



PRIFYSGOL CYMRU

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Practical and Analytical Investigation into Motorcycle Steering Dynamics and Rider Control

Postgraduate Research Project

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Abstract

A highly sophisticated data-acquisition setup on a 'Ducati Panigale 1199 S' to measure the physical rider input at the handlebars in conjunction with the attitude and overall performance of the motorcycle is described. It is used to collect data from simple manoeuvres and comprehensive track sessions with different riders of distinct riding ability and varying experience. The observed dynamic behaviour of the entire vehicle-operator system is subsequently analysed in terms of fundamental steering theory, steady-turning performance and transient-state manoeuvrability, as well as rider control and handling techniques. In addition, the recorded measurements are used to compute variants of the total steering torque from its components that are based on steady-state and transient-state equations of existing mathematical models in order to investigate the factors that control the primary steering mechanism and its interaction with the dynamic response of the motorcycle to rider inputs. The calculation of these steering torque components requires innovative and new methodology that is developed for this purpose. The results are analysed and validated by means of comparison with the measured steering torque that is actually applied by rider. Moreover, the existing equations are enhanced by the derivation of additional steering torque components. Ultimately, the computed steering torque results can accurately model and describe the physical steering input of the rider that was measured at moderate to high velocities. Lastly, a multibody software package is used to simulate the dynamic behaviour of virtual models and the steering torque input is validated.

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Table of Symbols

ε	Rake Angle
ϑ	Chassis Pitch Angle
δ	Steering Angle
$\dot{\delta}$	Steering Velocity
$\ddot{\delta}$	Steering Acceleration
φ	Roll Angle
$\dot{\varphi}$	Roll Velocity
$\ddot{\varphi}$	Roll Acceleration
ψ	Yaw Angle
$\Omega, \dot{\psi}$	Yaw Velocity
$\ddot{\psi}$	Yaw Acceleration
τ	Steering Torque (Components)
I_{Wf}, i_{fy}	Front Wheel Spin Inertia
ω_f	Front Wheel Angular Velocity
m_f, M_f	Front Frame Mass
b_f, h_f, e_f, l_z, e	Front Frame Mass Centre Offset
g	Gravitational Acceleration
a_n, t	Normal Trail
N, N_f, N_r, Z_f	Normal Force
$V, \dot{x}_1$	Forward Velocity
r_f, R_f	Front Wheel Radius
K	Steering Damper Damping Coefficient
$\dot{v}$	Lateral Acceleration
k	Horizontal Distance from Front to Rear Frame Mass Centre
$I_{fz}, I_{xf}, I_{yf}, I_{zf}$	Front Frame Principle Inertias
j	Vertical Distance from Front Mass Centre to Rear Contact Patch
F_s, F_f, F_r, Y_f	Lateral Force
t_f, t_r, q_f, q_r	Cross-Sectional Tyre Radius
ρ_f, ρ_r	Torus Centre Circle Radius
m, M_T	Total Mass
M_U	Unsprung Mass

M_S	Sprung Mass
k_{Susp}	Suspension Spring Stiffness
c_{Susp}	Suspension Damping Coefficient
c_{crit}	Suspension Critical Damping Coefficient
ζ	Suspension Damping Ratio
$\dot{x}_{Angle}, \dot{y}_{Angle}, \dot{z}_{Angle}$	Local Angular Rates
λ	Sideslip Angle
k_λ	Tyre Cornering Stiffness
k_ϕ	Tyre Camber Stiffness
μ	Kinematic Pitch Angle
d, l_1	Fork Offset
P_f, P_r	Contact Patch
S, S_f, S_r, X_f	Longitudinal Force
G, G_f, G_r	Mass Centre
θ	Kinematic Determinant
Δ	Kinematic Steering Angle
Δ^*	Effective Steering Angle
ξ	Steering Ratio
R_c, R_{cO}, R_{c_r}	Turning Radius
C, C_O	Turning Centre Point
γ	Trajectory Angle
τ_f	Front Load Transfer Angle
F_{Susp}	Suspension Force
N_{tr}	Load Transfer Force
p	Wheelbase
b	Horizontal Distance from Overall Mass Centre to Rear Contact Patch
$Braking_{ON/OFF}$	Boolean Condition
x_{Susp}	Suspension Position
$\dot{x}_{Susp}$	Suspension Velocity
$\ddot{x}_{Susp}$	Suspension Acceleration
M_y	Tyre Rolling Resistance Moment
d_f	Tyre Rolling Resistance

f_w	Tyre Rolling Resistance Coefficient
M_z	Tyre Yawing Moment
k_{sa}	Tyre Self-Aligning Stiffness
k_{tw}	Tyre Twisting Stiffness
β, β'	Toroidal and Poloidal Front Contact Patch Angles
$C_1, C_2, C_3, C_4, C_5, A, B, C, D, E$	Kinematic Variables (Capitalised)
x_{Pf}, y_{Pf}	Local Front Contact Patch Coordinates
Δh	Front Axle Lowering
a_n^*	Normal Cornering Trail
a_1, a_2, a_3	Normal Cornering Trail Components
$X_{Pf}, X_{Pr}, Y_{Pf}, Y_{Pr}$	Global Contact Patch Coordinates
θ	Turn Curvature
X, Y, Z	Global Reference Frame Coordinates
x, y, z	Local Rear Frame Coordinates
x_f, y_f, z_f	Local Front Frame Coordinates
X_Q, Y_Q, Z_Q	Global Q-Joint Coordinates
x_Q, h_Q	Horizontal and Vertical Distance from Point Q to Rear Contact Patch
d_Q	Vertical Displacement of Point Q along Steering Axis
x_{Gf}, y_{Gf}, z_{Gf}	Local Front Frame Mass Centre Coordinates
X_{Gf}, Y_{Gf}, Z_{Gf}	Global Front Frame Mass Centre Coordinates
I_{CXZf}, I_{CZf}	Global Front Frame Products of Inertia
τ_z	Torque
I_y	Inertia
ω_x, ω_y	Angular Velocity
L_y	Angular Momentum

1. Introduction

The field of motorcycle engineering relies heavily on a thorough understanding of the dynamic behaviour that is associated with single-track vehicles. It is needed to determine the desired handling and performance characteristics of motorcycles, as well as the required safety standards regarding their use by variedly skilled riders. This applies to the industry, for example at the design stage of production motorcycles that can be tailored to vastly different terrains, as well as the motorsport sector, where the effective setup of racing motorcycles to suit the demands of a specific circuit and rider is key. Whilst some motorcycles are comprised of very powerful engines and comprehensive technology, which can directly contribute to the governing factors that control their behaviour and response to external inputs, the fact that all conventional two-wheeled vehicles are inherently unstable, subjected to gyroscopic effects and capable of moving out of plane due to multiple degrees of freedom causes their intrinsic dynamics to be highly complex when compared to those of other common land vehicles, such as automobiles. Furthermore, the role of the operator and their means of control over the vehicle are substantially different, for example due to the sensitivity of a motorcycle to the positioning of its rider. However, the primary method for manoeuvring a motorcycle is the physical application of a torque to the front wheel at the handlebars. The investigation of this main control mechanism is the subject of this study and the definitive research objectives are extrapolated from the current literature on motorcycle dynamics in general and rider control in particular. The project is carried out both practically and analytically by means of recording the interaction of the rider-induced steering torque input and the dynamic response of a motorcycle, as well as numerically analysing it. This serves the purpose of increasing the knowledge and understanding of motorcycle dynamics, especially in transient state, as well as constructively adding to the academic discourse by validating and enhancing existing mathematical models. In addition, the research is aimed at identifying, quantifying and assessing the factors that control the dynamic behaviour and steering performance of motorcycles to allow for an advanced evaluation of different riding styles, as well as motorcycle geometry, setup and chassis design.

2. Literature Review

2.1. Overview

The earliest studies on single-track vehicles date back as far as the invention of the first bicycle in the 19th century. A detailed historical summary of the literature on their dynamics is given by (Schwab & Meijaard, 2013), as well as (Kooijman & Schwab, 2011). (Sharp, 1978) and (Sharp, 1985) offer admittedly dated but more motorcycle-specific reviews on the prevailing dynamics theory and mathematical modelling capabilities at the time, with respect to general steering behaviour, oscillatory disturbances that can be observed at certain speeds and excitation frequencies in straight-line running, as well as the effects or impact of motorcycle design and technology. A more recent and comprehensive review on single-track vehicle modelling techniques and control considerations is presented by (Limebeer & Sharp, 2006). Partly in joint efforts, the same authors have significantly contributed to the understanding and knowledge of motorcycle dynamics before, during and since that time with papers like (Sharp, 1971), (Sharp, 1994), (Sharp, 2001), (Sharp & Limebeer, 2001), (Limebeer, et al., 2001), (Sharp, et al., 2004) and (Sharp, 2010) that primarily focus on modelling motorcycle stability and control. Another relevant source relating to this is (Weir, et al., 1979), especially since it offers an in-depth presentation of the mathematical model and derivations that lay the foundation for its investigation of the capsize, weave and wobble modes in conjunction with rider control considerations and resulting handling implications.

2.2. Model Developments

Extensive theoretical models of bicycle and motorcycle dynamics have been developed at a number of universities over the years, for example as part of master's or doctorate theses by (Koenen, 1983), (Giles, 1985), (Styles, 2004), (Rowell, 2007) and (Ooms, 2011). Fundamental references in current literature are the books by (Foale, 2002) and (Cossalter, 2006). The latter was written by one of the most prominent motorcycle dynamicists, who is a professor of applied mechanics and held a chair for vehicle dynamics at the University of Padova. In fact, some of the most relevant research that was conducted in the field and significant publications of recent times have originated from his

consortium at the aforementioned university. Examples of such that entail the development of highly sophisticated modelling techniques by taking advantage of evermore advanced computational methods on top of incorporating unique and complex multibody codes to study the dynamic behaviour of motorcycles through simulations are (Cossalter, et al., 1999), (Cossalter & Lot, 2002), (Cossalter, et al., 2004), (Massaro & Lot, 2007), (Cossalter, et al., 2011), (Massaro, et al., 2012) or (Massaro, et al., 2013). The multibody models that were derived at the University of Padova have also led to the creation of so-called lap time simulators, such as 'FastBike' which is presented by (Cossalter, et al., 2003). This particular type of software allows for a prediction of the fastest times that a specific motorcycle model can achieve in pre-defined conditions and over a full lap or track section of any given race circuit by defining an ideal path within adjustable limits and computing its dynamic attitude or trim via multiple iterations. Ultimately, these simulations can be used for the assessment of different design and setup configurations in order to establish and reach the best performance compromise possible, as is demonstrated by (Cossalter, et al., 2008) and (Cossalter, et al., 2013). Thus, lap time simulators are increasingly being put to use by manufacturers and motorsport teams that compete at the World Championship level of racing.

2.3. Handling Analysis

The physical steering input that is required from the rider for a certain manoeuvre and its interaction with the dynamic motorcycle response has long been of great interest to researchers, manufacturers and road traffic safety organisations alike. Following on from fundamental research on single-track vehicles in steady turning by (Fu, 1966) and since their equations of motion enabled the first derivation of a formula to calculate steering torque by (Sharp, 1971), the assessment of motorcycle design based on manoeuvrability and handling performance, as well as the definition of ideal handling quality, have been the subject of copious studies. The first evaluations of transient-state handling characteristics of motorcycles, as well as those of the influence of different riding techniques, were already conducted in the mid-1970s by virtue of datalogging and measuring steering torque on instrumented reference motorcycles during simple manoeuvres, like lane changes or slalom tests.

Whilst (Taguchi, 1975) and (Rice, 1979) only give an account of experimentally acquired results, a comprehensive report for the 'National Highway Traffic Safety Administration' of the U.S. Department of Transportation (DOT) by (Rice & Kunkel, 1976) also includes simulated results. A similar approach which is aimed at comparing recorded measurements to the obtained results of a linearized mathematical model was chosen by (Weir & Zellner, 1978). The two authors have carried out extensive investigations into motorcycle stability and the effects of design alterations in conjunction with steering torque measurements, which also led to a report for the DOT by (Weir, et al., 1979) and considerations regarding the conducted handling test procedures by (Wier & Zellner, 1980). Practical research in this particular area was also carried out for Japanese motorcycle manufacturers by (Aoki, 1980), (Sugizaki & Hasegawa, 1988) and (Kuroiwa, et al., 1995). More recently, and by collecting data with much more advanced sensory equipment than was available for use on reference motorcycles in the aforementioned studies, experimental transient-state validations of analytical transfer functions that were derived from the mathematical model by (Sharp, 1971) and a computational multibody model by (Cossalter & Lot, 2002) were performed at the University of Padova with slalom manoeuvres at different speeds and cone spacings. The work is presented by (Biral, et al., 2003) and the authors claim that the calculated transfer functions from the obtained measurements of physical steering inputs and the directly resulting response behaviour of the motorcycle show a "fair to good agreement" (p. 2) with the computed model predictions. The equations of motion that were originally presented by (Sharp, 1971) were later revised, for example by (Sharp, 1994) himself or (Ueda, 2004), to include additional factors and describe the system more accurately. Other mathematical models that allow for an exploration of the rider-applied steering torque have been derived as well, such as the analytical approach that was developed by (Cossalter, et al., 1999) and describes the individual components that the total steering torque is made up of in steady-state cornering conditions. The nature of this overall steering torque and its constituents was investigated further and the results of the model were experimentally validated with obtained data from multiple reference motorcycles in turns of varying radii and at different cornering speeds by (Cossalter, et al., 2007) and (Cossalter, et al., 2011).

2.4. Virtual Riders

Generally, a numerical vehicle model for motorcycles requires an accurate virtual rider model for a simulation of the full system dynamics. Initial advancements based on the ‘Optimal Manoeuvre Method’ and using PID architecture were presented by (Lot, et al., 2008) and (Massaro, 2011), before exceptionally good correlation of the calculated overall steering torque with results of real-world tests was achieved by (Massaro, et al., 2012). To do so, the non-linear motorcycle model from (Cossalter, et al., 2011) is provided with previously collected data of roll angle and forward velocity to establish and dictate the path that sets a target profile for the virtual controller. It is then given predictive input control over discrete time steps at which the non-linear multibody model is linearized about a quasi-static state condition with assigned values for vehicle roll, speed and acceleration. Another example of a virtual rider model is (Ajmi, et al., 2017). However, the multibody code derived herein lacks validation through experimental testing and makes use of a commercially sold multibody dynamics programme for comparison instead, namely ‘MSC ADAMS/View’.

2.5. Multibody Software

Readily available multibody simulation packages, like ‘MSC ADAMS/View’ and ‘Simscape Multibody’, allow for the creation and study of complex systems on the basis of geometric constraints and inertial properties that are assigned to individual objects or entire assemblies without the need for any fundamental derivations of their equations of motion by the end user. One such system of a motorcycle and a virtual rider was built and presented by (Sequenzia, et al., 2015). It is used to assess the influence of rider movement on the performance of a motorcycle and thereby shows that the capabilities of modern multibody software are not just limited to facilitating the numerical analysis of certain vehicles alone, but also how it can be capitalised on for studies of the dynamic interaction between a human controller and vehicle. In addition to the previously mentioned multibody code ‘FastBike’, which is used for simulation services by (Dynamotion S.r.l., 2018), several other software packages that are designed to meet the needs of motorcycle engineers, such as ‘VI-BikeRealTime’ or ‘VI-Motorcycle’, are available and can equally be used for

lap time simulations, design parameter optimisation and research purposes. Nowadays, these products also take advantage of real-time solvers and even account for interactions between a motorcycle and the human body (VI-grade GmbH, 2020). Nonetheless, a study of the effect of passively exerted steering impedance by riders on the stability of motorcycles in straight-running conditions, which is presented by (Cossalter, et al., 2011), demonstrates how also having access to an actual riding simulator that is linked to a multibody simulation environment can be of great benefit.

2.6. Riding Simulators

Most dynamic simulators make use of virtual reality and provide the operator with physical cues to create an experience that is similar to the perceived sensation when controlling the actual vehicle, such as the 'MORIS' motorcycle simulator which is introduced by (Ferrazzin, et al., 2003) and aims at reducing the required time, effort and costs that are associated with the development of a vehicle prototype during the design stage. A more recent publication by (Terada, et al., 2019) describes the creation and industrial use of a motorcycle simulator by the manufacturer 'Kawasaki'. Its purpose is to partly enable the determination of handling characteristics and the potential steering feel of new models in steady-state cornering conditions, without the need for real-world testing, based on a simplified version of the governing equations from (Cossalter, et al., 1999). A state-of-the-art riding simulator for research purposes that is significantly more complex is described and evaluated by (Westerhof, 2018) and (Westerhof, et al., 2019). Whilst its underlying multibody code is an adoption of the mathematical model by (Sharp & Alstead, 1980) within 'SimScape Multibody', the interface between a riding simulator and its vehicle dynamics model can also incorporate motorcycle-specific multibody software, such as 'VI-BikeRealTime'. An example of one that was developed by the 'Würzburg Institute of Traffic Sciences', in partnership with the 'FZD TU Darmstadt' and 'BMW Motorrad', is the DESMORI simulator (Stedmon, et al., 2017). Its system architecture includes the aforementioned simulation package by 'VI-grade' and this rider-in-the-loop platform has been used for research into coupling effects between a rider and the vehicle by (Scherer, et al., 2019), as well as into models for rider presence by (Will, 2017).

3. Research Objectives

Despite the indicated extent of research in the field of motorcycle dynamics with respect to rider control inputs, the range of recent studies is limited and very little scientific work has been reported to determine how to best control a motorcycle by taking certain advantage of its intrinsic steering dynamics. Furthermore, very few investigations have considered the transient effects on factors that influence the steering torque of the rider. This leaves a gap for innovative research which combines the use of up-to-date technology with new methodology that allows for the numerical analysis of accurate measurements.

3.1. Evaluating Handling Effort and Riding Techniques

The initial goal for this project is to measure the amount of steering torque that is physically applied at the handlebars and simultaneously record the exact attitude, as well as the changing intrinsic variables and common parameters, of a modern high-performance motorcycle during various dynamic scenarios. These can range from simple manoeuvres, like travelling on a circular path at constant speed, lane changes or slaloms, to full flying laps at several race circuits, which are furthermore executed by different riders of distinct riding ability and varying experience. This serves the purpose of capturing the dynamic behaviour of an entire vehicle-operator system for its subsequent analysis in terms of fundamental steering theory, steady-turning performance and transient-state manoeuvrability, as well as rider control and handling techniques, based on the given chassis geometry and setup configurations. Thus, the primary research question for this project is summed up as follows.

“What is the steering effort that is required of a rider and how is it controlled?”

3.2. Computing Steering Torque of a Steady-State Model

In addition, the aim of the project is to utilise the equations that are derived and presented by (Cossalter, et al., 1999) and to implement the previously collected data for a calculation of the total steering torque from its components, which are illustrated in **Figure 1**. The schematic diagram shows the steered front frame assembly of a motorcycle and it depicts the steering torque components that are induced by gravity, the centrifugal effect, a normal load, lateral force and longitudinal force that act at the contact patch of the front tyre,

along with tyre rolling resistance and yawing moments, as well as a gyroscopic effect that arises from the angular momentum of the front wheel during steady-state cornering conditions.

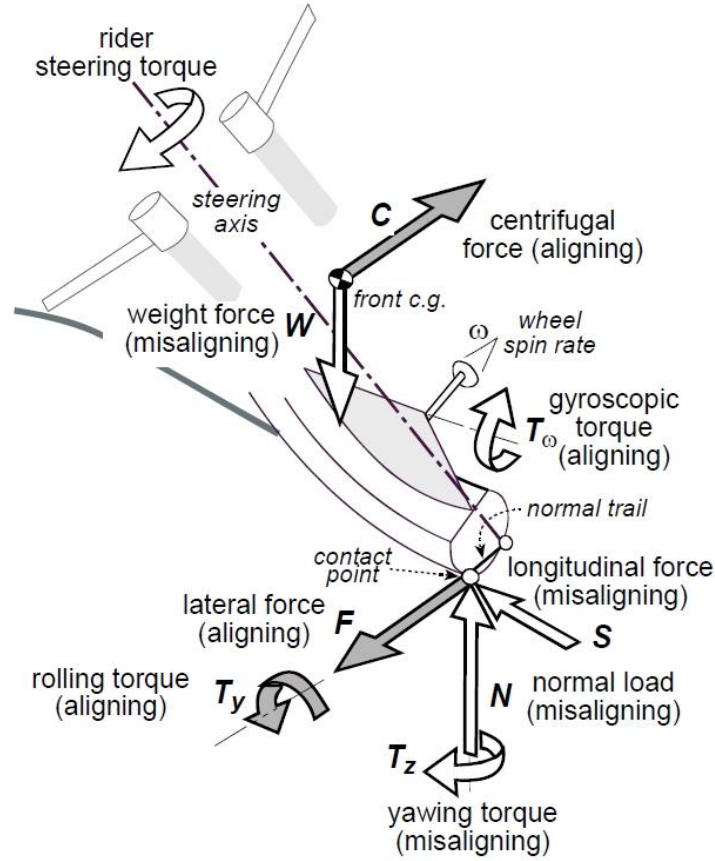


Figure 1 Graphic representation of forces and moments that control steering torque in steady-state cornering conditions (Cossalter, et al., 2011)

The current literature could thus be built on by comparing this calculated steering torque with the measured steering torque and thereby further validating the provided steady-state equations. Similarly to the approach that was chosen for a virtual rider by (Massaro, et al., 2012), another underlying research question of this project is, therefore, the following.

“Can applying a quasi-static model that is based on derivations for the calculation of steering torque in steady-state cornering equilibria show sufficient correlation with dynamic measurements of the transient case, when it is supplied with previously recorded data from a real motorcycle?”

3.3. Analysing, Validating and Enhancing Steady-State Equations

In contrast to the aforementioned paper, and provided that the sum of the calculated results matches the computed steering torque channel which is obtained from physical measurements on the reference motorcycle, the main objective is to assess the different steering torque components individually and to analyse them collectively for an improved understanding of transient-state motorcycle dynamics. In the case that this newly calculated steering torque does not successfully mirror the measured and proven steering torque that is required of a rider to control the motorcycle, be it in a steady or transient situation, the applied dynamics model will undeniably require revision or enhancement to suffice for the desired investigation of its components by the author. This is partly done through the derivation of further algebraic equations that directly account for additional dynamic influences.

At this stage, the author has already identified an interaction between the rider input and a gyroscopic component that could apply a further torque to the handlebars due to the angular momentum of the front wheel when considering a transient scenario. The then present change in vehicle roll, in other words its roll rate, would create a misaligning torque that opposes the counter-steering ambitions of the rider during the turn-in phase and contributes to the observable self-adjustment of the front frame assembly when the rider lets out the previously applied torque which initially instigated this roll motion due to the associated momentum of the motorcycle. This foreseeably results in the necessity of making an alteration to the equation of the gyroscopic steering torque component from (Cossalter, et al., 1999), since it is derived for steady-state cornering conditions and only accounts for the torque component that is caused by a turning motion of the vehicle, in other words its yaw rate. The following extension is therefore proposed and to be checked for correctness, alongside the abovementioned theory, as part of this research project:

$$\begin{aligned}\tau_{Gyroscopic} &= \tau_{Yaw} + \tau_{Roll} \\ &= (\sin \varepsilon * \cos \varphi * \cos \delta - \sin \varphi * \sin \delta) * I_{Wf} * \omega_f * \dot{\psi} - \cos \varepsilon * \cos \delta * I_{Wf} * \omega_f * \dot{\varphi}\end{aligned}$$

3.4. Computing Steering Torque of a Transient-State Model

Alternatively, the numerical analysis of steering torque and its composition can also be carried out by means of computing them with extracts from a different model altogether, such as the steer torque equation that was derived by (Sharp, 1971). This second-order differential equation is comprised of 16 separate terms that are listed and described in **Figure 2**.

No.	1971 Paper Notation	Description
1	$-M_f e g \phi$	Moment from weight of front frame from roll.
2	$t Z_f \phi$	Moment of normal force on the tyre from the ground from roll.
3	$-M_f e g \sin \epsilon \delta$	Moment from weight of front frame steer.
4	$t Z_f \sin \epsilon \delta$	Moment of normal force on the tyre from the ground from steer.
5	$M_f e \dot{x}_1 \dot{\psi}$	Moment from front frame mass centre lateral acceleration from yaw.
6	$i_{fy}/R_f \sin \epsilon \dot{x}_1 \dot{\psi}$	Gyroscopic torque from yaw.
7	$-i_{fy}/R_f \cos \epsilon \dot{x}_1 \dot{\phi}$	Gyroscopic torque from roll.
8	$K \dot{\delta}$	Steering damper moment.
9	$M_f e \dot{v}$	Moment from lateral acceleration of front frame.
10	$M_f e k \ddot{\psi}$	Moment from lateral acceleration of front frame from yaw.
11	$I_{fz} \cos \epsilon \ddot{\psi}$	Front frame inertia cross-coupling torque from yaw.
12	$M_f e j \ddot{\phi}$	Moment from lateral acceleration of front frame from roll.
13	$I_{fz} \sin \epsilon \ddot{\phi}$	Front frame inertia cross-coupling torque from roll.
14	$M_f e^2 \ddot{\delta}$	Steer mass inertia torque.
15	$I_{fz} \ddot{\delta}$	Steer inertia torque.
16	$t Y_f$	Aligning moment from side force through mechanical trail.

Figure 2 The individual outputs from the Sharp 1971 steer torque equation (Styles, 2004)

Compared to the set of equations from the mathematical model by (Cossalter, et al., 1999), it includes several additional factors which are mostly dynamic by nature and thus characteristic of the transient state. Whilst the large number of unknowns and differentials could pose a serious challenge with regards to a calculation of these steering torque components from the ground up, the aim of this research project is to determine them by incorporating the processed dataset, along with any essential formulae from (Cossalter, 2006) where necessary.

3.5. Analysing, Validating and Enhancing Transient-State Equations

As was the case with the previous model, the listed factors seem incomplete and thus they require the derivation of a 17th component in order to include influence of a longitudinal force at the front contact patch. On the basis of an extension to the original equation, the results may be used to validate the model by (Sharp, 1971) and compare it with the model by (Cossalter, et al., 1999) that is of a different fidelity.

3.6. Calculating and Validating Steering Torque of a Multibody Software

Similarly, another important research objective is the validation of a leading commercial multibody software package by means of comparing its predicted results for the required steering torque of the virtual rider on a model that closely resembles the reference motorcycle with those of the previously collected data.

Ultimately, the gained knowledge and increased understanding of transient-state dynamics from this research project shall lay the foundation for a publication on motorcycle steering behaviour and rider control in the context of handling performance. Moreover, it will be applied to professional challenges in the future in order to further help riders in developing their handling techniques and to improve the performance capabilities of motorcycles, as well as of software solutions and simulators.

4. Methodology

4.1. Data Collection

The primary data for this research project is recorded by using the university-owned 'Ducati Panigale 1199 S' that was previously transformed into a 'Dynamic Vehicle Response Measurement Platform' (DVRMP) for the purpose of academic research in the field of motorcycle dynamics and as part of the Bachelor's dissertation by the author himself (Przibylla, 2018). The DVRMP is a highly sophisticated data acquisition setup that uses built-in and aftermarket sensors, an inertial measurement unit (IMU), a dual-antenna GPS, as well as custom-built and novel sensors that are based on strain gauge technology to measure physical rider input at the handlebars, i.e. steering torque, in conjunction with motorcycle attitude and its overall performance. The following list specifies the recorded variables and channels that are obtained from direct measurement on the motorcycle:

- 🦋 Engine speed
- 🦋 Gear position
- 🦋 Front and rear wheel speed
- 🦋 Front and rear suspension position
- 🦋 Throttle grip and throttle body position
- 🦋 Brake and clutch pressure
- 🦋 Ambient and engine coolant temperature
- 🦋 Global position and heading (dual-antenna GPS)
- 🦋 Attitude (roll, yaw, pitch, etc. from IMU)
- 🦋 Handlebar rotation (steering angle)
- 🦋 Force on the handlebars (steering torque)

Figure 3 shows the DVRMP and its associated datalogging equipment. A description of the exact working mechanisms behind the unique sensory layout, as well as a detailed derivation and explanation of the mathematical code for computing the steering torque from obtained strain gauge recordings, including proof of its exactness, are given by (Przibylla, 2018). Furthermore, the DVRMP was greatly improved by adding a pressure sensor to the hydraulic clutch system, which eliminates the need for a sensor strip on the clutch lever.



Figure 3 The DVRMP along with its aftermarket sensors and custom-built features

Initial data collection from simple manoeuvres was carried out at the University of Wales Trinity Saint David with lecturer Andrew Harrison riding the DVRMP, along with more comprehensive and extensive datasets from complete sessions on track that were recorded with Andrew Harrison, as well as the semi-professional and competitive road racer Samuel Mousley of 'UWTSD Orthrus Racing', who rode the DVRMP at Pembrey Circuit. Furthermore, additional datasets were collected at Donington Park Circuit with the help of Andrew Harrison and Michael Neeves, a test rider and journalist for the British motorcycle magazine 'MCN'.

4.2. Data Analysis

Initial data analysis for the verification of fundamental motorcycle steering theory and the comparison of the observed handling techniques is made within the 'Analyzer' application of the software 'WinARace' by the company '2D', which allows for visual examination of the recorded dataset and incorporation of calculated channels in a moving window, as shown in **Figure 4**.



Figure 4 ‘Analyzer’: Screenshot of data channels in a moving window

The screenshot above presents an overview that displays several channels of a dataset in four separate areas. The first one entails common motorcycle parameters and rider inputs, the second one the front and rear suspension position, the third one the roll and steering angle alongside the processed steering torque measurement and the fourth one a selection of IMU-outputs. In addition, the ‘Analyzer’ allows for the implementation of video footage or multi-dimensional plots, as shown by the examples in **Figure 5** and **Figure 6**.



Figure 5 ‘Analyzer’: Screenshot of data and on-board footage of the rider

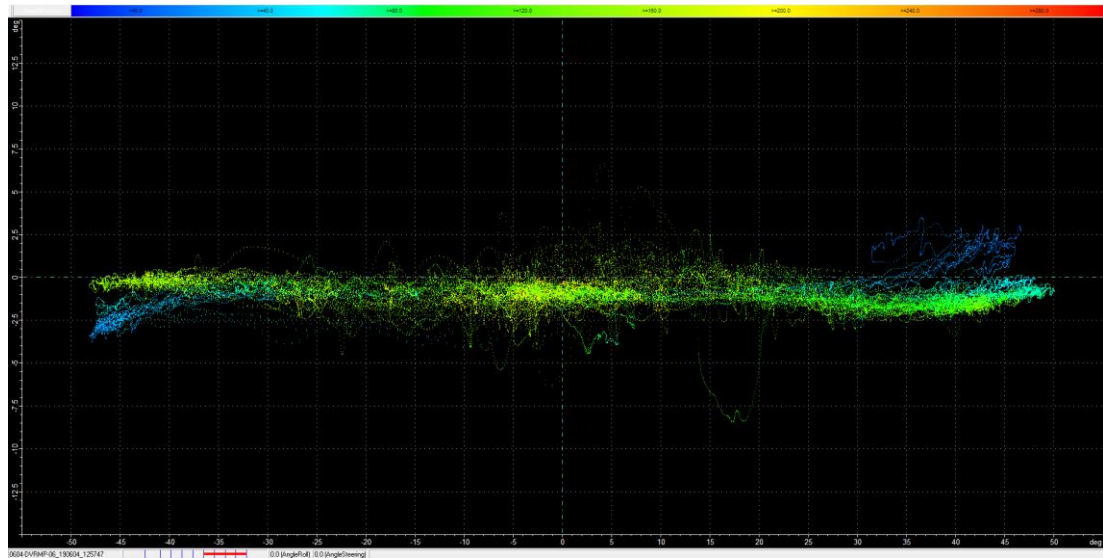


Figure 6 ‘Analyzer’: Screenshot of data for roll angle (x-axis) and steering angle (y-axis) against vehicle speed (colour scale)

4.3. Data Processing

For the implementation of the mathematical models which calculate the steering torque that is independent of the physically measured and real steering torque yet based on other dynamic factors of the recorded datasets, the required code is written in the numerical computing environment ‘MATLAB’. Its programming language is considerably more advanced and effective than the language that is used by the calculation file manager ‘CalcTool’ of ‘WinARace’. Even though the ‘2D’-software includes a plugin for bi-directional data exchange between ‘MATLAB’ and ‘WinARace’ (2D Debus & Diebold Meßsysteme GmbH, 2020), its export function is used to recreate the datasets in ‘Excel’ file format. Since ‘MATLAB’ allows for simple import and export transfer of numerical tables, all channels of the previously recorded dataset can be imported from ‘Excel’. The results can then be exported back into ‘Excel’ and from there to ‘WinARace’ for further analysis.

4.4. Computational Model

4.4.1. Determining the Inputs

To make use of the equations that are presented by (Cossalter, et al., 1999), the values of several constants and variables that describe the reference motorcycle and are not part of the previously recorded dataset have to be determined and used as inputs, like its chassis configuration or tyre properties, for example. With the exception of some attributes that require referencing from literature, the values of these static and dynamic parameters are identified by using established techniques together with newly developed and unique methods that are employed as follows. Whilst most geometrical dimensions can be physically measured on the reference motorcycle, the chosen approach is to model the entire chassis of the DVRMP as a two-dimensional sketch in the CAD package 'SOLIDWORKS', based on quoted geometry parameters of a structurally identical Ducati model that was used in the World Superbike Championship from the chassis and suspension program 'MotoSPEC', which are shown in **Figure 7**.

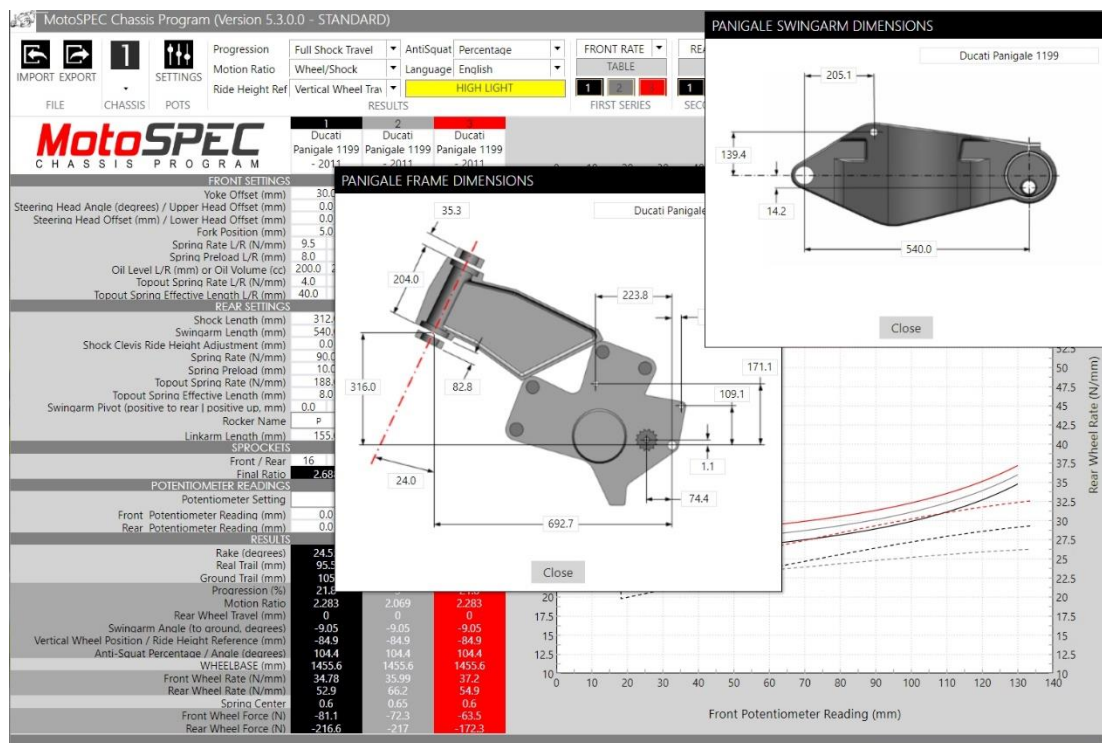


Figure 7 'MotoSPEC': Screenshot of the Ducati Panigale 1199 chassis geometry


```

1      %% Steering Torque Calculation for the DVRMP
2      clc
3      clear all
4
5      % Inputs
6      Data2D=readtable('Exit5.xlsx');
7      DataSW=readtable('DesignStudyDVRMP.xlsx');
8      LinForkTravel=linspace(min(DataSW.ForkTravel),max(DataSW.ForkTravel),117);
9      LinShockTravel=linspace(min(DataSW.ShockTravel),max(DataSW.ShockTravel),33);
10     [GridForkTravel,GridShockTravel]=meshgrid(LinForkTravel,LinShockTravel);
11     Gridepsilon=griddata((DataSW.ForkTravel),(DataSW.ShockTravel),(DataSW.epsilon),GridForkTravel,GridShockTravel);
12     GridxQ=griddata((DataSW.ForkTravel),(DataSW.ShockTravel),(DataSW.xQ),GridForkTravel,GridShockTravel);
13     GridhQ=griddata((DataSW.ForkTravel),(DataSW.ShockTravel),(DataSW.hQ),GridForkTravel,GridShockTravel);
14     GriddQ=griddata((DataSW.ForkTravel),(DataSW.ShockTravel),(DataSW.dQ),GridForkTravel,GridShockTravel);
15     Gridp=griddata((DataSW.ForkTravel),(DataSW.ShockTravel),(DataSW.p),GridForkTravel,GridShockTravel);
16     Gridb=griddata((DataSW.ForkTravel),(DataSW.ShockTravel),(DataSW.b),GridForkTravel,GridShockTravel);
17     Gridh=griddata((DataSW.ForkTravel),(DataSW.ShockTravel),(DataSW.h),GridForkTravel,GridShockTravel);
18     epsilon=interp2(GridForkTravel,GridShockTravel,Gridepsilon,(Data2D.Susp_F),(Data2D.Susp_R));
19     xQ=interp2(GridForkTravel,GridShockTravel,GridxQ,(Data2D.Susp_F),(Data2D.Susp_R));
20     hQ=interp2(GridForkTravel,GridShockTravel,GridhQ,(Data2D.Susp_F),(Data2D.Susp_R));
21     dQ=interp2(GridForkTravel,GridShockTravel,GriddQ,(Data2D.Susp_F),(Data2D.Susp_R));
22     p=interp2(GridForkTravel,GridShockTravel,Gridp,(Data2D.Susp_F),(Data2D.Susp_R));
23     b=interp2(GridForkTravel,GridShockTravel,Gridb,(Data2D.Susp_F),(Data2D.Susp_R));
24     h=interp2(GridForkTravel,GridShockTravel,Gridh,(Data2D.Susp_F),(Data2D.Susp_R));

```

Figure 9 ‘MATLAB’: Screenshot of the project code including lookups

Other relevant inputs and constants which are needed to calculate the forces and moments that act around the front assembly of the motorcycle are displayed in **Figure 10**. Some of them must be determined differently, such as the spin inertia of the front wheel. Experiments are conducted to specify this intrinsic property and **Figure 11** shows the front wheel accelerating down a ramp, which is repeatedly timed to determine an average time taken by the wheel to cover a certain distance and height on the ramp. Thus, the front wheel spin inertia can be calculated by following the procedure from (Bradley, 1996).

```

26     % Constants (Determined)
27     StaticRakeAngle=23.22;
28     StaticWheelbase=1.43549;
29     StaticSagFront=33.6;
30     StaticSagRear=4.3;
31     DynamicRakeAngle=23.10;
32     DynamicWheelbase=1.43088;
33     DynamicSagFront=48;
34     DynamicSagRear=8.9;
35     Iwf=0.604;
36     RFront=0.30335;
37     RRear=0.32818;
38     tFront=0.06258;
39     tRear=0.1076;
40     rhoFront=RFront-tFront;
41     rhoRear=RRear-tRear;
42     ForkOffset=0.03;
43     MT=264;
44     MUFront=22.5;
45     MSFrontStatic=0.4883*MT-MUFront;
46     kSuspFront=2*10000;
47     cSuspFront=0.4*2*sqrt((kSuspFront/cosd(DynamicRakeAngle).^2)*MSFrontStatic)*cosd(DynamicRakeAngle).^2;

```

Figure 10 ‘MATLAB’: Screenshot of the project code including determined constants



Figure 11 Picture of experimental procedure to determine the front wheel spin inertia

Whilst the radii of the tyres can be calculated from measurements of their circumference, which are easily taken with the help of a piece of string or wire that is wound around the centre, their cross-sectional radii are approximated from physical measurements of the chord and rise of their entire contact area by applying the general equation for the radius of an arc. It is illustrated in

Figure 12 and given by the formula: $R = \frac{L^2}{8 \cdot H} + \frac{H}{2}$

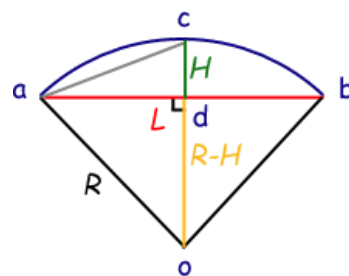


Figure 12 Generic sketch of arc lengths and geometry (La Rocque, n.d.)

