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School of Engineering

Con-Rod Accelerations, Forces & Displacements

Advanced Engine Design

(M6X00410)

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1. Introduction

I. Aims

- Measure and model the Ford 1.6 GDI Piston/Connecting Rod assembly (moving masses)
- Interpret cylinder pressure data and apply relevant calculations for Connecting Rod analysis
- Theoretically increase engine speed to determine the effects on the Connecting Rod

II. Objectives

The original GDI pressure data will be processed in Microsoft Excel. Calculations that are relevant to the connecting rod (forces, accelerations and displacements) will be applied to the mean cylinder pressure across an entire engine cycle. The resulting data will be exported to MATLAB where 3D plots can be produced to aid in analysis and interpretation. The focus of this investigation will be on the effects of inertia loads through increasing the engine speed past that of the production engine. Finally, a Titanium connecting rod replacement will be applied from theory, to the model, to further understand the effects of different materials.

2. Reciprocating Assembly - Measurements & 3D Modelling

The information in *table 1* was obtained through physical measurements and Ford datasheets (Ford, 2019).

<i>Component</i>	<i>Measurement</i>	<i>Figure</i>	<i>Units</i>
Cylinder	<i>Bore</i>	0.079	<i>m</i>
	<i>Stroke</i>	0.0814	<i>m</i>
Crankshaft	<i>Rod Pin Radius</i>	0.0407	<i>m</i>
Connecting Rod	<i>Rod Length (bearings, C to C)</i>	0.134	<i>m</i>
Piston	<i>Piston Mass (without pin)</i>	0.2517	<i>kg</i>
	<i>Gudgeon Pin Mass</i>	0.08	<i>kg</i>
	<i>Piston Ring Mass (combined)</i>	0.016	<i>kg</i>
	<i>Piston Assembly mass (total)</i>	0.3477	<i>kg</i>

Table 1 - Ford 1.6 GDI Measurements

A model of the reciprocating assembly was then created in Solidworks.

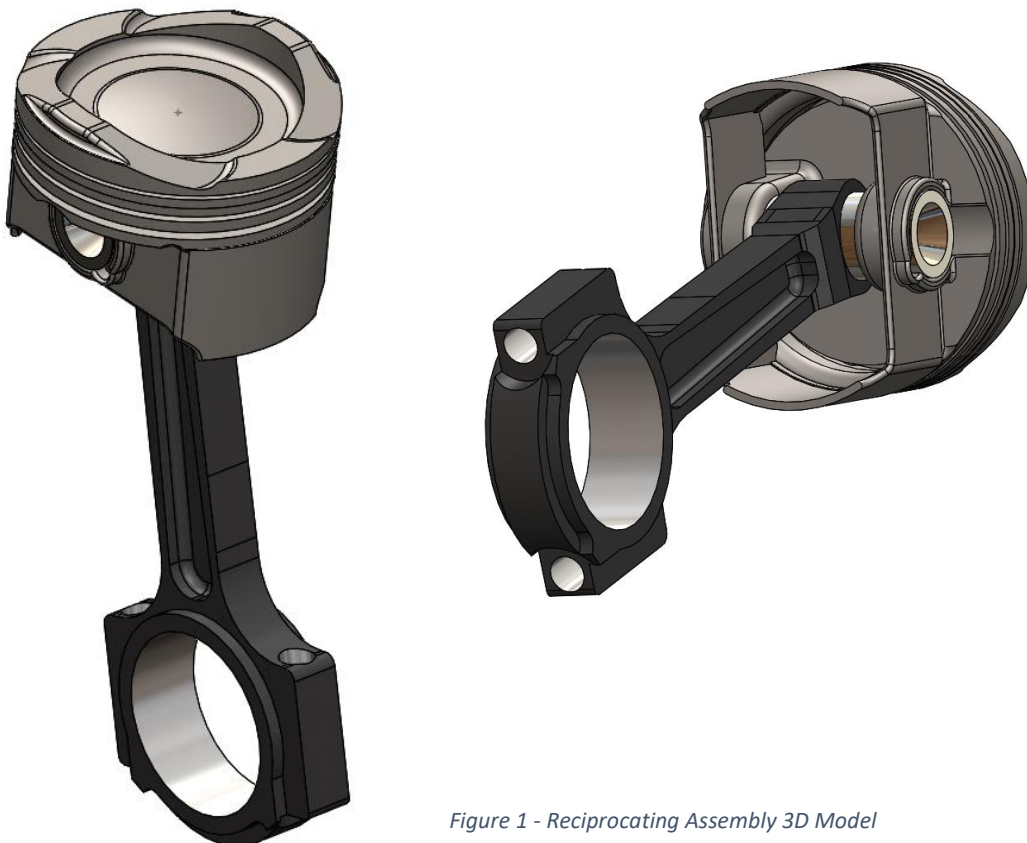


Figure 1 - Reciprocating Assembly 3D Model

3. Modified Engine Characteristics

The calculations during this investigation will compare the standard production GDI engine with a theoretically modified version. The production engine pressure data ceases at 6000rpm.

