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

The resulting matrix is called the transposed matrix of A and denoted A^T

- Example 1

- Find the transpose of the matrix

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Addition and subtraction of matrices

- In general if A and B are both $m \times n$ matrices, with elements a_{ij} and b_{ij} respectively, then their sum is a matrix C , also $m \times n$, such that the elements of C are

$$c_{ij} = a_{ij} + b_{ij} \quad i = 1, 2, \dots, m \quad j = 1, 2, \dots, n$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

- For the following matrices find, where possible, $A + B$, $A - B$, $B - A$, $2A$.

$$1. \quad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$2. \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$3. \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$1. \quad A + B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \quad A - B = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \quad B - A = \begin{bmatrix} 0 & -1 \\ -2 & -3 \end{bmatrix} \quad 2A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

$$2. \quad A + B = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 8 & 9 & 10 \end{bmatrix} \quad A - B = \begin{bmatrix} 0 & 1 & 2 \\ 5 & 6 & 7 \\ 6 & 7 & 8 \end{bmatrix} \quad B - A = \begin{bmatrix} 0 & -1 & -2 \\ -5 & -6 & -7 \\ -6 & -7 & -8 \end{bmatrix}$$

$$2A = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}$$

$$3. \quad \text{None of } A + B, \quad A - B, \quad B - A, \text{ are defined.} \quad 2A = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}$$

Simple matrix properties

- Matrix addition is commutative: $A + B = B + A$
- Matrix addition is associative: $A + (B + C) = (A + B) + C$
- The distributive law holds: $k(A + B) = k A + k B$
- $(A + B)^T = A^T + B^T$
- $(A - B)^T = A^T - B^T$
- $(A^T)^T = A$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \text{ so that } (A^T)^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = A$$

Exercise

For the matrices $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ verify that

$$(i) \ 3(A + B) = 3A + 3B \quad (ii) \ (A - B)^T = A^T - B^T.$$

$$(i) \quad A + B = \begin{bmatrix} 2 & 1 \\ 2 & 5 \end{bmatrix}; \quad 3(A + B) = \begin{bmatrix} 6 & 3 \\ 6 & 15 \end{bmatrix}; \quad 3A = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix};$$

$$3B = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}; \quad 3A + 3B = \begin{bmatrix} 6 & 3 \\ 6 & 15 \end{bmatrix}.$$

$$(ii) \quad A - B = \begin{bmatrix} 0 & 3 \\ 4 & 3 \end{bmatrix}; \quad (A - B)^T = \begin{bmatrix} 0 & 4 \\ 3 & 3 \end{bmatrix}; \quad A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix};$$

$$B^T = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}; \quad A^T - B^T = \begin{bmatrix} 0 & 4 \\ 3 & 3 \end{bmatrix}.$$

MATLAB



Polynomial division

$$\frac{x^4 + x^3 + 7x^2 - 6x + 8}{x^2 + 2x + 8} = x^2 - x + 1$$

Command Window

```
>> u = [1 1 7 -6 8];  
v = [1 2 8];  
>> [q,r] = deconv(u,v)
```

```
q =
```

```
    1    -1     1
```

```
r =
```

```
    0     0     0     0     0
```

```
fx >>
```

Solve this using MATLAB:

$$\frac{x^2 + 5x + 6}{x + 2}$$

Command Window

r =

0 0 0 0 0

>> u = [1 5 6];

>> v = [1 2];

>> [q,r] = deconv(u,v)

q =

1 3

r =

0 0 0

$$\frac{x^4+5x^3+3x^2-x+10}{x^2+2x+8} =$$

$$= x^2 + 3x - 11 \frac{-3x + 98}{x^2 + 2x + 8}$$

Quadratic Equations

- Find the roots of this equation: $x^2 - 3x - 2 = 0$

Command Window

```
>> solve(x^2-3*x-2==0)
```

```
ans =
```

```
3/2 - 17^(1/2)/2  
17^(1/2)/2 + 3/2
```

```
fx >>
```

- Find the roots of this equation: $2x^2 + x - 6 = 0$

Command Window

```
>> solve(2*(x^2)+x-6==0)
```

```
ans =
```

```
-2
```

```
3/2
```

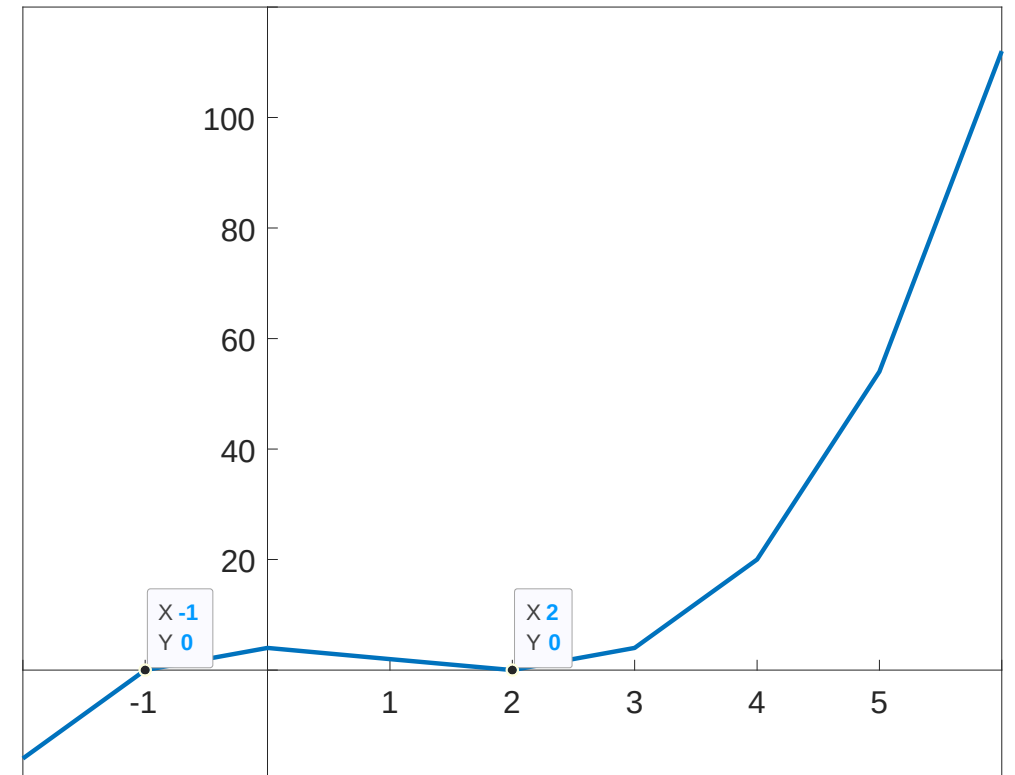
```
fx >>
```

Graphs of polynomial functions

- Find the root of the polynomial : $x^3 - 3x^2 + 4 = 0$

Command Window

```
>> x=-2:6;  
>> y=x.^3-3*x.^2+4;  
>> plot(x,y)|  
>> grid on  
>> ax = gca;  
>> ax.XAxisLocation = 'origin';  
>> ax.YAxisLocation = 'origin';  
fx >> |
```



Graphs of polynomial functions

- Find the root of the polynomial : $x^3+3x^2-x-3=0$