The mass properties and static weight distribution of the entire motorcycle with its rider are determined by employing scales and following the procedure that is described by (Cossalter, 2006). The resulting measurements are then used in combination to calculate the static sprung mass that is supported by the front suspension as follows:

$$M_{S\ Front\ Static} = Weight\ Distribution_{Front} * M_T - M_{U\ Front}$$

Without reliable data for the exact damping characteristics of the telescopic front forks of the DVRMP or access to the suspension dynamometer at the university amid the 'COVID-19' pandemic, the static sprung mass can be used to calculate a constant suspension damping coefficient. The chosen approach is based on assuming a typical damping ratio of 0.4, which describes the ratio between the actual suspension damping coefficient and a critical damping coefficient. The latter is defined as the damping mode that enables a specific mass that is located on a spring to return to its original position as quickly as possible yet without overshooting it. The desired characteristic for motorcycle suspension is therefore an underdamped setup. While accounting for the given chassis geometry, based on the concept of reduced spring stiffness and damping that is described by (Cossalter, 2006), the suspension damping coefficient is calculated as follows:

$$\zeta = \frac{c}{c_{Crit}} \Rightarrow c_{Susp\ Front} = \zeta * c_{Crit\ Front}$$

$$c_{crit} = 2 * \sqrt{k_{Spring} * m} \Rightarrow c_{crit\ Front} = 2 * \sqrt{\left(\frac{k_{Susp\ Front}}{\cos^2 \varepsilon}\right) * M_{S\ Front\ Static}}$$

$$\therefore c_{Susp\ Front} = \zeta * c_{Crit\ Front} * \cos^2 \varepsilon$$

The values of further constants that are specific to the reference motorcycle and that usually require the application of highly specialised testing equipment for accurate determination are listed in **Figure 13**. Owing to time constraints and the lack of access to the university workshops, the inertial properties of the front frame assembly and the position of its mass centre with respect to the principle axes and origin of its reference coordinate system could not be measured by employing spatial mechanisms or pendulum methods, such as the ones that are suggested by (Da Lio, et al., 1999), (Moore, et al., 2009) or (Tang & Shangguan, 2011). Instead, the respective values for a ‘Ducati 848’ model, which has a very similar front-frame design, are referenced from (Gallego, 2011).

```

49 % Constants (Referenced)
50 - Mf=30.7;
51 - bf=0.024;
52 - hf=0.461;
53 - Ixf=1.23;
54 - Iyf=1.40;
55 - Izf=0.44;
56 - cSteeringDamper=6.78;
57 - kCamberFront=1.33;
58 - kCorneringFront=12.17;
59 - kCamberRear=1.31;
60 - kCorneringRear=11.33;
61 - kTwistingFront=0.035;
62 - kSelfAligningFront=-0.475;
63 - CResRolling=0.06757;
64 - mdFront=CResRolling*RFront;

```

Figure 13 ‘MATLAB’: Screenshot of the project code including referenced constants

For the same reason, other constants are initially referenced as well, such as the relevant stiffness values of the tyres. These are quoted for the chosen front and rear tyre pressures of 2.275 bar from the plotted results of original tests that were conducted by (Massaro, et al., 2013). Depending on the results that are ultimately obtained from the project code, these values may potentially require revision in the form of further experimental testing of the actual tyres on site. For this purpose, the author proposes the application of a custom-built rig that can support the DVRMP on the car dynamometer of the university, whilst possibly still allowing for limited multi-directional movement of the motorcycle. **Figure 14** shows the base of this rig, which is bolted into the ground. It can have another piece attached to its rose joint that resembles a

bracket and is connected to the motorcycle via a spindle through the back end of its swingarm, where the rear wheel would otherwise be mounted. This allows for the front wheel of the DVRMP to be driven on the drum of the dynamometer. Furthermore, the position of the drum can be adjusted and its rotational speed can be controlled by the operator.



Figure 14 *Picture of the bottom part of a rig for possible step-tests on a car dynamometer*

Even though this setup does not replicate true riding conditions, for example due to the absence of any centrifugal effect, it could conceivably be used to record data from a number of step-tests that are aimed at establishing tyre characteristics or twisting moments at different speeds and in combination with the utilisation of anti-slip tape, lasers or a high-speed video camera. Appropriate safety measures for these tests are certainly required and possible solutions could include securing the DVRMP by clamping it in place or suspending it from the chains of a car lift.

Before any significant calculations are carried out with the results of the design studies in ‘SOLIDWORKS’ or the channels of the recorded dataset, important modifications and additions are made, as is shown in **Figure 15**.

```

66 % Conversions
67 - mAngleRake=epsilon*(pi/180);
68 - mxQ=xQ/1000;
69 - mhQ=hQ/1000;
70 - mdQ=dQ/1000;
71 - mp=p/1000;
72 - mb=b/1000;
73 - mh=h/1000;
74 - mAnglePitchIMU=(Data2D.AnglePitch)*(pi/180);
75 - mAngleRoll=(Data2D.AngleRoll)*(pi/180);
76 - mAngleSteering=-(Data2D.Steering)*(pi/180);
77 - mVelocityForward=(Data2D.V_GPS)/3.6;
78 - mVelocityFront=(Data2D.V_Front57)/3.6;
79 - mAngleVelocityFront=mVelocityFront/RFront;
80 - mAngleVelocityYaw=((Data2D.AngleRateY)*(pi/180)).*sin(mAngleRoll)./cos(mAnglePitchIMU)+(((Data2D.AngleRateZ)*(pi/180)).*cos(mAngleRoll)).*sin(mAnglePitchIMU);
81 - mAngleVelocityRoll=((Data2D.AngleRateX)*(pi/180))+(((Data2D.AngleRateY)*(pi/180)).*sin(mAngleRoll)).*tan(mAnglePitchIMU);
82 - mAngleVelocitySteering=gradient(mAngleSteering);
83 - mAngleAccelYaw=((Data2D.AngAccelY)*(pi/180)).*sin(mAngleRoll)./cos(mAnglePitchIMU)+(((Data2D.AngAccelZ)*(pi/180)).*cos(mAngleRoll)).*sin(mAnglePitchIMU);
84 - mAngleAccelRoll=((Data2D.AngAccelX)*(pi/180))+(((Data2D.AngAccelY)*(pi/180)).*sin(mAngleRoll)).*tan(mAnglePitchIMU);
85 - mAngleAccelSteering=gradient(mAngleVelocitySteering);
86 - mAccelLateral=(Data2D.AccelLateral);
87 - mSuspPosFront=(Data2D.Susp_F)/1000;
88 - mSuspVelFront=gradient(mSuspPosFront);
89 - mSuspAccFront=gradient(mSuspVelFront);
90 - mDynamicSagFront=DynamicSagFront/1000;
91 - mFrontBrakeON=(Data2D.F_Brake)>0;

```

Figure 15 ‘MATLAB’: Screenshot of the project code including conversions and transformations

On top of converting every input to SI units, the code calculates the angular rates that are crucial for the subsequent computation. It partly utilises the resulting equations from a typical transformation matrix to define the global yaw and roll rates of the motorcycle based on IMU-measurements of its attitude and angular rates around the local vehicle-frame axes x, y and z. The necessity for this transformation is further explained by (CHRobotics LLC., n.d.) and the quoted matrix yields the equations as follows.

$$D(\varphi, \vartheta, \psi) = \begin{bmatrix} 1 & \sin \varphi * \tan \vartheta & \cos \varphi * \tan \vartheta \\ 0 & \cos \varphi & -\sin \varphi \\ 0 & \sin \varphi / \cos \vartheta & \cos \varphi / \cos \vartheta \end{bmatrix}$$

$$\begin{pmatrix} \dot{\varphi} \\ \dot{\vartheta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \dot{x}_{Angle} + \dot{y}_{Angle} * \sin \varphi * \tan \vartheta + \dot{z}_{Angle} * \cos \varphi * \tan \vartheta \\ \dot{y}_{Angle} * \cos \varphi - \dot{z}_{Angle} * \sin \varphi \\ \dot{y}_{Angle} * \sin \varphi / \cos \vartheta + \dot{z}_{Angle} * \cos \varphi / \cos \vartheta \end{pmatrix}$$

4.4.2. Determining the Variables

A number of variables are needed to ultimately calculate the steering torque for both the steady and the transient state. Some of these variables, such as the kinematic steering angle, sideslip angles or the forces that are acting at the front and rear contact patches, are depicted in **Figure 16**. The schematic diagram that is displayed herein is representative of a motorcycle in steady-state cornering equilibrium. Thus, it prescribes a constant path radius at a constant yaw rate around a theoretical turning centre point.

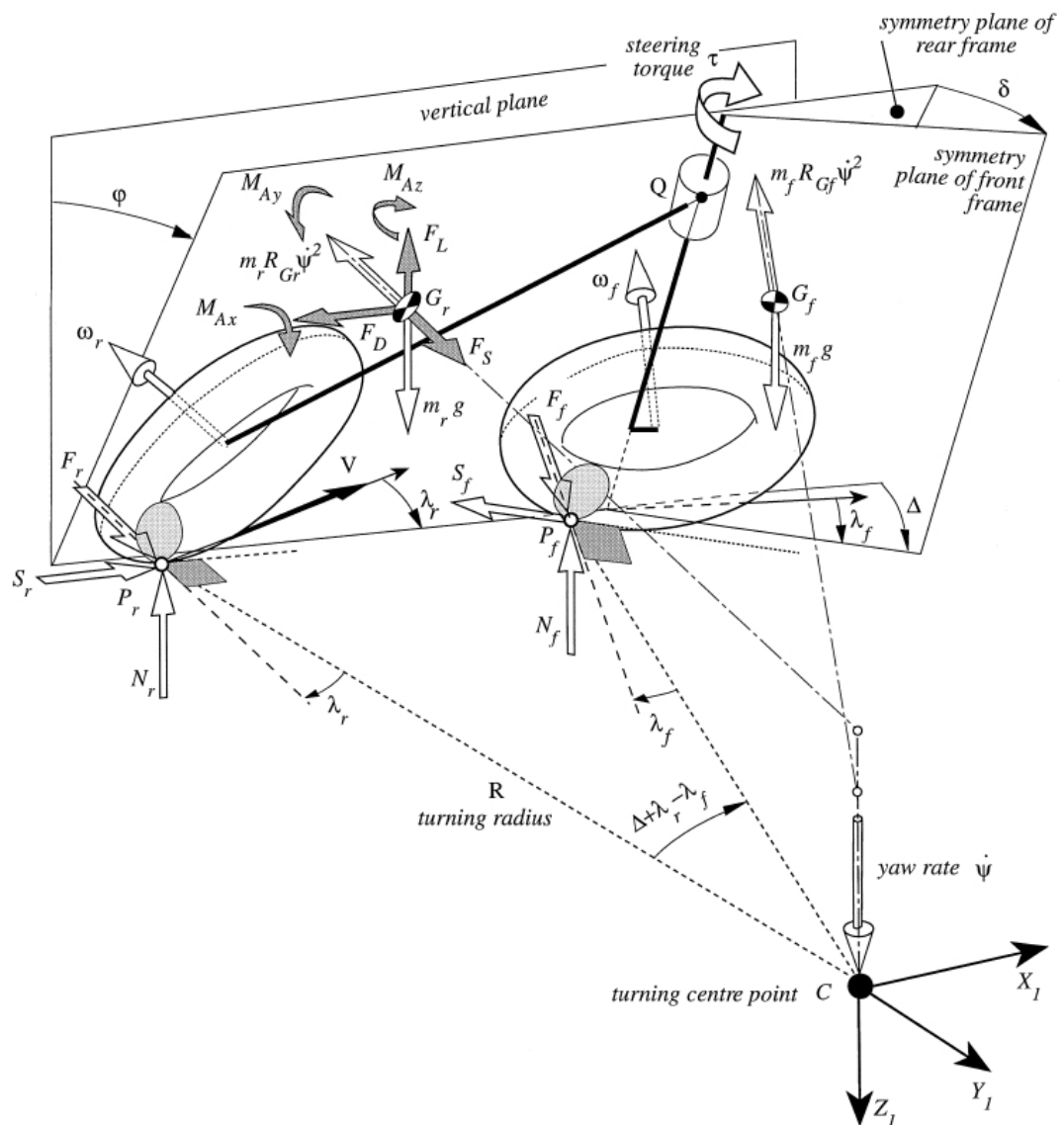


Figure 16 Graphic representation of a motorcycle in steady turning
(Cossalter, et al., 1999)

The relative position of this turning centre point, when compared to an ideal turning centre, is controlled by the sideslip angles which are inversely proportional to the cornering stiffness and in turn depend on the camber stiffness of the tyres and the roll angle of the motorcycle as shown below (Cossalter, 2006).

$$\lambda_f \approx \frac{1-k_{\phi_f}}{k_{\lambda_f}} * \varphi \qquad \lambda_r \approx \frac{1-k_{\phi_r}}{k_{\lambda_r}} * \varphi$$

Figure 17 reveals how the abovementioned equations are used to determine the required sideslip angles to arrive at further variables, along with various additional factors that are of significance for the computational model.

```

93 % Variables
94 - mAngleSideslipFront=((1-kCamberFront)/kCorneringFront).*mAngleRoll;
95 - mAngleSideslipRear=((1-kCamberRear)/kCorneringRear).*mAngleRoll;
96 - mtheta=atan((sin(mAngleSteering).*tan(mAngleRoll)-sin(mAngleRake).*cos(mAngleSteering))./cos(mAngleRake));
97 - mAnglePitchKinematicPaper=-(((ForkOffset.*sin(mAngleRake)).*(1-cos(mAngleSteering))+tRear-tFront+rhoFront).*(cos(mAngleRake+mAnglePitchKinematicPaper))./(cos(mAngleRake+mAnglePitchKinematicPaper)));
98 - mAngleSteeringKinematicPaper=atan((sin(mAngleSteering).*cos(mAngleRake+mAnglePitchKinematicPaper))./(cos(mAngleRake+mAnglePitchKinematicPaper)));
99 - mAngleSteeringEffectivePaper=mAngleSteeringKinematicPaper+mAngleSideslipRear-mAngleSideslipFront;
100 - mSteeringRatioPaper=mAngleSteeringEffectivePaper./mAngleSteeringKinematicPaper;
101 - mpminusb=mp-mpminusb;
102 - mMSFrontDynamic=((mp-mpminusb)./mp).*MT-MUFront;
103 - mSuspForceFrontStatic=kSuspFront*mDynamicSagFront;
104 - mSuspForceFrontDynamic=mMSFrontDynamic.*mSuspAccFront+cSuspFront.*mSuspVelFront+kSuspFront.*mSuspPosFront-mSuspF;
105 - mSuspForceFrontBraking=((mSuspForceFrontDynamic.*mFrontBrakeON)./(mh./mp)).*cos(mAngleRake+mAnglePitchKinematicPaper);
106 - mNf=(9.81*MUFront+(mSuspForceFrontStatic+mSuspForceFrontDynamic-mSuspForceFrontBraking)./cos(mAngleRake+mAnglePitchKinematicPaper));
107 - mFf=(kCorneringFront.*mAngleSideslipFront+kCamberFront.*mAngleRoll).*mNf;
108 - mSf=-mSuspForceFrontBraking./sin(mAngleRake+mAnglePitchKinematicPaper);
109 - mMyFront=mdFront.*mNf;
110 - mMzFront=(kSelfAligningFront.*mAngleSideslipFront+kTwistingFront.*mAngleRoll).*mNf;

```

Figure 17 ‘MATLAB’: Screenshot of the project code including tyre-related variables, forces and moments

It follows the calculation of the kinematic pitch angle, which becomes very useful for the computation the exact rake angle of the DVRMP from a combination of the previously mentioned design study results that are extracted from a two-dimensional sketch with the out-of-plane contributions from roll and steering motion that this pitch angle effectively describes. The code uses the following equation, which is proposed by (Cossalter, et al., 1999) and yields an approximate solution that is also said to be very close to the numerical value when the roll and steering angles are large.

$$\mu =$$

$$\frac{\left[l_1 * \sin \varepsilon * (1 - \cos \delta) + q_r - q_f + \rho_f * (\cos \varepsilon * \cos \theta - \cos \delta * \sin \varepsilon * \sin \theta - 1) \right] * \cos \varphi + (l_1 * \sin \delta + \rho_f * \sin \delta * \sin \theta) * \sin \varphi + q_f - q_r}{\left[l_1 * \cos \varepsilon * (1 - \cos \delta) - p \right] * \cos \varphi - \rho_f * (\sin \varepsilon * \cos \varphi * \sin \delta + \cos \delta * \sin \varphi) * \cos \theta * \sin \delta}$$

Where:

$$\theta = \tan^{-1} \left(\frac{\sin \delta * \tan \varphi - \sin \varepsilon * \cos \delta}{\cos \varepsilon} \right)$$

This kinematic pitch angle subsequently allows for the calculation of the kinematic steering angle that represents the angle between the symmetry planes of the front and rear frame at the intersection with the road plane. Its coincidence with the physical steering angle dictates whether the steering mechanism of the motorcycle is attenuated ($\Delta < \delta$) or amplified ($\Delta > \delta$) and it is controlled by the roll, steering, rake and kinematic pitch angles as follows (Cossalter, 2006).

$$\Delta = \tan^{-1} \left[\frac{\sin \delta * \cos(\varepsilon + \mu)}{\cos \varphi * \cos \delta - \sin \varphi * \sin \delta * \sin(\varepsilon + \mu)} \right]$$

However, the effective steering angle, which is linked to the position of the actual turning centre that is the point of intersection of the front and rear wheel turning radii, also depends on the sideslip angles and is calculated by using the following equation (Cossalter, 2006).

$$\Delta^* = \Delta + \lambda_r - \lambda_f$$

Hence, the effective steering angle is only identical to the kinematic steering angle when both sideslip angles are equal and thus the steering behaviour of a motorcycle is distinguished into neutral ($\xi = 1$), understeering ($\xi < 1$) and

oversteering ($\xi > 1$) character by the ratio of the two previously calculated steering angles (Cossalter, 2006).

$$\xi = \frac{\Delta^*}{\Delta}$$

The schematic diagrams in **Figure 18** further depict this influence of different front and rear sideslip angles on the position of the turning centre point.

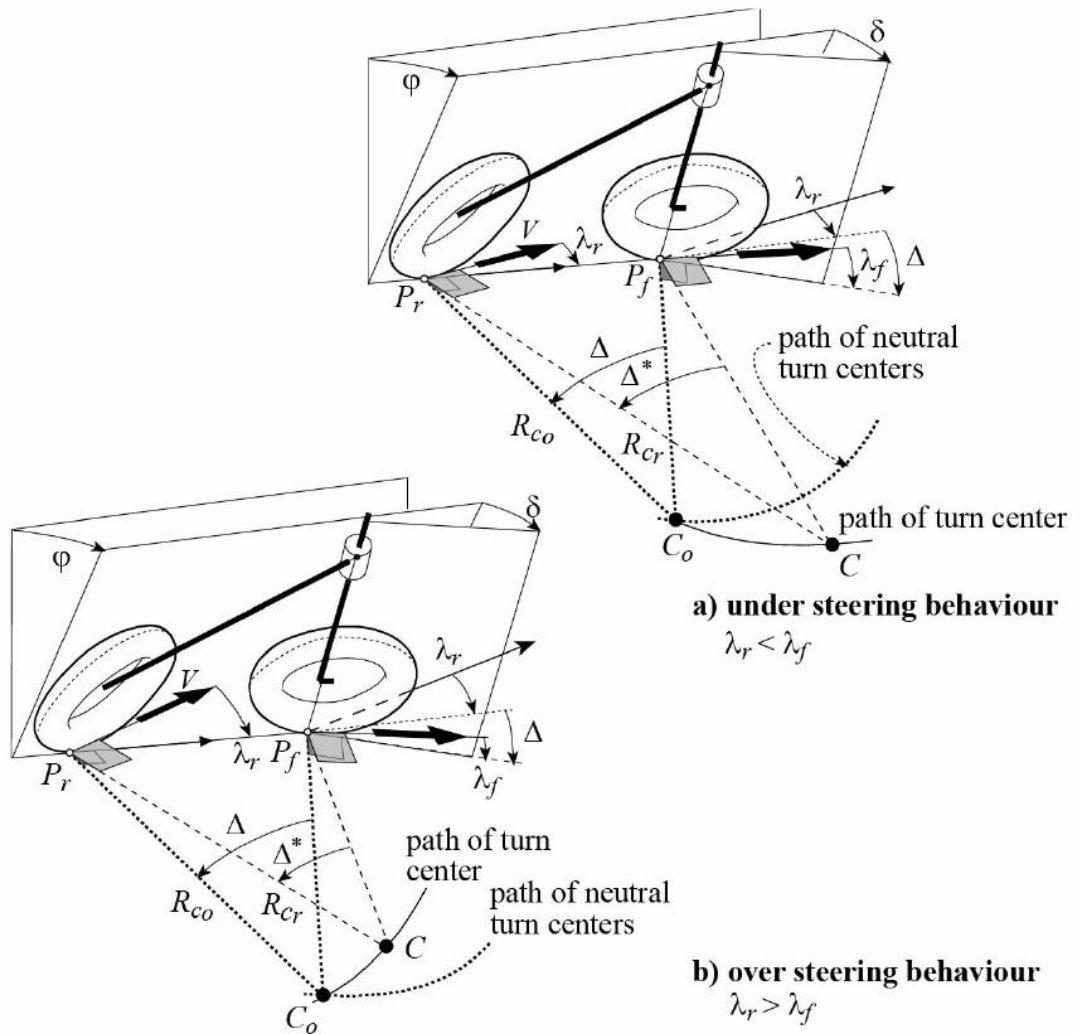


Figure 18 Graphic representation of the under- and oversteering behaviour of motorcycles (Cossalter, 2006)

The steering behaviour of a motorcycle fundamentally depends on the amount of lateral force that is needed at the contact patches of its front and rear tyre, to what extent this lateral force can be generated by the camber thrust of the respective tyre and whether it also requires the presence of a sideslip component to ensure the steady-state cornering equilibrium that is displayed in **Figure 19** (Cossalter, 2006).

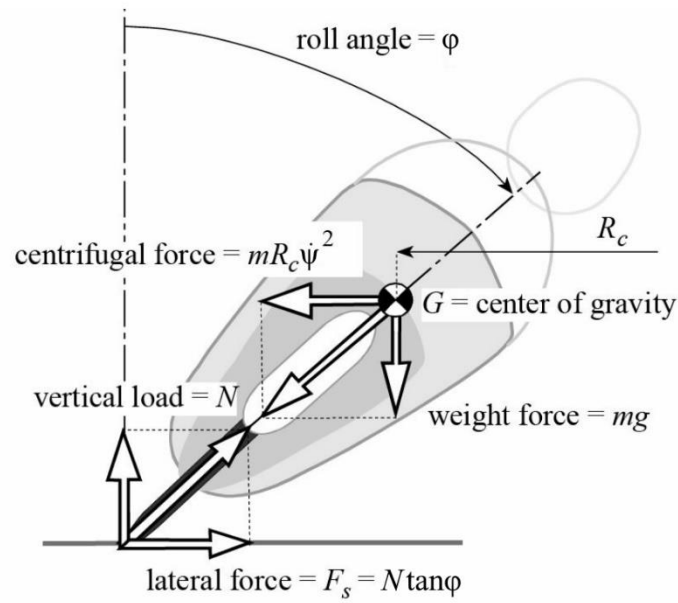


Figure 19 Graphic representation of the equilibrium of forces that act on the centre of gravity (Adaptation of Cossalter, 2006)

Since the lateral force is essentially related to the vertical load via the tangent of the roll angle, the normal force on the tyre must be determined beforehand. Regarding the front end of the motorcycle, this can be achieved by virtue of combining the mass properties and suspension characteristics of the DVRMP with varying dimensions from the previously mentioned design studies and processed channels of the obtained dataset. The following approach is based on calculating the dynamically changing vertical load on the front contact patch from the sum of the static load of the unsprung mass and the dynamic load of the sprung mass, whereby the latter is determined from the recorded fork travel.

During the application of the front brake, however, the additional compression of the front suspension depends on a resultant suspension force of both the normal load transfer force and the longitudinal front braking force, as depicted in **Figure 20** (Cossalter, 2006).

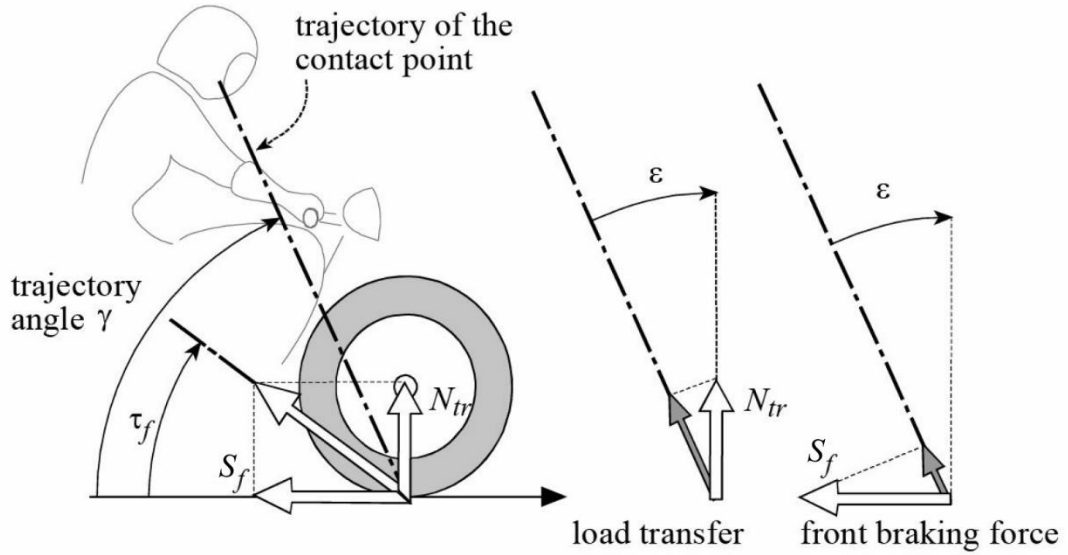


Figure 20 Graphic representation of the forces that cause the front suspension to compress during braking (Adaptation of Cossalter, 2006)

Moreover, if the rear brake is not used simultaneously, the ratio of the sum of the components that further compress the front suspension to the front braking force solely depends on the present rake angle and the load transfer angle (Cossalter, 2006).

$$\frac{F_{Susp Front Dynamic}}{S_f} = \frac{N_{tr} * \cos \varepsilon + S_f * \sin \varepsilon}{S_f} = \frac{S_f * \frac{h}{p} * \cos \varepsilon + S_f * \sin \varepsilon}{S_f} = \frac{h}{p} * \cos \varepsilon + \sin \varepsilon$$

Assuming that the rider only uses the front brake, a vital set of equations to compute the braking force, the load transfer force and subsequently the

dynamic vertical load on the front tyre can be derived from this given ratio. The first step of the suggested process involves rearranging the previous equation and including a Boolean value (1 or 0) that is determined by the application of the front brake to calculate the front braking force component that acts along the suspension inclination angle, as shown below.

$$\frac{F_{Susp Front Dynamic}}{S_f} = \frac{h}{p} * \cos \varepsilon + \sin \varepsilon \Rightarrow S_f = \frac{F_{Susp Front Dynamic}}{\left(\frac{h}{p} * \cos \varepsilon + \sin \varepsilon\right)}$$

$$\therefore F_{Susp Front Braking}$$

$$= S_f * \sin(\varepsilon + \mu) = \left\{ \frac{F_{Susp Front Dynamic} * Braking_{ON/OFF}}{\left[\frac{h}{p} * \cos(\varepsilon + \mu) + \sin(\varepsilon + \mu)\right]} \right\} * \sin(\varepsilon + \mu)$$

However, the equation above requires prior computation of the total dynamic front suspension force that is essentially defined by the following second-order differential equation, which derives from the three fundamental terms for the dynamic force, damping force and spring force of a generic mass-spring-damper system.

$$F_{Susp Front Dynamic}$$

$$= M_{s Front Dynamic} * \ddot{x}_{Susp Front} + c_{Susp Front} * \dot{x}_{Susp Front} + k_{Susp Front} * x_{Susp Front} - F_{Susp Front Static}$$

The dynamic part of the equation involves the varying sprung mass and relies on design study measurements to account for the changing weight distribution.

$$M_{s Front Dynamic} = \left[\frac{p - (p - b)}{p} \right] * M_T - M_{U Front}$$

Since the front suspension force in the original equation purely arises from an additional compression of the fork that is caused by dynamic factors, the static suspension force from the initial spring compression, i.e. the dynamic sag and installed or additional preload, must be subtracted from the equation of the dynamic front suspension force. It is calculated as shown below.

$$F_{Susp Front Static} = k_{Susp Front} * (x_{Susp Front Dynamic Sag} + x_{Susp Front Preload})$$

Ultimately, the resulting front suspension braking force directly yields the longitudinal braking force when it is resolved back into the horizontal plane as follows.

$$F_{Susp Front Braking} = S_f * \sin(\varepsilon + \mu)$$

$$\therefore S_f = \frac{F_{Susp Front Braking}}{\sin(\varepsilon + \mu)}$$

Furthermore, it allows for the computation of the load transfer force, once the given ratio has been rearranged as shown below. Hence, the front suspension braking force is subtracted from the total dynamic front suspension force and the remainder is then resolved back into the vertical plane.

$$\frac{F_{Susp Front Dynamic}}{S_f} = \frac{N_{tr} * \cos \varepsilon + S_f * \sin \varepsilon}{S_f} \Rightarrow N_{tr} = \frac{F_{Susp Front Dynamic} - S_f * \sin \varepsilon}{\cos \varepsilon}$$

$$\therefore N_{tr} = \frac{F_{Susp Front Dynamic} - F_{Susp Front Braking}}{\cos(\varepsilon + \mu)}$$

From this load transfer force, the static front suspension force and the static weight force of the front unsprung mass, it is now possible to compute the total dynamic load on the front. To account for the degree of freedom of the motorcycle to move out of plane, it must further be resolved into its true vertical

component by the cosine of the roll angle to ultimately calculate the dynamic normal force that acts on the front contact patch.

$$N_f = \left\{ M_{U\ Front} * g + \left[\frac{F_{Susp\ Front\ Static}}{\cos(\varepsilon + \mu)} \right] + N_{tr} \right\} * \cos \varphi$$

As previously mentioned, the magnitude of this dynamic normal force is of fundamental importance for the calculation of the lateral force that acts on the contact patch. It is computed based on a linear steady-state model and the following equation by (Cossalter, 2006), which involves the sideslip angle, and the camber thrust of the tyre.

$$F_f = \left(k_{\lambda_f} * \lambda_f + k_{\varphi_f} * \varphi \right) * N_f$$

In addition, the dynamic normal force also controls the moments which act around the x-, y- and z-axis on the contact patch of the tyre (Cossalter, 2006). Thus, the front tyre rolling resistance moment and front tyre yawing moment are also computed from it by accounting for the rolling resistance coefficient and wheel radius, as well as the self-aligning and twisting stiffness of the tyre. The necessary constants and equations are referenced from (Cossalter, et al., 1999) and (Massaro, et al., 2013) respectively.

$$M_{y\ Front} = d_f * N_f = f_w * r_f * N_f$$

$$M_{z\ Front} = \left(k_{sa_f} * \lambda_f + k_{tw_f} * \varphi \right) * N_f$$

Beyond establishing the magnitude of the forces and moments that are generated at the front contact patch, the determination of its exact location and perpendicular distance from the steering axis is also of great relevance. For this purpose, more kinematic variables are computed, as shown in **Figure 21**.

```

112 % Variables 2
113 mbetaDashPaper=mAngleRake+atan((sin(mAngleSteering).*tan(mAngleRoll)-sin(mAngleRake+mAnglePitchKinematicPaper)).*
114 mA=ForkOffset.*sin(mAngleRake).*(1-cos(mAngleSteering))+tRear-tFront;
115 mB=rhoFront.*(cos(mAngleRake).*cos(mbetaDashPaper-mAngleRake)-cos(mAngleSteering).*sin(mAngleRake).*sin(mbetaDash
116 mC=ForkOffset.*sin(mAngleSteering)+rhoFront.*sin(mAngleSteering).*sin(mbetaDashPaper-mAngleRake);
117 mD=mp-ForkOffset.*cos(mAngleRake).*(1-cos(mAngleSteering));
118 mE=rhoFront.*(sin(mAngleRake).*cos(mbetaDashPaper-mAngleRake)+cos(mAngleSteering).*cos(mAngleRake).*sin(mbetaDash
119 mbetaDashBook=mAngleRake+atan((sin(mAngleSteering).*tan(mAngleRoll)-sin(mAngleRake).*cos(mAngleSteering))./cos(m
120 mC1=ForkOffset.*sin(mAngleRake).*(1-cos(mAngleSteering))+tRear-tFront;
121 mC2=rhoFront.*(cos(mAngleRake).*cos(mbetaDashBook-mAngleRake)-cos(mAngleSteering).*sin(mAngleRake).*sin(mbetaDash
122 mC3=ForkOffset.*sin(mAngleSteering)+rhoFront.*sin(mAngleSteering).*sin(mbetaDashBook-mAngleRake);
123 mC4=mp-ForkOffset.*cos(mAngleRake).*(1-cos(mAngleSteering));
124 mC5=rhoFront.*(sin(mAngleRake).*cos(mbetaDashBook-mAngleRake)+cos(mAngleSteering).*cos(mAngleRake).*sin(mbetaDash
125 mAnglePitchKinematicBook=(mC1+mC2).*cos(mAngleRoll)+mC3.*sin(mAngleRoll)+tFront-tRear)./(mC4+mC5).*cos(mAngleR
126 mAngleSteeringKinematicBook=atan((sin(mAngleSteering).*cos(mAngleRake+mAnglePitchKinematicBook))./cos(mAngleRol
127 mAngleSteeringEffectiveBook=mAngleSteeringKinematicBook+mAngleSideslipRear-mAngleSideslipFront;
128 mSteeringRatioBook=mAngleSteeringEffectiveBook./mAngleSteeringKinematicBook;
129 mxPf=(mC1+mC2).*sin(mAnglePitchKinematicBook)+(mC4+mC5).*cos(mAnglePitchKinematicBook);
130 myPf=(-(mC1+mC2).*cos(mAnglePitchKinematicBook)+(mC4+mC5).*sin(mAnglePitchKinematicBook)).*sin(mAngleRoll)+mC3.*
131 mwB=mxPf+myPf.*tan(mAngleSteeringKinematicBook);
132 mdeltaH=RFront.*(1-sqrt(1-sin(mAngleSteering).^2.*sin(mAngleRake).^2))-ForkOffset.*sin(mAngleRake).*(1-cos(mAngl
133 mTrailNormal=RFront.*sin(mAngleRake)-ForkOffset;
134 mTrailA1=-(myPf.*cos(mAngleRoll)+tRear.*sin(mAngleRoll)).*cos(mAngleRake+mAnglePitchKinematicBook);
135 mTrailA2=(mxPf.*cos(mAngleRake+mAnglePitchKinematicBook)-mp.*cos(mAngleRake)+RRear.*sin(mAngleRake)-mTrailNormal
136 mTrailA3=myPf.*sin(mAngleRake+mAnglePitchKinematicBook)+(mxPf.*cos(mAngleRake+mAnglePitchKinematicBook)-mp.*cos(t
137 mTrailNormalCornering=sqrt(mTrailA1.^2+mTrailA2.^2+mTrailA3.^2);