In the case of the theoretically modified engine, the maximum engine speed will be increased to 9000rpm (in 1000rpm increments) with no further engine modifications being made. The proposed modification would never be implemented to a real engine without also increasing the cylinder pressures e.g. increasing turbo pressure, fuel quantity, improvements to cylinder head/valvetrain. However, the theoretical model guarantees a single controlled variable is manipulated, which in turn provides the investigation with a more representative comparison to the sole effects of engine speed on the connecting rod.

I. Peak Pressure in Modified Engine

Figure 2 was generated to illustrate the location of peak average gas pressures per engine speed for the production engine. It can be observed that overall peak pressure occurs at 6000rpm. A 2nd order polynomial curve was applied to extend the trend to 9000rpm. The resultant trend is reasonable considering no other engine modifications have been employed.

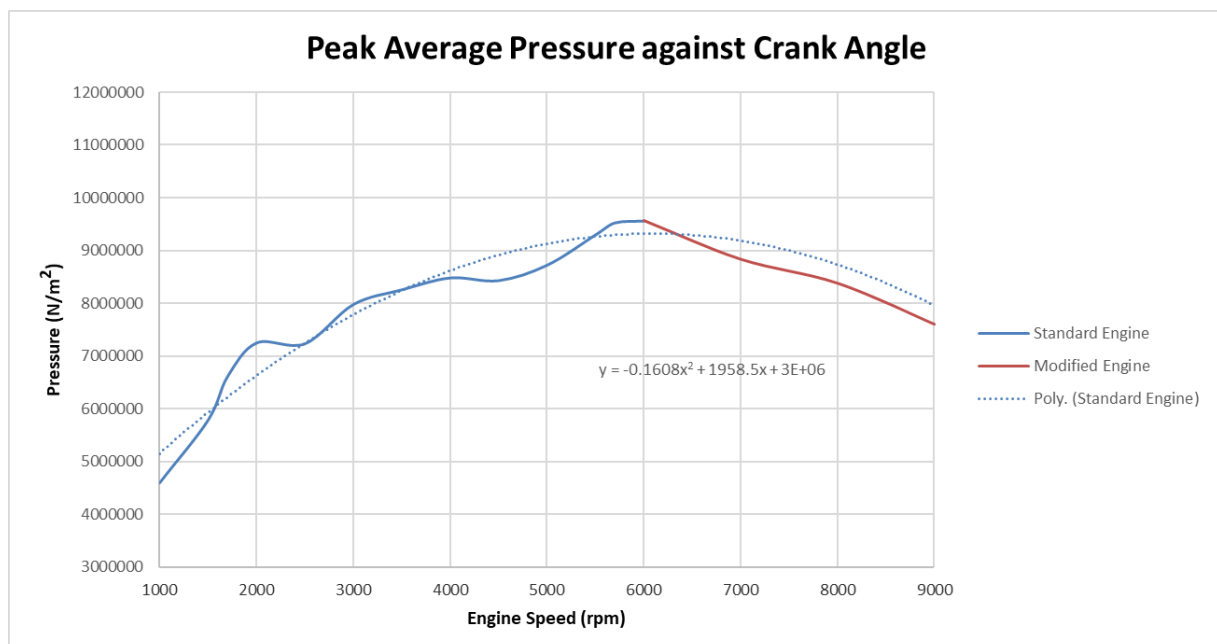


Figure 2 - Peak Average Pressure against Crank Angle

4. Gas Forces

Cylinder pressure is applied equally in all directions. The resultant pressure pushing down on the piston, and in turn through the rod (compression), can be assumed to act across the area of the cylinder bore. This accounts for the pressure being exerted onto the piston rings, as well as the piston crown. A very small amount of gas pressure will apply to the side of the piston (between ring inner edge and piston ring groove) but is so small in magnitude it can be neglected in this instance.

From first principles,

$$Pressure = \frac{Force}{Area}$$

Therefore,

$$F_{gas} = \frac{P_{ave} \times \pi \times B^2}{4}$$

Where F_{gas} = Instantaneous Gas Force (N), P_{ave} = Average Cylinder Pressure (N/m^2),

B = Cylinder Bore Diameter (m)

