

1) A line charge density is given by $\rho_L = 5x^2 \sin(2\pi x)$. Find the total charge along the line segment from $x = -2$ to $x = 3$.

Given:

$$\rho_L = 5x^2 \sin(2\pi x)$$

Limits $x = -2$, to $x = 3$.

$$Q = \int_L \rho_L dL$$

$$Q = \int_{-2}^3 5x^2 \sin(2\pi x) dx \quad \text{--- ①}$$

Integrating by Parts

$$\int u dv = uv - \int v du$$

$$Q = 5 \int_{-2}^3 x^2 \sin(2\pi x) dx$$

$$u = x^2, \quad dv = \sin(2\pi x) dx$$

$$du = ? \quad v = ?$$

$$\begin{array}{l} \text{Diff } u = x^2 \\ du = 2x dx \end{array} \left| \begin{array}{l} \text{Integrate } dv = \sin(2\pi x) dx. \\ \text{Integrating both sides.} \\ \int dv = \int \sin(2\pi x) dx \end{array} \right.$$

$$v = -\frac{1}{2\pi} \cos(2\pi x)$$

$$\left[dx^n = nx^{n-1} dx \right]$$

$$\left[\int \sin ax dx = -\frac{1}{a} \cos ax \right]$$

$$\left[\int u dv = uv - \int v du \right]$$

$$\int x^2 \sin(2\pi x) dx = x^2 \left(\frac{-1}{2\pi} \cos(2\pi x) \right) - \int \frac{-1}{2\pi} \cos(2\pi x) \cdot (2x) dx$$

$$u \quad v \quad - \int v \quad dv$$

$$= -\frac{x^2}{2\pi} \cos(2\pi x) + \int \frac{2x}{2\pi} \cos(2\pi x) \cdot dx$$

$$\int x^2 \sin(2\pi x) dx = -\frac{x^2}{2\pi} \cos(2\pi x) + \frac{1}{\pi} \int x \cdot \cos(2\pi x) dx. \quad \text{--- (2)}$$

$\int u \cdot dv$

To Simplify.

$$\int x \cdot \cos(2\pi x) dx$$

$u = x$
Differentiate

$$du = dx$$

$$dv = \cos(2\pi x) dx$$

Integrating both the sides

$$\int dv = \int \cos(2\pi x) dx.$$

$$v = \frac{1}{2\pi} \sin(2\pi x)$$

$$\int x \cos(2\pi x) dx = x \cdot \left(\frac{1}{2\pi}\right) \sin(2\pi x) - \int \frac{1}{2\pi} \sin(2\pi x) dx$$

$$\int dv = v$$

$$\int \cos ax dx = \frac{1}{a} \sin ax$$

$$\int u dv = uv - \int v \cdot du$$

$$\int x \cos(2\pi x) dx = \frac{x}{2\pi} \sin(2\pi x) - \frac{1}{2\pi} \int \sin(2\pi x) dx$$

$$= \frac{x}{2\pi} \sin 2\pi x - \frac{1}{2\pi} \left[\frac{-1}{2\pi} \cos(2\pi x) \right]$$

$$\int x \cos(2\pi x) dx = \frac{x}{2\pi} \sin 2\pi x + \frac{1}{4\pi^2} \cos 2\pi x \quad \text{--- (3)}$$

Sub. eq (3) in eq (2).

$$\textcircled{2} \Rightarrow \int x^2 \sin(2\pi x) dx = \frac{-x^2}{2\pi} \cos 2\pi x + \frac{1}{\pi} \left[\frac{x}{2\pi} \sin 2\pi x + \frac{1}{4\pi^2} \cos 2\pi x \right]$$

$$= \frac{-x^2}{2\pi} \cos(2\pi x) + \frac{x}{2\pi^2} \sin(2\pi x) + \frac{1}{4\pi^3} \cos(2\pi x) \quad \text{--- (4)}$$

$$Q = 5 \left[\frac{-x^2}{2\pi} \cos(2\pi x) + \frac{x}{2\pi^2} \sin(2\pi x) + \frac{1}{4\pi^3} \cos(2\pi x) \right]_{-2}^3$$

$$Q = 5 \left[\left[\frac{-3^2}{2\pi} \cos(2\pi \times 3) + \frac{3}{2\pi^2} \sin(2\pi \times 3) + \frac{1}{4\pi^3} \cos(2\pi \times 3) \right] - \left[\frac{-(-2)^2}{2\pi} \cos(2\pi \times (-2)) + \frac{(-2)}{2\pi^2} \sin(2\pi \times (-2)) + \frac{1}{4\pi^3} \cos(2\pi \times (-2)) \right] \right]$$

$$Q = 5 \left[\left[-\frac{9}{2\pi} \cos(6\pi) + \frac{3}{2\pi^2} \sin(6\pi) + \frac{1}{4\pi^3} \cos(6\pi) \right] - \left[\frac{-4}{2\pi} \cos(-4\pi) - \frac{1}{\pi^2} \sin(-4\pi) + \frac{1}{4\pi^3} \cos(-4\pi) \right] \right]$$

$$Q = 5 \left[\left[-\frac{9}{2\pi} (1) + \frac{3}{2\pi^2} (0) + \frac{1}{4\pi^3} (1) \right] - \left[\frac{-2}{\pi} (1) - \frac{1}{\pi^2} (0) + \frac{1}{4\pi^3} (1) \right] \right]$$

$$Q = 5 \left[\left[-\frac{9}{2\pi} + \frac{1}{4\pi^3} \right] - \left[\frac{-2}{\pi} + \frac{1}{4\pi^3} \right] \right] = 5 \left[-\frac{9}{2\pi} + \frac{1}{4\pi^3} + \frac{2}{\pi} - \frac{1}{4\pi^3} \right]$$

$$= 5 \left[\frac{-9}{2\pi} + \frac{2}{\pi} \right]$$

$$= -3.978 \text{ C}$$