```

Figure 21 ‘MATLAB’: Screenshot of the project code including additional kinematic variables

The following set of referenced equations for the kinematic variables that are used by (Cossalter, et al., 1999) and (Cossalter, 2006) is identical between the two sources, with the exception of the underlying formula for the angular position of the contact patch. In one instance, it already includes the kinematic pitch angle, since it is calculated beforehand by (Cossalter, et al., 1999).

$$C_1 \text{ \& } A = d * \sin \varepsilon * (1 - \cos \delta) + t_r - t_f$$

$$C_2 \text{ \& } B = \rho_f * [\cos \varepsilon * \cos(\beta' - \varepsilon) - \cos \delta * \sin \varepsilon * \sin(\beta' - \varepsilon) - 1]$$

$$C_3 \text{ \& } C = d * \sin \delta + \rho_f * \sin \delta * \sin(\beta' - \varepsilon)$$

$$C_4 \text{ \& } D = p - d * \cos \varepsilon * (1 - \cos \delta)$$

$$C_5 \text{ \& } E = \rho_f * [\sin \varepsilon * \cos(\beta' - \varepsilon) + \cos \delta * \cos \varepsilon * \sin(\beta' - \varepsilon)]$$

Where:

$$\beta' = \varepsilon + \tan^{-1} \left[\frac{\sin \delta * \tan \varphi - \sin(\varepsilon + \mu) * \cos \delta}{\cos(\varepsilon + \mu)} \right]$$

From these kinematic variables, the location of the front contact patch with respect to a local coordinate system for the motorcycle that has its origin at the centre of the rear contact patch can be calculated with the following equations from (Cossalter, 2006).

$$x_{Pf} = (C_1 + C_2) * \sin \mu + (C_4 + C_5) * \cos \mu$$

$$y_{Pf} = [-(C_1 + C_2) * \cos \mu + (C_4 + C_5) * \sin \mu] * \sin \varphi + C_3 * \cos \varphi - (t_f - t_r)$$

The vertical lowering of the front wheel centre due to the presence of a steering angle is isolated from the overall kinematic pitch and given in the same source.

$$\Delta h = R_f * \left(1 - \sqrt{1 - \sin^2 \delta * \sin^2 \varepsilon}\right) - d * \sin \varepsilon * (1 - \cos \delta)$$

Finally, the normal trail within the plane that passes through the centre of the motorcycle is calculated by using the simple formula below. Together with the previously determined contact patch coordinates, it allows for the dynamic normal trail during cornering to be calculated as shown by (Cossalter, 2006).

$$a_n = R_f * \sin \varepsilon - d$$

$$a_n^* = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Where:

$$a_1 = -(y_{Pf} * \cos \varphi + t_r * \sin \varphi) * \cos(\varepsilon + \mu)$$

$$a_2 = [x_{Pf} * \cos(\varepsilon + \mu) - p * \cos \varepsilon + R_r * \sin \varepsilon - a_n] * \cos \varphi - [\rho_r * \cos \varphi + t_r] * \sin(\varepsilon + \mu)$$

$$a_3 = y_{Pf} * \sin(\varepsilon + \mu) + [x_{Pf} * \cos(\varepsilon + \mu) - p * \cos \varepsilon * R_r * \sin \varepsilon - a_n - \rho_r * \sin(\varepsilon + \mu)] * \sin \varphi$$

4.4.3. Determining Steering Torque using Steady-State Equations

The mathematical model of a motorcycle in steady turning motion around a fixed turning centre that is presented by (Cossalter, et al., 1999) is based on the condition of a steady-state cornering equilibrium. Thus, the forward velocity of the motorcycle is related to the yaw velocity and turn curvature with the following equation from (Cossalter, et al., 1999).

$$V = \frac{\dot{\psi}}{\theta}$$

Therefore, the turn curvature can be calculated from the recorded dataset and the computed channels for the forward velocity and yaw velocity of the DVRMP with the equation below:

$$\theta_{yaw} = \frac{\dot{\psi}}{V}$$

Alternatively, since the curvature is also defined as the reciprocal of the path radius of the rear contact patch, it may be expressed with the following equation from (Cossalter, et al., 1999), once the relative contact patch positions of the front and rear tyre have been determined from the set of kinematic variables with respect to a global frame of reference that has its origin at the turning centre point.

$$\theta_{Pr} = \frac{\cos \lambda_r * \sin(\Delta - \lambda_f) + \sin|\lambda_r| * \cos(\Delta - \lambda_f)}{(X_{Pf} - X_{Pr}) * \cos(\Delta - \lambda_f) + (Y_{Pf} - Y_{Pr}) * \sin(\Delta - \lambda_f)}$$

Where:

$$X_{Pf} - X_{Pr} = (D + E) * \cos \mu + (A + B + \rho_r) * \sin \mu - [B + A + \rho_r * (1 - \cos \mu)] * \sin \mu * \sin \varphi$$

$$Y_{Pf} - Y_{Pr} = C * \cos \varphi + [B - A - \rho_r * (1 - \cos \mu)] * \cos \mu * \sin \varphi + [E + D + \rho_r * \sin \mu] * \sin \mu * \sin \varphi$$

As shown in **Figure 22**, both approaches are used to separately compute a rear turning radius. One radius is calculated with kinematic equations and the other one is calculated from the yaw rate that is based on IMU-measurements.

```

139  %% Motorcycle Path
140 - mXPfminusXPr=(mD+mE).*cos(mAnglePitchKinematicPaper)+(mA+mB+rhoRear).*sin(mAnglePitchKinematicPaper)-(mB+mA+rhoRear);
141 - mYPfminusYPr=mC.*cos(mAngleRoll)+(mB-mA-rhoRear.*(1-cos(mAnglePitchKinematicPaper))).*cos(mAnglePitchKinematicPaper);
142
143 - mTurnCurvaturePr=(cos(mAngleSideslipRear).*sin(mAngleSteeringKinematicPaper-mAngleSideslipFront)+sin(abs(mAngleSideslipFront)).*cos(mAngleSteeringKinematicPaper-mAngleSideslipFront)).*sin(mAngleSteeringKinematicPaper-mAngleSideslipFront);
144 - mTurnRadiusRearPr=1./mTurnCurvaturePr;
145 - mTurnRadiusRearPr(isinf(mTurnRadiusRearPr)|isnan(mTurnRadiusRearPr))==0;
146 - mAngleVelocityPr=mVelocityForward./mTurnRadiusRearPr;
147 - mTurnCurvatureYaw=mAngleVelocityYaw./mVelocityForward;
148 - mTurnRadiusRearYaw=1./mTurnCurvatureYaw;
149 - mTurnRadiusRearYaw(isinf(mTurnRadiusRearYaw)|isnan(mTurnRadiusRearYaw))==0;

```

Figure 22 ‘MATLAB’: Screenshot of the project code including the path determination

The equation that is used in both cases purely relies on the reciprocal relationship between radius and curvature.

$$R_{cr} = \frac{1}{\theta}$$

Thus, the computed turning radii describe the dynamically changing distance from the contact patch of the rear tyre to a turning centre point that is essentially no longer fixed but floats in the road plane instead. Nevertheless, the orientation of the global frame of reference with its origin at this turning centre point remains unchanged and true to its original convention regardless of the direction of the turn due to the yaw velocity changing sign instead. By definition, the road or track underneath the motorcycle is level, the global axes X and Y of the reference frame lie in the road plane. The global X-axis is oriented parallel to the local x-axis of the rear-frame coordinate system of the motorcycle that has its origin at the contact patch of the rear tyre; the global Z-axis is perpendicular to the road plane, directed downwards and thus parallel to the local z-axis of the motorcycle, as depicted in **Figure 23** (Cossalter, et al., 1999).

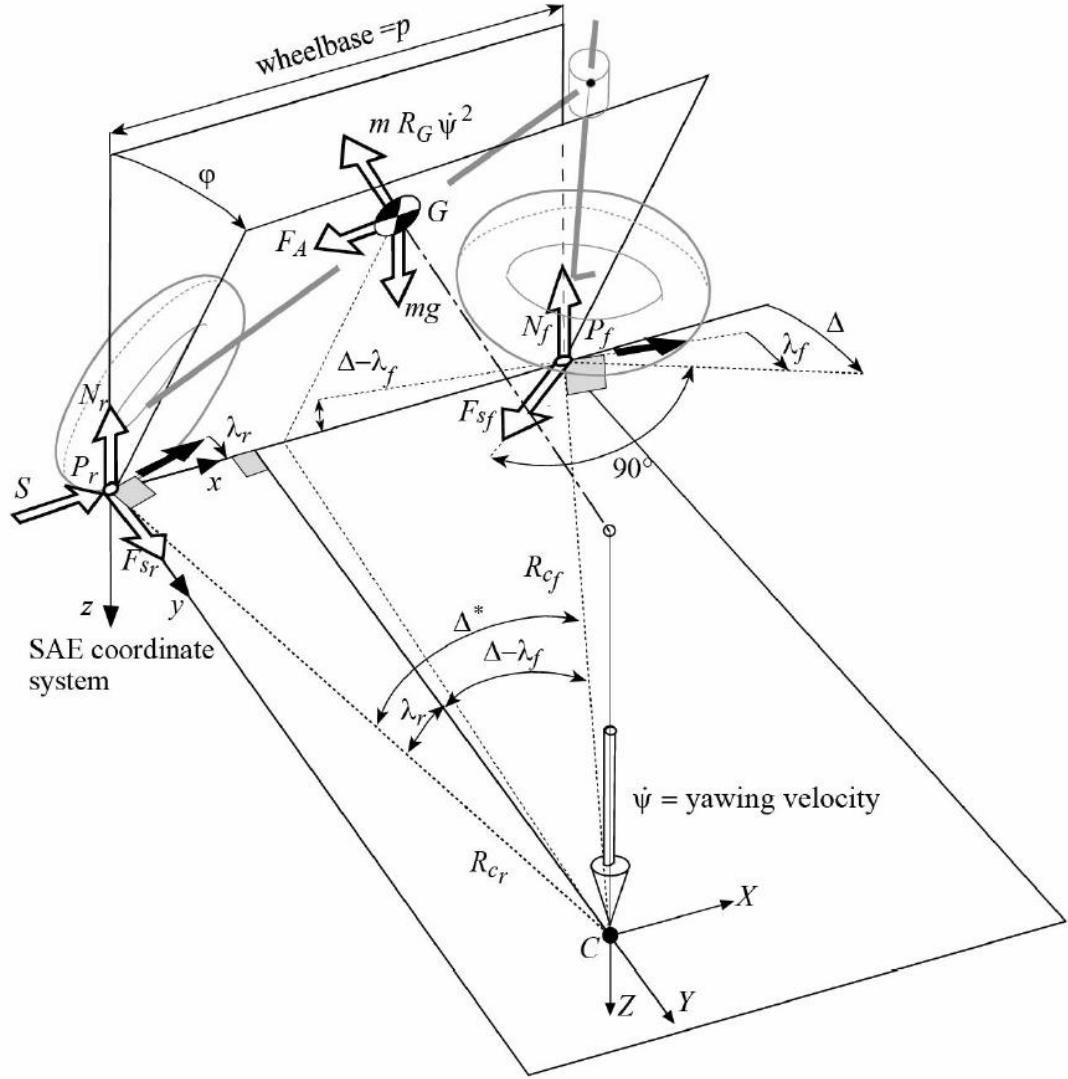


Figure 23 Graphic representation of the local coordinate system and the global frame of reference (Adaption of Cossalter, 2006)

Furthermore, the schematic diagram reveals that the turning radius of the rear contact patch and the rear sideslip angle can be used to express the position of the rear contact patch with respect to the global reference frame with the following equations from (Cossalter, 2006).

$$X_{Pr} = -R_{c_r} * \sin \lambda_r$$

$$Y_{Pr} = -R_{c_r} * \cos \lambda_r$$

Hence, any relevant parameters of the motorcycle can be related to the global reference frame through their local coordinates with the equations above, as shown in **Figure 24** and described in the following.

```

151 % Coordinates
152 - mXPr=-mTurnRadiusRearYaw.*sin(mAngleSideslipRear);
153 - mYPr=-mTurnRadiusRearYaw.*cos(mAngleSideslipRear);
154 - mXPf=mXPfminusXPr+mXPr;
155 - mYPf=mYPfminusYPr+mYPr;
156 - mXQ=mXQ+mXPr;
157 - mYQ=mhQ.*sin(mAngleRoll)+mYPr;
158 - mZQ=-mhQ.*cos(mAngleRoll)+mdeltah;
159 - mxGf=mXQ+(hf+mdQ).*sin(mAngleRake+mAnglePitchKinematicPaper)+bf.*cos(mAngleRake+mAnglePitchKinematicPaper).*cos(
160 - myGf=(mhQ-(hf+mdQ)).*cos(mAngleRake+mAnglePitchKinematicPaper)+bf.*sin(mAngleRake+mAnglePitchKinematicPaper).*cos
161 - mzGf=-((mhQ-(hf+mdQ)).*cos(mAngleRake+mAnglePitchKinematicPaper)+bf.*sin(mAngleRake+mAnglePitchKinematicPaper)).*c
162 - mXGf=mXGf+mXPr;
163 - mYGf=mYGf+mYPr;
164 - mZGf=mZGf;
165
166 % Conversions 2
167 - mICXZf=Mf.*mXGf.*mZGf+(Ixf.*cos(mAngleSteering).^2+Iyf.*sin(mAngleSteering).^2-Izf).*cos(mAngleRake+mAnglePitchK
168 - mICYZf=Mf.*mYGf.*mZGf+cos(mAngleSteering).*sin(mAngleSteering).*sin(mAngleRake+mAnglePitchKinematicPaper).*(Ixf-

```

Figure 24 'MATLAB': Screenshot of the project code including derivations of local and global coordinates

From the previous equations for the distances between the front and rear contact patch follows that the global coordinates of the contact patch of the front tyre can be calculated as follows.

$$X_{Pf} = X_{Pf} - X_{Pr} + X_{Pr}$$

$$Y_{Pf} = Y_{Pf} - Y_{Pr} + Y_{Pr}$$

In addition to the global coordinates of the front contact patch, those of the point Q at which the steering axis intersects a perpendicular plane that passes through the centre of the rear wheel are also of major importance for a couple of reasons. Firstly, it is a point of reference at which the external forces and moments generate a component of torque along the steering axis and the steady-state equilibrium condition for the front and rear frame around the steering axis at this point gives the steering torque that is exerted by the rider and applied to the handlebars (Cossalter, et al., 1999). Secondly, the point Q

also serves as the origin of the front frame coordinate system, which is oriented as shown in **Figure 25** and that the static position of the front frame mass centre relates back to.

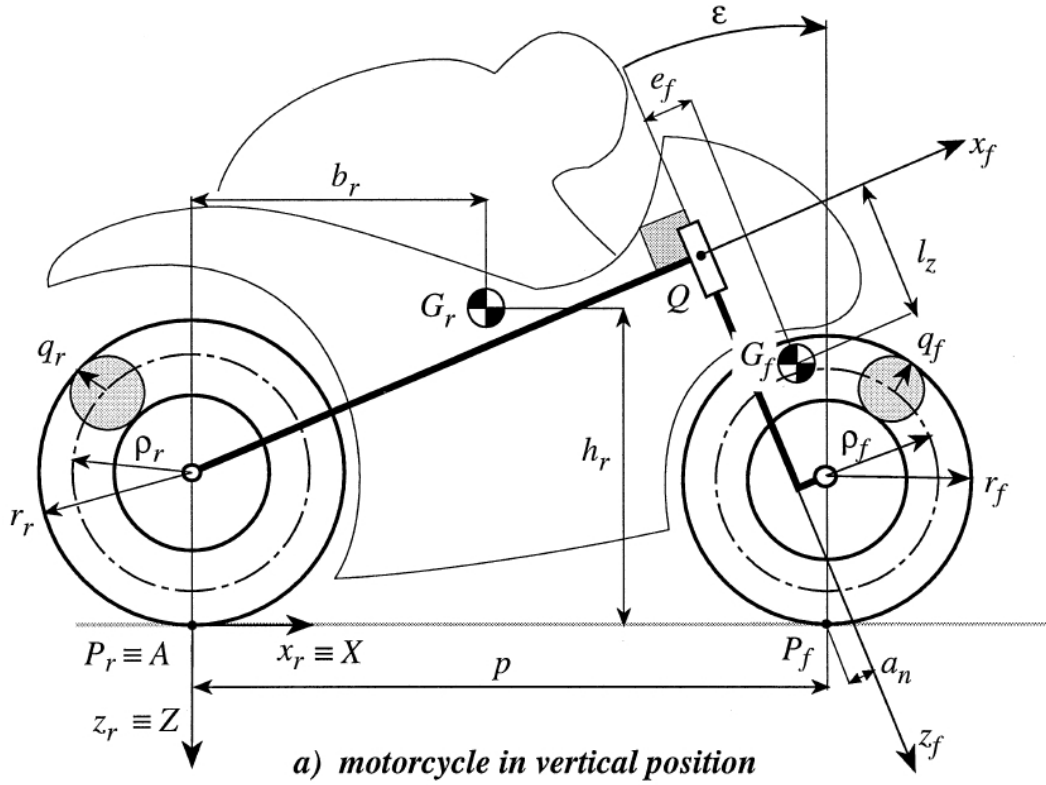


Figure 25 Graphical representation of the motorcycle with its local coordinate systems and their global orientations (Cossalter, et al., 1999)

Its local position with respect to the rear frame coordinate system, i.e. the horizontal and vertical distance between the rear contact patch and the point Q, is initially determined from a 'SOLIDWORKS' design study and then expressed in its global coordinates via the following equations from (Cossalter, 2006), whereby the lowering of the front wheel axle due to steering is added as a further vertical displacement of the point Q along the Z-axis.

$$X_Q = x_Q + X_{Pr}$$

$$Y_Q = h_Q * \sin \varphi + Y_{Pr}$$

$$Z_Q = -h_Q * \cos \varphi + \Delta h$$

Furthermore, the local position of the point Q is used to approximate the dynamic coordinates of the front frame mass centre relative to the rear frame coordinate system. **Figure 26** shows a schematic diagram of the dimensions and angles that are assumed to control the front frame mass centre position.

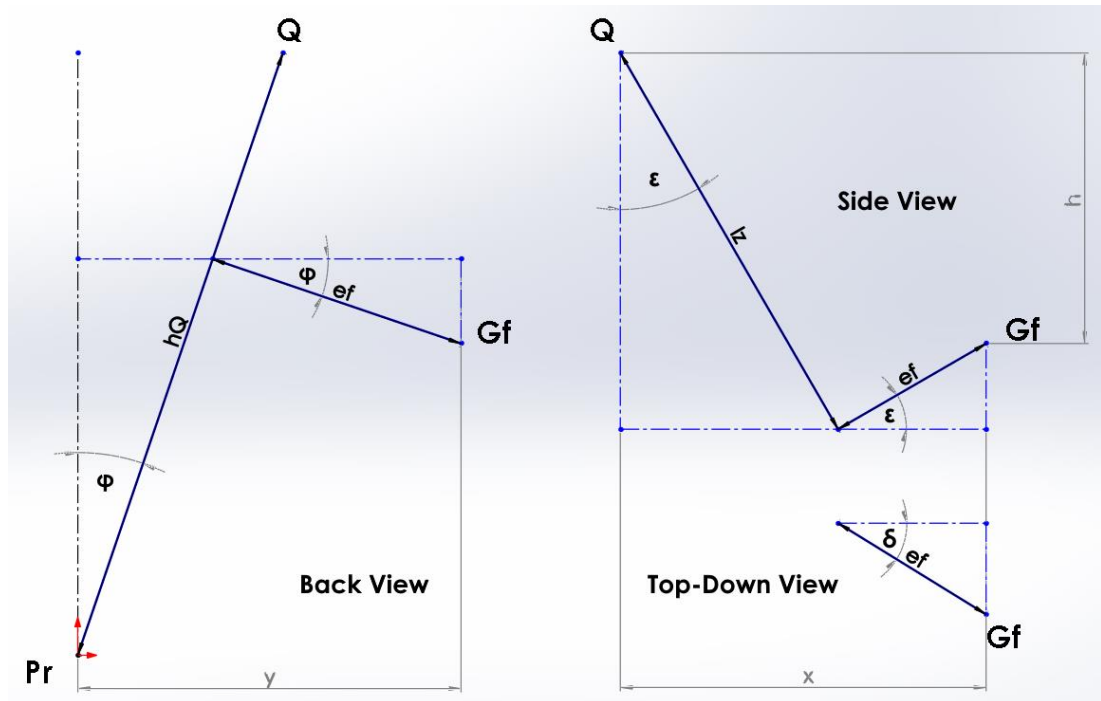


Figure 26 Sketch of dimensions and angles for the derivation of local front frame mass centre coordinates

Whilst it does not necessarily coincide with the position of the mechanical steering joint of the motorcycle, the full 'SOLIDWORK' sketch of the DVRMP reveals that the position of point Q moves along and down the steering axis in correspondence with the chassis when the forks are compressed. However, when the rear suspension compresses, the point Q is shifted upwards along the steering axis to a great extent. Therefore, this extra displacement away

from the steering joint is also recorded with a design study. Moreover, it is included in the derivation of the following equations for the approximate local coordinates of the front frame mass centre, since its static position is measured within the front frame coordinate system and is thereby directly related to the position of point Q.

$$x_{Gf} = x_Q + (l_z + d_Q) * \sin(\varepsilon + \mu) + e_f * \cos(\varepsilon + \mu) * \cos \delta$$

$$y_{Gf} = [h_Q - (l_z + d_Q) * \cos(\varepsilon + \mu) + e_f * \sin(\varepsilon + \mu) * \cos \delta] * \sin \varphi + e_f * \sin \delta * \cos \varphi$$

$$z_{Gf} = -\{[h_Q - (l_z + d_Q) * \cos(\varepsilon + \mu) + e_f * \sin(\varepsilon + \mu) * \cos \delta] * \cos \varphi - \Delta h - e_f * \sin \delta * \sin \varphi\}$$

The dynamic position of the front frame mass centre with respect to the rear contact patch is subsequently related to the global frame of reference via the previously determined global coordinates of the rear contact patch itself.

$$X_{Gf} = x_{Gf} + X_{Pr}$$

$$Y_{Gf} = y_{Gf} + Y_{Pr}$$

$$Z_{Gf} = z_{Gf}$$

The equations of motion for a motorcycle during steady turning are derived from the underlying equilibrium condition and are expressed with respect to the global reference frame by (Cossalter, et al., 1999). As a consequence of this approach, the equations of the angular momentum around the turning centre point require the products of inertia of the front frame with respect to the global coordinate system to be determined. Since the inertial properties of the

front frame are defined with respect to the principle axes of its local coordinate system and due to the assumption that the mass centre of the front frame lies in a symmetry plane, the inertia tensor is diagonal and only the products of inertia with respect to the X-Z and Y-Z plane are calculated by using the following equations from (Cossalter, et al., 1999).

$$I_{CXZf} = m_f * X_{Gf} * Z_{Gf} + (I_{xf} * \cos^2 \delta + I_{yf} * \sin^2 \delta - I_{zf}) * \cos(\varepsilon + \mu) * \sin(\varepsilon + \mu) \\ * \cos \varphi + (I_{yf} - I_{xf}) * \cos(\varepsilon + \mu) * \cos \delta * \sin \delta * \sin \varphi$$

$$I_{CZf} = m_f * Y_{Gf} * Z_{Gf} + \cos \delta * \sin \delta * \sin(\varepsilon + \mu) * (I_{xf} - I_{yf}) * (\cos^2 \varphi - \sin^2 \varphi) \\ + \cos \varphi * \sin \varphi \\ * \{I_{xf} * [\cos^2 \delta * \sin^2(\varepsilon + \mu) - \sin^2 \delta] + I_{yf} \\ * [\sin^2 \delta * \sin^2(\varepsilon + \mu) - \cos^2 \delta] + I_{zf} * \cos^2(\varepsilon + \mu)\}$$

For the mathematical model of the motorcycle that is described by (Cossalter, et al., 1999) as a system of six rigid bodies, Newton's equations of motion yield three force equilibrium equations and three moment equilibrium equations, from which the steady-state steering torque equations are ultimately determined by considering the equations that are related to the front frame (Cossalter, 2006). However, in the original model, only position variables are solved prior to the steady turning equations, as the tyre-related and kinematic equations that allow for a calculation of the turn curvature, forward velocity, as well as the angular velocities of the front and rear wheel, must be solved simultaneously, whereby the total number of equations (13) is equal to the number of unknowns (Cossalter, et al., 1999). These unknown variables comprise five reaction forces, the forward velocity, front and rear sideslip angles, longitudinal slip, turn curvature, yaw velocity, as well as angular front and rear wheel velocity. Owing to the tyre and kinematic formulae, these equations are non-linear and are solved numerically for assigned values of roll angle and steering angle by initially setting the slip quantities to zero, which changes the steady turning equations to a linear system with six equations and six unknowns. In the next step of the original procedure, the sideslip angles

and longitudinal slip are estimated by using the tyre formulae which allows for both position variables and velocity coefficients to be calculated separately. This marks the beginning of an iterative process in which the linear system for the six unknowns of normal and lateral force on the front and rear tyre, the longitudinal force at the rear contact patch and the squared yaw velocity are recalculated, while a new estimate is repeatedly made for the slip quantities that is then compared with the previous one until the residuals are below a pre-established threshold (Cossalter, et al., 1999). In contrast to the method that is described above and that defines the original model by (Cossalter, et al., 1999), the project code simply utilises the recorded dataset to determine the necessary values for the dynamic variables and it only computes the reaction forces that are required for the set of algebraic steady-state steering torque equations which is shown in **Figure 27**.

```

170 %% Steering Torque Components (V.C. 1999/2006):
171
172 % Gravity Torque
173 mTorqueGravity=9.81*Mf.*(mYQ-mYGf).*sin(mAngleRake+mAnglePitchKinematicPaper)+(mXQ-mXGf).*cos(mAngleRake+mAngle
174 % Inertia Torque
175 mTorqueInertiaPaper=mAngleVelocityYaw.^2.*Mf.*(mYGf.*mXQ-mXGf.*mYQ).*cos(mAngleRake+mAnglePitchKinematicPaper).
176 mTorqueInertiaPaperAlt=mAngleVelocityYaw.^2.*Mf.*(mYGf.*mXQ-mXGf.*mYQ).*cos(mAngleRake+mAnglePitchKinematicPaper
177 mTorqueInertiaBook=- (mAngleVelocityYaw.^2.*Mf.*(mXQ.*mYGf-mXGf.*mYQ).*cos(mAngleRake+mAnglePitchKinematicPaper)
178 mTorqueInertiaBookAlt=- (mAngleVelocityYaw.^2.*Mf.*(mXQ.*mYGf-mXGf.*mYQ).*cos(mAngleRake+mAnglePitchKinematicPap
179 % Normal Torque
180 mTorqueNormal=- (mYQ-mYPf).*sin(mAngleRake+mAnglePitchKinematicPaper)+(mXQ-mXPf).*cos(mAngleRake+mAnglePitchKine
181 % Lateral Torque
182 mTorqueLateral=(mYQ-mYPf).*sin(mAngleSteeringKinematicPaper)+(mXQ-mXPf).*cos(mAngleSteeringKinematicPaper)).*co
183 % Lateral Torque Alternative
184 mTorqueLateralAlt=mTrailNormalCornering.*mFf;
185 % Longitudinal Torque
186 mTorqueLongitudinal=- ((mYPf-mYQ).*cos(mAngleSteeringKinematicPaper)+(mXQ-mXPf).*sin(mAngleSteeringKinematicPape
187 % Tyre Torque
188 mTorqueTyre=-mMzFront.*cos(mAngleRake+mAnglePitchKinematicPaper).*cos(mAngleRoll)+(cos(mAngleRake+mAnglePitchKin
189 % Yaw Torque
190 mTorqueYaw=(sin(mAngleRake+mAnglePitchKinematicPaper).*cos(mAngleRoll).*cos(mAngleSteering)-sin(mAngleRoll)).*sin
191 % Roll Torque (Transient)
192 mTorqueRoll=- (cos(mAngleRake+mAnglePitchKinematicPaper).*cos(mAngleSteering)).*Iwf.*mAngleVelocityFront.*mAngleVe
193 % Gyroscopic Torque (Transient)
194 mTorqueGyroscopic=mTorqueYaw+mTorqueRoll;
195
196 % Total StTorque
197 mStTorqueVC99=mTorqueGravity+mTorqueInertiaPaper+mTorqueNormal+mTorqueLateral+mTorqueLongitudinal+mTorqueTyre+mT
198 % Total StTorque Alternative
199 mStTorqueVC99Alt=mTorqueGravity+mTorqueInertiaPaper+mTorqueNormal+mTorqueLateralAlt+mTorqueLongitudinal+mTorqueT

```

Figure 27 ‘MATLAB’: Screenshot of the project code including the steady-state steering torque equations

The steering torque that a rider must apply to the handlebars to ensure the equilibrium of the motorcycle in a turn is evaluated from the equilibrium of moments around the steering axis, whereby the torque on the front frame

creates an equal and opposite reaction on the rear frame and vice versa (Cossalter, 2006). Thus, it is equal to the total sum of the torque components that are generated along the steering axis by the external forces that act on the front frame and the torque that the rear frame exerts on the front frame at point Q (Cossalter, et al., 1999). When the six equations for the equilibrium of forces and moments during steady turning are written in component form and related to the global frame of reference, the steering torque can be calculated with equations from (Cossalter, et al., 1999) or (Cossalter, 2006) and is given by the summation of seven or six terms respectively. In both cases, the different terms of torque describe the disaligning influence on the steered front frame assembly from its associated weight force, the normal load on the front wheel and the tyre yawing moment, as well as the aligning influence on the front frame from the centrifugal effect on it, the lateral force at the front contact patch and the gyroscopic effect of the front wheel due to yaw rate. In addition, the more complex model by (Cossalter, et al., 1999) also includes the disaligning influence from a longitudinal force at the front contact patch and the aligning influence from the rolling resistance moment of the tyre. Furthermore, the two sets of equations differ in terms of sign convention and the point of reference for the distances in the torque equations, which is taken as the point at which the normal trail intersects the steering axis by (Cossalter, 2006), as opposed to point Q by (Cossalter, et al., 1999). Due to an increased model fidelity, the following equations from (Cossalter, et al., 1999) are referenced and used for the project code, with the exception of the equation for the gyroscopic effect of the front wheel that is taken from (Cossalter, 2006) for the purpose of changing the sign. This change is also necessary for the torque components from the normal load and the longitudinal force to ensure their consistency with the given sign convention of the model by (Cossalter, et al., 1999). Subsequently, the total steering torque is computed as the resultant of the following steady-state torque components.

$$\tau_{Total} = \tau_{Gravity} + \tau_{Inertia} + \tau_{Normal} + \tau_{Lateral} + \tau_{Longitudinal} + \tau_{Tyre} + \tau_{Yaw}$$

Where:

$$\tau_{Gravity}$$

$$= g * m_f * [(Y_Q - Y_{Gf}) * \sin(\varepsilon + \mu) + (X_Q - X_{Gf}) * \cos(\varepsilon + \mu) * \sin \varphi]$$

$$\tau_{Inertia}$$

$$\begin{aligned} = & \dot{\psi}^2 * m_f * (Y_{Gf} * X_Q - X_{Gf} * Y_Q) * \cos(\varepsilon + \mu) * \cos \varphi - \dot{\psi}^2 * m_f * Z_Q \\ & * [Y_{Gf} * \sin(\varepsilon + \mu) + X_{Gf} * \cos(\varepsilon + \mu) * \sin \varphi] + \dot{\psi}^2 \\ & * [I_{CXYZf} * \sin(\varepsilon + \mu) + I_{CXZf} * \cos(\varepsilon + \mu) * \sin \varphi] \end{aligned}$$

$$\tau_{Normal}$$

$$= -[(Y_Q - Y_{Pf}) * \sin(\varepsilon + \mu) + (X_Q - X_{Pf}) * \cos(\varepsilon + \mu) * \sin \varphi] * N_f$$

$$\tau_{Lateral}$$

$$\begin{aligned} = & [(Y_Q - Y_{Pf}) * \sin \Delta + (X_Q - X_{Pf}) * \cos \Delta] * \cos(\varepsilon + \mu) * \cos \varphi * F_f - Z_Q \\ & * [\cos \Delta * \sin(\varepsilon + \mu) - \cos(\varepsilon + \mu) * \sin \varphi * \sin \Delta] * F_f \end{aligned}$$

$$\tau_{Longitudinal}$$

$$\begin{aligned} = & -\{[(Y_{Pf} - Y_Q) * \cos \Delta + (X_Q - X_{Pf}) * \sin \Delta] * \cos(\varepsilon + \mu) * \cos \varphi * S_f - Z_Q \\ & * [\cos \Delta * \cos(\varepsilon + \mu) * \sin \varphi + \sin \Delta * \sin(\varepsilon + \mu)] * S_f\} \end{aligned}$$

$$\tau_{Tyre}$$

$$\begin{aligned} = & -M_{z\ Front} * \cos(\varepsilon + \mu) * \cos \varphi + [\cos(\varepsilon + \mu) * \sin \varphi * \cos \Delta + \sin(\varepsilon + \mu) * \sin \Delta] \\ & * M_{y\ Front} \end{aligned}$$

$$\tau_{Yaw}$$

$$= [\sin(\varepsilon + \mu) * \cos \varphi * \cos \delta - \sin \varphi * \sin \delta] * I_{Wf} * \omega_f * \dot{\psi}$$

In addition to the given equation for the gyroscopic effect of the front wheel due to yaw rate, a new equation for the gyroscopic effect due to roll rate in transient state is derived by relating the angular momentum of the front wheel to the recorded roll velocity and steering angle. This gyroscopic torque is then resolved along the steering axis, which yields the following equation for the additional steering torque component.

$$\begin{aligned}\tau_{Roll} \\ = -\cos(\varepsilon + \mu) * \cos \delta * I_{Wf} * \omega_f * \dot{\phi}\end{aligned}$$

Whilst the gyroscopic torque component due to yaw rate has an aligning effect on the steered front frame assembly of the motorcycle in steady turning, the gyroscopic torque component due to roll rate acts in the opposite direction around the steering axis and thus has a disaligning influence on the front frame (Cossalter, 2006). Hence, a combined term for the resultant steering torque component from the gyroscopic effects of the front wheel can be written as follows.

$$\begin{aligned}\tau_{Gyroscopic} &= \tau_{Yaw} + \tau_{Roll} \\ &= [\sin(\varepsilon + \mu) * \cos \varphi * \cos \delta - \sin \varphi * \sin \delta] * I_{Wf} * \omega_f * \dot{\psi} \\ &\quad + [-\cos(\varepsilon + \mu) * \cos \delta * I_{Wf} * \omega_f * \dot{\phi}]\end{aligned}$$

Along with a simplified formula for the torque component from the lateral force on the front contact patch, it is proposed for an alternative torque summation.

$$\begin{aligned}\tau_{Total\ Alternative} \\ = \tau_{Gravity} + \tau_{Inertia} + \tau_{Normal} + \tau_{Lateral\ Alternative} + \tau_{Longitudinal} + \tau_{Tyre} + \tau_{Gyroscopic}\end{aligned}$$

Where:

$$\tau_{Lateral\ Alternative} = a_n^* * F_f$$

4.4.4. Determining Steering Torque using Transient-State Equations

The mathematical model of a motorcycle in free control that was developed by (Sharp, 1971) for the analysis of typical stability characteristics and their dependence on various parameters is different to the previously discussed model by (Cossalter, et al., 1999) in a number of ways. First and foremost, it describes a motorcycle that travels at a constant forward velocity and that has the freedom to sideslip, yaw and roll. Nevertheless, only small perturbations from straight line running are considered. Secondly, the equations of motion are derived by the application of Lagrange's equation and the model makes use of several simplifying assumptions, whereby the wheels are treated as rigid discs that roll without longitudinal slip and the motorcycle has no freedom to pitch (Sharp, 1971). For the analysis of steady turning in the original model, the resulting linear differential equations of motion in homogeneous form are reduced to a set of first order differential equations by the introduction of new variables. Their solutions are assumed to take on an exponential form. Since the exponential function is nontrivial, this technique allows for the solutions to be substituted back into the original equations, which thus makes them a set of algebraic equations. Furthermore, these equations can be expressed in matrix form and they are ultimately solved numerically and simultaneously by assigning values to the forward velocity and roll angle of the motorcycle (Sharp, 1971). The initial derivation of the following and unreduced equation for the total steering torque that is applied by the rider at the handlebars, based on the kinetic and potential energies of the entire system, is given in the appendix of (Sharp, 1971).

