

Moments

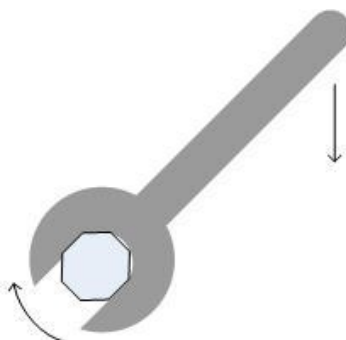
Objectives

On completion of this unit you should be able to:

1. Calculate the Magnitude of a Moment
2. Calculate Counter-balances required for Equilibrium

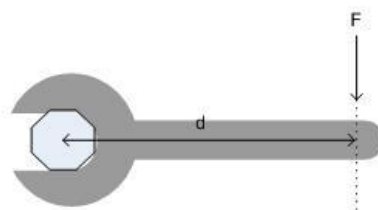
Introduction

A moment is a 'turning force'. For example, consider a spanner and a nut; a downward force on the handle produces a turning force on the nut.



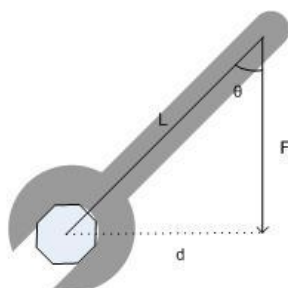
Calculating Moments

A moment is defined as the force applied (F) multiplied by the perpendicular distance (d) from the line of action of the force to the pivot.



$$\text{Moment} = F.d$$

If the line of action of the force isn't perpendicular to the spanner handle, then the perpendicular distance will need to be calculated. For example:



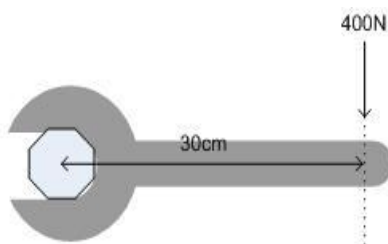
$$d = L \sin \theta$$
$$\text{Moment} = F L \sin \theta$$

Units of Measurement

The force is measured in Newtons (N), the distance in metres (m), and the moment in Newton metres (Nm).

Example 1

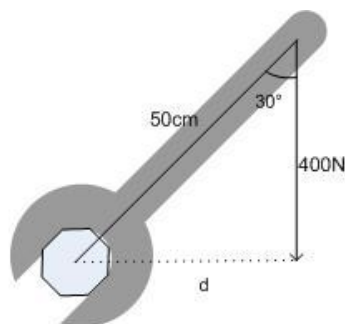
A spanner is used on a nut. The spanner is 30cm long, used horizontally, and has a force of 400N applied vertically. Calculate the moment.



It is first necessary to convert the distance into metres, so $d = 0.3\text{m}$

$$\begin{aligned}\text{Moment} &= F.d \\ &= 400\text{N} \times 0.3\text{m} \\ &= 120\text{Nm}\end{aligned}$$

The apprentice believes he will have better luck moving the nut if he uses a longer spanner. He chooses one 50cm in length, and uses it at an angle of 30° to the vertical. Is he any better off? (The same vertical force is applied.)



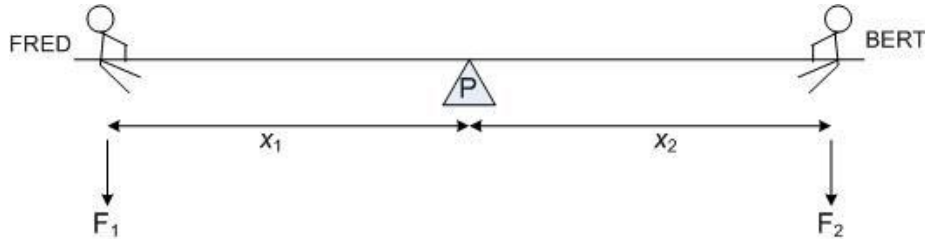
First calculate the perpendicular distance.
 $d = 0.5\text{m} \times \sin 30^\circ$

$$\begin{aligned}\text{Moment} &= F.d \\ &= 400\text{N} \times 0.5\text{m} \times \sin 30^\circ \\ &= 100\text{Nm}\end{aligned}$$

So in fact the turning force is less, even though the spanner is bigger.