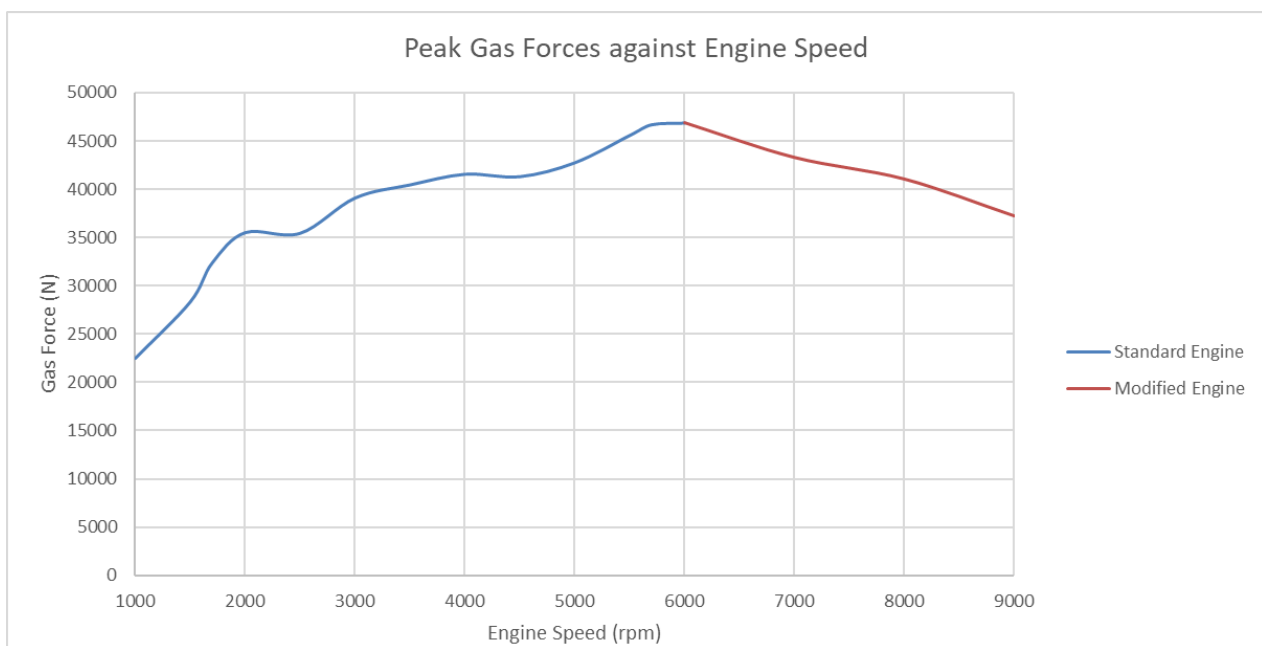


Figure 3 - Peak Gas Forces against Engine Speed

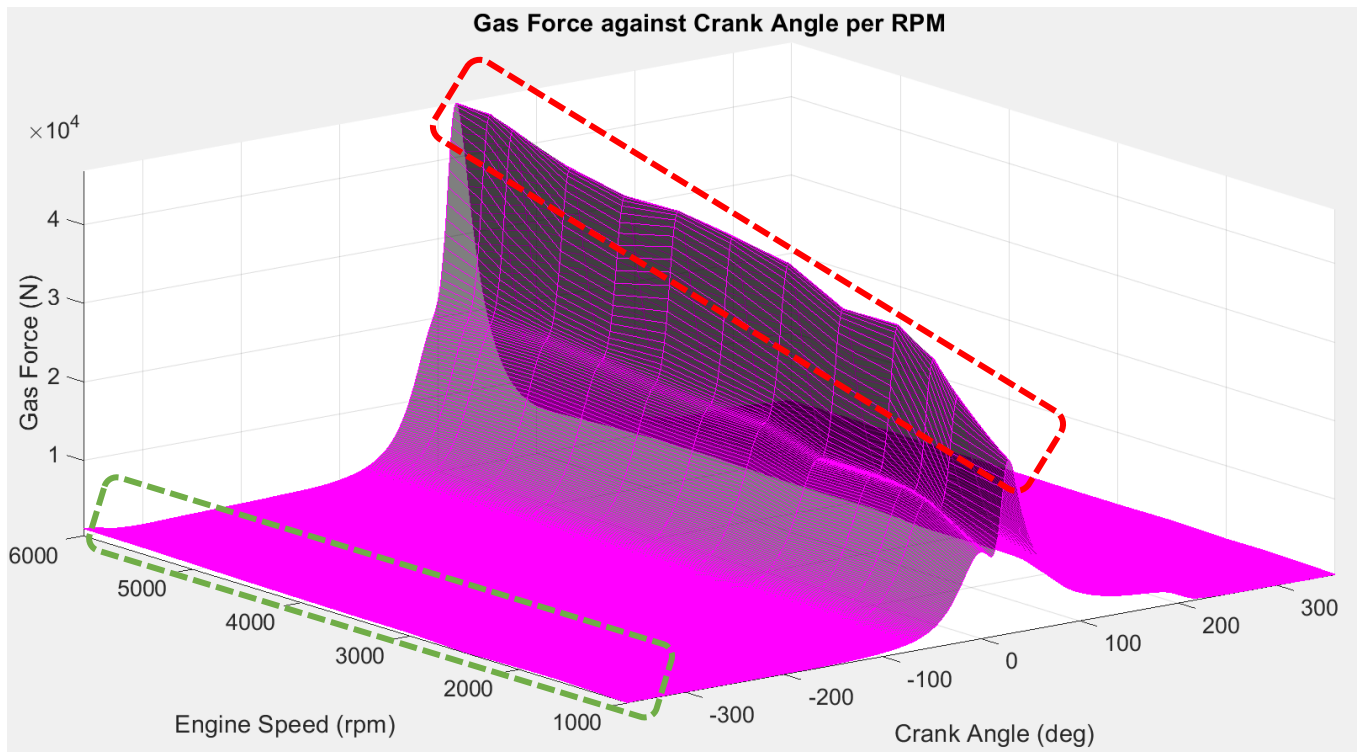


Figure 4 - Gas Force against Crank Angle per RPM

- Peak gas forces (---) act after Top Dead Centre (TDC) on the combustion stroke.
- The smallest gas force (---) can be found close to TDC on the exhaust stroke.