$$\begin{aligned}
 & \tau_{Total} \\
 &= M_f * e * \dot{v} + (I_{fz} + M_f * e^2) * \ddot{\delta} + K * \dot{\delta} + (t * Z_f - M_f * e * g) * \sin \varepsilon * \delta \\
 &+ (M_f * e * j + I_{fz} * \sin \varepsilon) * \ddot{\varphi} - \frac{i_{fy}}{R_f} * \cos \varepsilon * \dot{x}_1 * \dot{\varphi} \\
 &+ (t * Z_f - M_f * e * g) * \varphi + (M_f * e * k + I_{fz} * \cos \varepsilon) * \ddot{\psi} \\
 &+ \left(M_f * e + \frac{i_{fy}}{R_f} * \sin \varepsilon \right) * \dot{x}_1 * \dot{\psi} + t * Y_f
 \end{aligned}$$

As with the steady-state model, the method that is applied to the project code simply utilises the original steering torque equation in conjunction with the recorded dataset and the previously determined inputs and variables, as well as local coordinates, which is shown in **Figure 29**.

```

201 %% Steering Torque Components (R.S. 1971):
202
203 %1 Weight Moment from Roll
204 mTorque1=-Mf*bf*9.81.*mAngleRoll;
205 %2 Normal Force Moment from Roll
206 mTorque2=-mTrailNormalCornering.*mNf.*mAngleRoll;
207 %3 Weight Moment from Steer
208 mTorque3=-Mf*bf*9.81.*sin(mAngleRake+mAnglePitchKinematicBook).*mAngleSteering;
209 %4 Normal Force Moment from Steer
210 mTorque4=-mTrailNormalCornering.*mNf.*sin(mAngleRake+mAnglePitchKinematicBook).*mAngleSteering;
211 %5 Front Frame Moment from Yaw
212 mTorque5=Mf*bf.*mVelocityForward.*mAngleVelocityYaw;
213 %6 Gyroscopic Torque from Yaw
214 mTorque6=Iwf/RFront.*sin(mAngleRake+mAnglePitchKinematicBook).*mVelocityFront.*mAngleVelocityYaw;
215 %7 Gyroscopic Torque from Roll
216 mTorque7=-Iwf/RFront.*cos(mAngleRake+mAnglePitchKinematicBook).*mVelocityFront.*mAngleVelocityRoll;
217 %8 Steering Damper Moment
218 mTorque8=cSteeringDamper.*mAngleVelocitySteering;
219 %9 Front Frame Moment from Lateral Acceleration
220 mTorque9=Mf*bf.*mAccelLateral;
221 %10 Front Frame Moment from Yaw
222 mTorque10=Mf*bf.*(mxGf-mb).)*mAngleAccelYaw;
223 %11 Front Frame Inertia Torque from Yaw
224 mTorque11=Izf.*cos(mAngleRake+mAnglePitchKinematicBook).*mAngleAccelYaw;
225 %12 Front Frame Moment from Roll
226 mTorque12=Mf*bf.*-mzGf.*mAngleAccelRoll;
227 %13 Front Frame Inertia Torque from Roll
228 mTorque13=Izf.*sin(mAngleRake+mAnglePitchKinematicBook).*mAngleAccelRoll;
229 %14 Steer Mass Inertia Torque
230 mTorque14=Mf*bf^2.*mAngleAccelSteering;
231 %15 Steer Inertia Torque
232 mTorque15=Izf.*mAngleAccelSteering;
233 %16 Lateral Force Moment from Trail
234 mTorque16=mTrailNormalCornering.*mFf;
235 %17 Longitudinal Force Moment from Roll
236 mTorque17=tFront.*cos(mAngleRake+mAnglePitchKinematicBook).*sin(mAngleRoll).*mSf;
237
238 % Total StTorque
239 mStTorqueRS71=mTorque1+mTorque2+mTorque3+mTorque4+mTorque5+mTorque6+mTorque7+mTorque8+mTorque9+mTorque10+mTorque
240 % Total StTorque Alternative
241 mStTorqueRS71Alt=mTorque1+mTorque2+mTorque3+mTorque4+mTorque5+mTorque6+mTorque7+mTorque8+mTorque9+mTorque10+mTor

```

Figure 28 ‘MATLAB’: Screenshot of the project code including the transient-state steering torque components

For extended analysis, the resultant equation is broken down into its constituent terms that represent the sixteen individual torque components, of which some are highly dynamic and specific to the transient state, as follows.

$$\tau_{Total} =$$

$$\tau_1 + \tau_2 + \tau_3 + \tau_4 + \tau_5 + \tau_6 + \tau_7 + \tau_8 + \tau_9 + \tau_{10} + \tau_{11} + \tau_{12} + \tau_{13} + \tau_{14} + \tau_{15} + \tau_{16}$$

Where:

$$\tau_1 \langle \text{Weight Moment from Roll} \rangle$$

$$= M_f * e * g * \varphi \stackrel{\text{def}}{=} m_f * e_f * g * \varphi$$

$$\tau_2 \langle \text{Normal Force Moment from Roll} \rangle$$

$$= t * Z_f * \varphi \stackrel{\text{def}}{=} a_n * N_f * \varphi \cong a_n^* * N_f * \varphi$$

$$\tau_3 \langle \text{Weight Moment from Steer} \rangle$$

$$= -M_f * e * g * \sin \varepsilon * \delta \stackrel{\text{def}}{=} -m_f * e_f * g * \sin \varepsilon * \delta \cong -m_f * e_f * g * \sin(\varepsilon + \mu) * \delta$$

$$\tau_4 \langle \text{Normal Force Moment from Steer} \rangle$$

$$= t * Z_f * \sin \varepsilon * \delta \stackrel{\text{def}}{=} a_n * N_f * \sin \varepsilon * \delta \cong a_n^* * N_f * \sin(\varepsilon + \mu) * \delta$$

$$\tau_5 \langle \text{Front Frame Moment from Yaw} \rangle$$

$$= M_f * e * \dot{x}_1 * \dot{\psi} \stackrel{\text{def}}{=} m_f * e_f * V * \dot{\psi}$$

$$\tau_6 \langle \text{Gyroscopic Torque from Yaw} \rangle$$

$$= \frac{i_{fy}}{R_f} * \sin \varepsilon * \dot{x}_1 * \dot{\psi} \stackrel{\text{def}}{=} \frac{I_{wf}}{R_f} * \sin \varepsilon * V * \dot{\psi} \cong \frac{I_{wf}}{R_f} * \sin(\varepsilon + \mu) * V * \dot{\psi}$$

$$\tau_7 \langle \text{Gyroscopic Torque from Roll} \rangle$$

$$= -\frac{i_{fy}}{R_f} * \cos \varepsilon * \dot{x}_1 * \dot{\varphi} \stackrel{\text{def}}{=} -\frac{I_{wf}}{R_f} * \cos \varepsilon * V * \dot{\varphi} \cong -\frac{I_{wf}}{R_f} * \cos(\varepsilon + \mu) * V * \dot{\varphi}$$

$$\tau_8 \langle \text{Steer Damper Moment} \rangle$$

$$= K * \delta$$

$$\tau_9 \langle \text{Front Frame Moment from Lateral Acceleration} \rangle$$

$$= M_f * e * \dot{v} \stackrel{\text{def}}{=} m_f * e_f * \dot{v}$$

$$\tau_{10} \langle \text{Front Frame Moment from Yaw} \rangle$$

$$= M_f * e * k * \ddot{\psi} \stackrel{\text{def}}{=} m_f * e_f * (x_{Gf} - x_{Gr}) * \ddot{\psi} \cong m_f * e_f * (x_{Gf} - b) * \ddot{\psi}$$

$$\tau_{11} \langle \text{Front Frame Inertia Torque from Yaw} \rangle$$

$$= I_{fz} * \cos \varepsilon * \ddot{\psi} \stackrel{\text{def}}{=} I_{zf} * \cos \varepsilon * \ddot{\psi} \cong I_{zf} * \cos(\varepsilon + \mu) * \ddot{\psi}$$

$$\tau_{12} \langle \text{Front Frame Moment from Roll} \rangle$$

$$= M_f * e * j * \ddot{\phi} \stackrel{\text{def}}{=} m_f * e_f * (-z_{Gf}) * \ddot{\phi}$$

$$\tau_{13} \langle \text{Front Frame Inertia Torque from Roll} \rangle$$

$$= I_{fz} * \sin \varepsilon * \ddot{\phi} \stackrel{\text{def}}{=} I_{zf} * \sin \varepsilon * \ddot{\phi} \cong I_{zf} * \sin(\varepsilon + \mu) * \ddot{\phi}$$

$$\tau_{14} \langle \text{Steer Mass Inertia Torque} \rangle$$

$$= M_f * e^2 * \ddot{\delta} \stackrel{\text{def}}{=} m_f * e_f^2 * \ddot{\delta}$$

$$\tau_{15} \langle \text{Steer Inertia Torque} \rangle$$

$$= I_{fz} * \ddot{\delta} \stackrel{\text{def}}{=} I_{zf} * \ddot{\delta}$$

$$\tau_{16} \langle \text{Lateral Force Moment from Trail} \rangle$$

$$= t * Y_f \stackrel{\text{def}}{=} a_n * F_f \cong a_n^* * F_f$$

Since the original model by (Sharp, 1971) is only concerned with a motorcycle that travels at a constant velocity and due to it omitting the rolling resistance and longitudinal slip of the tyres by making the assumption that the wheels are thin discs and in point-contact with the ground, the influence of any longitudinal forces on the steered front frame assembly is completely neglected. Therefore, an additional steering torque component that arises from the braking force that acts in line with the front wheel and at the effective trail of the tyre is derived by accounting for its cross-sectional radius, as depicted in **Figure 29**.

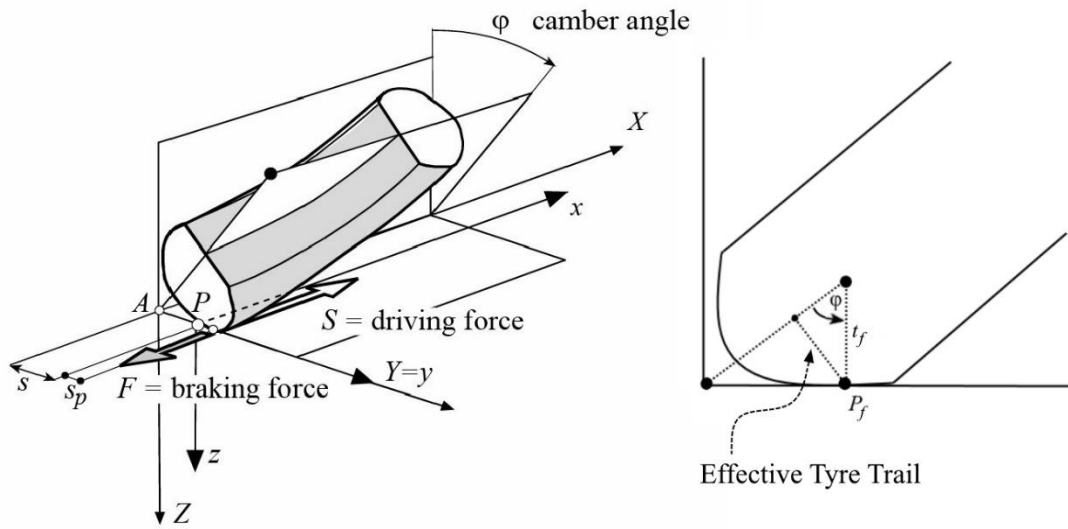


Figure 29 Graphic representation of tyres (Adaption of Cossalter, 2006)

Resolving this braking torque along the steering axis yields the following equation. It is proposed as an added term for an alternative torque summation.

$$\tau_{17} \text{ (Longitudinal Force Moment from Roll)}$$

$$= t_f * \cos(\varepsilon + \mu) * \sin \phi * S_f$$

$$\tau_{Total \text{ Alternative}} =$$

$$\tau_1 + \tau_2 + \tau_3 + \tau_4 + \tau_5 + \tau_6 + \tau_7 + \tau_8 + \tau_9 + \tau_{10} + \tau_{11} + \tau_{12} + \tau_{13} + \tau_{14} + \tau_{15} + \tau_{16} \\ + \tau_{17}$$

4.5. Multibody Software Simulations

On top of computing and assessing the steering torque based on equations of the steady- and transient-state model, a state-of-the-art multibody software package that is designed for the analysis of dynamic motorcycle and rider behaviour is employed for subsequent simulations. The chosen program is called 'VI-BikeRealTime' and it is part of a range of products by the company 'VI-grade'. It takes advantage of multibody technology from 'MSC ADAMS' and real-time solvers to enable motorcycle engineers to model and improve their products with regards to handling, ride, comfort, durability, safety, reliability and more, throughout the design process (VI-grade GmbH, 2020). Furthermore, this particular software application appears to be the best choice for the purpose of the research project, since it offers a number of benefits over alternative simulation tools. In addition to rendering the need for any validation of the models that are derived from detailed assemblies obsolete, these benefits include an interface that allows for direct input of the basic model-defining parameters and one that is linked to 'MATLAB Simulink' for software-in-the-loop studies, along with a built-in tool for designing two- or three-dimensional paths, as well as a postprocessing environment for animation and plotting of the simulation results (VI-grade GmbH, 2020). Thus, a virtual motorcycle model of the DVRMP is created within 'VI-BikeRealTime'. The program offers different templates for new motorcycle models, which are labelled as 'standard', 'conceptual', 'direct shock' or 'progressive', yet it also provides a pre-built default model. By working through tutorials and running full simulations on sample roads, this standard model is found to be working and it is subsequently adopted for the generation of a modified model that closely resembles the DVRMP. The alterations are made to the model by adjusting the input parameters within setup pages, of which there are 12 different ones. These include a page for the characteristics of the suspension internals, one for the geometric and inertial properties of the front suspension, which is shown in **Figure 30**, as well as one for the properties of the rear suspension and one for the main frame, which can include the fuel tank, the rider and a pillion. Whilst many values can be determined from earlier measurements and the 'SOLIDWORKS' sketch of the DVRMP chassis, most of the mass properties of its components and detailed suspension characteristics remain unknown for

the previously stated reasons that are related to the 'COVID-19' pandemic. Thus, some parameter inputs are identical to those of the default parent model.

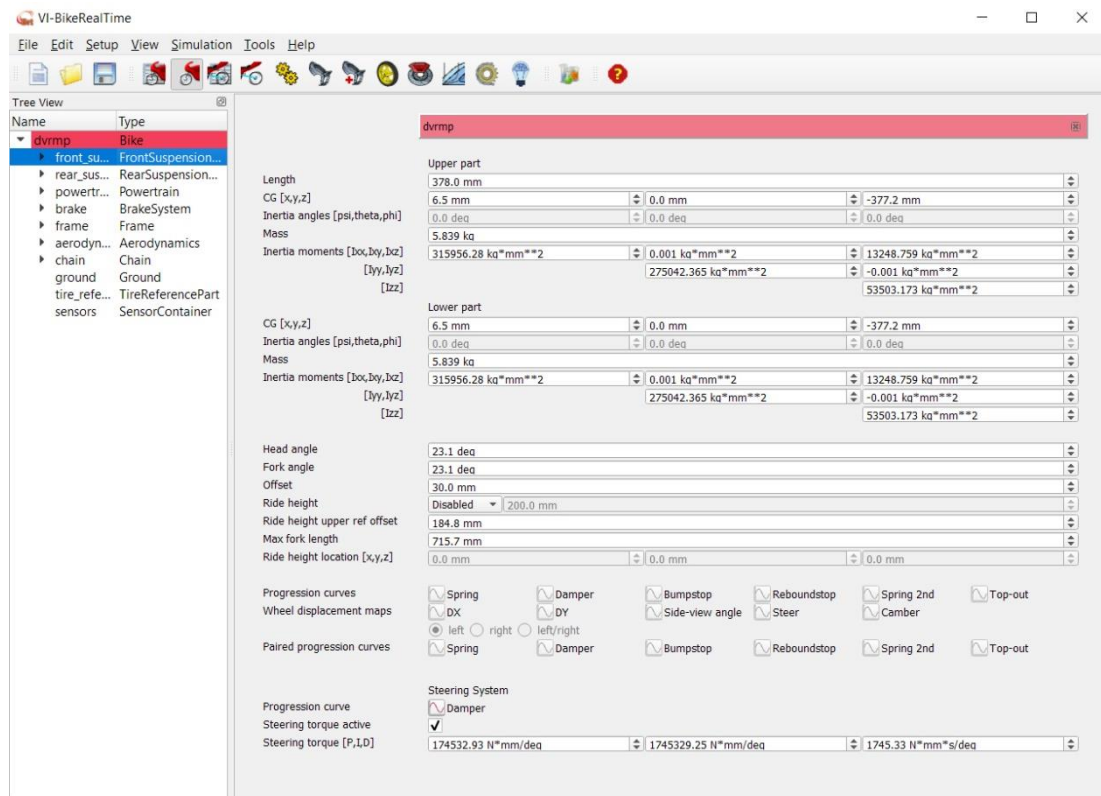


Figure 30 'VI-BikeRealTime': Screenshot of a setup page for the front suspension parameters of the motorcycle model

Furthermore, the virtual model is based on setup pages for the properties of its chain, the wheels and the brake system, as well as aerodynamic and powertrain specifications, for which the required values can be established in numerous different ways. The wheel spin inertia is measured by using the previously described method, the chain pitch value and internal gear ratios are referenced from (Ducati Motor Holding S.p.A., 2012), the maximum brake pressure is quoted from a dataset by exploiting the recordings from a CAN-channel for front brake pressure, which are previously calibrated with readings from an aftermarket sensor for measuring pressure within a hydraulic system; the frontal area of the DVRMP is estimated by using a technique of counting scaled squares in a head-on picture of it, as is suggested by (Bradley, 1996),

the exact engine characteristics of the DVRMP are accounted for by quoting its dynamic torque output throughout the entire engine speed range from data of a power run on the rolling road of the 'MAHA' dynamometer at the university. Despite missing more precise information for several other parameters, such as the friction coefficients of the brakes, aerodynamic drag coefficients and pressure centres or the specific tyre characteristics, the finished motorcycle model can be used to carry out various simulations with 'VI-BikeRealTime'. The different types of viable simulations range from dynamic simulations that are customisable with the built-in tool 'VI-EventBuilder' and stationary test rig simulations for the assessment of suspension subsystem design to extensive lap time and dynamic performance limit simulations, as well as simulations of predetermined manoeuvres. Out of the aforementioned options, the dynamic performance limit simulation is used to allow for the comparison of a processed dataset from the DVRMP at Donington Park Circuit with the computed results of 'VI-BikeRealTime' from a virtual model that does exactly the same lap. For this purpose, previously recorded GPS data of the DVRMP is exported and used to create a path within 'Google Earth', as shown in **Figure 31**. This path can then be saved as a file that has the right format for 'VI-Road'.

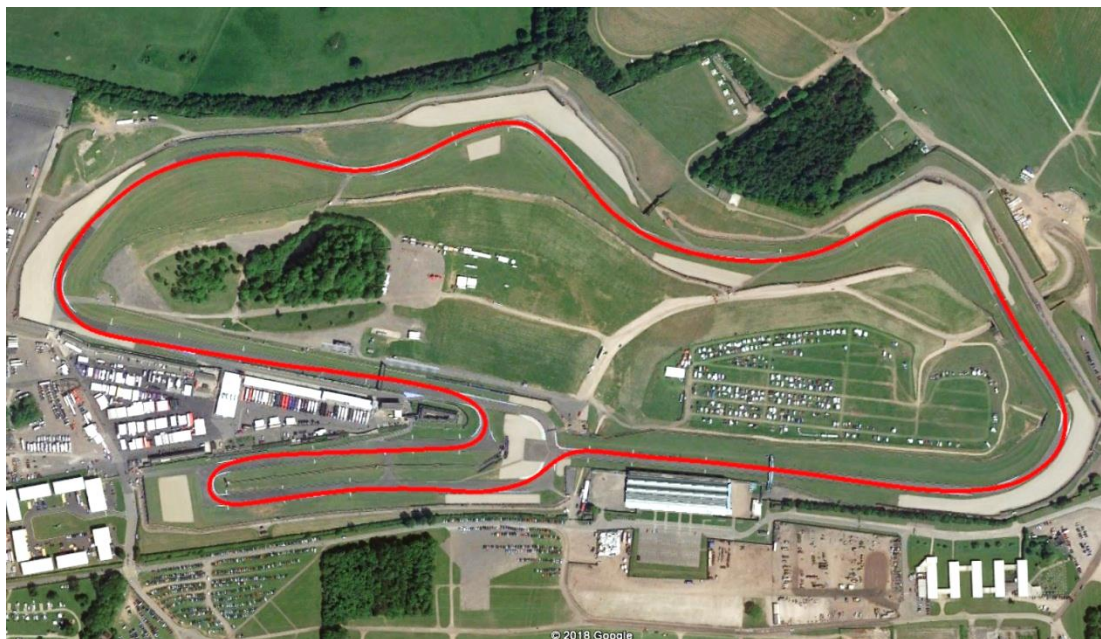


Figure 31 'Google Earth': Screenshot of Donington Park Circuit and the exact path of a full lap by the DVRMP

'VI-Road' is a built-in tool of 'VI-BikeRealTime' for creating road and path profiles. It allows for the GPS-derived path to be directly imported and used as the centreline of a new road file, on which an additional target path is generated with a 'corner-cutting' function, as displayed in **Figure 32**. Both lines are then smoothed with a spline algorithm which eliminates the potential noise that could result from an irregular arc. Once the origin is transformed to intersect the paths at the start-finish line of the track, files of the obtained data are exported from 'VI-Road'. The simulation uses a static solver to compute a velocity profile along the desired path and verifies its feasibility with a dynamic solver. If the velocity profile is found to be unfeasible, an internal optimiser modifies it accordingly and this process is iteratively repeated until a feasible velocity profile is found (VI-grade Engineering Software & Services, 2019).

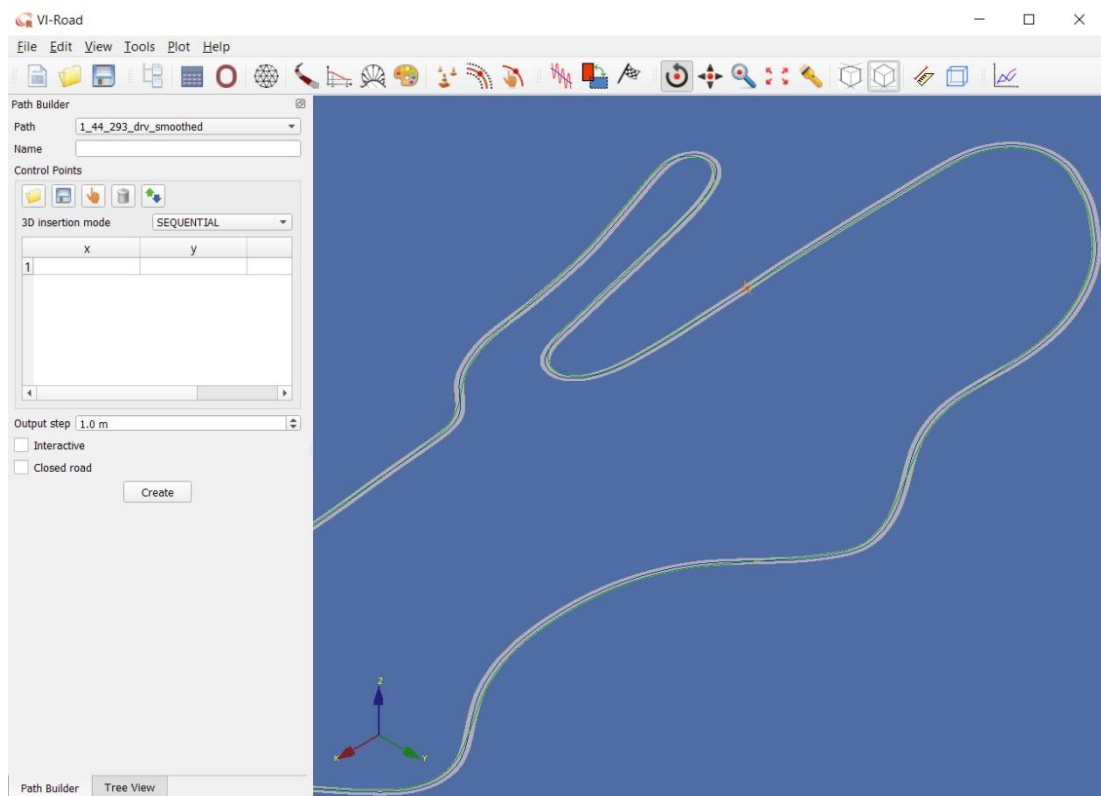


Figure 32 'VI-Road': Screenshot of road and path profiles for the circuit

Subsequently, the results become available for animation and plotting within the post-processing tool 'VI-Animator', as shown in **Figure 33** and **Figure 34**.

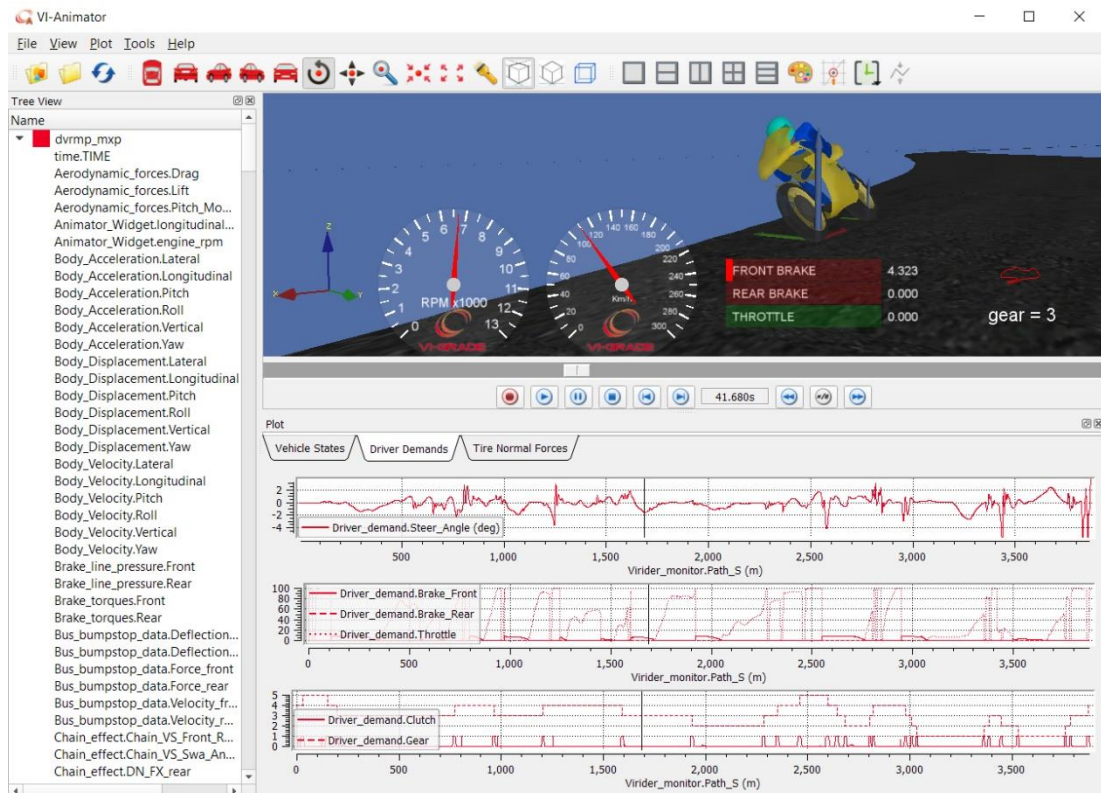


Figure 33 'VI-Animator': Screenshot of results for a performance limit run

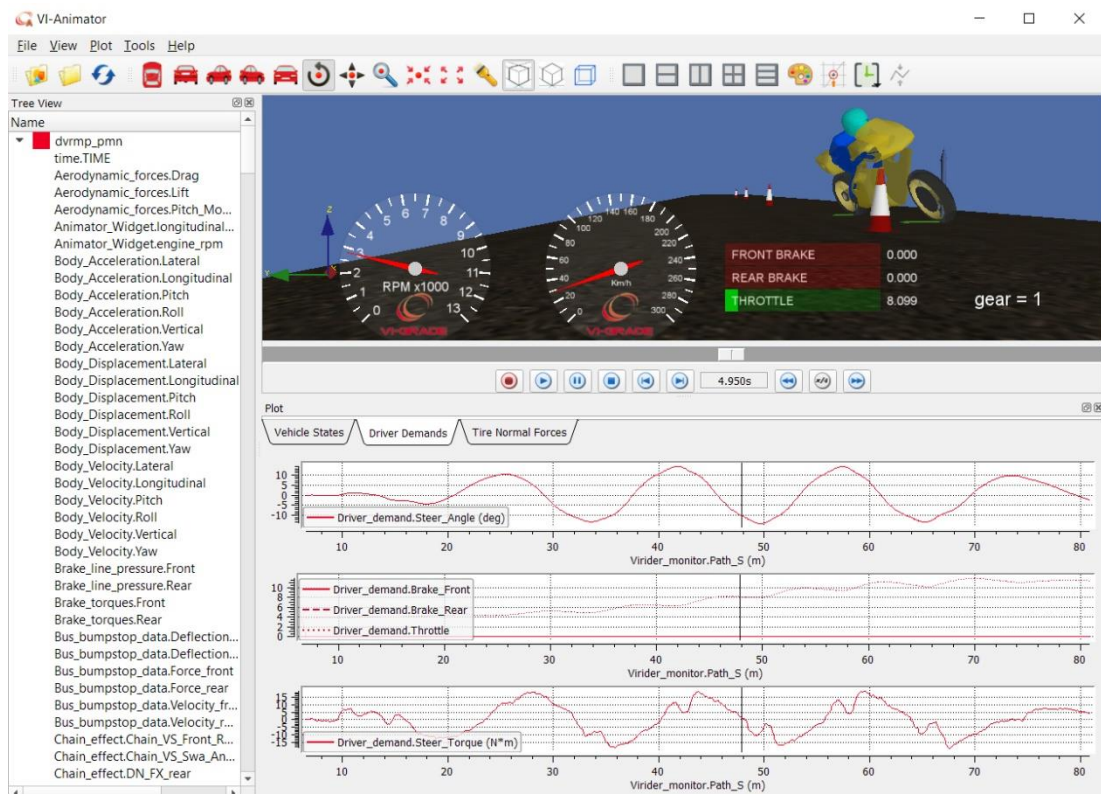


Figure 34 'VI-Animator': Screenshot of results for a slalom manoeuvre

In addition to the simulations of full flying laps at Donington Park Circuit, a predetermined manoeuvre simulation is carried out with a slalom course that has a cone spacing of 7 m. In contrast to the previously described working mechanisms of the dynamic performance limit simulation, it uses a genetic algorithm to generate target trajectories that are submitted and verified during the dynamic integration, whilst a constant velocity is set as the longitudinal dynamics target. Furthermore, a cone interference check is used for each iteration to verify whether the target trajectory is feasible. When the full set of trajectories has been verified, the velocity is increased and a new cycle begins. The simulation is complete when a single combination of target trajectory and limit or maximum velocity is identified which simultaneously fulfils the condition that no cones are hit (VI-grade Engineering Software & Services, 2019). Therefore, the slalom manoeuvre simulation can be set up in a manner that enables it to closely represent the slalom tests that were previously conducted with the DVRMP by setting the initial gear and velocity, as well as the upper speed limit, to match the recorded dataset. Ultimately, this allows for further validation of the computed results of multibody software simulations with respect to the slow-speed behaviour and steering dynamics of motorcycles by virtue of comparing them with the processed results of practically obtained measurements.

5. Results and Discussion

5.1. Primary Data

5.1.1. Fundamental Steering Theory

The recorded datasets from the DVRMP and its unique sensory equipment, which allows for a direct measurement of the rider-induced steering torque at the handlebars, offer valuable insights into the dynamic interaction between a motorcycle and the physical steering inputs of its rider. By bridging the gap between man and machine, this makes it possible to analyse their synergy and assess fundamental aspects of motorcycle dynamics theory. An example of the empirically observable behaviour of the motorcycle and a rider is given in **Figure 35**, which shows an excerpt of a dataset that was acquired when journalist Michael Neeves rode the DVRMP at Donington Park Circuit and it reveals several enlightening facts about key aspects of motorcycle handling.

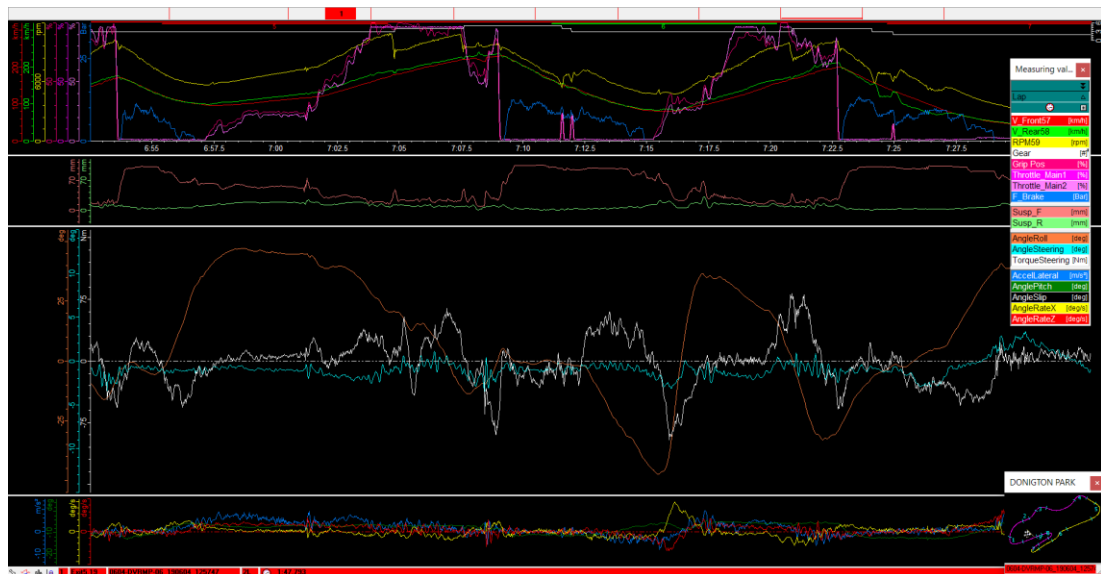


Figure 35 'Analyzer': Screenshot of data from the DVRMP at Donington Park Circuit

First and foremost, the graphs which are plotted in the third and fourth area from the top show that the physical steering torque input of the rider always instigates a roll motion of the motorcycle in the opposite direction to the applied torque. This proves the general theory of counter-steering correct, since the

lateral acceleration and the yaw rate of the motorcycle around its local z-axis both depend on the direction of the roll angle and increase almost linearly with its magnitude, which is shown in **Figure 36** and **Figure 37**. The accumulated data points for these plots represent the same four laps of the DVRMP at Donington Park Circuit.