See-saws

We use a similar idea of moments when dealing with see-saws. Here moments are taken about a pivot, P.



In the diagram above,

$$\text{Anti-clockwise moment} = F_1 x_1$$

$$\text{Clockwise moment} = F_2 x_2$$

If Fred and Bert are to play together nicely, then the see-saw should balance. This would require the clockwise moments and the anti-clockwise moments to cancel each other out, or in other words, the moments would be of equal magnitude.

In this case, equilibrium would occur if

$$F_1 x_1 = F_2 x_2$$

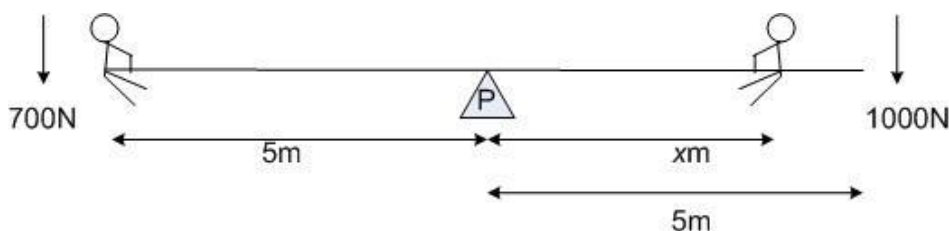
At equilibrium, the sum of all the clockwise moments is equal to the sum of all the anticlockwise moments. Expressed mathematically, we have

$$\sum M_{Cw} = \sum M_{Acw}$$

where M_{Cw} represent the clockwise moments, and M_{Acw} represent the anti-clockwise moments.

Example 2

A monkey is sitting on the end of a 10m long see-saw. The monkey weighs 700N. A second monkey weighing 1000N decides to play on the see saw. How far from the end must he sit so that the see-saw will balance? Assume that the pivot of the see saw is in the centre.



$$\begin{aligned}\text{Clockwise moments} &= x \times 1000 = 1000x \text{ Nm} \\ \text{Anti-clockwise moments} &= 700 \times 5 = 3500 \text{ Nm}\end{aligned}$$

For equilibrium, Clockwise moment = Anti-clockwise moment

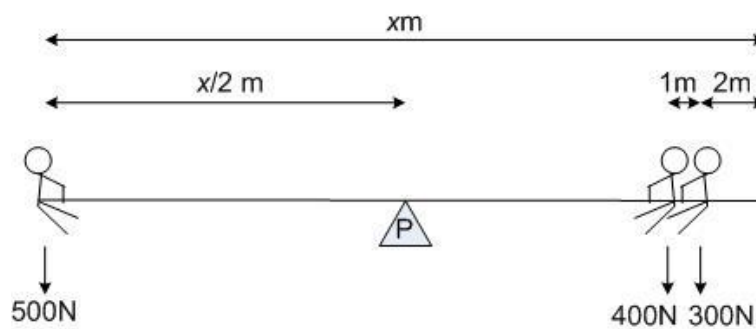
$$1000x = 3500$$

$$x = 3500/1000 = 3.5\text{m}$$

This means that the second monkey must sit 1.5m from the end (since $3.5\text{m} + 1.5\text{m} = 5\text{m}$ i.e. the length of the see-saw).

