

Ways forward for Carlos

The specific aim for working with Carlos was to generate and consolidate factual knowledge, underpinned by sound conceptual knowledge of addition. The strategies for achieving this aim were as follows:

- *Providing strong visual/spatial images of number facts alongside written representations of the sums.* This strategy was devised to draw on Carlos' spatial ability, and to provide him with an immediate boost to his confidence. An old set of Colour Factor rods was located at the back of the school's mathematics stock cupboard, and cannibalised by writing the numerical value on each rod. The teacher then used the rods, alongside the relevant sums, to illustrate the pattern of the basic addition bonds to ten for Carlos. The advantage of using the Colour Factor rods for this exercise, rather than Multi-link or similar individual cubes, was that the rods represented each number as a composite unit (so, for example, the 6 block was a length of wood equivalent to six single units, but the block was not marked as individual units and so could not be counted one by one). This encouraged Carlos to identify each number as a composite unit rather than as a collection of single units. After some initial work with the rods, Carlos' strong spatial sense enabled him to play a 'guess the missing number game' with the rods (see Figure 6.1).
- *Setting achievable targets.* Carlos' work with the rods was linked to targets for him to memorise and use key number facts. Carlos' parents were involved in these targets. They continued using the rods with him at home, gradually moving from playing the guessing game with the rods to playing it with flash cards with sums written on them.
- Support at home and school thus gave Carlos *opportunities to practise and use his developing knowledge*. Developing his knowledge of number patterns and relationships was important here, so that his factual knowledge was connected to his growing conceptual understanding of the operation of addition.

Using the rods again, the teacher illustrated to Carlos the commutativity of addition and the ways in which addition was the inverse of subtraction. A simple computer program 'Function Machine' helped Carlos to understand this inverse relationship further.

Stage 1: Using the Rods

10	
6	4

Stage 2: The Guessing Game with the Rods

10	
?	4

Stage 3: Facts flashcards – two versions

$6 + ? = 10$
$6 + 4 = 10$

Ways forward for Amy

The specific aim was for Amy to understand the *mathematical meaning* of the number facts she was trying to memorise so diligently. The strategies for achieving this aim were as follows:

- *Working with Amy to identify a variety of visual and oral representations of addition facts.* For example, Amy was first asked to find different ways of picturing $4+6$. Supported initially by her teacher, she was able to draw the sum on a number line and collect sets of four and six items to combine together. This provided Amy with explicit models of addition as both *aggregation* (Haylock (2000:208) defines this as 'two or more quantities combined into a single quantity') and *augmentation* (Haylock defines this as where 'a quantity is increased by an amount and the operation of addition is required in order to find the ... increased value'). On another occasion she was asked to 'read' $4+6$ in a variety of different ways, using mathematical language such as 'add', 'plus', and 'more than'.
- *Involving parents.* The class teacher set up a meeting with Amy's parents to outline and discuss the situation. Following this meeting, it was agreed that her parents would continue to give Amy regular practice of the facts that she knew, alongside developing and supporting the teacher's other strategies.
- *Experimenting with number patterns.* Amy was asked to explore different number patterns, drawing on and extending her existing knowledge. One day she took home the start of the number pattern

$$10+1=$$

$$11+1=$$

$$12+1=$$

She was asked to complete and extend the pattern to 25 and to come back to school able to discuss how and why the pattern worked. Another pattern task involved finding as many different ways as possible of solving the puzzle $? + ? = 15$ and discussing any patterns she saw. Later pattern work included an emphasis on subtraction as the inverse of addition. This work was valuable in developing Amy's understanding of number relationships. It also provided creative and enjoyable ways in which Amy could use, apply and extend her existing knowledge. Amy and her parents extended this work at home using a calculator to enable the exploration of addition patterns with large numbers.

- *strengthening knowledge of two digit place value.* For Amy this involved a particular focus on developing her knowledge of the order of the numbers between 11 and 99.
- *achievable targets* for Amy were set in all of these areas, and monitored by Amy herself, her parents and the class teacher.

Ways forward for Ravinder

The specific aims for working with Ravinder were to extend her knowledge of the different methods for mental calculation, and to consolidate and extend her conceptual knowledge and understanding of addition. The teaching and learning strategies for achieving this aim were as follows:

- *Working with a more able peer explaining and discussing mental methods.* The teacher set up daily five minute discussion sessions for Ravinder and a friend who was confident in using a wide range of mental strategies. The pair were given a small list of calculations, asked to work out the sums and then explain their calculation to one another. These sessions had two aims: firstly, they aimed to provide Ravinder with unpressurised opportunities to listen to a range of methods; secondly, they aimed to give her regular practice in explaining her own methods to develop her use of mathematical language. At regular intervals the teacher also joined in with these sessions to provide visual images of the strategies being described. For example, she drew representations of the partitioning of two-digit numbers and of calculations using an empty number line.
- *Setting achievable targets.* This strategy was used to help Ravinder to develop a wider range of factual knowledge, including knowledge of key addition bonds to 10 and then 20. After a meeting with the teacher, Ravinder's parents were involved in these targets. Each week she was set a small number of facts to practise at home. Later she also took home more complex sums and was asked to work with her parents and older siblings to find ways of using her developing factual knowledge to find the answers.
- *Developing knowledge of place value.* This work with Ravinder focused on developing her knowledge of three aspects of two-digit place value: number names, number order and partitioning. Her teacher provided additional oral practice of the number names between 10 and 100, linking this practice to the ways in which the numbers were written. The computer program 'Counter' enabled the teacher to devise a variety of ways of counting with the numbers on the computer screen. A calculator programmed with the constant function to keep adding one provided a simpler version of the same idea. Linking this counting practice to a number line enabled Ravinder to develop her understanding of number order. She also worked at partitioning two-digit numbers into tens and units. This work included use of a variety of visual, tactile and symbolic resources, including the computer game 'Highs and Lows' which provided an effective and motivating learning context.