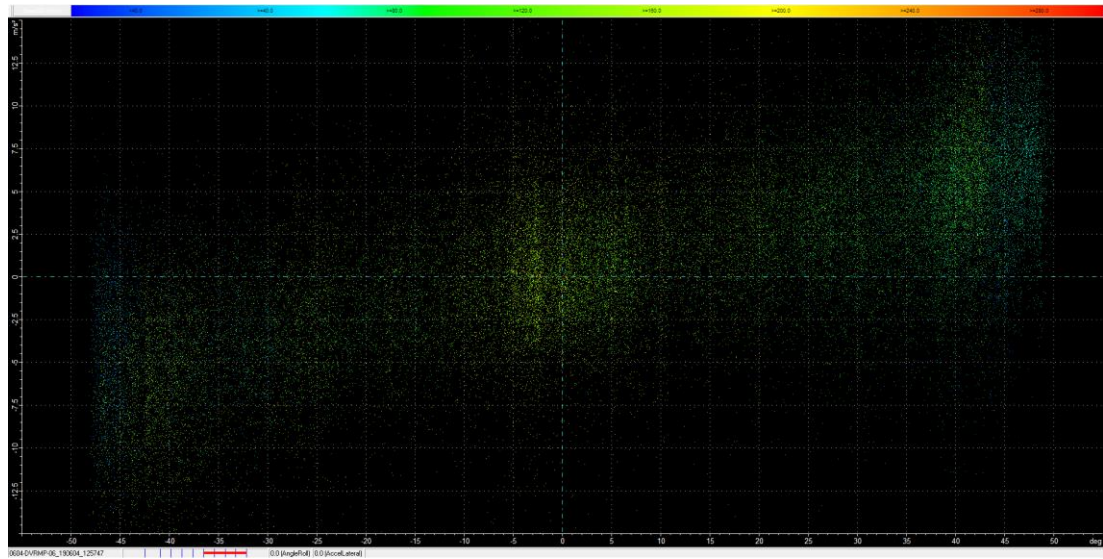


Figure 36 'Analyzer': Screenshot of data for roll angle (x-axis) and lateral acceleration (y-axis) against vehicle speed (colour scale)

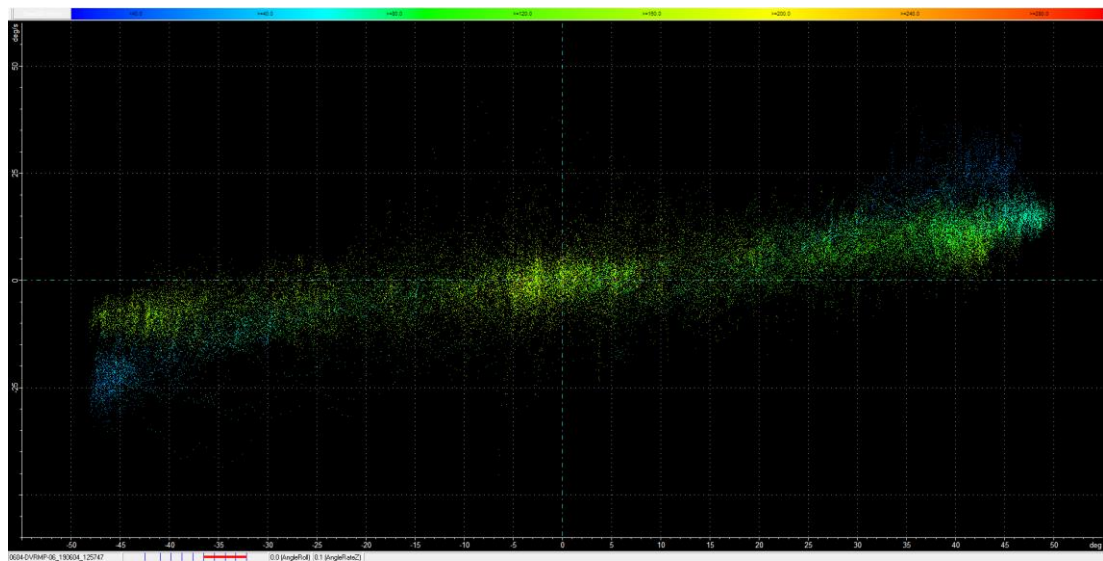


Figure 37 'Analyzer': Screenshot of data for roll angle and local yaw rate against vehicle speed

The observation of counter-steering might come as a surprise to anyone who is yet to ride a motorcycle and one could infer that it must be a highly complex task to maintain a steady turn, let alone manoeuvre the motorcycle in a controlled manner. However, counter-steering is characteristic for most single-track vehicles, including bicycles, that are inherently unstable when stationary and effectively operating their control mechanism eventually only requires a subconscious effort from the rider to whom the correct technique quickly becomes instinctive (Foale, 2002). Furthermore, the underlying mechanism for manoeuvring a single-track vehicle can be understood as follows. Some riders can balance a stationary motorcycle by keeping the connecting line of its contact patches in the road plane underneath the combined centre of gravity through steering or moving their body from side to side, however, the lateral forces and gyroscopic effect that are produced at the wheels of single-track vehicles only enable them to gain a degree of intrinsic stability in the presence of non-zero velocities. As the speed increases, so does a centrifugal effect which acts on the combined centre of gravity and is caused by the lateral forces at the contact patch of the front and rear tyre. This results in the necessity to balance the motorcycle by inducing a roll angle and thus creating a cornering equilibrium that prevents the motorcycle from changing its trajectory or falling over. By way of example, a steering action to the right causes it to start turning to the right, yet the resulting lateral force at the tyres simultaneously creates a centrifugal effect that causes it to start rolling to the left. In addition, the steering action coupled with the angular momentum of the front wheel produces a gyroscopic effect that further creates a precessional tendency of the motorcycle to roll to the left. Initially, the weight force due to gravity also augments the roll towards the left until the rider refrains from applying a distinct steering input and the cornering forces at the tyres can build up to balance the gravitational moment completely. This is achieved when the vehicle has reached a stable cornering condition (Foale, 2002). Therefore, in steady turning the resultant vector of a weight force that pulls the motorcycle downwards and the centrifugal effect that tries to stand the motorcycle back up is fully balanced by a resultant vector of the normal loads and lateral forces that act at the front and rear contact patch of its tyres. The displayed data channels in **Figure 35** corroborate this by also showing that the lateral

acceleration and yaw rate increase significantly once the rider lets out the previously applied torque that initially instigated the roll rate, which subsequently allows the steered front frame assembly to self-adjust and the cornering forces that are necessary for steady turning to build up at the contact patches.

5.1.2. Steady-Turning Performance

In steady turning, the concomitant equilibrium of forces and moments around the steering axis suggests that the individual steering torque components would theoretically cancel each other out altogether (Cossalter, 2006). The screenshot in **Figure 38** displays an enlarged selection of the previously presented dataset excerpt in **Figure 35**. It focusses on the track section from ‘Coppice’ to ‘Starkey’s Straight’ at Donington Park Circuit. The recorded measurements clearly show that the motorcycle essentially reaches a cornering equilibrium mid-turn, while the residual steering torque is indeed very close to zero throughout the period of stable roll angle and throttle application that practically result in a constant velocity and yaw rate of the motorcycle. Furthermore, the data channel for the steering angle in **Figure 38** reveals that it takes on a negative value throughout the whole turn.

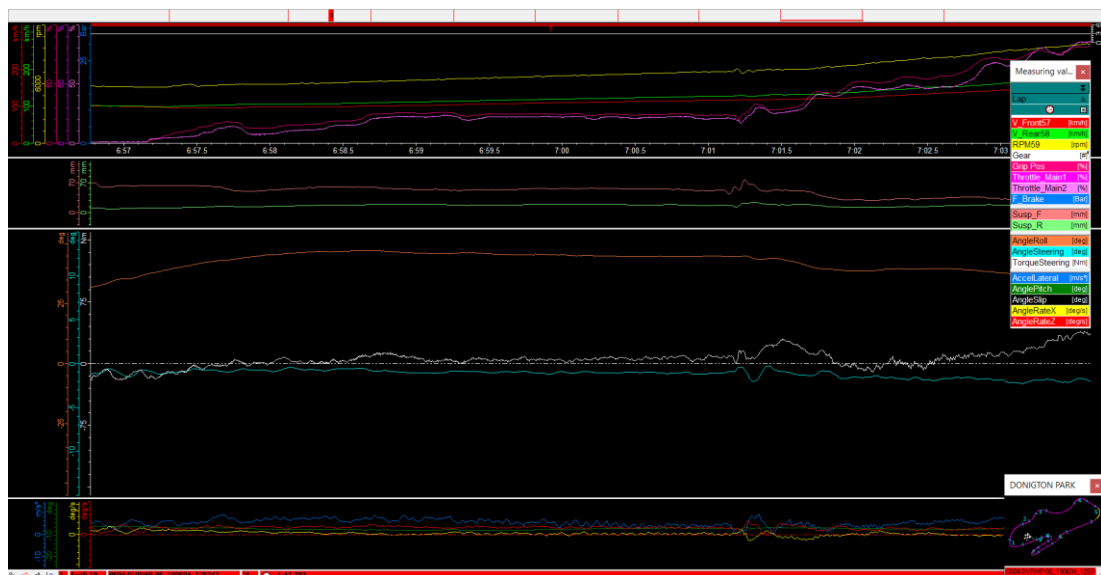


Figure 38 ‘Analyzer’: Screenshot of data from the DVRMP between ‘Coppice’ and ‘Starkey’s Straight’ at Donington Park Circuit

To further investigate this, plots of data for the roll angle and steering angle, as well as for the local yaw rate and steering angle of the DVRMP against its forward velocity from the previously mentioned laps at Donington Park Circuit are shown in **Figure 39** and **Figure 40** respectively.

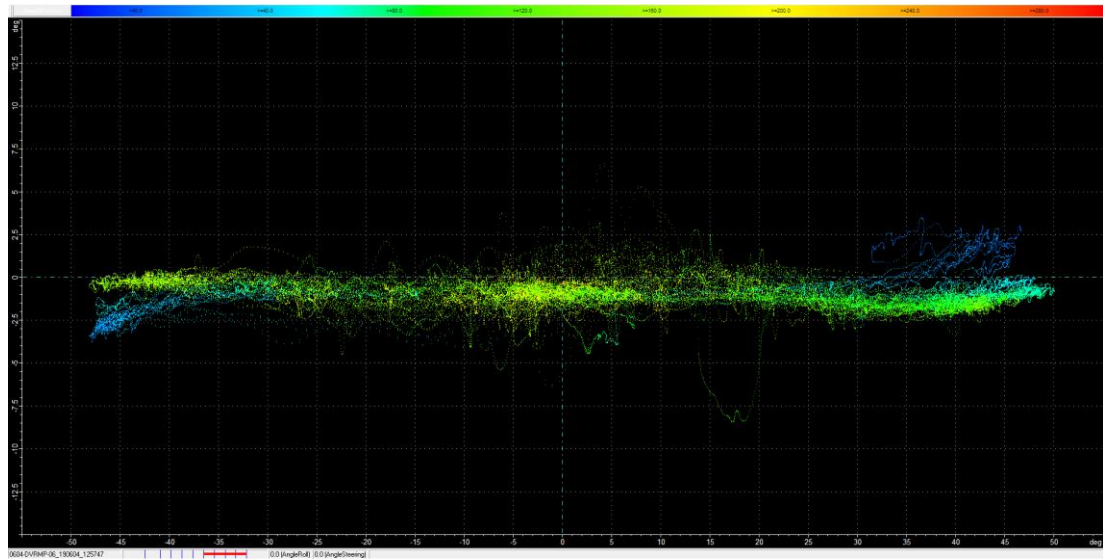


Figure 39 'Analyzer': Screenshot of data for roll angle rate and steering angle against vehicle speed

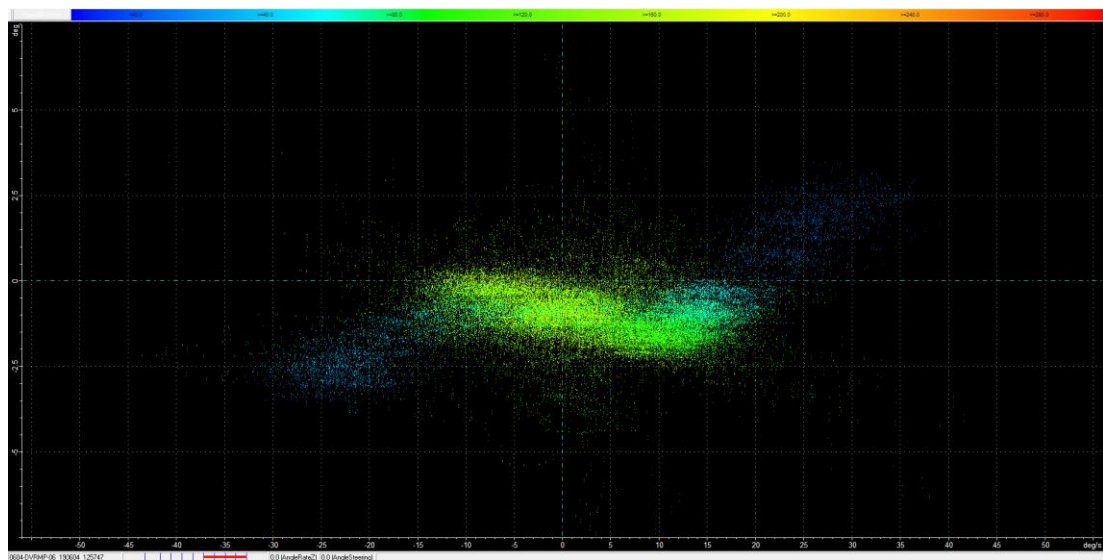


Figure 40 'Analyzer': Screenshot of data for local yaw rate and steering angle against vehicle speed

The two plots reveal that the direction and magnitude of the observed steering angle primarily depend on the speed of the motorcycle and that the steering angle seldom if ever becomes greater than 2.5 degrees over the course of these four flying laps. Moreover, the steering angle is largest at slow speeds, where it is always directed towards the turn and clearly accompanied by a much greater yaw rate than at higher forward velocities. Whilst both the lateral acceleration and yaw rate arise with the roll angle of the motorcycle, the previously presented plot in **Figure 37** also shows that the yaw rate of the motorcycle is inversely proportional to the vehicle speed. Since the occurrence of a higher yaw rate at a lower forward velocity implies that the motorcycle is on a much tighter radius and trajectory around the theoretical turning centre, this shows that its magnitude equally depends on the steering angle. Nevertheless, the observed steering angle eventually changes sign as the speed increases, especially throughout right-hand turns of the circuit. This is typical of a motorcycle with a very strong oversteering tendency that compensates for having a rear sideslip angle which is much greater than the front sideslip angle by counter-steering ($\xi < 0$) after the motorcycle surpasses the critical speed (Cossalter, et al., 2007). According to (Cossalter, et al., 2007), a general tendency to oversteer is highly favourable in comparison to an understeering behaviour during cornering, as the rider would be obliged to increase the roll angle and thereby the steering angle in order to consequently generate more lateral force at the front contact patch. This is considered dangerous, since the extra force that is needed for a cornering equilibrium could potentially lead to the front wheel braking traction with the road, whereby its sudden loss of adherence would inevitably result in an accident due to the rider having ultimately lost control of the motorcycle. The extra force that is needed for a cornering equilibrium with an oversteering motorcycle, on the other hand, could cause the maximum friction force between the rear tyre and the road to be exceeded. This means that the rear wheel slides out instead, yet a skilled rider is expected to have a better chance of controlling the motorcycle and avoiding a fall by applying a counter-steering input. In general, neutral or modest over-steering behaviour is said to be preferred for good handling (Cossalter, 2006). To substantiate this, the recorded measurements of roll angle and steering torque are plotted in **Figure 41**.

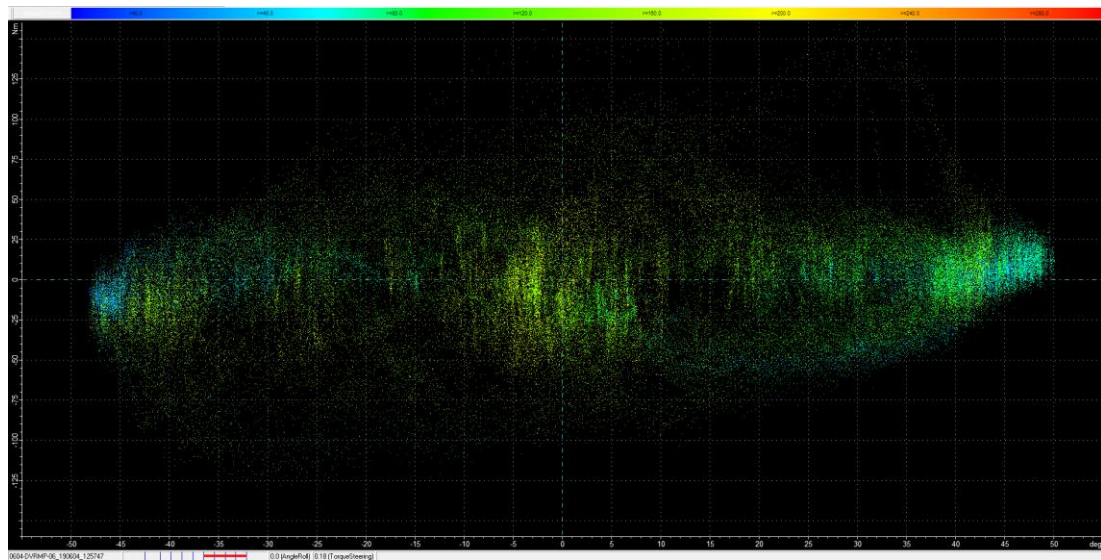


Figure 41 'Analyzer': Screenshot of data for roll angle and steering torque against vehicle speed

The visible concentration of the accumulated data points near the zero datum for steering torque throughout the entire range of the observed roll angle, particularly at the two extremes, proves that the motorcycle almost reaches a cornering equilibrium in every manoeuvre. In addition, the data point concentrations allow for the cornering performance of the DVRMP to be assessed, based on the direction in which a residual steering torque still has to be applied by the rider in order to maintain a constant turn. The plot also suggests that the DVRMP has an oversteering tendency during cornering, since the residual torque is predominantly applied in a positive direction, which means that the rider keeps the motorcycle from tightening the turn by pushing on the outer or pulling on the inner handlebar relative to the position of the turn centre. However, this further demonstrates that the steering angle is actually controlled by the sum of the steering torque components and merely an effect instead of a distinct input by the rider, since it partly points in the opposite direction to the applied torque at the recorded speeds. This implies that a motorcycle has the natural and underlying disposition to achieve a cornering equilibrium regardless of its characteristic steering behaviour. Nonetheless, the observable trend of a predominantly positive steering torque that is depicted in **Figure 41** also indicates that the rider must prevent the steering

angle from increasing any further. It signifies that the motorcycle has predominantly surpassed the transition speed, beyond which the capsize mode returns to an unstable condition and the motorcycle is thus at risk of falling, since the weight force would ultimately gain prevalence over the diminishing centrifugal effect should the rider stop applying a positive steering torque. Datasets from slow-speed manoeuvres or the out- and in-laps at race tracks, including the plot of data for roll angle and steering torque against vehicle speed from the in-lap of the presented exit at Donington Park Circuit which is shown in **Figure 42**, highlight that this residual steering torque is a consequence of the elevated cornering speeds that the DVRMP carries during the flying laps around the circuit, as the capsize stability condition is reversed at slower speeds and the rider has to apply a negative steering torque.

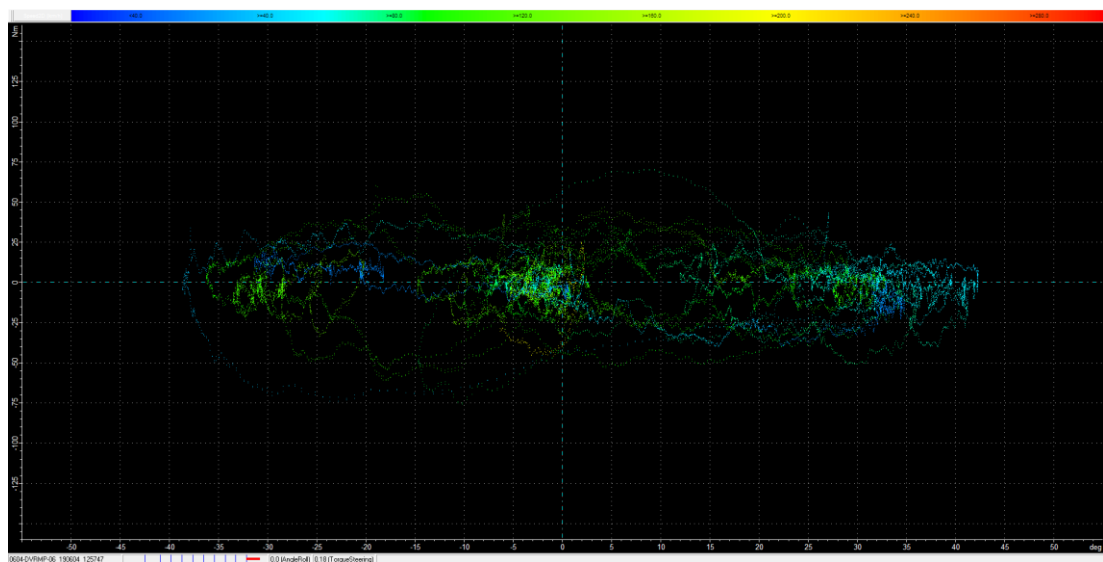


Figure 42 ‘Analyzer’: Screenshot of data for roll angle and steering torque against vehicle speed from an in-lap at Donington Park Circuit

Thus, a preferable driving zone for increased safety lies below the transition speed and is characterised by a negative steering torque and positive steering torque gradient, which means that it is applied in the opposite direction to the turn and that it further approaches zero as the lateral acceleration increases, while the motorcycle exhibits a positive steering angle and oversteering behaviour (Cossalter, et al., 2007).

5.1.3. Transient-State Manoeuvrability

In order to avoid confusion between manoeuvrability and handling, the two attributes can be distinguished as suggested by (Cossalter, 2006). Whilst manoeuvrability is described as the maximum performance that a motorcycle can achieve in terms of its ability to do complex manoeuvres in the shortest time possible, handling quantifies the best performance that a rider can get from the motorcycle when also considering their human performance limits. Therefore, the handling of a motorcycle depends on its manoeuvrability, as well as the physical and psychological effort that is required of the rider to make it perform fast. However, manoeuvrability and handling can similarly be understood as measures of safety, since a different motorcycle and rider could potentially accomplish the same manoeuvres without having to reach their limits and thereby use up all of the possible safety margins. It is concluded that manoeuvrability should thus not be seen as an entirely intrinsic property of the motorcycle and that the overall performance of the motorcycle-rider system has to be assessed instead, which would require a specific rider. Since it is equally clear that some motorcycles are, in fact, better than others, the proposed solution is to employ computational methods and assume an ideal rider that is capable of performing an optimal manoeuvre, which is established through simulations and characterised by both the best trajectory and the necessary rider input to minimise the performance objective for a given motorcycle (Cossalter, 2006). However, the chosen approach for this research project is to extrapolate the dynamics of motorcycle manoeuvrability and handling from the recorded datasets, before a further investigation into the contributing factors is carried out by utilising the steady-state and transient-state steering torque equations of the two mathematical models. As the motorcycle must adopt a certain roll angle to create the lateral forces which control its trajectory and are necessary to reach the cornering equilibrium for a desired turning radius at the given speed, the highest achievable roll rate of a motorcycle could almost be regarded as equivalent to its manoeuvrability, even though tyre adherence and steering behaviour would have an impact on it by controlling the required roll angle, as previously discussed. Therefore, manoeuvrability primarily depends on the resistance of the motorcycle to roll around its local x-axis, which is governed by intrinsic properties after all and

closely linked to its stability. Thus, the lateral dynamics of a motorcycle can be related to the observable behaviour of its steering and vehicle subsystems, which are defined by (Cossalter, 2006) as the mechanism that transforms steering torque into a lateral force that acts on the front wheel and the rigid body of the vehicle on which wheel-to-ground contact forces are exerted, as is illustrated by the diagram in **Figure 43**.

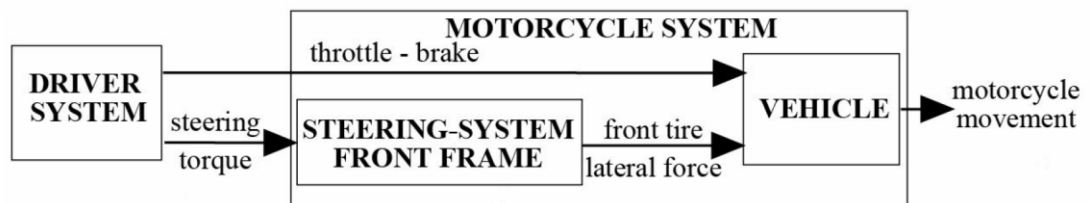


Figure 43 Schematic diagram of the motorcycle subsystems for lateral-directional control (Adaptation of Cossalter, 2006)

Over the course of the same exit from the pits at Donington Park Circuit that the previously presented dataset excerpts originate from, the maximum roll rate of the DVRMP was locally measured by the IMU at 106.7 deg/s through ‘The Esses’, together with a forward velocity of approximately 80 km/h, as shown in **Figure 44**. The data channels reveal that this roll rate appears in conjunction with a changing steering angle and steering torque input that peak slightly in advance at -3.4 deg and -105.3 Nm respectively. This can be explained by the rider trying to augment the magnitude of the steering angle with the applied steering torque and thus causing an additional build-up of lateral force at the contact patch of the front tyre, whereby the resulting increase in centrifugal effect on the centre of gravity of the motorcycle exceeds the balancing effect of the weight force, which ultimately picks up the motorcycle and causes it to return to the stable vertical position and even roll in the opposite direction until a new cornering equilibrium is reached. In addition, a gyroscopic effect of the front wheel translates the resulting steering rate into a precessional torque which is transmitted through the chassis as an overturning moment that also contributes to a roll motion (Cossalter, 2006).

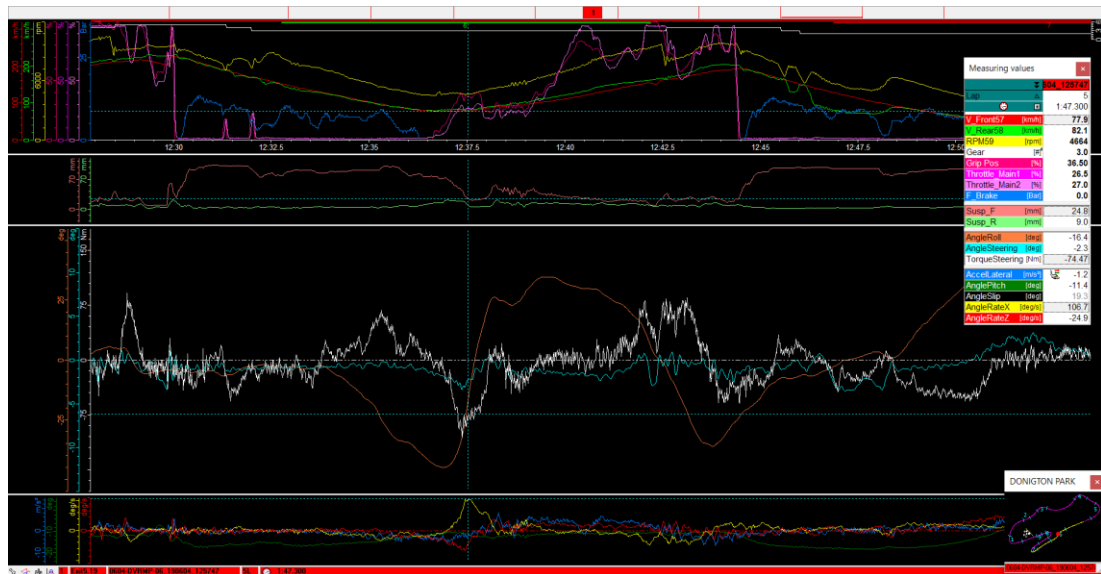


Figure 44 'Analyzer': Screenshot of data from the DVRMP through 'The Esses' at Donington Park Circuit

It is commonly accepted that this induced roll rate gives rise to another gyroscopic effect which is caused by the angular momentum of the front wheel and that balances most of the counter-steering input of the rider by acting in opposition to the applied steering torque (Foale, 2002). Furthermore, this precessional torque initially steers the front wheel into the turn, thereby assisting the rider when entering it, and eventually steers the front wheel out of the turn when the roll rate changes sign, thus helping the rider exit it and return to rectilinear motion (Cossalter, 2006). However, the applied steering torque and this gyroscopic effect due to roll rate deserve further examination, since it is not only the steering rate that could be adding to the roll motion. The phenomenon of gyroscopic effects is explained by (Foale, 2002) as follows. Basic laws of physics dictate that any rotating object is subject to the conservation of momentum, whereby every one of its particles has its own instantaneous linear momentum. When an external torque is applied to the object around any of the two axes that are perpendicular to its spin axis, the original trajectory and instantaneous linear momentum of every rotating particle will be altered in a different direction. This results in the object gaining an angular velocity around a third axis that is perpendicular to the spin axis, as depicted in **Figure 45**.

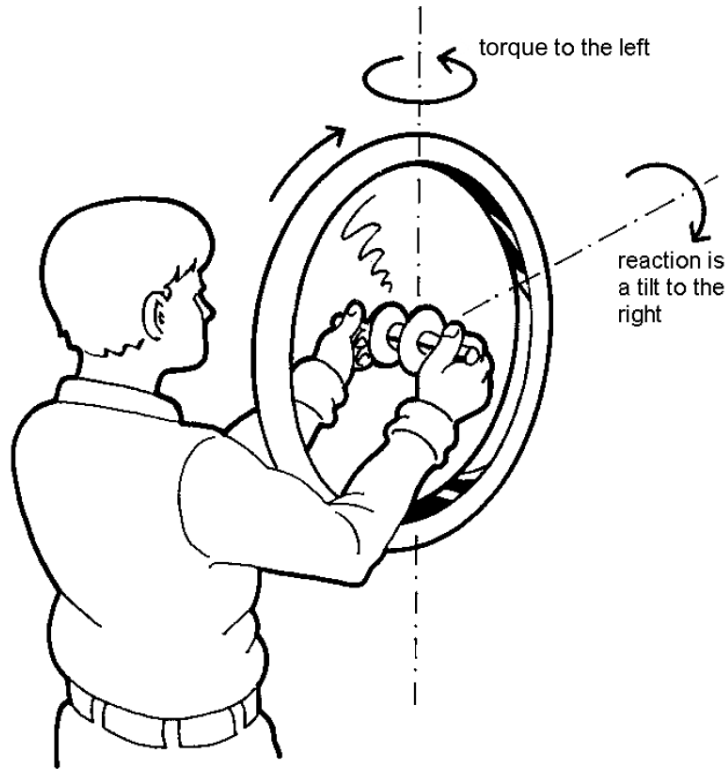


Figure 45 Graphical representation of gyroscopic precession with a bicycle wheel (Adaptation of Foale, 2002)

This effect is called gyroscopic precession and, if a spinning object precesses around one of these axes, it equally applies a gyroscopic torque around the other axis that is perpendicular to its spin axis. The overall momentum of all its particles is considered the angular momentum of a rotating object. The angular momentum depends on the intrinsic inertia of the object around its spin axis, as well as the magnitude and direction of its angular velocity. By definition, the gyroscopic torque is a function of this angular momentum and the angular rate of precession as follows (Foale, 2002).

$$\tau_z = I_y * \omega_y * \omega_x = L_y * \omega_x$$

This implies that the angular momentum of a rotating object controls its resistance to precession and further suggests that the gyroscopic precession of the front wheel from the applied steering torque is a steering mechanism

that directly contributes to the change in attitude. In any case, the observable roll rate of the motorcycle is expected to increase with the applied steering torque. **Figure 46** shows recorded data for roll rate and steering torque against vehicle speed from the previous four laps at Donington Park Circuit.

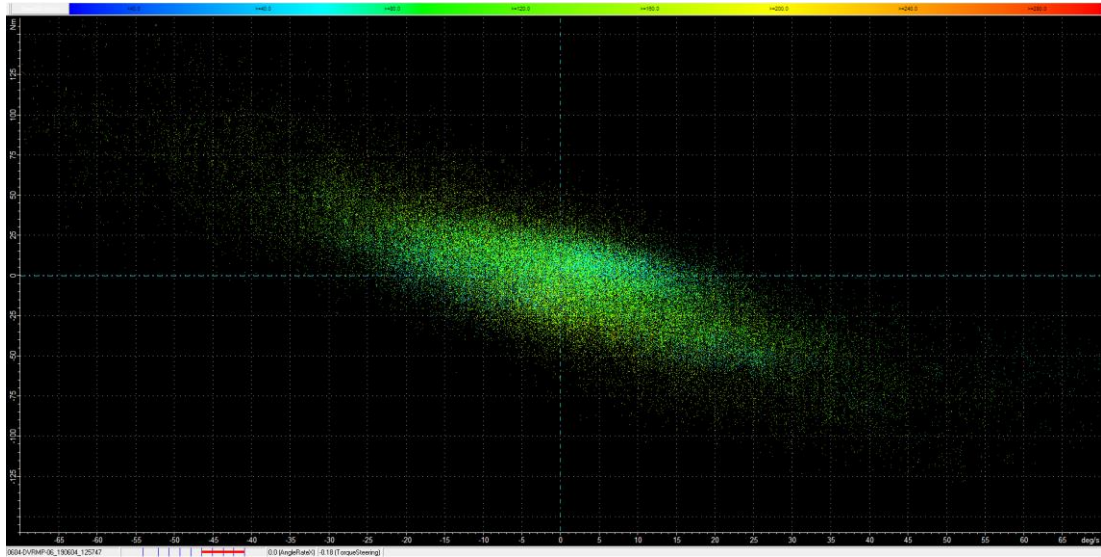


Figure 46 ‘Analyzer’: Screenshot of data for roll rate and steering torque against vehicle speed

The plot proves this assumption correct and further reveals an almost linear relationship between the physical rider input and the transient roll response of the motorcycle. In addition to showing that a stronger steering torque results in a higher roll rate, the plot above indicates that the steering torque that is required to achieve a certain roll rate increases with the forward velocity of the motorcycle. This could be a consequence of the higher angular velocity of the front wheel, which would suggest that the rider is mostly defying the angular momentum of the front wheel and using the gyroscopic effect to manoeuvre the motorcycle. **Figure 47** shows data that was recorded on the DVRMP through the ‘Craner Curves’ at Donington Park Circuit. It reveals that the highest steering torque of the same exit as before was measured at 167.2 Nm with a speed of almost 170 km/h, which results in a peak roll rate of 80.0 deg/s. A comparison of this data against the presented excerpt in **Figure 44** confirms that the required steering torque for a given manoeuvre increases with speed.

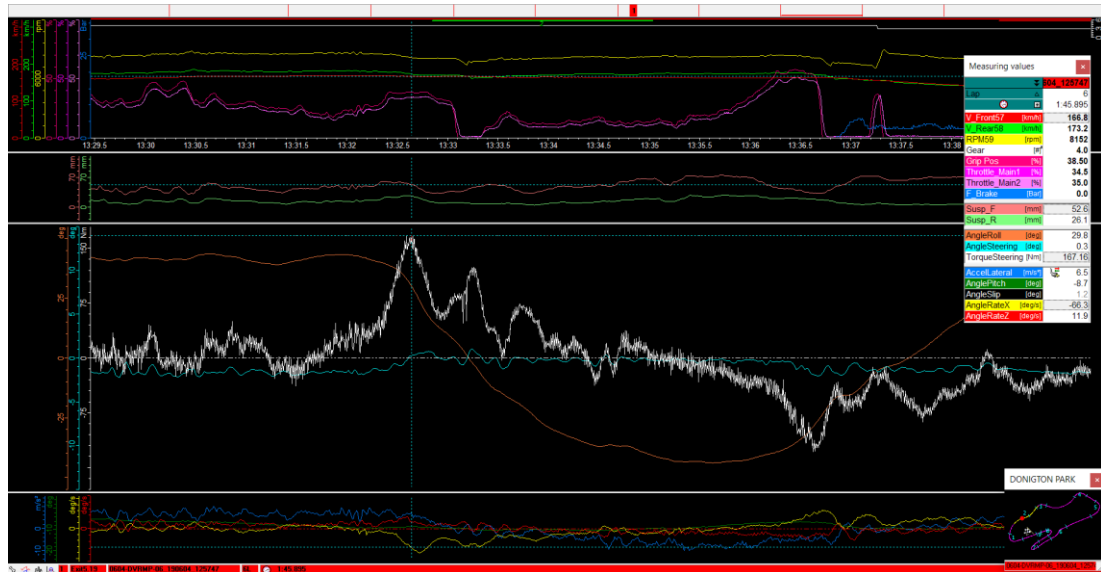


Figure 47 'Analyzer': Screenshot of data from the DVRMP through 'Craner Curves' at Donington Park Circuit

Furthermore, the displayed data channels show that the accompanying change in steering angle gradually diminishes with higher speeds. Together with the plot of data for steering angle and steering torque against vehicle speed that is displayed in **Figure 48**, this supports the theory that the angular momentum of the front wheel governs the handling effort of the rider.

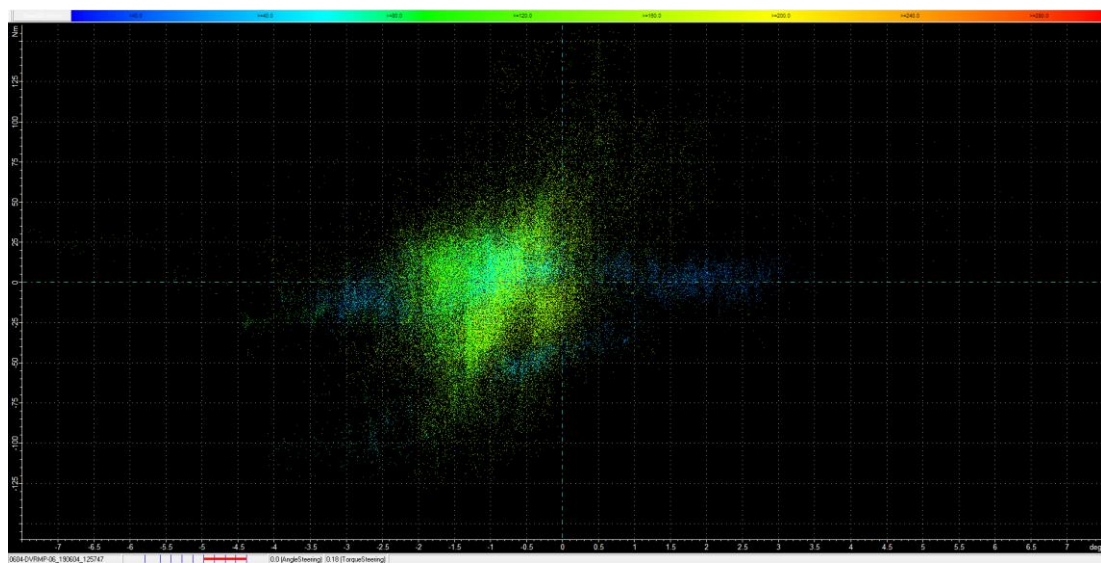


Figure 48 'Analyzer': Screenshot of steering angle and steering torque against vehicle speed

Nevertheless, it is conceivable that the lateral force at the contact patch of the tyre builds up more sharply at elevated speeds and that it equally requires a smaller steering angle to give rise to the centrifugal effect that pushes the motorcycle over. Whilst the lateral tyre force in transient state is believed to increase exponentially with the forward velocity, this is merely a consequence of the relaxation length of the tire that is caused by its elastic deformation. Thus, it describes the delay with which the lateral force builds up to its steady-state value. Therefore, the maximum lateral force that is generated by the tyre remains controlled by the roll angle and sideslip angle that it is subjected to, as well as its intrinsic stiffness properties (Cossalter, 2006). Consequently, a smaller steering angle results in a decreased lateral force and a reduced centrifugal effect. This is further substantiated by the previously gained insights into the relationship between the steering angle and yaw rate, since the latter is inversely proportional to the turning radius and squared in the formula for centrifugal effects. **Figure 49** displays data points for roll rate and steering angle against vehicle speed from the same four laps as before.