Example 3

A see-saw is x metres in length. A monkey weighing 500N sits on one side, on the other side, one monkey weighing 300N sits 2m from the end, and another weighing 400N sits 3m from the end. Assuming that the see saw is pivoted in the centre, and that it is in equilibrium, find the length of the see saw.



$$\text{Anticlockwise moments} = 500 \times x/2 = 250x \text{ Nm}$$

$$\begin{aligned}\text{Clockwise moments} &= 300 \times (x/2 - 2) + 400 \times (x/2 - 1) \\ &= 150x - 600 + 200x - 400 \\ &= (350x - 1000) \text{ Nm}\end{aligned}$$

For equilibrium, Clockwise moments = Anti-clockwise moments

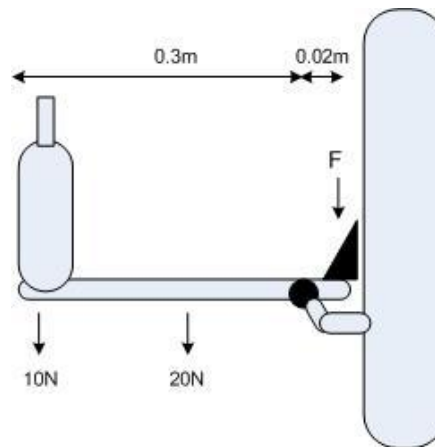
$$\begin{aligned}350x - 1000 &= 250x \\ 350x - 250x &= 1000 \\ 100x &= 1000\end{aligned}$$

$$x = 1000/100 = 10\text{m}$$

So the length of the see saw is 10m.

Example 4

A drop down table, as you might find on a train or plane, supports a bottle of wine.



Calculate the force applied by the strut (F) for the table to be used as in the diagram. Assume that the force is applied at the inner edge of the table, which is 0.02m from the pivot, and that the table weight of 20N acts through the centre of the table.

$$\text{Clockwise moments} = F \times 0.02 = 0.02 F \text{ Nm}$$

$$\text{Anti-clockwise moments} = 10 \times 0.3 + 20 \times 0.15 = 6\text{Nm}$$

To maintain equilibrium, Clockwise moments = Anti-clockwise moments

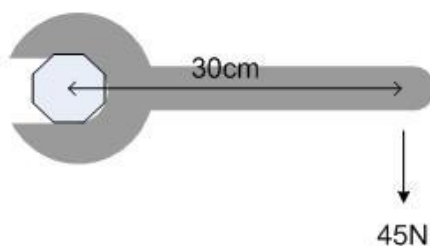
$$0.02 F = 6$$

$$F = 6/0.02 = 300\text{N}$$

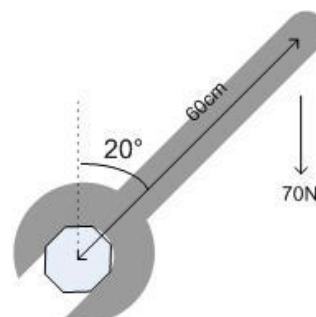
Exercises

1. An apprentice wishes to loosen a nut. Which option provides the greatest moment?

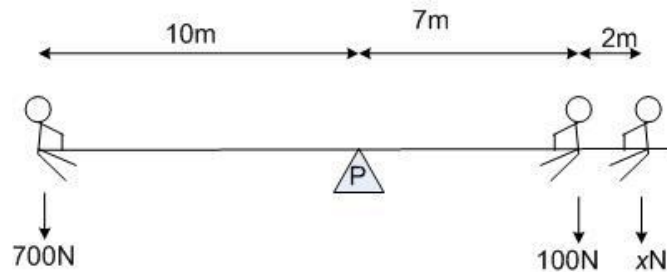
i)



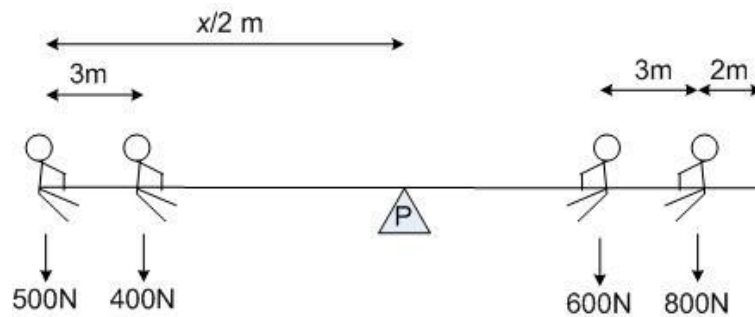
ii)