Figure 3 shows the modified engine decreases in gas force as expected due to no additional gas force compensation

5. Inertia Forces

From Newtons second law of motion for constant mass systems,

$$Force = m \frac{\Delta v}{\Delta t} = ma$$

Where m = mass, v = Velocity, t = Time, a = Acceleration

The constant mass component of this equation is the piston assembly.

I. Piston Acceleration

$$a_{piston} = R\omega^2 \cos\theta + \frac{R}{L} \times R\omega^2 \cos 2\theta$$

Where a_{piston} = Piston Acceleration (m/s^2), ω = Angular Velocity (Rad/s)

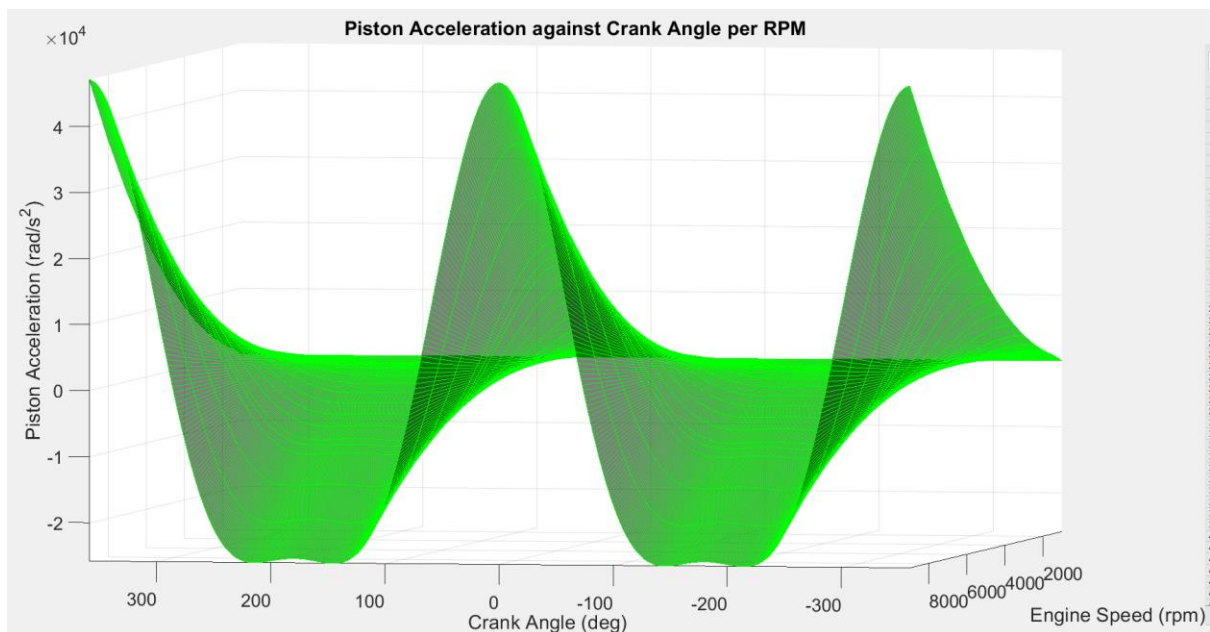


Figure 5 - Piston Acceleration against Crank Angle per RPM

II. 1st Order

The inertia forces can be separated into two primary components, 1st and 2nd order. 1st order Inertia Forces are a function of crank angle and take place at engine speed. These forces can be balanced at the crankshaft through counterweights.

$$1^{st} Order = R\omega^2 \cos\theta \times M_{piston}$$

Where M_{piston} = Piston Mass (kg)

The first order component derives from the first term in the piston acceleration equation and is multiplied by the piston mass to become a force.