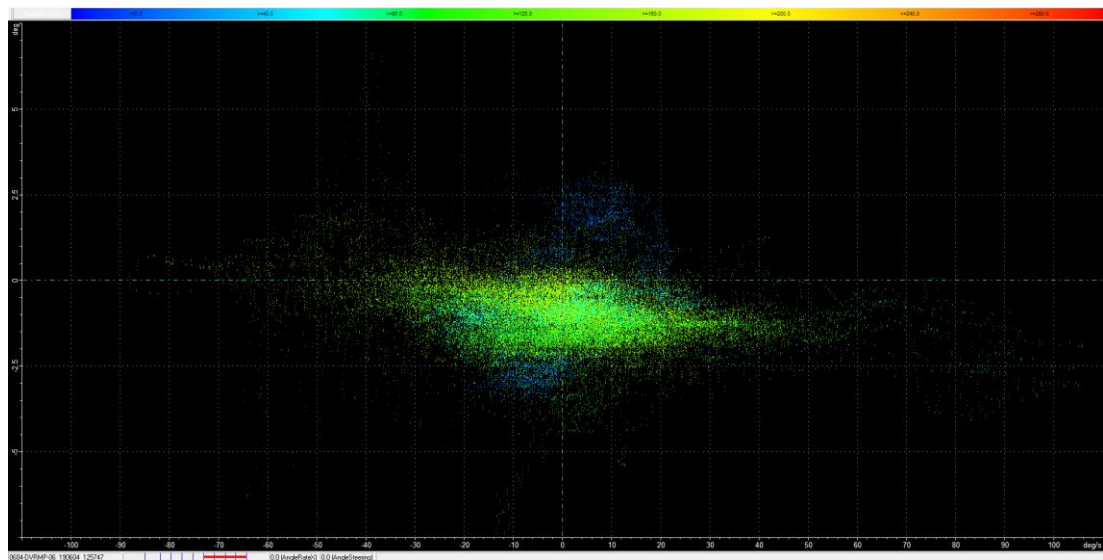


Figure 49 ‘Analyzer’: Screenshot of data for roll rate and steering angle against vehicle speed

This plot shows that instances of extremely high roll rate are concurrent with rather insignificant steering angles. Moreover, it reveals that comparatively

large steering angles, which can be observed at slow speeds, are not equivalent to a high roll rate and that the steered front frame assembly can even point in the direction of the turn during maintained roll motion. This proves that any measurements of steering angle alone are inconclusive and unsuitable for an analysis of the physical steering input at the handlebars. The recording of steering torque, however, clearly allows for the steering action or steering demand and thus the handling or control of a motorcycle by the rider to be assessed. In spite of that, the previous analysis has shown that the steering mechanism is very complex and of a variable nature, which makes it difficult to distinguish to what extent certain regimes of the system are effectively controlled by the steering torque at any given moment in time. On top of that, its determinants are manifold and thus the interaction of the different steering torque components has to be investigated in conjunction with their quantitative relations by using the equations of mathematical models in order to gain a deeper understanding of the governing dynamics behind manoeuvrability and handling.

5.1.4. Rider Control and Handling Techniques

The rider can control a motorcycle by physically applying a force to the handlebars that generates a steering torque, by operating the throttle or the brakes of the motorcycle, which causes it to accelerate, maintain its speed or decelerate, as well as by changing their body position on the motorcycle (Cossalter, 2006). Whilst steering torque can be considered the primary measure for controlling a motorcycle, body movements of the rider always represent a secondary control input, since they only cause small accelerations of the vehicle frame, together with very limited variations in its moments of inertia and the combined centre-of-gravity position, especially when the ratio of vehicle mass to rider mass is high. Furthermore, the frequency range of handlebar control is significantly higher than that of body control, which means that the rider can alter the applied steering torque much more quickly than they can shift their body on the seat (Cossalter, et al., 2007). Nevertheless, the rider undeniably has the ability to transfer their body weight towards the turn centre to some extent, in order to reduce the centrifugal effect on the motorcycle and thus the roll angle that is required to ensure the cornering equilibrium for a

certain turning radius at the given velocity. In addition, the displacement of the upper body and the leg of the rider creates an aerodynamic yawing moment that facilitates entering and rounding a turn (Cossalter, 2006). Ultimately, this technique enables the rider to achieve higher cornering speeds with any given roll angle of the motorcycle and, hence, an increased maximum velocity through any given turn. Conversely, since the same cornering speeds can be achieved with a lessened roll angle, it increases the safety margin for the rider by allowing for further adjustment and tightening of the trajectory if necessary. Velocity changes with regards to the longitudinal forces that arise at the front or rear contact patch during acceleration and braking also control the steering input that is required of the rider due to a coupling between in-plane and out-of-plane dynamics that strongly affects the behaviour of a motorcycle. Unless the motorcycle is perfectly straight, the braking force that is generated between the front tyre and the road surface by the application of the front brake creates a misaligning torque around the steering axis which causes the motorcycle to reduce its roll angle if the rider does not adapt by applying a resistive steering torque that cancels out this effect (Cossalter, et al., 2011). **Figure 50** displays an enlarged selection of the previously presented dataset excerpt in **Figure 35** and it covers the section from ‘Starkey’s Straight’ to the ‘Melbourne Hairpin’.

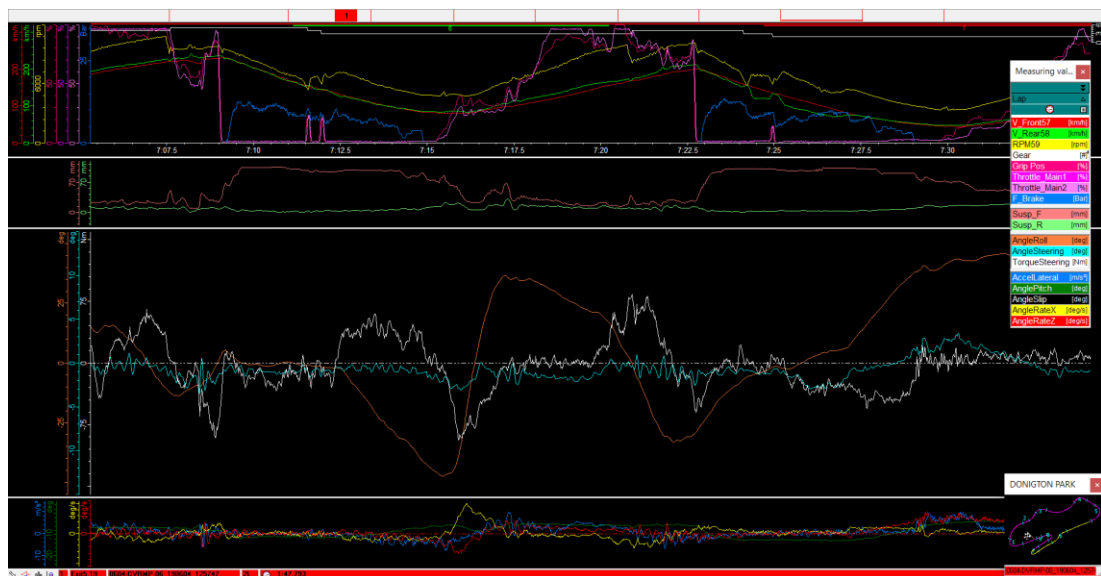


Figure 50 ‘Analyzer’: Screenshot of data from the DVRMP between ‘Starkey’s Straight’ and ‘Melbourne Hairpin’ at Donington Park Circuit

Between minute 7:12.5 and 7:15, the data channels that are plotted in the third area and the recorded measurements of the applied front brake pressure prove that applying the front brakes has a misaligning influence on the front wheel, as it opposes the counter-steering ambition of the rider, which necessitates the need for a higher steering torque input in order to maintain a certain roll rate. However, between minute 7:22.5 and 7:25, the dataset excerpt also shows that the braking force can be of great assistance to the rider in standing the motorcycle back up, as it clearly adds to any steering torque that is applied in a positive direction. From minute 7:25 onwards, the dataset excerpt that is presented above reveals yet another way in which braking can essentially become a steering input. If the motorcycle is not perfectly straight, and especially when a sudden application of the front brakes causes the weight on the rear wheel to diminish toward zero due to the resulting load transfer, the braking force at the front contact patch and the inertial force of deceleration, which acts at the centre of gravity, generate a moment that tends to cause the motorcycle to yaw, as depicted in **Figure 51** (Cossalter, 2006).

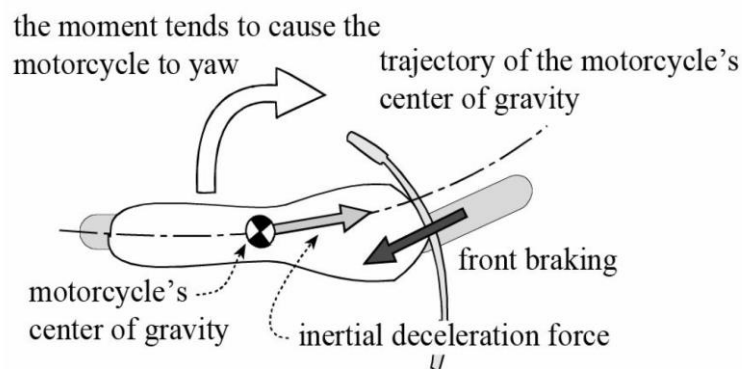


Figure 51 Graphical representation of the yawing moment that is induced by the front brakes (Cossalter, 2006)

This extreme load transfer is displayed in **Figure 50**, with the graph of the rear wheel speed dropping below the graph of the front wheel speed and the rear suspension travel reaching a static minimum or zero value during the braking phase for the 'Melbourne Hairpin'. The dataset excerpt implies that the shock

absorber is fully extended and the rear wheel loses contact with the ground, since the latter is slowed down more than the front wheel as a consequence of the negative torque from the engine. Moreover, the steering angle drastically becomes increasingly negative despite the applied brake pressure. This suggests that the rear frame of the motorcycle is stepping out and yawing in a positive direction by rotating around the steering joint. At minute 7:27, the rear wheel starts to regain traction on the ground, which is depicted by the measurements of the rear wheel speed converging with those of the front wheel speed and the graph of the recorded engine speed momentarily picking up in amplitude, since the grip of the rear tyre transfers a back torque to the engine as it recovers speed. Simultaneously, the steering angle begins to adjust despite the continuously applied steering torque and a subsequent steering torque amplification seems necessary to oppose the aforementioned misaligning torque that is induced by the front braking force. The increase in steering torque appears to bring about a sharp rise in roll rate, which allows the cornering forces to build up and the motorcycle to finally gain a substantial yaw velocity. Whilst the displayed graphs show that the point of rear wheel lift-off from the ground ultimately limits the amount of front braking that can be utilised, the positive rotation of the chassis around the front frame places the motorcycle differently on the road and this implies that it has already accomplished some part of the yaw motion that is required to make the turn. When comparing the two previously discussed turn entries of the DVRMP into 'The Esses' and 'Melbourne Hairpin', it becomes apparent that the gradient of the achieved deceleration of the motorcycle is very similar in both examples, even though the varyingly applied front brake pressure is vastly different for the two distinct braking techniques. Furthermore, the second approach seems to require less physical effort, since the magnitude of the recorded steering torque is predominantly lower, however, the psychological effort that could be required to accurately modulate the brake pressure and control the yawing moment may be higher. An additional measure to balance this effect is the application of the rear brake. Within the available grip limit, a rear braking force can be used to generate a moment that tends to align the motorcycle, as it works in the opposite direction to the yawing moment that is generated by the front braking force, as depicted in **Figure 52**.

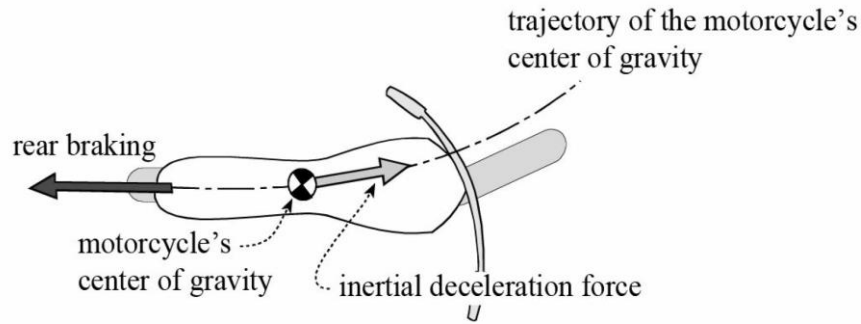


Figure 52 Graphical representation of the straightening moment that is induced by the rear brake (Cossalter, 2006)

Thus, it can be seen that a proper utilisation of the front and rear brake has a stabilising effect on the motorcycle (Cossalter, 2006). This essentially means that longitudinal accelerations can be considered further steering control inputs. In theory, when a motorcycle travels at a constant speed through a turn, it follows a constant circular path unless the rider makes an adjustment to the attitude of the motorcycle. Applying a positive steering torque decreases the roll angle and widens the path, whereas applying a negative steering torque increases the roll angle which tightens the path. Depending on its velocity, the motorcycle may, however, follow the same path despite carrying a different roll angle, which, therefore, means that the path can also be adjusted by accelerating or decelerating the motorcycle. Whilst the previously presented dataset excerpt has shown that decelerating by applying the front brakes can create a misaligning torque that must be physically resisted by the rider to avoid a sudden change in roll angle, applying the rear brake does not directly generate an equivalent torque component around the steering axis, since the braking force acts at the rear contact patch. The rear braking action may add to the overall steering torque by generating a moment in the plane of the motorcycle that causes an increase in the normal force on the front contact patch, which is similar to the effect of load transfer, yet any extra steering torque from the additional normal force is most likely cancelled out by the arising steering torque from the accompanying increase in lateral force at the front contact patch, as is the case with the front brake application (Cossalter, et al., 2011). Therefore, the path of the motorcycle through any given turn can

also be tightened by applying the rear brake, however, without the need to counteract an increasing steering torque, which equally means that the front frame assembly is allowed to self-adjust and restore a cornering equilibrium. On the contrary, gradually applying the throttle widens the path of a motorcycle by increasing its velocity. The steering torque is not significantly affected under acceleration due to the fact that the driving force is generated at the contact patch of the rear tyre and the load transfer reversed, which means that the coupling between in-plane and out-of-plane dynamics is significantly weaker than during braking and the change in roll angle is less noticeable by the rider (Cossalter, et al., 2011). Nonetheless, the reduction in load on the front wheel means that the main steering control mechanism is impeded, since the front wheel easily lifts off the ground, which makes all of its cornering forces disappear and merely leaves the gyroscopic effect without wheel-to-road interaction and the shift of body weight as means of controlling the attitude of the motorcycle. **Figure 53** shows an excerpt of a dataset that was acquired when road racer Samuel Mousley rode the DVRMP at Pembrey Circuit.

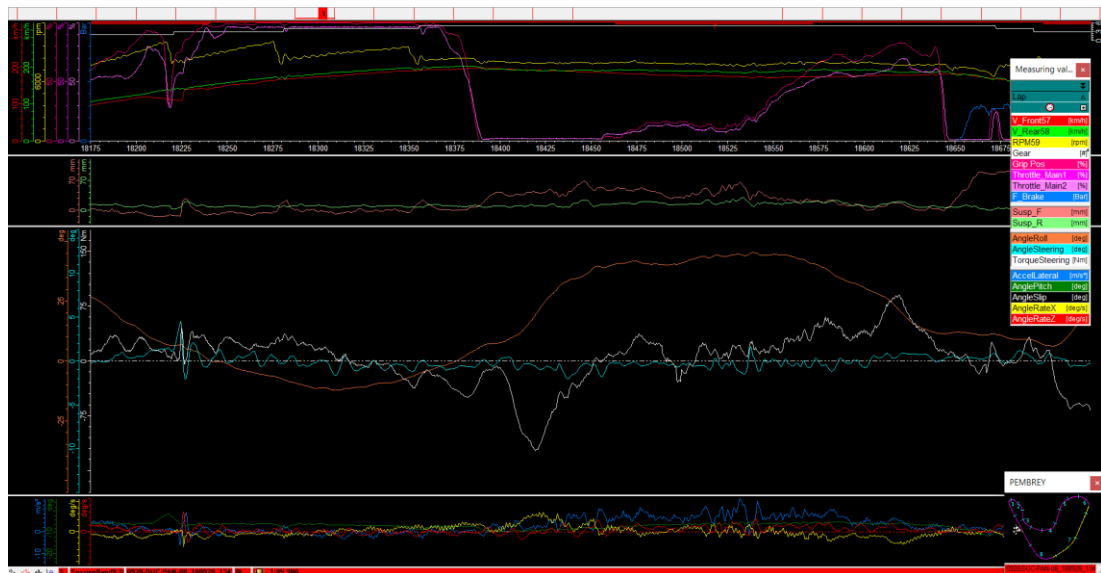


Figure 53 'Analyzer': Screenshot of data from the DVRMP with Sam between 'Brooklands Hairpin' and 'Honda' at Pembrey Circuit

It focusses on the track section from the 'Brooklands Hairpin' through to 'Honda' and the recorded measurements of the front suspension travel clearly

show the reduced load on the front during the exit from 'Brooklands Hairpin' and onto the 'Speedway Straight', as well as the front wheel losing contact with the road at metre 18205, which is further depicted by the graphs of the front and rear wheel speed. Since the front wheel is not driven by the engine, its speed drops below that of the rear wheel speed while it spins freely in the air. Moreover, the displayed data plot reveals that the throttle can also be used as a steering input that allows to increase the yaw angle, despite the generated driving force during acceleration generally widening the path of the motorcycle by raising the forward velocity and reducing the roll angle. This technique is used by Sam and **Figure 53** explicitly shows how it is applied, yet even more so when comparing it to **Figure 54**, which displays a dataset excerpt that was also acquired at Pembrey Circuit and that focusses on the same track section, however, with lecturer Andrew Harrison riding the DVRMP.

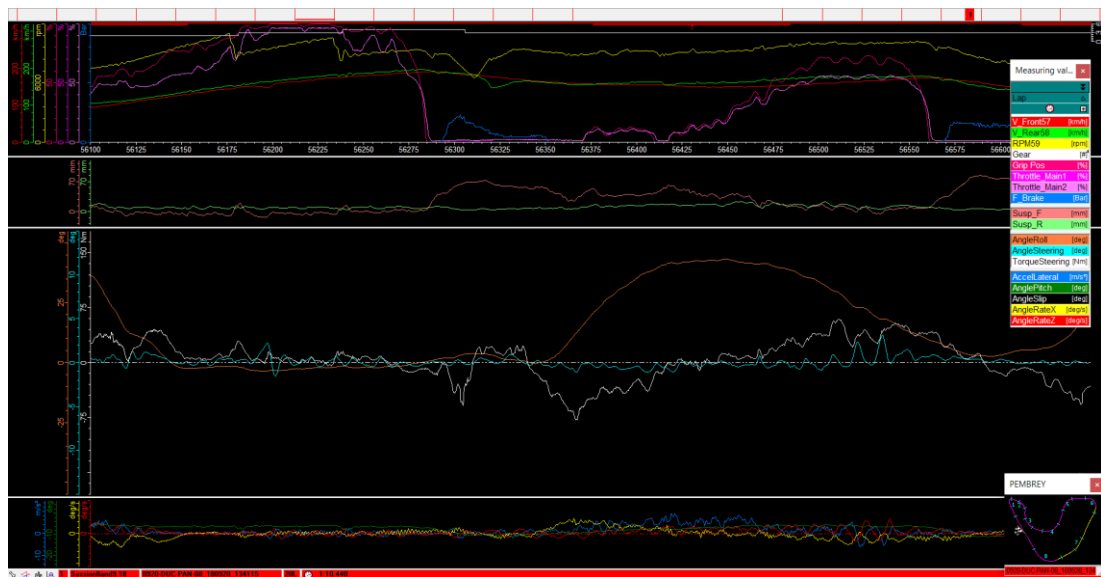


Figure 54 Analyzer': Screenshot of data from the DVRMP with Andy between 'Brooklands Hairpin' and 'Honda' at Pembrey Circuit

At the centre of the two data plots lies turn number 7, which is called 'Woodlands' and it basically resembles a kink in the track that leads off from the 'Speedway Straight' onto 'Honda', as shown by footage that is presented in **Figure 55**.



Figure 55 ‘Analyzer’: Screenshot of footage from the DVRMP with Sam riding through ‘Woodlands’ at Pembrey Circuit

Therefore, ‘Woodlands’ is taken at a relatively high velocity, which can be seen in both **Figure 53** and **Figure 54**. In contrast to Andy, Sam carries significantly more speed into the turn as he applies a very distinct and pronounced steering torque input on the approach, which causes the DVRMP to adopt the required attitude and position more quickly than from the prolonged and lessened input by Andy. Subsequently, Sam opens the throttle at metre 18455 while exhibiting a much larger roll angle. The displayed measurements for the wheel speeds show that the rear wheel starts to spin up considerably more than the front wheel, which results in the motorcycle being subjected to a higher yaw rate. This can be explained by the longitudinal driving force at the rear contact patch increasing the thrust as well as the sideslip of the rear tyre to the point where the total friction force approaches its limit value. Thus, the rear frame of the motorcycle moves outwards due to the tyre losing traction and grip on the road. Whilst essentially generating additional oversteer that ultimately allows for higher cornering speeds, the technique of steering with the throttle also bears the risk of causing a dangerous dynamic phenomenon that is called a highside. Especially after the limit value of the friction force is surpassed and the rear

tyre loses all of the remaining grip at once, if the rider suddenly stops accelerating to prevent the rear wheel from sliding out even more, it can regain traction very abruptly and the large sideslip angle that is still present generates a lateral force impulse which is not fully balanced by the centrifugal force, as depicted in **Figure 56**. In the worst case, this may result in the motorcycle being violently twisted and thrown upwards (Cossalter, 2006).

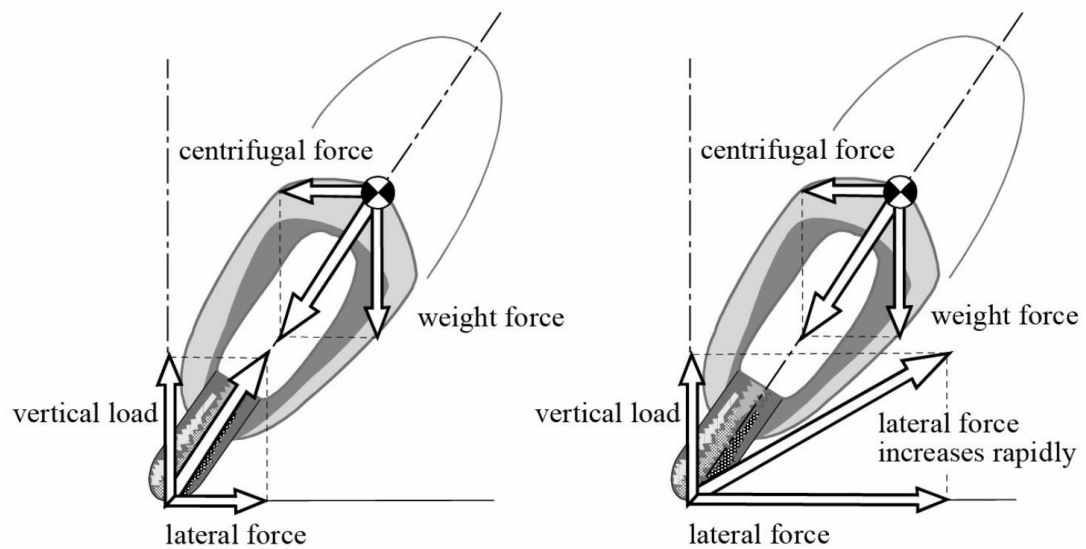


Figure 56 Graphical representation of the cornering equilibrium and the imbalance of forces that leads to a highside (Cossalter, 2006)

5.2. Processed Results

5.2.1. Steering Torque Components of the Steady-State Model

In steady-state cornering conditions, the torque that is applied by the rider is equal to the sum of the influences of mass forces which act at the mass centre of the front frame, reaction forces that arise at the front contact patch, the gyroscopic effect of the front wheel due to yaw rate and the twisting torque on the front tyre, if its rolling resistance and any longitudinal forces are neglected, as is shown in **Figure 57** (Cossalter, 2006).

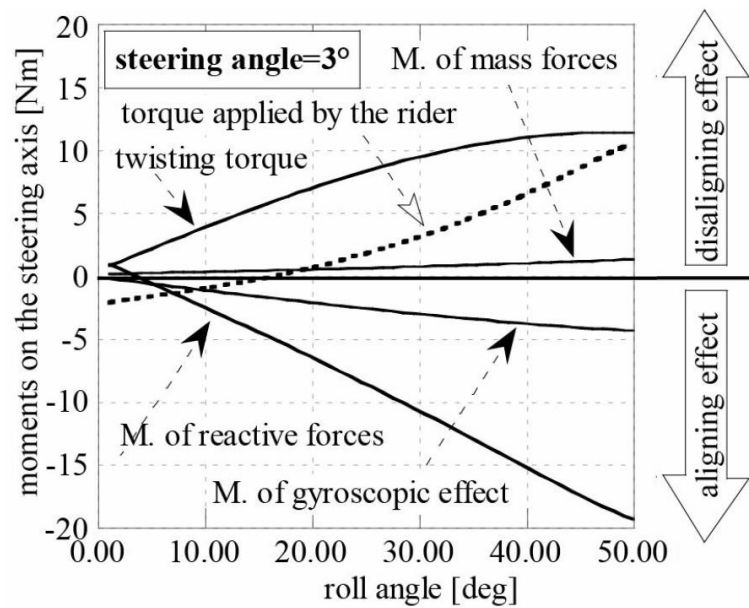


Figure 57 Steering torque that is applied by the rider in steady-state cornering conditions (Cossalter, 2006)

By indicating how the applied torque changes with the variation in roll angle, the dotted graph suggests that the steering torque is negative at slow speeds and that it becomes positive as the velocity increases, which is discussed in the previous chapter and confirmed by the plots of the recorded data for roll angle and steering torque against vehicle speed (see **Figure 41** and **Figure 42**). The maximum manoeuvrability is said to be achieved when the torque that is necessary for ensuring the equilibrium of the rear frame and front frame of the motorcycle in steady turning is zero or very close to it. However, it seems

more plausible that this represents ideal handling conditions (Cossalter, 2006). In addition, it can be seen that the total steering torque is, in fact, equal to the sum of the aforementioned moments when their signs are changed. Moreover, both the total moment of the mass forces which comprise the weight force and the centrifugal force that are associated with the front frame, as well as the total moment of the reactive forces which entail the normal force and lateral force that arise at the front contact patch, depend on moments of opposite sign, as shown in **Figure 58** (Cossalter, 2006).

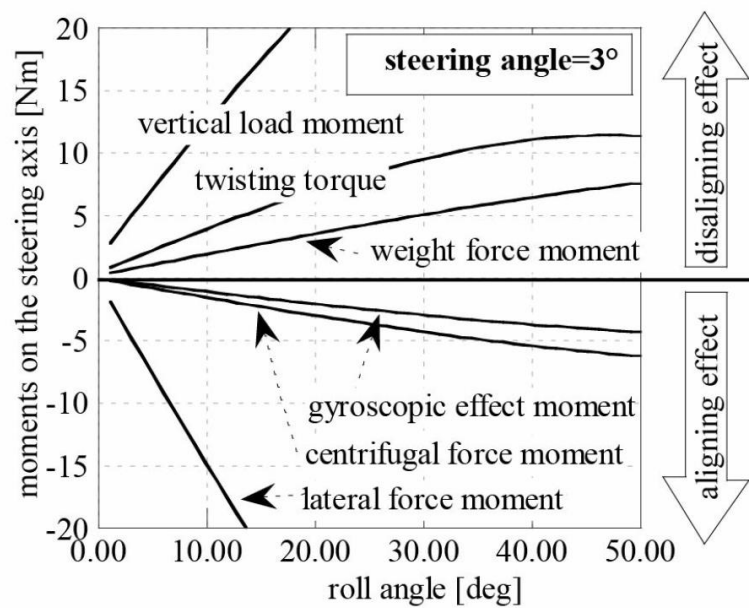


Figure 58 Moments that are exercised around the steering axis in steady-state cornering conditions (Cossalter, 2006)

The two previously presented plots can be used to validate the results of the project code by comparing the displayed graphs for the magnitude of each moment to the calculated values of the equivalent steering torque components that were computed with the steady-state equations from (Cossalter, et al., 1999) and (Cossalter, 2006). For this purpose, the resulting steering torque components of the two approaches that are based on determining the turning radius from either kinematic variables or from processed IMU-measurements for global yaw rate are disclosed in **Figure 59** and **Figure 60** respectively.

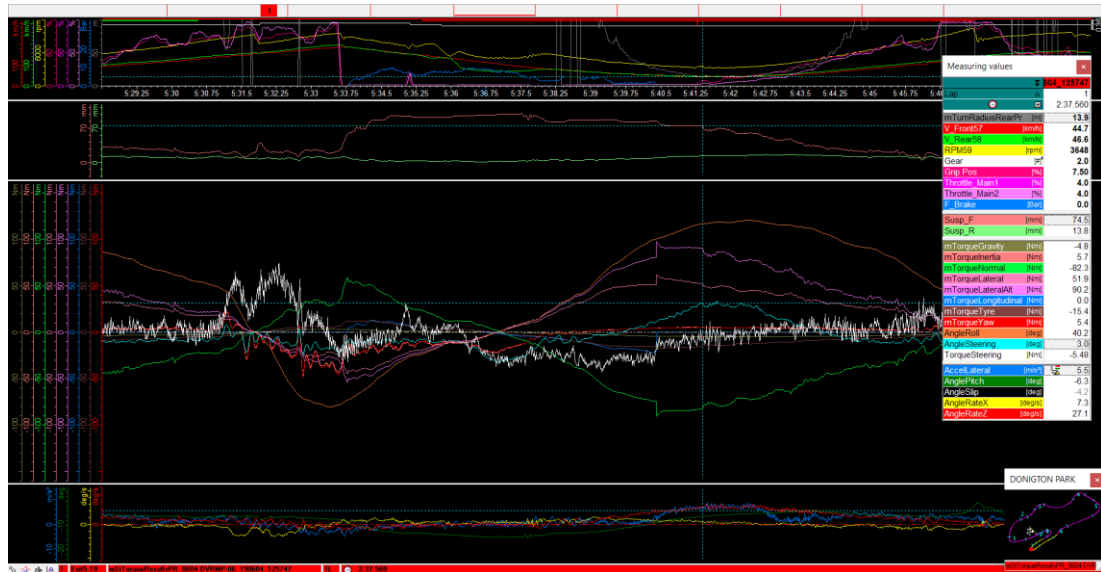


Figure 59 'Analyzer': Screenshot of data with computed results for steering torque components with the turning radius based on kinematic variables

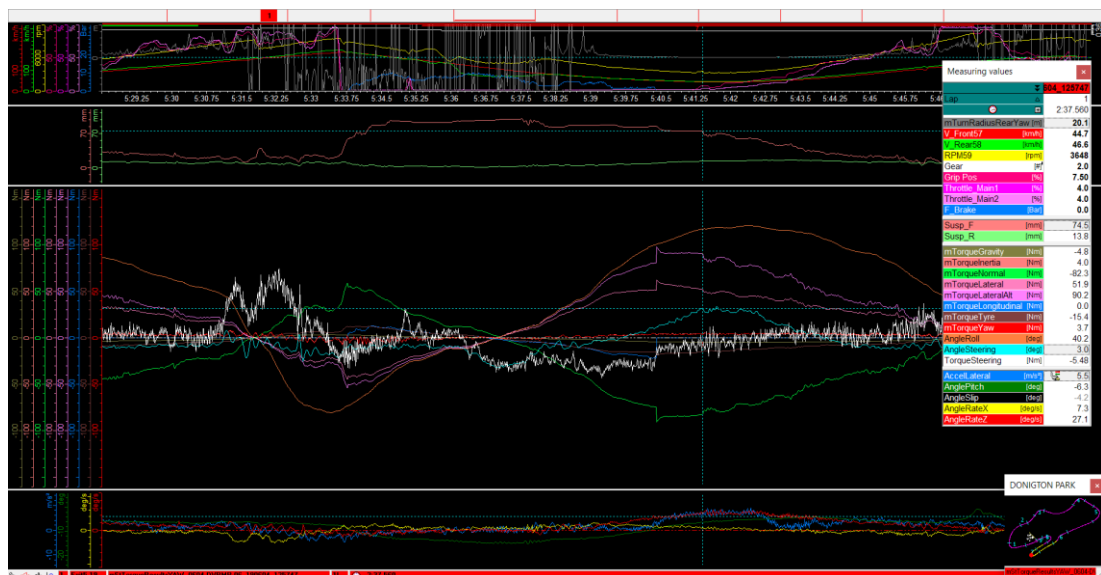


Figure 60 'Analyzer': Screenshot of data with computed results for steering torque components with the turning radius based on IMU-measurements

For ease of comparison with the previously presented plots, the marker is placed at a point where the motorcycle exhibits a steering angle of 3 deg and a roll angle of 40 deg. Whilst **Figure 59** shows that the kinematic turning radius can cause an error in the inertial torque and the gyroscopic torque due to yaw

rate for negative roll angles, **Figure 60** proves that all IMU-based results are free from this error and that every component behaves as expected, except for the lateral one. The observable discrepancy between the lateral torque component and the normal torque component could be related to the method for calculating the lateral force and the resulting lateral torque component within the original mathematical model by (Cossalter, et al., 1999). Therefore, an alternative lateral torque component is calculated with the same lateral force by using a different formula for the normal trail when cornering, which is referenced from (Cossalter, 2006) and presented in the methodology section of this report, since it is this perpendicular distance from the front contact patch to the steering axis that acts as the torque arm with which practically all of the lateral force generates a moment on the front frame, as depicted in **Figure 61**.

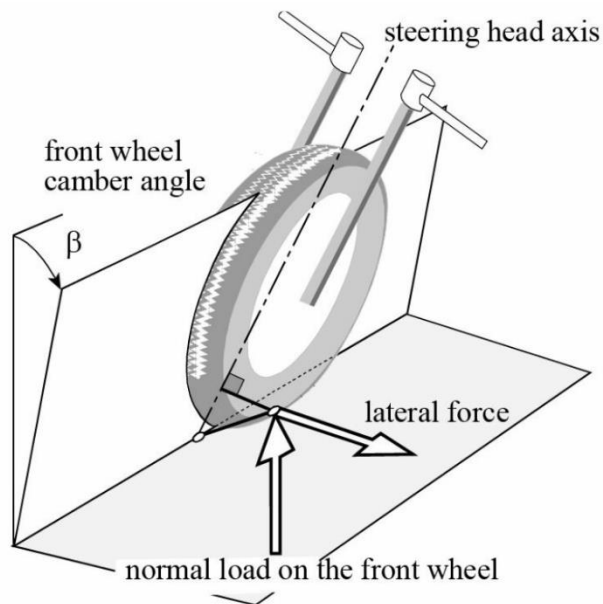


Figure 61 Graphical representation of the lateral force and normal load that generate a moment around the steering axis (Cossalter, 2006)

The graphs of the alternative steering torque component due to the lateral force, which are also plotted in **Figure 59** and **Figure 60**, indicate an improved correlation with the trend in **Figure 57**, as the total moment of the reactive forces is subject to an increasingly more overbalancing lateral force moment.

Hence, the results are used to compute a corrected total steering torque, which is plotted in **Figure 62**, alongside the total steering torque of the original torque components with the project code that utilises the turning radius which is based on IMU-measurements.

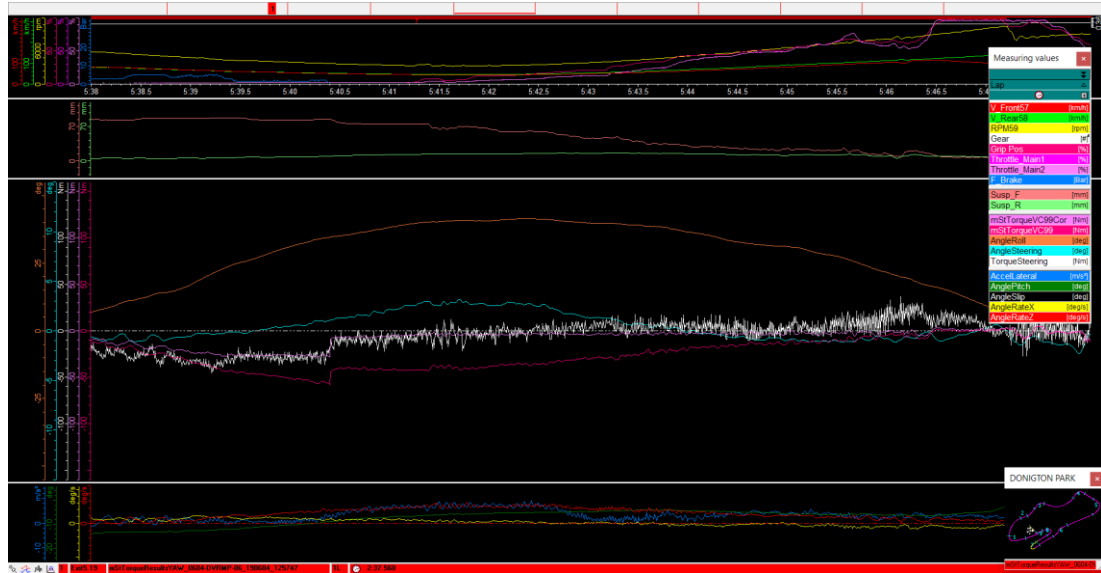


Figure 62 ‘Analyzer’: Screenshot of data with computed results for the total corrected and total original steering torque

The results from both steering torque computations that are plotted above also include the torque component which is induced by the longitudinal force at the front contact patch due to the application of the front brakes. Compared with the original steering torque, it can be seen that the corrected steering torque matches the measured steering torque considerably more closely throughout the whole dataset excerpt, which is an enlarged selection of the previously displayed data from the DVRMP at the ‘Melbourne Hairpin’ of Donington Park Circuit. Whilst the difference between the two results is only marginal at first, it becomes gradually more marked as the roll angle increases and it diminishes with the reduction in roll angle towards the exit of the turn. This is caused by the previously addressed shortcoming in the magnitude of the lateral torque component, which creates a significant imbalance between the opposing torque components due to the lateral and normal force at the front contact patch that offsets the total steering torque during steady turning.