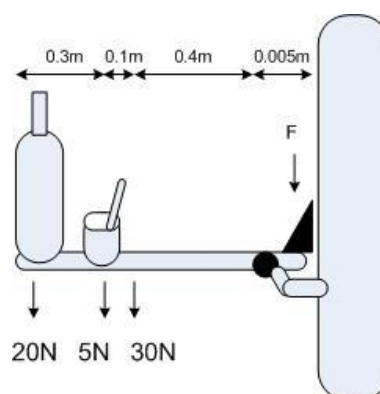
2. A tower crane is lifting a load of 2000N. Its jib is 100m long and weighs 300N, acting through the centre. The counter balance weight is placed 10m from the pivot on the other side of the crane. Calculate the required counter balance weight so that the crane does not topple over.
3. i) Calculate the weight of the third monkey so that the see-saw balances.



- ii) Find the length of the see-saw, assuming the see-saw balances.



4. Find the force on the spigot (F).



Solutions

1. Option 1:

$$\text{Moment} = F.d = 45\text{N} \times 0.3\text{m} = 13.5\text{Nm}$$

Option 2:

$$d = 0.6\text{m} \times \sin 20^\circ = 0.205\text{m}$$

$$\text{Moment} = F.d = 70\text{N} \times 0.6 \times \sin 20^\circ = 14.36\text{Nm}$$

So option 2 provides the greatest moment.

2. Moments of weight on jib = $2000\text{N} \times 100\text{m} + 300\text{N} \times 50\text{m} = 215,000\text{Nm}$

$$\text{Moments of counterbalance of jib} = x\text{N} \times 10\text{m} = 10x \text{ Nm}$$

$$\begin{aligned}\text{At equilibrium:} \quad 10x &= 215,000 \\ \text{So } x &= 21,500\text{N}\end{aligned}$$

The weight of the counterbalance required is 21,500N

3. i) Clockwise moments = $100\text{N} \times 7\text{m} + x\text{N} \times 9\text{m} = (700 + 9x) \text{ Nm}$

$$\text{Anti-clockwise moments} = 700\text{N} \times 10\text{m} = 7000 \text{ Nm}$$

$$\begin{aligned}\text{At equilibrium:} \quad 700 + 9x &= 7000 \\ \text{So } 9x &= 6300 \\ \text{And } x &= 700\end{aligned}$$

So the weight of the third monkey is 700N.

$$\begin{aligned}\text{ii) Clockwise moments} &= 600\text{N} \times (x/2 - 5)\text{m} + 800\text{N} \times (x/2 - 2)\text{m} \\ &= (700x - 4600) \text{ Nm}\end{aligned}$$

$$\begin{aligned}\text{Anti-clockwise moments} &= 500\text{N} \times x/2 \text{ m} + 400\text{N} \times (x/2 - 3)\text{m} \\ &= (450x - 1200) \text{ Nm}\end{aligned}$$

$$\begin{aligned}\text{At equilibrium:} \quad 700x - 4600 &= 450x - 1200 \\ \text{So } 250x &= 3400 \\ \text{And } x &= 13.6 \text{ m}\end{aligned}$$

4. Clockwise moments = $0.005 \times F \text{ Nm}$

$$\begin{aligned}\text{Anti-clockwise moments} &= 30\text{N} \times 0.4\text{m} + 5\text{N} \times 0.5\text{m} + 20\text{N} \times 0.8\text{m} \\ &= 12 + 2.5 + 16 = 30.5 \text{ Nm}\end{aligned}$$

$$\begin{aligned}\text{At equilibrium:} \quad 0.05F &= 30.5 \\ \text{So } F &= 30.5/0.05 = 610 \text{ N}\end{aligned}$$