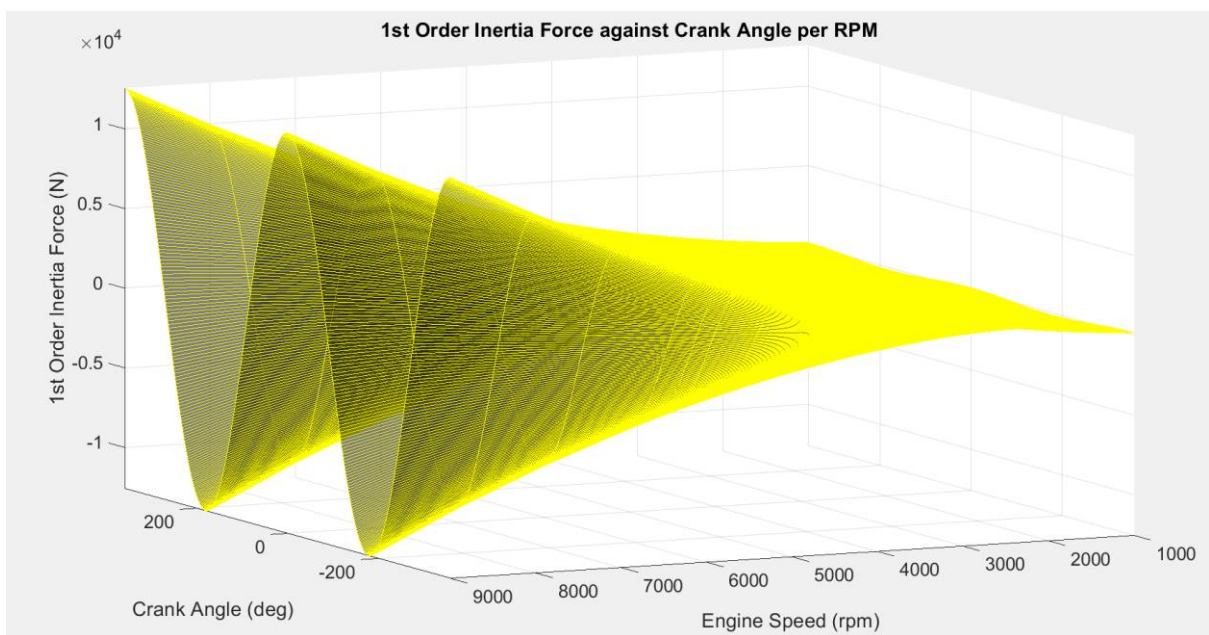


Figure 6 - 1st Order Inertia Force against Crank Angle per RPM

III. 2nd Order

2nd order inertia forces are a function of Connecting Rod angle and take place at twice engine speed. These forces are in phase and must be balanced with counterrotating shafts (relative to engine rotation), at twice engine speed (Stone, 2012).

$$2^{nd} Order = \frac{R}{L} \times R\omega^2 \cos 2\theta \times M_{piston}$$

The 2nd order component derives from the second term in the piston acceleration equation and is multiplied by the piston mass to become a force.

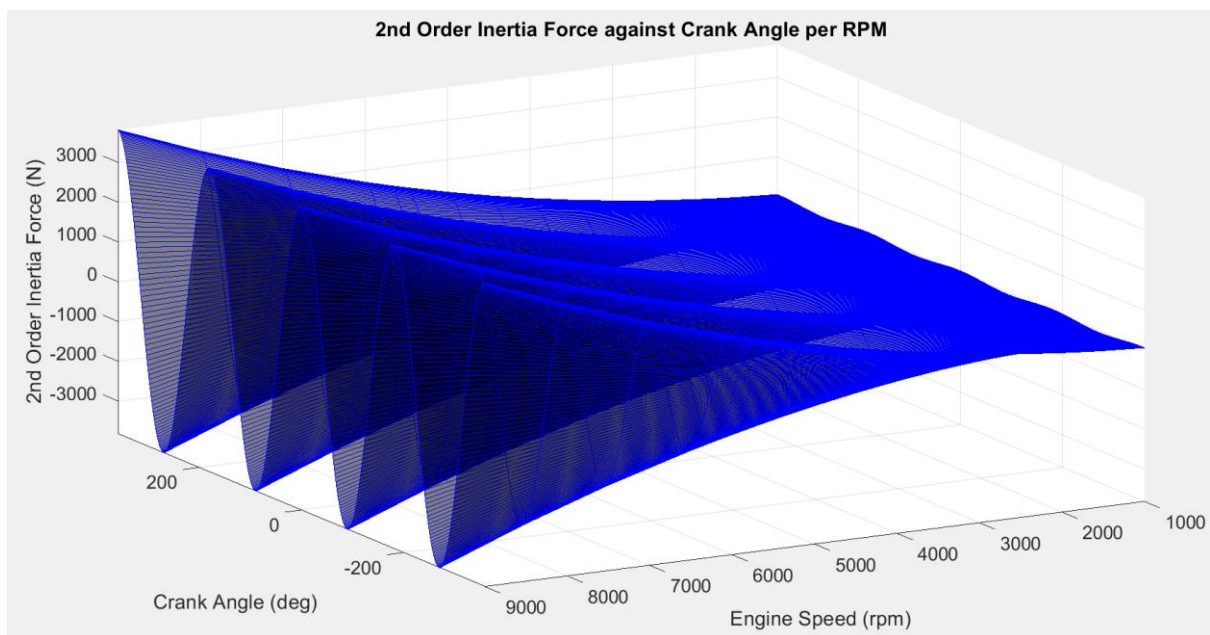


Figure 7 - 2nd Order Inertia Force against Crank Angle per RPM

IV. Separated Inertia Forces



Figure 8 - 1st and 2nd Order Inertia Forces against Crank Angle per RPM

Overlaying both inertia traces onto one plot provides evidence of the different frequencies and amplitudes they operate at.

V. Combined Inertia Force Components

Amalgamating both 1st and 2nd order Inertia Forces into one equation gives the resultant Inertia Force acting on the Connecting Rod.