The exceptionally good correlation of the corrected steering torque with the measured steering torque, however, reveals that the individual steering torque components cancel each other out almost entirely during steady turning. Thus, it appears that applying steady-state equations to a transient dataset and treating each data point as representative of a quasi-static scenario is a legitimate and effective method to calculate the actual steering torque input of the rider. To further verify this, the computed results for the total corrected steering torque are plotted in **Figure 63**, along with the recorded data from the DVRMP for a comprehensive track section at Donington Park Circuit that ranges from ‘McLean’s’ to the ‘Melbourne Hairpin’.

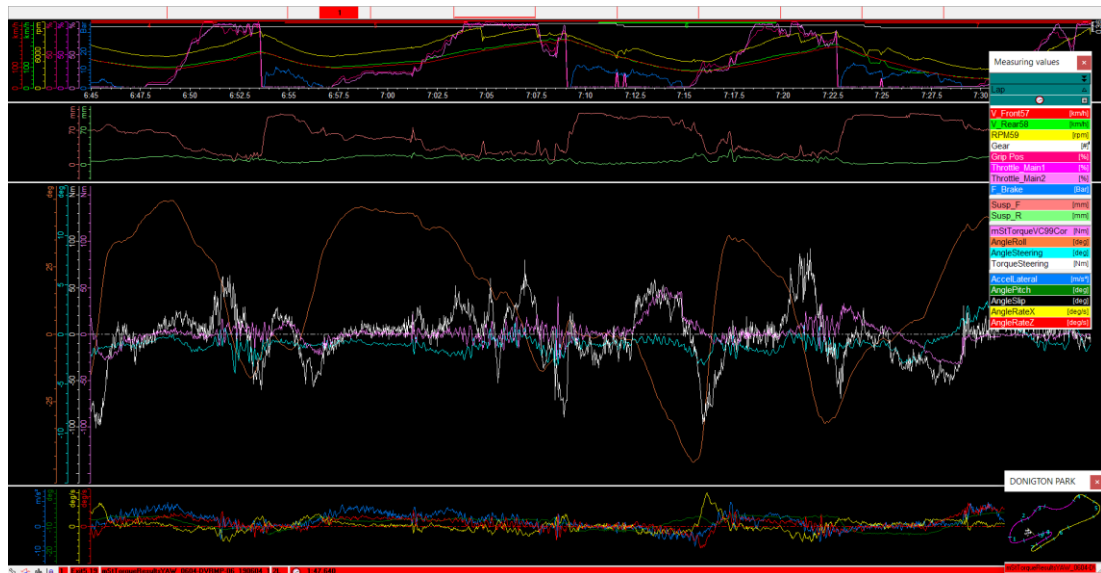


Figure 63 ‘Analyzer’: Screenshot of data with computed results for the total corrected and total alternative steering torque

It becomes evident that the corrected steering torque solely matches the measured steering torque during cornering equilibria, where it is very close to zero. In fact, it only deviates notably from the zero datum under the application of the front brakes, which is due to the longitudinal torque component that arises with the front braking force. The previously presented plots in **Figure 59** and **Figure 60** prove that all of the steering torque components continuously vary in magnitude and change sign with the direction of the roll angle. This implies that these diverse components with aligning and disaligning influence

on the front frame practically cancel out altogether and essentially balance at all times. The only exception to this seems to be the longitudinal torque component, once the braking force at the contact patch of the front tyre is accounted for. Thus, the steering torque components that are computed with the steady-state equations fail to sufficiently describe the physical rider input at the handlebars which controls the dynamic attitude and trajectory of the motorcycle. Therefore, a gyroscopic torque component due to roll rate, which is not included in the computation of the total original or corrected steering torque, is derived and added to the steady-state equations for the calculation of a total alternative steering torque. As a consequence, this total alternative steering torque becomes more representative of the transient state, since it involves the influence of the gyroscopic effect due to a changing roll angle. **Figure 64** shows the same dataset excerpt as **Figure 63**, yet with the computed results for the total alternative steering torque, as well as its gyroscopic torque components that are induced by the gyroscopic effects of the front wheel due to the yaw rate and the roll rate of the motorcycle. These two torque components are of opposite sign and are added together to form a combined gyroscopic steering torque component, which is displayed in the same plot.

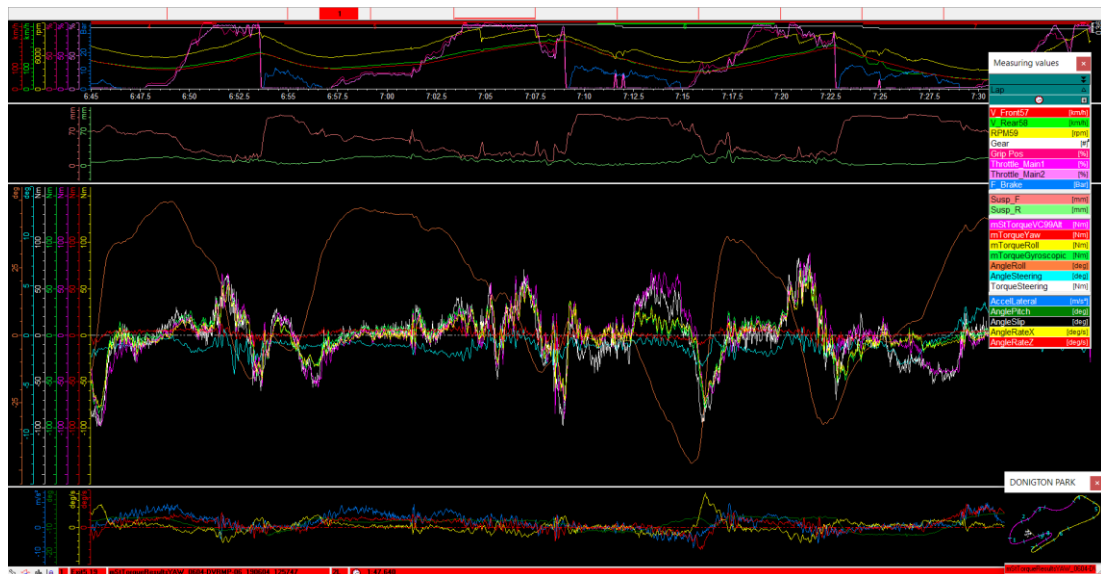


Figure 64 ‘Analyzer’: Screenshot of data with computed results for the total alternative steering torque and gyroscopic torque components

Figure 64 reveals that the results for the total alternative steering torque match the measured steering torque very precisely over the entire section of the track. The few instances where it deviates slightly from the steering torque that is actually applied by the rider occur under braking and this is most likely caused by minor inaccuracies or inconsistencies of the longitudinal torque component. Since the only difference between the total corrected steering torque and the total alternative steering torque is the addition of a newly calculated gyroscopic torque due to roll rate, the results prove that this torque is not balanced by any other steering torque component and so the gyroscopic effect of the front wheel clearly dominates the handling effort that is required of the rider. Moreover, all of the presented dataset excerpts reveal that the measured steering torque consistently precedes the recorded roll rate. Therefore, the new gyroscopic torque component basically describes the torque which is required to generate the observed roll rate with the front wheel and that is equally exerted on the rear frame of the motorcycle. Ultimately, the high level of correspondence between this gyroscopic torque and the measured steering torque, as shown in **Figure 65**, suggests that the rider almost exclusively uses the gyroscopic effect of the front wheel to control the attitude of the motorcycle, as the remaining torque components essentially cancel each other out.

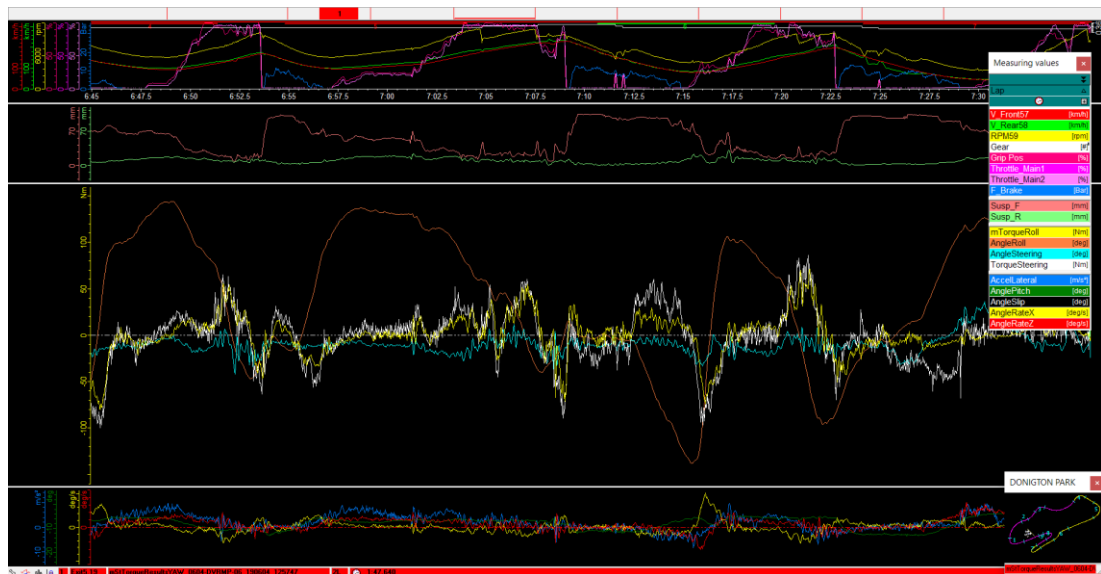


Figure 65 'Analyzer': Screenshot of data with computed results for the gyroscopic torque component due to roll rate

5.2.2. Steering Torque Components of the Transient-State Model

The steering torque equation of the transient-state model also allows for the computation of many additional torque components that are highly dynamic and thus not accounted for by the previously discussed results for the steering torque components of the steady-state model. **Figure 66** lists these sixteen transient-state components from the original steer torque equation of the mathematical model by (Sharp, 1971) that are computed with the project code.

No.	1971 Paper Notation	Description
1	$-M_f e g \phi$	Moment from weight of front frame from roll.
2	$t Z_f \phi$	Moment of normal force on the tyre from the ground from roll.
3	$-M_f e g \sin \epsilon \delta$	Moment from weight of front frame steer.
4	$t Z_f \sin \epsilon \delta$	Moment of normal force on the tyre from the ground from steer.
5	$M_f e \dot{x}_1 \dot{\psi}$	Moment from front frame mass centre lateral acceleration from yaw.
6	$i_{fy}/R_f \sin \epsilon \dot{x}_1 \dot{\psi}$	Gyroscopic torque from yaw.
7	$-i_{fy}/R_f \cos \epsilon \dot{x}_1 \dot{\phi}$	Gyroscopic torque from roll.
8	$K \delta$	Steering damper moment.
9	$M_f e \dot{v}$	Moment from lateral acceleration of front frame.
10	$M_f e k \ddot{\psi}$	Moment from lateral acceleration of front frame from yaw.
11	$I_{fx} \cos \epsilon \ddot{\psi}$	Front frame inertia cross-coupling torque from yaw.
12	$M_f e j \ddot{\phi}$	Moment from lateral acceleration of front frame from roll.
13	$I_{fx} \sin \epsilon \ddot{\phi}$	Front frame inertia cross-coupling torque from roll.
14	$M_f e^2 \ddot{\delta}$	Steer mass inertia torque.
15	$I_{fx} \ddot{\delta}$	Steer inertia torque.
16	$t Y_f$	Aligning moment from side force through mechanical trail.

Figure 66 The individual outputs from the Sharp 1971 steer torque equation (Styles, 2004)

The results for the computed torque components from a dataset of the DVRMP at Donington Park Circuit are displayed in **Figure 67**. Whilst a majority of the additional components are of negligible magnitude, it can be seen that the significant ones closely resemble those of previously presented results for the computed steering torque components of the steady-state model in **Figure 60**.

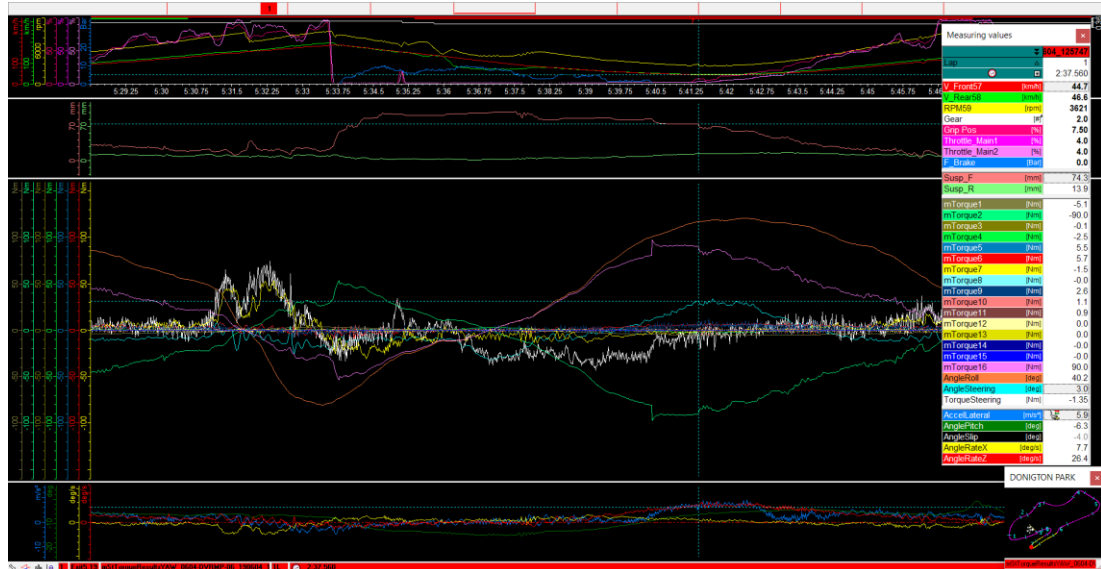


Figure 67 'Analyzer': Screenshot of data with computed results for the transient-state steering torque components

The main differences between the results that are displayed above and the previously examined torque components are the presence of a gyroscopic torque component due to roll rate and the absence of any torque components that are induced by the rolling resistance or twisting moment of the front tyre and a longitudinal force, which arises at its contact patch when the front brakes are applied. Thus, an equation for the additional steering torque component due to the longitudinal force, which is presented in the methodology section of this report, is derived and added to compute an alternative total steering torque. Whilst **Figure 68** shows the computed results for the original total steering torque and the aforementioned alternative total steering torque for an enlarged selection of the previously displayed data from the DVRMP at the 'Melbourne Hairpin', **Figure 69** shows results for the latter and the added torque component between 'McLean's' and the 'Melbourne Hairpin'.

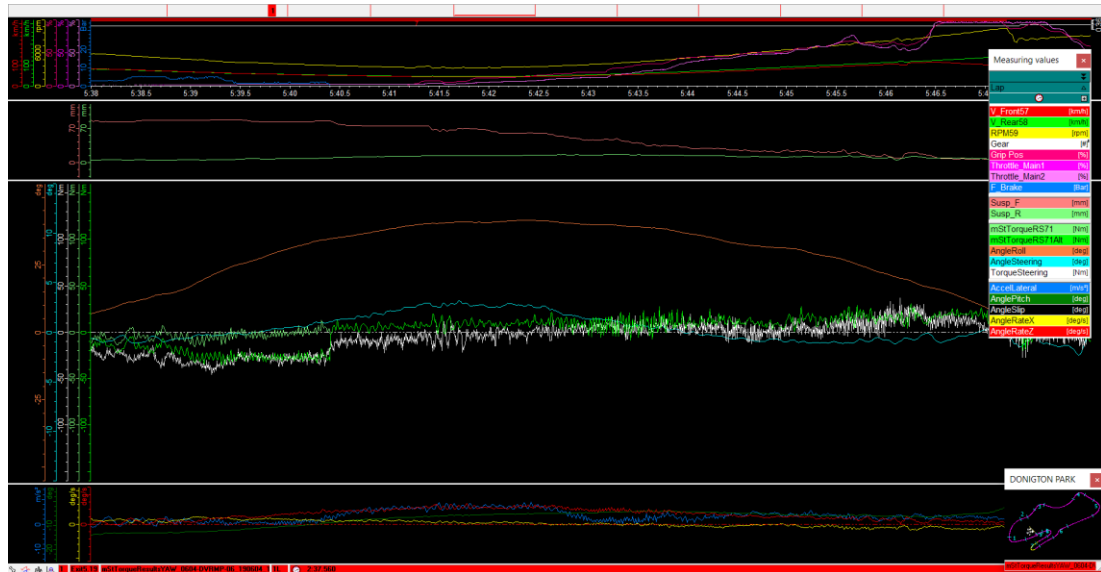


Figure 68 ‘Analyzer’: Screenshot of data with computed results for the original total and alternative total steering torque

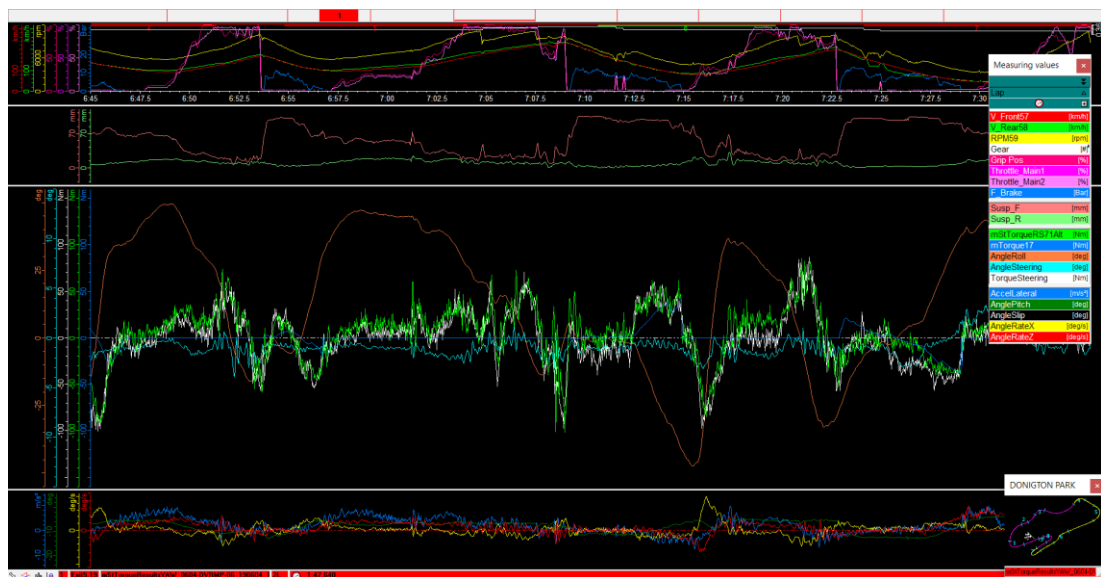


Figure 69 ‘Analyzer’: Screenshot of data with computed results for the alternative total steering torque and longitudinal torque component

Compared with the original total steering torque, the alternative total steering torque matches the measured steering torque much more closely during the application of the front brakes and both plots show that the torque components of the transient-state model also cancel each other out during steady turning.

In addition, the alternative total steering torque of the components of the transient-state model matches the measured steering torque as precisely as the alternative total steering torque of the components of the steady-state model, which includes the added gyroscopic torque from roll rate that is technically a transient-state component. Nevertheless, it is calculated slightly differently to the gyroscopic torque due to roll rate that is computed with the equation of the transient-state model and plotted in **Figure 70**.

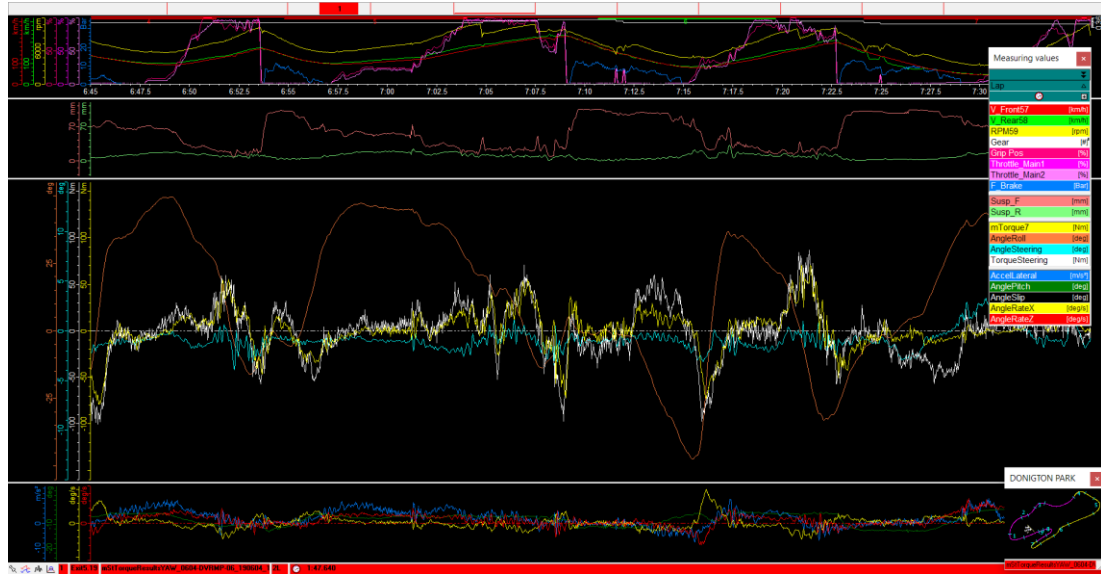


Figure 70 ‘Analyzer’: Screenshot of data with computed results for the gyroscopic torque due to roll rate

The graphs of the recorded data and the computed results which are displayed above corroborate the previous finding that the rider predominantly uses the gyroscopic torque to control the motorcycle, since even the fifteen steering torque components of the original transient-state model actually cancel each other out altogether and leave the gyroscopic effect to remain unbalanced. At this stage, it seems sensible to analyse the alternative total steering torque and its constituent components with respect to their direct influence on the steered front frame, as opposed to their contribution to the steering torque that must be applied by the rider. For this purpose, the total alternative steering torque is plotted against a reversed scale within the dataset excerpt that is shown in **Figure 71**, since each individual torque component originally had

their sign changed. The displayed dataset excerpt covers a large track section of Donington Park Circuit that ranges from the start-finish line on the 'Wheatcroft Straight' to 'McLean's'.

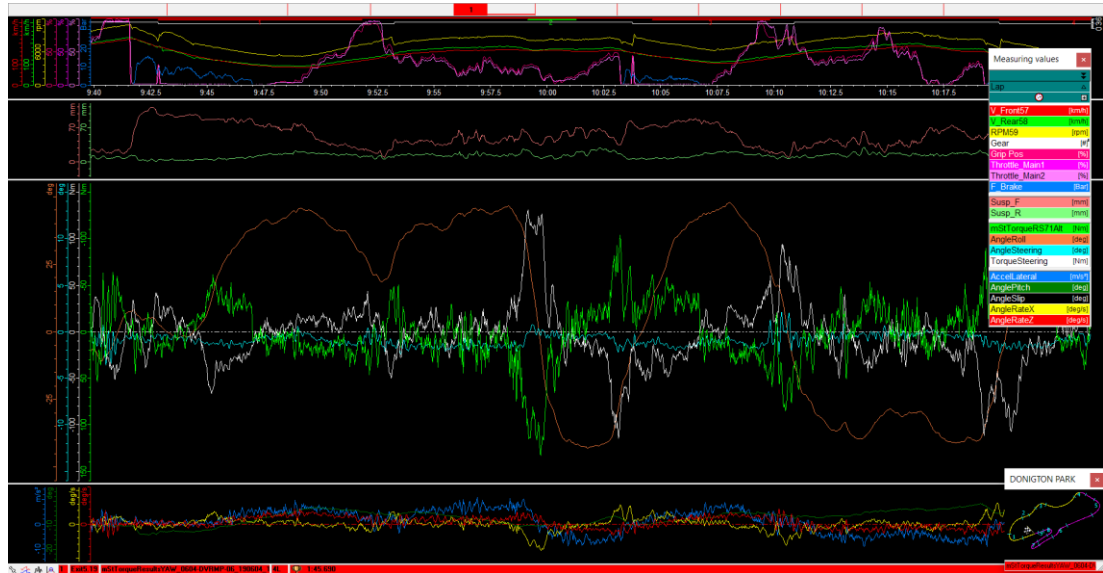


Figure 71 'Analyzer': Screenshot of data from the DVRMP between the 'Wheatcroft Straight' and 'McLean's' at Donington Park Circuit

In contrast to the previous plots of the computed results for the steering torque of the steady-state or transient-state model, the alternative total steering torque no longer represents the torque that is applied by the rider to balance the sum of its components. Instead, it reveals the total torque that these components apply on the steered front frame. Bearing in mind that the gyroscopic torque component due to roll rate basically describes the torque that is applied to the front wheel by the rider and mostly causes the recorded roll rate, it only effectively contributes to the plotted graph of the total alternative steering torque in **Figure 71** when the rider reduces the physically applied and measured steering torque input due to the ever-present momentum of the motorcycle that is associated with the originally induced roll rate by the rider. To further substantiate this theory, an enlarged selection of the previously presented dataset excerpt that covers the track section between 'Redgate' and the 'Old Hairpin' at Donington Park Circuit is displayed in **Figure 72**. It clearly shows that the recorded roll rate arises from the measured steering torque

5.2.3. Steering Torque Input at Slow Speed

Despite the exceptionally good agreement with the measured steering torque throughout the previously presented dataset excerpts, the computed results for the alternative total steering torque of the steady-state and transient-state model, which include the additionally derived steering torque components of the gyroscopic effect due to roll rate and the longitudinal force at the front contact patch, both fail to successfully describe the total steering torque that must be applied by the rider at forward velocities below 40 km/h, as shown in **Figure 73**. Therefore, a substantial analysis of the sum of the steering torque components is inhibited and most computed results of the project code merely allow for a conjectural interpretation of the steering dynamics at slow speeds.

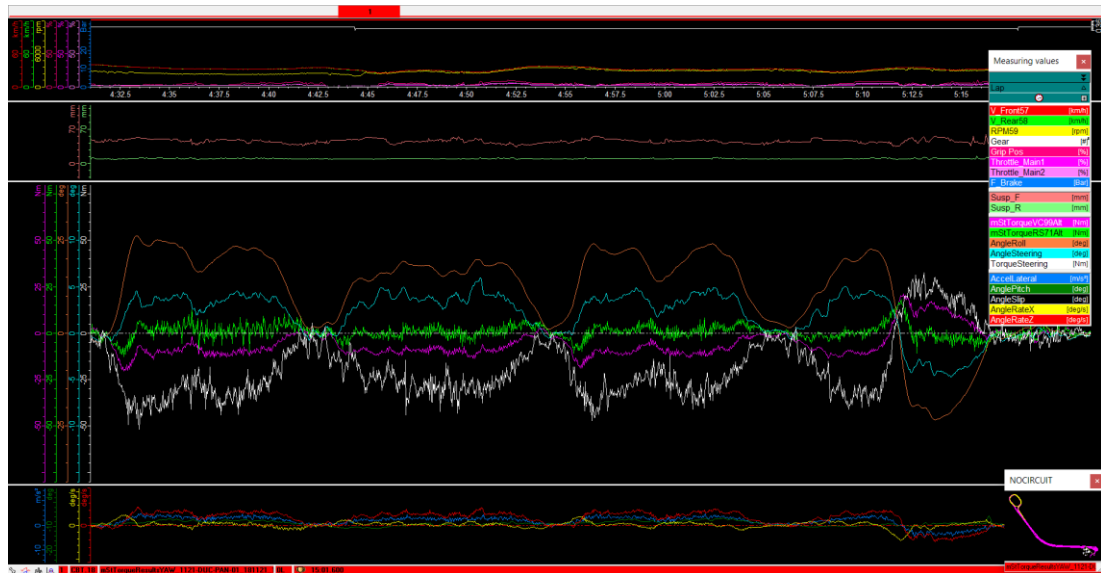


Figure 73 ‘Analyzer’: Screenshot of data with computed steady-state and transient-state results for the corrected total and alternative total steering torque

The first observation is that the motorcycle does not seem to reach a cornering equilibrium by itself and that the required steering torque to ensure its balance is very pronounced. Furthermore, it is consistently applied in a negative direction, which suggests that the capsized condition is stable and that the total steering torque that acts on the front frame assembly is disaligning. This also explains why the measured steering torque in the presented dataset excerpt

from the DVRMP barely shows that the rider is using a counter-steering input. However, when examining the graph more closely, it can be noted that a reduction in the negatively exerted steering torque by the rider still leads to a negative roll rate of the motorcycle. Thus, the steering mechanism at slow speeds may not be fundamentally different to its previously discussed and observed nature at more elevated speeds. In order to investigate the use of the gyroscopic effect of the front wheel and its control over the attitude of a motorcycle at slow speeds, the computed results for the corrected total steering torque and the gyroscopic torque component due to roll rate are plotted alongside a presented dataset excerpt in **Figure 74**. The displayed measurements were recorded when Andrew Harrison rode the DVRMP through slalom courses and figures of eights at the university.

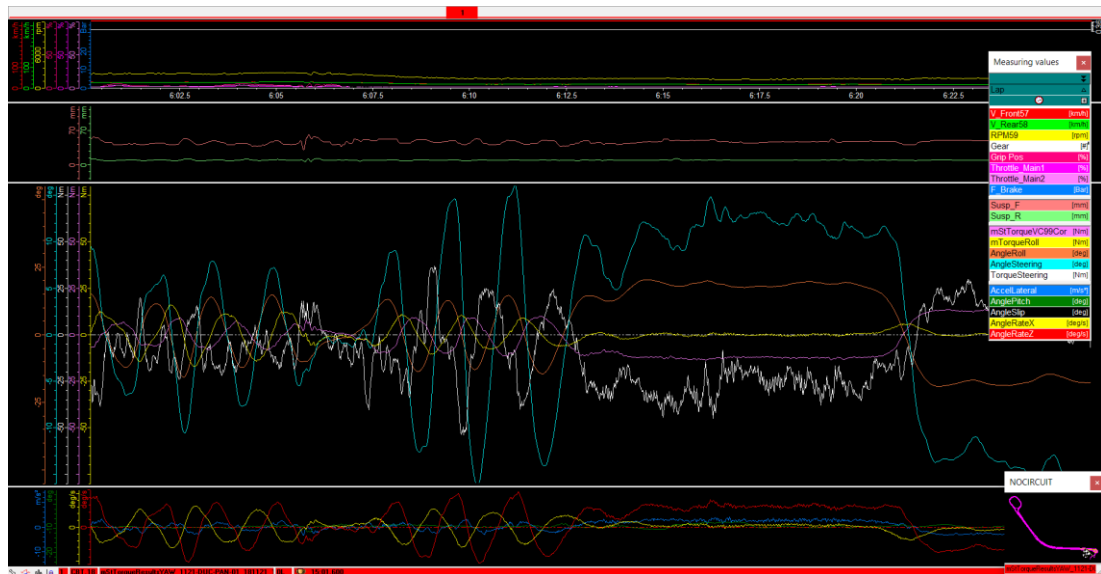


Figure 74 ‘Analyzer’: Screenshot of data with computed steady-state results for the corrected total steering torque and gyroscopic torque component due to roll rate

A conventional counter-steering torque input obviously instigates the initial roll rate that marks the beginning of the manoeuvre. Throughout the displayed slalom manoeuvres, however, the gyroscopic steering torque due to roll rate is mostly of opposite sign to the measured steering torque and thus compliant with it. Since the gyroscopic torque component essentially represents the

component shows a significant discrepancy when it is compared with the corrected total steering torque of the steady-state model. Whilst the shift of the graph of the alternative steering torque towards the graph of the gyroscopic torque component is evident, the difference in magnitude is primarily due to the absence of the steering torque component that arises from a twisting moment of the front tyre. Therefore, the observable build-up of the corrected total steering torque of the steady-state model in **Figure 74** is most likely caused by this twisting moment, which depends on the roll angle and is generally less balanced by the aligning torque components than it is at higher forward velocities. Thus, the steering torque component due to the twisting moment of the front tyre should further be added to the computed total alternative steering torque of the transient-state model, which is otherwise considered to represent the actual steering torque less accurately than the computed alternative total steering torque of the steady-state model in this case. Ultimately, the rider appears to change the direction of the motorcycle by letting off the applied steering torque which initially balances the disaligning moment that is induced by the front tyre and seems to prevail at slow speeds. This allows it to create a surplus of positive torque around the steering axis, which instigates a positive roll rate that builds up enough momentum to carry the motorcycle past the upright position and into an opposite roll angle, which resembles a deliberate and strong counter-steering action by the rider.

5.3. Simulation Results

Commercial multibody simulation software, such as ‘VI-BikeRealTime’ by the company ‘VI-grade’, can be used to assess the performance and dynamic behaviour of a specific motorcycle by enabling the operator to run simulations that are based on fully customisable virtual models and road profiles. In order to validate the steering torque that is calculated by the software, a virtual model of the DVRMP is created and used for a performance limit run which is aimed at determining the fastest lap time that the motorcycle model can achieve along a predefined path. For this purpose, the performance limit simulation is based on real GPS-data from the DVRMP, with which a road profile of a full lap at Donington Park Circuit was generated. The exported results for the calculated steering torque demand of the rider are plotted within the ‘Analyzer’ and presented in **Figure 76**, along with the measured steering torque from the DVRMP for a flying lap at Donington Park Circuit for ease of comparison.

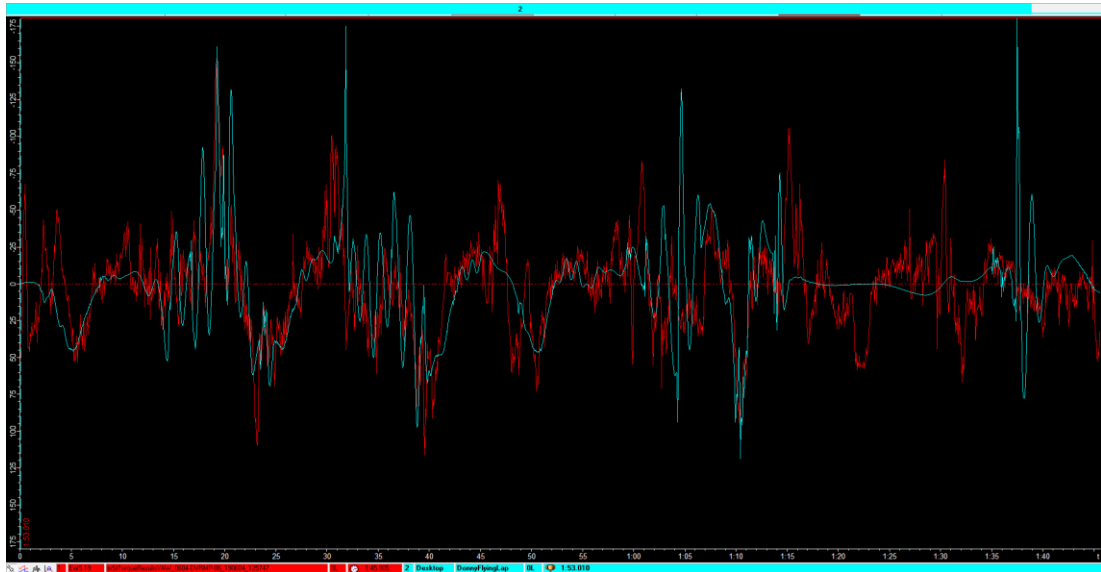


Figure 76 ‘Analyzer’: Screenshot of the measured steering torque (red) and initially simulated steering torque (blue) for a full flying lap at Donington Park Circuit

The results for the calculated steering torque from ‘VI-BikeRealTime’ that are displayed above follow the general trend of the measured steering torque. Whilst it is mostly of a similar magnitude to the torque that is actually applied

by the rider, the graph reveals that it fluctuates considerably more between the positive and negative extremes. Thus, the calculated steering input makes the virtual rider seem very indecisive. The apparent reason for this is the fact that the virtual rider is based on a proportional integral derivative (PID) controller. This means that it only has predictive control over the motorcycle due to an error-based feedback loop, whereby a correction factor that depends on the differences between the proportional, integral and derivative terms is applied to the input. These three terms, which define the architecture of the controller for the steering torque that is applied to the virtual motorcycle model during the simulation, can be individually adjusted from one run to the next. The results for the applied steering torque of the virtual rider from a simulation of another flying lap of the DVRMP at Donington Park Circuit are displayed in **Figure 77**.