Since,

$$F_{Inertia} = a_{piston} \times M_{piston}$$

Then,

$$F_{inertia} = R\omega^2 \cos\theta + \frac{R}{L} \times R\omega^2 \cos 2\theta \times M_{piston}$$

Where $F_{inertia}$ = Inertia Force (N)

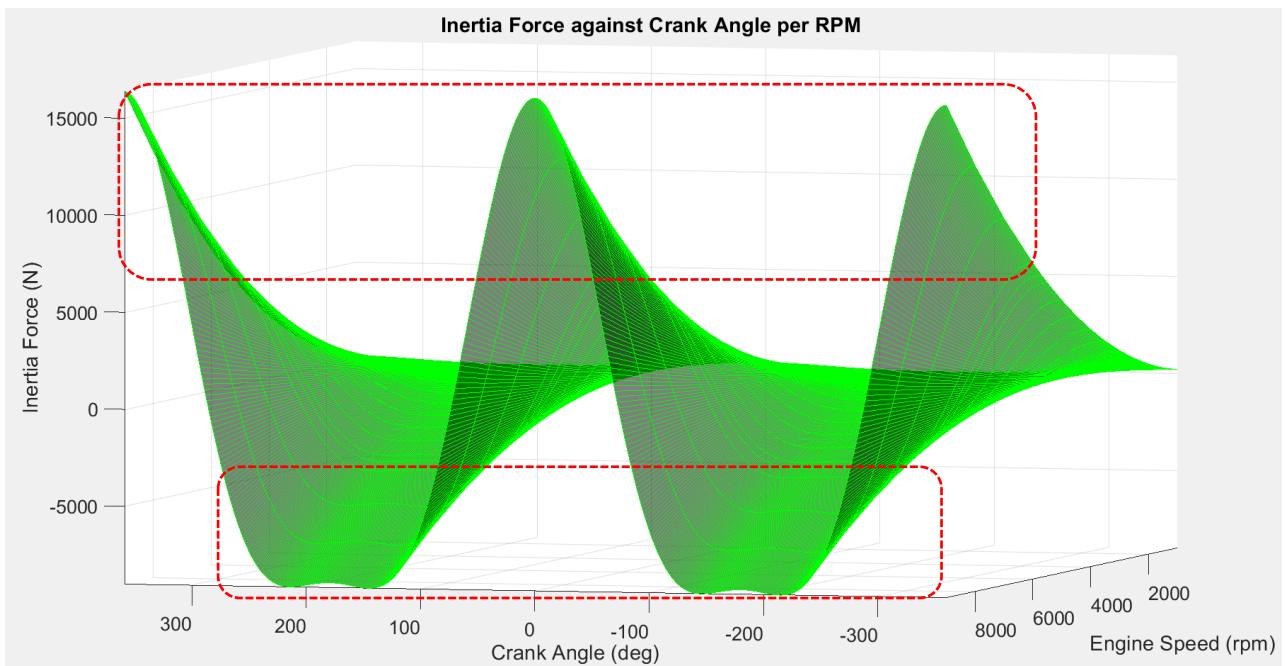


Figure 9 - Inertia Force against Crank Angle per RPM

Figure 9 Proves Inertia loads are a function of piston accelerations. Peak inertia forces acting on the Connecting rod arise at TDC. The modified engine (---) sees the largest rate of change overall. This demonstrates a nonlinear trend in inertia loads through engine speed which in turn confirms that a reduction in reciprocating mass is the primary constituent for reducing inertia loads at high RPM.

6. Net Forces

The net forces acting on the Connecting Rod can be calculated by subtracting the Inertia Loads from the Gas Forces.

$$F_{Net} = F_{gas} - F_{inertia}$$

Where F_{net} = Net Force

For a more accurate representation, the Connecting Rod angle (Phi) should be included.

For Phi,

$$\varphi = \sin^{-1} \left(\sin \theta \times \frac{R}{L} \right)$$

Where φ = Connecting Rod Angle ($^{\circ}$)