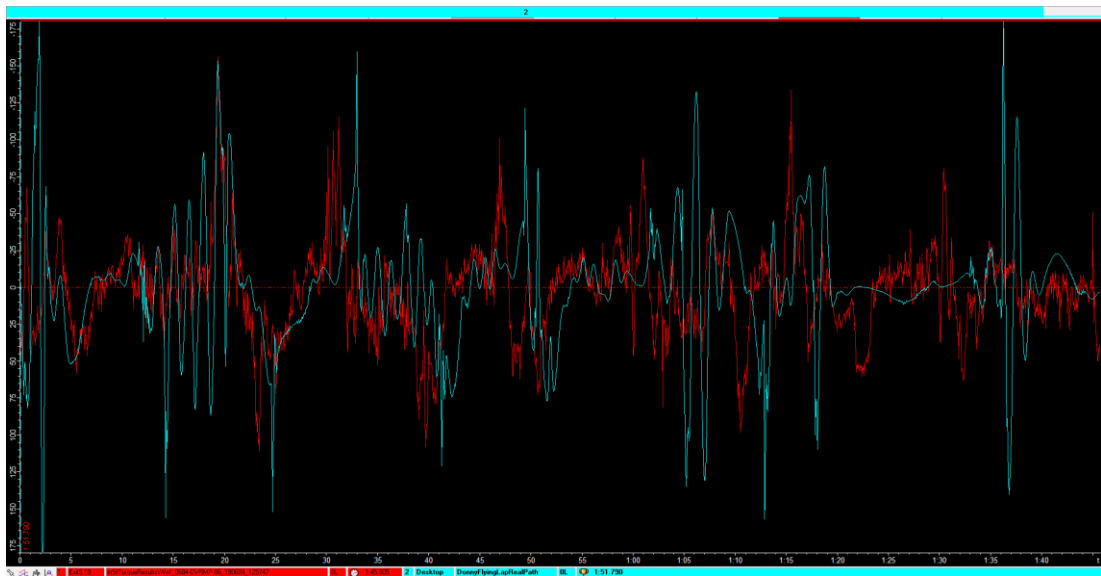


Figure 77 ‘Analyzer’: Screenshot of the measured steering torque and differently simulated steering torque for a full flying lap at Donington Park Circuit

The variables of the PID controller have been adjusted for this run and the plot which is displayed above reveals that their tuning significantly influences the steering torque, as the inputs are of a much higher amplitude than before. Ultimately, the controller appears to have no effect on the manoeuvrability of the motorcycle model or its performance, since the simulation can also be run

with the PID-controller disabled. However, its operating method implies that the calculated steering torque of a performance limit run is inconclusive with regards to the physical input that could be expected from a real rider, who is capable of assessing the required steering input based on human sensory reception and experience. This could pose a restriction to the assessment of handling characteristics with the calculated steering torque from performance limit simulations. Hence, the solution to this issue may be the implementation of a predetermined manoeuvre simulation which is used to calculate the model and rider behaviour through a slalom course that has a cone spacing of 7 m. The target velocity and selected gear for the exercise are referenced from a dataset that was recorded when Andy rode the DVRMP through a similar slalom manoeuvre at the university grounds. **Figure 78** shows the results for the steering angle and rider demands that were recorded by carrying out this predetermined manoeuvre simulation in 'VI-BikeRealTime'.



Figure 78 'VI-Animator': Screenshot of results for a slalom manoeuvre

6. Conclusion

The main goal of this research project was to establish the amount of steering torque that is physically applied at the handlebars of a motorcycle and to determine the factors that control it dynamically. This was achieved by directly recording measurements on a modern high-performance motorcycle during various dynamic scenarios that involved different riders and the use of a 'Dynamic Vehicle Response Measurement Platform' (DVRMP), which is a unique data-acquisition setup that had previously been developed by the author on the basis of a 'Ducati Panigale 1199 S'. The DVRMP allows for the recording of data that captures the dynamic behaviour of an entire vehicle-operator system and thereby enables a detailed analysis of the interaction between the rider-induced steering torque input and the dynamic response of a motorcycle that can increase the knowledge and understanding of transient motorcycle dynamics. In addition, this research was aimed at identifying, quantifying and assessing the factors that control the dynamic behaviour and steering performance of a motorcycle to ultimately enable an advanced evaluation of different riding styles, as well as motorcycle geometry, setup and chassis design. Initially, the primary datasets from the DVRMP were used for the analysis of fundamental steering theory, steady-turning performance and transient-state manoeuvrability, as well as rider control and handling techniques, based on the given chassis geometry and setup configurations.

Furthermore, this project was targeted at researching motorcycle dynamics by combined means of experimental testing and analytical methods. This involved the development of new methodology for the creation of computational models to calculate the steering torque from its components in steady-state cornering and transient-state conditions, since the steering torque measurements merely provide insight into the relationship between the total steering input of the rider and the resulting dynamic response of the motorcycle. For this purpose, equations of an existing quasi-static model by (Cossalter, et al., 1999), which is based on derivations for steady turning, as well as equations of an existing transient-state model by (Sharp, 1971) are used and supplied with the previously recorded data from the real motorcycle to establish whether a sufficient correlation of the calculated total steering torque with the directly

measured steering torque can ultimately be achieved. It became evident that some values for parameters that are initially used within the project code require refinement for an increased accuracy of the final results. Nevertheless, an exceptionally good correlation of the computed results for the steering torque components of both mathematical models with the recorded steering torque measurements is ultimately achieved for moderate to high speeds, once the derived gyroscopic torque component due to roll rate and the derived torque component due to the longitudinal force on the front contact patch are added to the total steering torque of the steady-state model and the total steering torque of the transient-state model respectively. Therefore, the project code provides a powerful tool for analytical investigations into the behaviour of a motorcycle-rider system in transient state, with particular focus on steering dynamics and handling performance by effectively calculating the numerous components that the steering torque is made up of, whilst it can continuously be supplied with new data from a variety of road tests, experiments or track sessions that make use of different riders and setup configurations on the motorcycle. For this research, the project code was employed to compute the steering torque components for various datasets from the DVRMP that were acquired during simple manoeuvres, like travelling on a circular path at constant speed, lane changes or slaloms, and full flying laps at race tracks, including Pembrey Circuit and Donington Park Circuit.

The concluding findings of the subsequent analysis that was carried out for this project can be summarised as follows. The general theory of counter-steering is proven correct, since the physical steering torque input of the rider always instigates a roll motion of the motorcycle in the opposite direction to the applied torque. Whilst this could be explained with lateral tyre forces and a centrifugal effect that acts on the centre of gravity, as well as the weight force of the motorcycle, the computed results for the steering torque components show that the rider predominantly uses the gyroscopic effect of the front wheel that results from its angular momentum and ultimately converts the steering torque into a roll motion of the motorcycle. Whilst the induced roll rate almost linearly depends on the magnitude of the applied steering torque, the physical effort that is required from the rider to achieve a certain roll rate increases with

the speed of the motorcycle due to the higher angular velocity of the front wheel resisting the gyroscopic precession. When the rider reduces the initially applied torque, the roll rate diminishes and the steered front frame self-adjusts due to the gyroscopic effect of the front wheel. This causes the motorcycle to start turning by allowing the cornering forces to build up at the contact patch of the front and rear tyre. Now the front wheel either points towards or away from the turn, depending on the roll angle and speed of the motorcycle, which also control the residual steering torque that must be applied by the rider to ensure that the motorcycle remains in a cornering equilibrium and on the desired path, together with the inherent over- or understeering tendency of the motorcycle. At a constant velocity, the motorcycle then follows a constant trajectory, unless the rider makes an adjustment to the speed or the roll angle of the motorcycle. Accelerating or applying a steering torque inwards the turn picks up the motorcycle and widens the path that it is on, except for a condition during which the rear tyre begins to slide out due to the applied driving force causing the available friction force of the road to be overcome. In this case, the lateral force is insufficiently balanced and the motorcycle essentially drifts, which increases its yaw angle and thus helps it turn. Nevertheless, the potential risks of causing the rear to slide out fully due to the open throttle or provoking a dangerous highside by abruptly closing the throttle again are ever-present under these circumstances. Decelerating or applying a steering torque outward the turn further increases the roll angle of the motorcycle and tightens the path that it is on. Whilst the application of the front brakes can oppose the counter-steering ambitions of the rider and result in the need for a higher steering torque input to create or maintain a certain roll rate during the entry into a turn, the braking force at the front contact patch can also assist the rider in standing the motorcycle back up or leaning it from one side to the other by reducing the steering torque that must be applied to the handlebars when the motorcycle is already leant over. Using the rear brake within the available grip limits allows the rider to tighten any given turn, without significantly affecting the steering torque of the rider, as the braking force arises at the contact patch of the rear tyre. In addition, it generates a moment which tends to align the motorcycle and it can thus be used to counteract a yawing moment of the chassis around the steering head while the front brakes are applied. However, this yawing

moment could also be exploited by allowing it to already move the motorcycle through a yaw angle before the brakes are even let out or the required roll angle and steering angle are adopted for the cornering forces to build up.

At slow speeds below 40 km/h, the computation fails to fully model the steering torque that is applied by the rider, yet it indicates that the twisting torque of the tyre mainly controls the steering input of the rider, as it is significantly less balanced at these speeds. The rider must still use a deliberate input to instigate a roll rate, however, a reduction in the physically applied residual torque that is needed to ensure the cornering equilibrium of a motorcycle during the stable capsize condition by the rider seems to be sufficient for generating an overall torque that is exerted on the front frame in a counter-steering manner, which ultimately conceals the counter-steering input that stands the motorcycle back up or even moves it past the upright position.

This observation is further corroborated by the results of simulations that were carried out with a commercial multibody software package of the company 'VI-grade' and performed to validate its calculated steering torque as part of this research project. In order to ensure that the simulated rider input and dynamic response of the virtual motorcycle model are representative and useful for a comparison with the recorded data of the DVRMP, a default model was altered to account for its specific characteristics as best as possible and its GPS-data from one lap at Donington Park Circuit was used to recreate the actual path of the DVRMP within the software package. Thus, a performance limit simulation could be performed and its results could be directly compared to actual measurements. It was found that the PID-architecture of the virtual controller that determines the steering torque input yields a realistic yet atypically and indecisively applied steering torque, unless it is used to run a predetermined manoeuvre simulation.

The successful computation of the different steering torque components by the project code equally means that the physical rider input can be accurately computed from recorded data without the need for any sensors that measure the applied force on the handlebars. This might be of particular interest to the motorsport sector, since official regulations regarding the conformity of certain

sensory equipment can be very strict and thus inhibit the assessment of motorcycle handling or rider performance. Ultimately, the techniques and results of this research provide transferrable skills and knowledge, as well as an increased understanding of transient-state steering dynamics and rider control, which can considerably broaden the horizon in the field of motorcycle engineering.

7. Further Work

Any assessment regarding the influence of certain structural aspects and design alterations is limited to the DVRMP, since only the reference motorcycle carries the unique datalogging equipment and sensors for measuring steering torque. Therefore, transferring an already developed and improved evolution of the strain gauge setup to other motorcycles for cross-referencing and building up a database of measurements from different motorcycles might be a sensible option for dealing with these limitations. Nevertheless, the current findings of this research project are already assisting the university with the ongoing planning and development of a stationary but highly realistic motorcycle simulator.

In addition, the author seeks to invent new technology that can improve the performance and handling quality of motorcycles. This could be done by applying the increased understanding and heightened technical abilities from this project to the extraction of potential performance gains by making justified changes to common motorcycle parts in order to improve current chassis design or adding innovative features, like a gyroscopic stabiliser as it is proposed by (Lot & Fleming, 2019), to actively control the attitude of motorcycles during the braking, cornering and acceleration phase.

Ultimately, the gained knowledge and increased understanding of transient-state dynamics from this research project will be applied to any professional challenges in the future in order to further help riders in developing their handling techniques and to improve the performance capabilities of current motorcycles, software solutions and simulators alike.

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9. Appendix

```
%% Steering Torque Calculation for the DVRMP
clc
clear all

% Inputs
Data2D=readtable('Exit5.xlsx');
DataSW=readtable('DesignStudyDVRMP.xlsx');
LinForkTravel=linspace(min(DataSW.ForkTravel),max(DataSW.ForkTravel),117);
LinShockTravel=linspace(min(DataSW.ShockTravel),max(DataSW.ShockTravel),33);
[GridForkTravel,GridShockTravel]=meshgrid(LinForkTravel,LinShockTravel);
Grیدهpsilon=griddata((DataSW.ForkTravel),(DataSW.ShockTravel),(DataSW.epsilon),GridForkTravel,GridShockTravel);
GridxQ=griddata((DataSW.ForkTravel),(DataSW.ShockTravel),(DataSW.xQ),GridForkTravel,GridShockTravel);
GridhQ=griddata((DataSW.ForkTravel),(DataSW.ShockTravel),(DataSW.hQ),GridForkTravel,GridShockTravel);
GriddQ=griddata((DataSW.ForkTravel),(DataSW.ShockTravel),(DataSW.dQ),GridForkTravel,GridShockTravel);
Gridp=griddata((DataSW.ForkTravel),(DataSW.ShockTravel),(DataSW.p),GridForkTravel,GridShockTravel);
Gridb=griddata((DataSW.ForkTravel),(DataSW.ShockTravel),(DataSW.b),GridForkTravel,GridShockTravel);
Gridh=griddata((DataSW.ForkTravel),(DataSW.ShockTravel),(DataSW.h),GridForkTravel,GridShockTravel);
epsilon=interp2(GridForkTravel,GridShockTravel,Grیدهpsilon,(Data2D.Susp_F),(Data2D.Susp_R));
xQ=interp2(GridForkTravel,GridShockTravel,GridxQ,(Data2D.Susp_F),(Data2D.Susp_R));
hQ=interp2(GridForkTravel,GridShockTravel,GridhQ,(Data2D.Susp_F),(Data2D.Susp_R));
dQ=interp2(GridForkTravel,GridShockTravel,GriddQ,(Data2D.Susp_F),(Data2D.Susp_R));
p=interp2(GridForkTravel,GridShockTravel,Gridp,(Data2D.Susp_F),(Data2D.Susp_R));
b=interp2(GridForkTravel,GridShockTravel,Gridb,(Data2D.Susp_F),(Data2D.Susp_R));
h=interp2(GridForkTravel,GridShockTravel,Gridh,(Data2D.Susp_F),(Data2D.Susp_R));

% Constants (Determined)
StaticRakeAngle=23.22;
StaticWheelbase=1.43549;
StaticSagFront=33.6;
StaticSagRear=4.3;
DynamicRakeAngle=23.10;
DynamicWheelbase=1.43088;
DynamicSagFront=48;
DynamicSagRear=8.9;
```

```

Iwf=0.604;
RFront=0.30335;
RRear=0.32818;
tFront=0.06258;
tRear=0.1076;
rhoFront=RFront-tFront;
rhoRear=RRear-tRear;
ForkOffset=0.03;
MT=264;
MUFront=22.5;
MSFrontStatic=0.4883*MT-MUFront;
kSuspFront=2*10000;
cSuspFront=0.4*2*sqrt((kSuspFront/cosd(DynamicRakeAngle).^2)
)*MSFrontStatic)*cosd(DynamicRakeAngle).^2;

% Constants (Referenced)
Mf=30.7;
bf=0.024;
hf=0.461;
Ixf=1.23;
Iyf=1.40;
Izf=0.44;
cSteeringDamper=6.78;
kCamberFront=1.33;
kCorneringFront=12.17;
kCamberRear=1.31;
kCorneringRear=11.33;
kTwistingFront=0.035;
kSelfAligningFront=-0.475;
CResRolling=0.06757;
mdFront=CResRolling*RFront;

% Conversions
mAngleRake=epsilon*(pi/180);
mxQ=xQ/1000;
mhQ=hQ/1000;
mdQ=dQ/1000;
mp=p/1000;
mb=b/1000;
mh=h/1000;
mAnglePitchIMU=(Data2D.AnglePitch)*(pi/180);
mAngleRoll=(Data2D.AngleRoll)*(pi/180);
mAngleSteering=-(Data2D.Steering)*(pi/180);
mVelocityForward=(Data2D.V_GPS)/3.6;
mVelocityFront=(Data2D.V_Front57)/3.6;
mAngleVelocityFront=mVelocityFront/RFront;
mAngleVelocityYaw=((Data2D.AngleRateY)*(pi/180)).*sin(mAngleRoll)./cos(mAnglePitchIMU))+((Data2D.AngleRateZ)*(pi/180)).*cos(mAngleRoll)./cos(mAnglePitchIMU));
mAngleVelocityRoll=((Data2D.AngleRateX)*(pi/180))+((Data2D.AngleRateY)*(pi/180)).*sin(mAngleRoll).*tan(mAnglePitchIMU))+((Data2D.AngleRateZ)*(pi/180)).*cos(mAngleRoll).*tan(mAnglePitchIMU));

```

```

mAngleVelocitySteering=gradient(mAngleSteering);
mAngleAccelYaw=((Data2D.AngAccelY)*(pi/180)).*sin(mAngleRoll)./cos(mAnglePitchIMU))+((Data2D.AngAccelZ)*(pi/180)).*cos(mAngleRoll)./cos(mAnglePitchIMU));
mAngleAccelRoll=((Data2D.AngAccelX)*(pi/180))+((Data2D.AngAccelY)*(pi/180)).*sin(mAngleRoll).*tan(mAnglePitchIMU))+((Data2D.AngAccelZ)*(pi/180)).*cos(mAngleRoll).*tan(mAnglePitchIMU));
mAngleAccelSteering=gradient(mAngleVelocitySteering);
mAccelLateral=(Data2D.AccelLateral);
mSuspPosFront=(Data2D.Susp_F)/1000;
mSuspVelFront=gradient(mSuspPosFront);
mSuspAccFront=gradient(mSuspVelFront);
mDynamicSagFront=DynamicSagFront/1000;
mFrontBrakeON=(Data2D.F_Brake)>0;

% Variables
mAngleSideslipFront=((1-kCamberFront)/kCorneringFront).*mAngleRoll;
mAngleSideslipRear=((1-kCamberRear)/kCorneringRear).*mAngleRoll;
mtheta=atan((sin(mAngleSteering).*tan(mAngleRoll))-sin(mAngleRake).*cos(mAngleSteering))./cos(mAngleRake));
mAnglePitchKinematicPaper=-(((ForkOffset.*sin(mAngleRake)).*(1-cos(mAngleSteering))+tRear-tFront+rhoFront.*(cos(mAngleRake).*cos(mtheta)-cos(mAngleSteering).*sin(mAngleRake).*sin(mtheta)-1)).*cos(mAngleRoll)+(ForkOffset.*sin(mAngleSteering)+rhoFront.*sin(mAngleSteering).*sin(mtheta)).*sin(mAngleRoll)+tFront-tRear)./(ForkOffset.*cos(mAngleRake).*(1-cos(mAngleSteering))-mp).*cos(mAngleRoll)-rhoFront.*(sin(mAngleRake).*cos(mAngleRoll).*sin(mAngleSteering)+cos(mAngleSteering).*sin(mAngleRoll)).*cos(mtheta).*sin(mAngleSteering)));
mAngleSteeringKinematicPaper=atan((sin(mAngleSteering).*cos(mAngleRake+mAnglePitchKinematicPaper))./(cos(mAngleRoll).*cos(mAngleSteering)-sin(mAngleRoll).*sin(mAngleSteering).*sin(mAngleRake+mAnglePitchKinematicPaper)));
mAngleSteeringEffectivePaper=mAngleSteeringKinematicPaper+mAngleSideslipRear-mAngleSideslipFront;
mSteeringRatioPaper=mAngleSteeringEffectivePaper./mAngleSteeringKinematicPaper;
mpminusb=mp-mb;
mMSFrontDynamic=((mp-mpminusb)./mp).*MT-MUFront;
mSuspForceFrontStatic=kSuspFront*mDynamicSagFront;
mSuspForceFrontDynamic=mMSFrontDynamic.*mSuspAccFront+cSuspFront.*mSuspVelFront+kSuspFront.*mSuspPosFront-mSuspForceFrontStatic;
mSuspForceFrontBraking=((mSuspForceFrontDynamic.*mFrontBrakeON)./(mh./mp)).*cos(mAngleRake+mAnglePitchKinematicPaper)+

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```

sin(mAngleRake+mAnglePitchKinematicPaper))).*sin(mAngleRake
+mAnglePitchKinematicPaper);
mNf=(9.81*MUFront+(mSuspForceFrontStatic+mSuspForceFrontDyn
amic-
mSuspForceFrontBraking)./cos(mAngleRake+mAnglePitchKinemat
icPaper)).*cos(mAngleRoll);
mFf=(kCorneringFront.*mAngleSideslipFront+kCamberFront.*mAn
gleRoll).*mNf;
mSf=-
mSuspForceFrontBraking./sin(mAngleRake+mAnglePitchKinematic
Paper);
mMyFront=mdFront.*mNf;
mMzFront=(kSelfAligningFront.*mAngleSideslipFront+kTwisting
Front.*mAngleRoll).*mNf;

% Variables 2
mbetaDashPaper=mAngleRake+atan((sin(mAngleSteering).*tan(mA
ngleRoll)-
sin(mAngleRake+mAnglePitchKinematicPaper).*cos(mAngleSteeri
ng))./cos(mAngleRake+mAnglePitchKinematicPaper));
mA=ForkOffset.*sin(mAngleRake).(1-
cos(mAngleSteering))+tRear-tFront;
mB=rhoFront.*(cos(mAngleRake).*cos(mbetaDashPaper-
mAngleRake)-
cos(mAngleSteering).*sin(mAngleRake).*sin(mbetaDashPaper-
mAngleRake)-1);
mC=ForkOffset.*sin(mAngleSteering)+rhoFront.*sin(mAngleStee
ring).*sin(mbetaDashPaper-mAngleRake);
mD=mp-ForkOffset.*cos(mAngleRake).(1-cos(mAngleSteering));
mE=rhoFront.*(sin(mAngleRake).*cos(mbetaDashPaper-
mAngleRake)+cos(mAngleSteering).*cos(mAngleRake).*sin(mbeta
DashPaper-mAngleRake));
mbetaDashBook=mAngleRake+atan((sin(mAngleSteering).*tan(mAn
gleRoll)-
sin(mAngleRake).*cos(mAngleSteering))./cos(mAngleRake));
mC1=ForkOffset.*sin(mAngleRake).(1-
cos(mAngleSteering))+tRear-tFront;
mC2=rhoFront.*(cos(mAngleRake).*cos(mbetaDashBook-
mAngleRake)-
cos(mAngleSteering).*sin(mAngleRake).*sin(mbetaDashBook-
mAngleRake)-1);
mC3=ForkOffset.*sin(mAngleSteering)+rhoFront.*sin(mAngleSte
ering).*sin(mbetaDashBook-mAngleRake);
mC4=mp-ForkOffset.*cos(mAngleRake).(1-
cos(mAngleSteering));
mC5=rhoFront.*(sin(mAngleRake).*cos(mbetaDashBook-
mAngleRake)+cos(mAngleSteering).*cos(mAngleRake).*sin(mbeta
DashBook-mAngleRake));
mAnglePitchKinematicBook=((mC1+mC2).*cos(mAngleRoll)+mC3.*s
in(mAngleRoll)+tFront-tRear)./(mC4+mC5).*cos(mAngleRoll));
mAngleSteeringKinematicBook=atan((sin(mAngleSteering).*cos(
mAngleRake+mAnglePitchKinematicBook))./(cos(mAngleRoll).*co
s(mAngleSteering)-

```

```

sin(mAngleRoll).*sin(mAngleSteering).*sin(mAngleRake+mAngle
PitchKinematicBook));
mAngleSteeringEffectiveBook=mAngleSteeringKinematicBook+mAn
gleSideslipRear-mAngleSideslipFront;
mSteeringRatioBook=mAngleSteeringEffectiveBook./mAngleSteer
ingKinematicBook;
mxPf=(mC1+mC2).*sin(mAnglePitchKinematicBook)+(mC4+mC5).*co
s(mAnglePitchKinematicBook);
myPf=(-
(mC1+mC2).*cos(mAnglePitchKinematicBook)+(mC4+mC5).*sin(mAn
glePitchKinematicBook)).*sin(mAngleRoll)+mC3.*cos(mAngleRol
l)-(tFront-tRear);
mwb=mxPf+myPf.*tan(mAngleSteeringKinematicBook);
mdeltah=RFront.*(1-sqrt(1-
sin(mAngleSteering).^2.*sin(mAngleRake).^2))-
ForkOffset.*sin(mAngleRake).*(1-cos(mAngleSteering));
mTrailNormal=RFront.*sin(mAngleRake)-ForkOffset;
mTrailA1=-
(myPf.*cos(mAngleRoll)+tRear.*sin(mAngleRoll)).*cos(mAngleR
ake+mAnglePitchKinematicBook);
mTrailA2=(mxPf.*cos(mAngleRake+mAnglePitchKinematicBook)-
mp.*cos(mAngleRake)+Rrear.*sin(mAngleRake)-
mTrailNormal).*cos(mAngleRoll)-
(rhoRear.*cos(mAngleRoll)+tRear).*sin(mAngleRake+mAnglePitc
hKinematicBook);
mTrailA3=myPf.*sin(mAngleRake+mAnglePitchKinematicBook)+(mx
Pf.*cos(mAngleRake+mAnglePitchKinematicBook)-
mp.*cos(mAngleRake)+Rrear.*sin(mAngleRake)-mTrailNormal-
rhoRear.*sin(mAngleRake+mAnglePitchKinematicBook)).*sin(mAn
gleRoll);
mTrailNormalCornering=sqrt(mTrailA1.^2+mTrailA2.^2+mTrailA3
.^2);

```

%% Motorcycle Path

```

mXPfminusXPr=(mD+mE).*cos(mAnglePitchKinematicPaper)+(mA+mB
+rhoRear).*sin(mAnglePitchKinematicPaper)-
(mB+mA+rhoRear.*(1-
cos(mAnglePitchKinematicPaper))).*sin(mAnglePitchKinematicP
aper).*sin(mAngleRoll);
mYPfminusYPr=mC.*cos(mAngleRoll)+(mB-mA-rhoRear.*(1-
cos(mAnglePitchKinematicPaper))).*cos(mAnglePitchKinematicP
aper).*sin(mAngleRoll)+(mE+mD+rhoRear.*sin(mAnglePitchKinem
aticPaper)).*sin(mAnglePitchKinematicPaper).*sin(mAngleRoll
);

```

```

mTurnCurvaturePr=(cos(mAngleSideslipRear).*sin(mAngleSteeri
ngKinematicPaper-
mAngleSideslipFront)+sin(abs(mAngleSideslipRear)).*cos(mAng
leSteeringKinematicPaper-
mAngleSideslipFront))./(mXPfminusXPr.*cos(mAngleSteeringKin
ematicPaper-

```

```

mAngleSideslipFront)+mYPfminusYPr.*sin(mAngleSteeringKinematicPaper-mAngleSideslipFront));
mTurnRadiusRearPr=1./mTurnCurvaturePr;
mTurnRadiusRearPr(isinf(mTurnRadiusRearPr)|isnan(mTurnRadiusRearPr))=0;
mYawRatePrMean=mVelocityForward./mTurnRadiusRearPr;
mTurnCurvatureYaw=mAngleVelocityYaw./mVelocityForward;
mTurnRadiusRearYaw=1./mTurnCurvatureYaw;
mTurnRadiusRearYaw(isinf(mTurnRadiusRearYaw)|isnan(mTurnRadiusRearYaw))=0;

% Coordinates
mXPr=-mTurnRadiusRearYaw.*sin(mAngleSideslipRear);
mYPr=-mTurnRadiusRearYaw.*cos(mAngleSideslipRear);
mXPf=mXPfminusXPr+mXPr;
mYPf=mYPfminusYPr+mYPr;
mXQ=mxQ+mXPr;
mYQ=mhQ.*sin(mAngleRoll)+mYPr;
mZQ=-mhQ.*cos(mAngleRoll)+mdeltah;
mxGf=mxQ+(hf+mdQ).*sin(mAngleRake+mAnglePitchKinematicPaper)+bf.*cos(mAngleRake+mAnglePitchKinematicPaper).*cos(mAngleSteering);
myGf=(mhQ-(hf+mdQ).*cos(mAngleRake+mAnglePitchKinematicPaper)+bf.*sin(mAngleRake+mAnglePitchKinematicPaper).*cos(mAngleSteering)).*sin(mAngleRoll)+bf.*sin(mAngleSteering).*cos(mAngleRoll);
;
mzGf=-(mhQ-(hf+mdQ).*cos(mAngleRake+mAnglePitchKinematicPaper)+bf.*sin(mAngleRake+mAnglePitchKinematicPaper).*cos(mAngleSteering)).*cos(mAngleRoll)-mdeltah-bf.*sin(mAngleSteering).*sin(mAngleRoll));
mXGf=mxGf+mXPr;
mYGf=myGf+mYPr;
mZGf=mzGf;

% Conversions 2
mICXZf=Mf.*mXGf.*mZGf+(Ixf.*cos(mAngleSteering).^2+Iyf.*sin(mAngleSteering).^2-Izf).*cos(mAngleRake+mAnglePitchKinematicPaper).*sin(mAngleRake+mAnglePitchKinematicPaper).*cos(mAngleRoll)+(Iyf-Ixf).*cos(mAngleRake+mAnglePitchKinematicPaper).*cos(mAngleSteering).*sin(mAngleSteering).*sin(mAngleRoll);
mICYZf=Mf.*mYGf.*mZGf+cos(mAngleSteering).*sin(mAngleSteering).*sin(mAngleRake+mAnglePitchKinematicPaper).*(Ixf-Iyf).*(cos(mAngleRoll).^2-sin(mAngleRoll).^2)+cos(mAngleRoll).*sin(mAngleRoll).*(Ixf.*cos(mAngleSteering).^2.*sin(mAngleRake+mAnglePitchKinematicPaper).^2-sin(mAngleSteering).^2+Iyf.*(sin(mAngleSteering).^2.*sin(mAngleRake+mAnglePitchKinematicPaper).^2-cos(mAngleSteering).^2)+Izf.*cos(mAngleRake+mAnglePitchKinematicPaper).^2);

```

```

%% Steering Torque Components (V.C. 1999/2006):

% Gravity Torque
mTorqueGravity=9.81*Mf.*( (mYQ-
mYGf).*sin(mAngleRake+mAnglePitchKinematicPaper)+(mXQ-
mXGf).*cos(mAngleRake+mAnglePitchKinematicPaper).*sin(mAngleRoll));

% Inertia Torque
mTorqueInertiaPaper=mAngleVelocityYaw.^2.*Mf.*(mYGf.*mXQ-
mXGf.*mYQ).*cos(mAngleRake+mAnglePitchKinematicPaper).*cos(
mAngleRoll)-
mAngleVelocityYaw.^2.*Mf.*mZQ.*(mYGf.*sin(mAngleRake+mAnglePitchKinematicPaper)+mXGf.*cos(mAngleRake+mAnglePitchKinematicPaper).*sin(mAngleRoll))+mAngleVelocityYaw.^2.*(mICYZf.*sin(mAngleRake+mAnglePitchKinematicPaper)+mICXZf.*cos(mAngleRake+mAnglePitchKinematicPaper).*sin(mAngleRoll));
mTorqueInertiaPaperAlt=mAngleVelocityYaw.^2.*Mf.*(mYGf.*mXQ-
-
mXGf.*mYQ).*cos(mAngleRake+mAnglePitchKinematicPaper).*cos(
mAngleRoll)-
mAngleVelocityYaw.^2.*Mf.*mZQ.*(mYGf.*sin(mAngleRake+mAnglePitchKinematicPaper)+mYGf.*cos(mAngleRake+mAnglePitchKinematicPaper).*sin(mAngleRoll))+mAngleVelocityYaw.^2.*(mICYZf.*sin(mAngleRake+mAnglePitchKinematicPaper)+mICXZf.*cos(mAngleRake+mAnglePitchKinematicPaper).*sin(mAngleRoll));
mTorqueInertiaBook=- (mAngleVelocityYaw.^2.*(Mf.*(mXQ.*mYGf-
mXGf.*mYQ).*cos(mAngleRake+mAnglePitchKinematicPaper).*cos(
mAngleRoll)+(mICYZf-
Mf.*mYGf.*mZQ).*sin(mAngleRake+mAnglePitchKinematicPaper)+(
mICXZf-
Mf.*mYGf.*mZQ).*cos(mAngleRake+mAnglePitchKinematicPaper).*sin(mAngleRoll)));
mTorqueInertiaBookAlt=-
(mAngleVelocityYaw.^2.*(Mf.*(mXQ.*mYGf-
mXGf.*mYQ).*cos(mAngleRake+mAnglePitchKinematicPaper).*cos(
mAngleRoll)+(mICYZf-
Mf.*mYGf.*mZQ).*sin(mAngleRake+mAnglePitchKinematicPaper)+(
mICXZf-
Mf.*mXGf.*mZQ).*cos(mAngleRake+mAnglePitchKinematicPaper).*sin(mAngleRoll)));
% Normal Torque
mTorqueNormal=- ( (mYQ-
mYPf).*sin(mAngleRake+mAnglePitchKinematicPaper)+(mXQ-
mXPf).*cos(mAngleRake+mAnglePitchKinematicPaper).*sin(mAngleRoll)).*mNf;

% Lateral Torque
mTorqueLateral=( (mYQ-
mYPf).*sin(mAngleSteeringKinematicPaper)+(mXQ-
mXPf).*cos(mAngleSteeringKinematicPaper)).*cos(mAngleRake+mAnglePitchKinematicPaper).*cos(mAngleRoll).*mFf-
mZQ.*(cos(mAngleSteeringKinematicPaper).*sin(mAngleRake+mAnglePitchKinematicPaper))-

```

```

cos(mAngleRake+mAnglePitchKinematicPaper).*sin(mAngleRoll).
.*sin(mAngleSteeringKinematicPaper)).*mFf;
% Lateral Torque Alternative
mTorqueLateralAlt=mTrailNormalCornering.*mFf;
% Longitudinal Torque
mTorqueLongitudinal=-((mYPf-
mYQ).*cos(mAngleSteeringKinematicPaper)+(mXQ-
mXPf).*sin(mAngleSteeringKinematicPaper)).*cos(mAngleRake+m
AnglePitchKinematicPaper).*cos(mAngleRoll).*mSf-
mZQ.*(cos(mAngleSteeringKinematicPaper).*cos(mAngleRake+mAn
glePitchKinematicPaper).*sin(mAngleRoll)+sin(mAngleSteering
KinematicPaper).*sin(mAngleRake+mAnglePitchKinematicPaper))
.*mSf);
% Tyre Torque
mTorqueTyre=-
mMzFront.*cos(mAngleRake+mAnglePitchKinematicPaper).*cos(mA
ngleRoll)+(cos(mAngleRake+mAnglePitchKinematicPaper).*sin(m
AngleRoll).*cos(mAngleSteeringKinematicPaper)+sin(mAngleRak
e+mAnglePitchKinematicPaper).*sin(mAngleSteeringKinematicPa
per)).*mMyFront;
% Yaw Torque
mTorqueYaw=(sin(mAngleRake+mAnglePitchKinematicPaper).*cos(
mAngleRoll).*cos(mAngleSteering)-
sin(mAngleRoll).*sin(mAngleSteering)).*Iwf.*mAngleVelocityF
ront.*mAngleVelocityYaw;
% Roll Torque(Transient)
mTorqueRoll=-
(cos(mAngleRake+mAnglePitchKinematicPaper).*cos(mAngleSteer
ing).*Iwf.*mAngleVelocityFront.*mAngleVelocityRoll);
% Gyroscopic Torque(Transient)
mTorqueGyroscopic=mTorqueYaw+mTorqueRoll;

% Total StTorque
mStTorqueVC99Cor=mTorqueGravity+mTorqueInertiaPaper+mTorque
Normal+mTorqueLateralAlt+mTorqueLongitudinal+mTorqueTyre+mT
orqueYaw;

% Total StTorque Alternative
mStTorqueVC99Alt=mTorqueGravity+mTorqueInertiaPaper+mTorque
Normal+mTorqueLateralAlt+mTorqueLongitudinal+mTorqueTyre+mT
orqueGyroscopic;

%% Steering Torque Components (R.S. 1971):

%1 Weight Moment from Roll
mTorque1=-Mf*bf*9.81.*mAngleRoll;
%2 Normal Force Moment from Roll
mTorque2=-mTrailNormalCornering.*mNf.*mAngleRoll;
%3 Weight Moment from Steer
mTorque3=-
Mf*bf*9.81.*sin(mAngleRake+mAnglePitchKinematicBook).*mAngl
eSteering;

```

```

%4 Normal Force Moment from Steer
mTorque4=-
mTrailNormalCornering.*mNf.*sin(mAngleRake+mAnglePitchKinematicBook).*mAngleSteering;
%5 Front Frame Moment from Yaw
mTorque5=Mf*bf.*mVelocityForward.*mAngleVelocityYaw;
%6 Gyroscopic Torque from Yaw
mTorque6=Iwf/RFront.*sin(mAngleRake+mAnglePitchKinematicBook).*mVelocityFront.*mAngleVelocityYaw;
%7 Gyroscopic Torque from Roll
mTorque7=-
Iwf/RFront.*cos(mAngleRake+mAnglePitchKinematicBook).*mVelocityFront.*mAngleVelocityRoll;
%8 Steering Damper Moment
mTorque8=cSteeringDamper.*mAngleVelocitySteering;
%9 Front Frame Moment from Lateral Acceleration
mTorque9=Mf*bf.*mAccelLateral;
%10 Front Frame Moment from Yaw
mTorque10=Mf*bf.*(mxGf-mb).*mAngleAccelYaw;
%11 Front Frame Inertia Torque from Yaw
mTorque11=Izf.*cos(mAngleRake+mAnglePitchKinematicBook).*mAngleAccelYaw;
%12 Front Frame Moment from Roll
mTorque12=Mf*bf.*-mzGf.*mAngleAccelRoll;
%13 Front Frame Inertia Torque from Roll
mTorque13=Izf.*sin(mAngleRake+mAnglePitchKinematicBook).*mAngleAccelRoll;
%14 Steer Mass Inertia Torque
mTorque14=Mf*bf^2.*mAngleAccelSteering;
%15 Steer Inertia Torque
mTorque15=Izf.*mAngleAccelSteering;
%16 Lateral Force Moment from Trail
mTorque16=mTrailNormalCornering.*mFf;
%17 Longitudinal Force Moment from Roll
mTorque17=tFront.*cos(mAngleRake+mAnglePitchKinematicBook).*sin(mAngleRoll).*mSf;

% Total StTorque
mStTorqueRS71=mTorque1+mTorque2+mTorque3+mTorque4+mTorque5+mTorque6+mTorque7+mTorque8+mTorque9+mTorque10+mTorque11+mTorque12+mTorque13+mTorque14+mTorque15+mTorque16;

% Total StTorque Alternative
mStTorqueRS71Alt=mTorque1+mTorque2+mTorque3+mTorque4+mTorque5+mTorque6+mTorque7+mTorque8+mTorque9+mTorque10+mTorque11+mTorque12+mTorque13+mTorque14+mTorque15+mTorque16+mTorque17;

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