Therefore,

$$Fr_{net} = \frac{F_{net}}{\cos \varphi}$$

Where Fr_{net} = Net force resolved with respect to Connecting Rod Angle

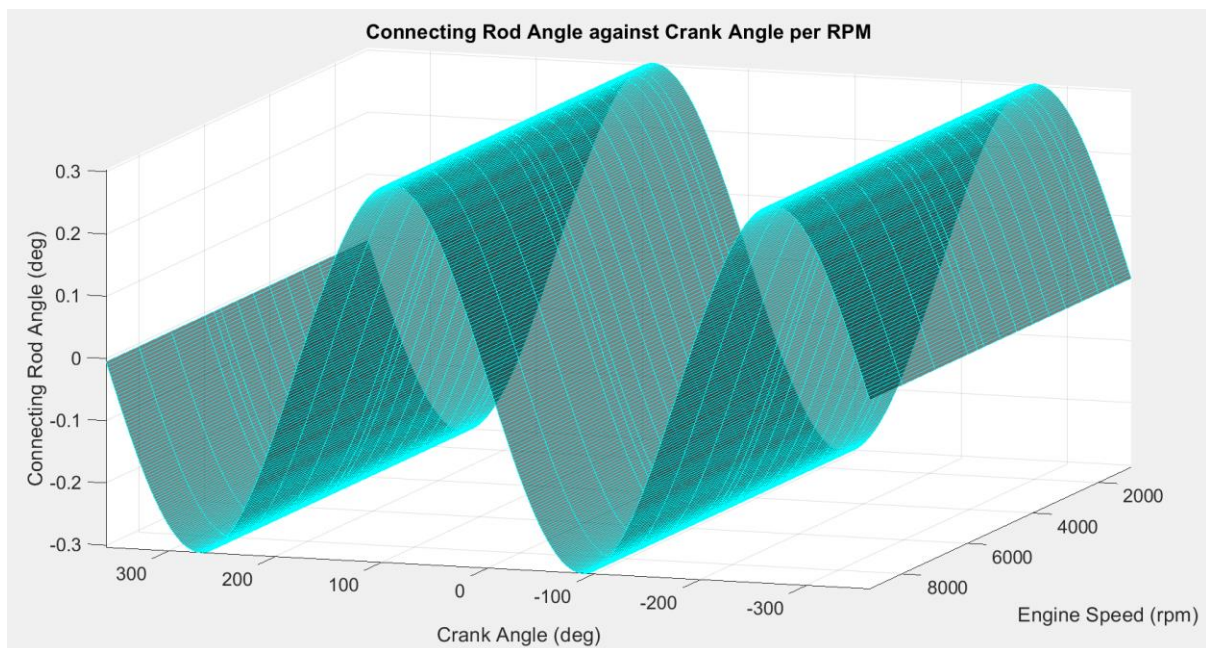


Figure 10 - Connecting Rod Angle against Crank Angle per RPM

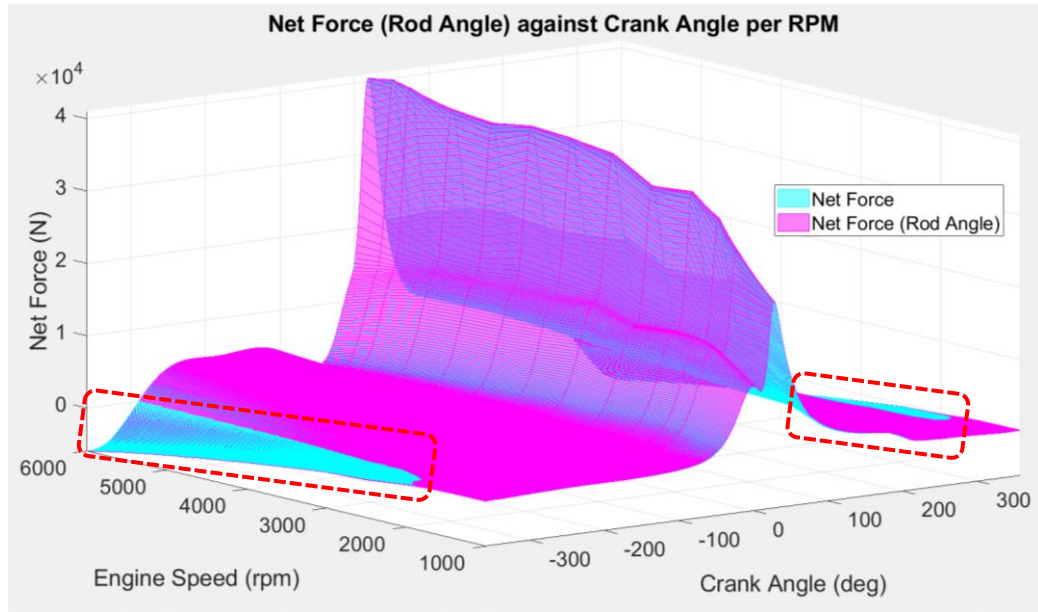


Figure 11 - Net Force (Rod Angle) against Crank Angle per RPM

Figure 11 illustrates the effect of rod angle on net force through the engine speed range of the production engine. Rod angles (φ) offset the net force, which increases net compressive and tensile loads around TDC. This is exaggerated with an increased engine speed (---). Although φ does influence the net forces, it must be noted that the maximum influence it has is only **197.5N @ 6000rpm**. This translates to an expansion of only **0.5%**.

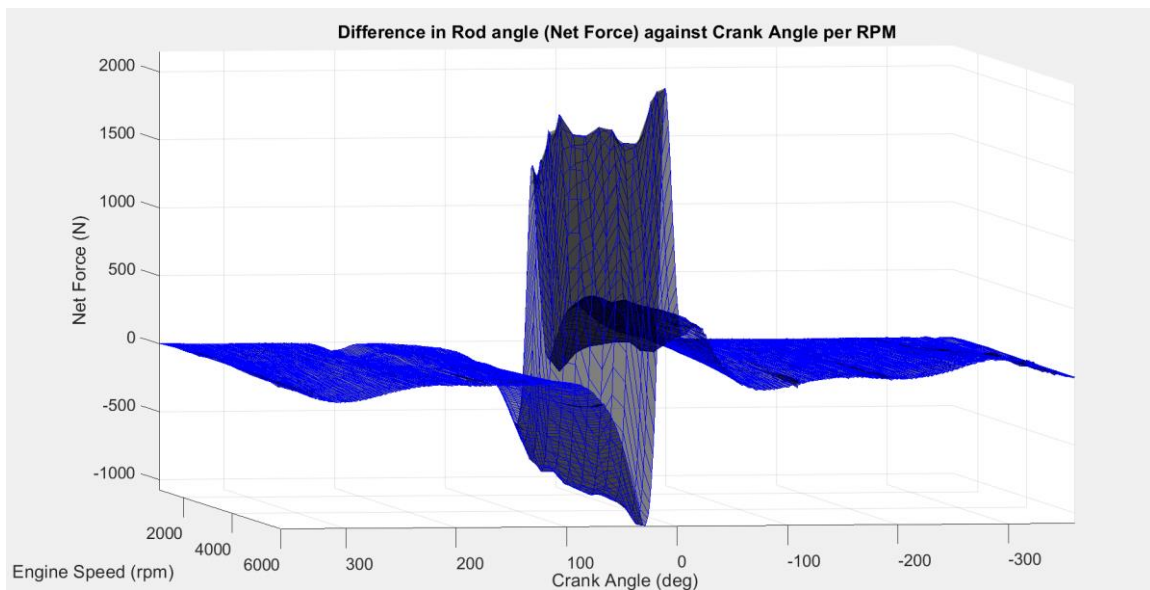


Figure 12 - Difference in Rod Angle (Net Force) against Crank Angle per RPM

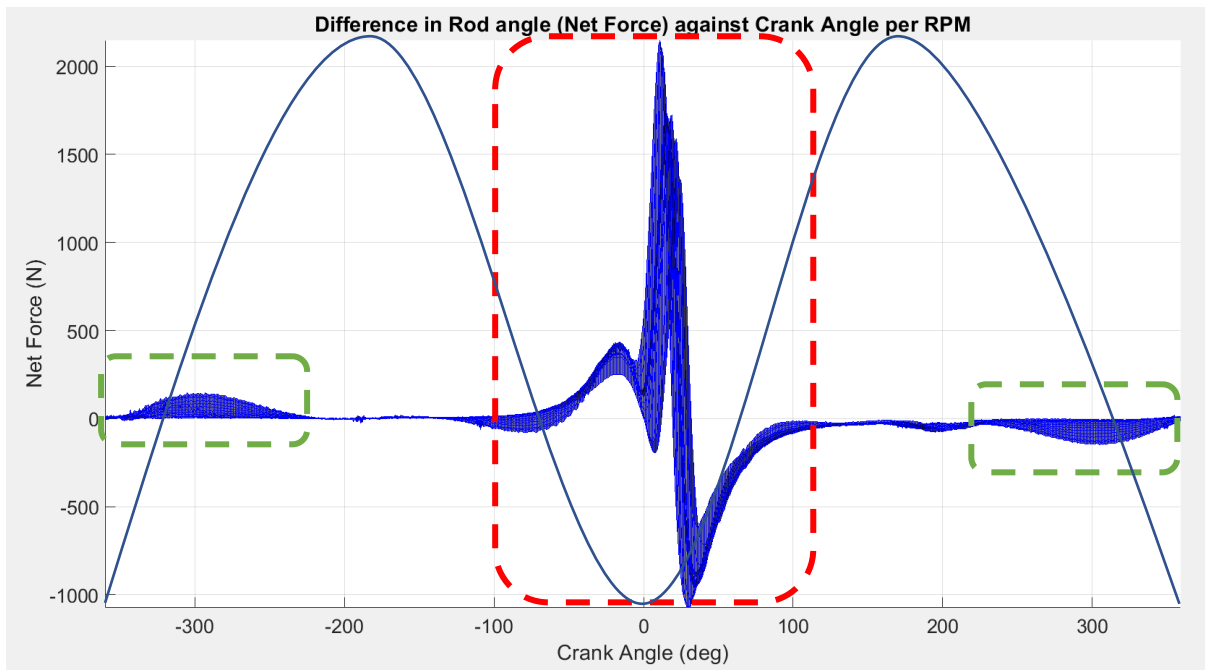


Figure 13 - Difference in Rod Angle (Net Force) against Crank Angle per RPM - XY View

Figures 12 and 13 Show the rate of change of net forces applied (resolved with respect to Connecting Rod angle). This further proves Cylinder pressure (---) has the greatest effect on net forces, whereas piston Inertia loads (---) contribute to a small amount around TDC exhaust. The change in direction of the piston (superimposed) can also be perceived from the trend.

I. Peak Net Compressive Force

The peak net compressive force acting on the Connecting Rod is found at the same point as peak gas force, for every measured engine speed. This is due to the gas force being far greater in magnitude than any opposing inertia forces during compression.

The position of these forces in terms of crank angle can be predicted from linear extrapolation.

Figure 14 illustrates a reasonable trend, where maximum peak pressure still occurs 15° ATDC.

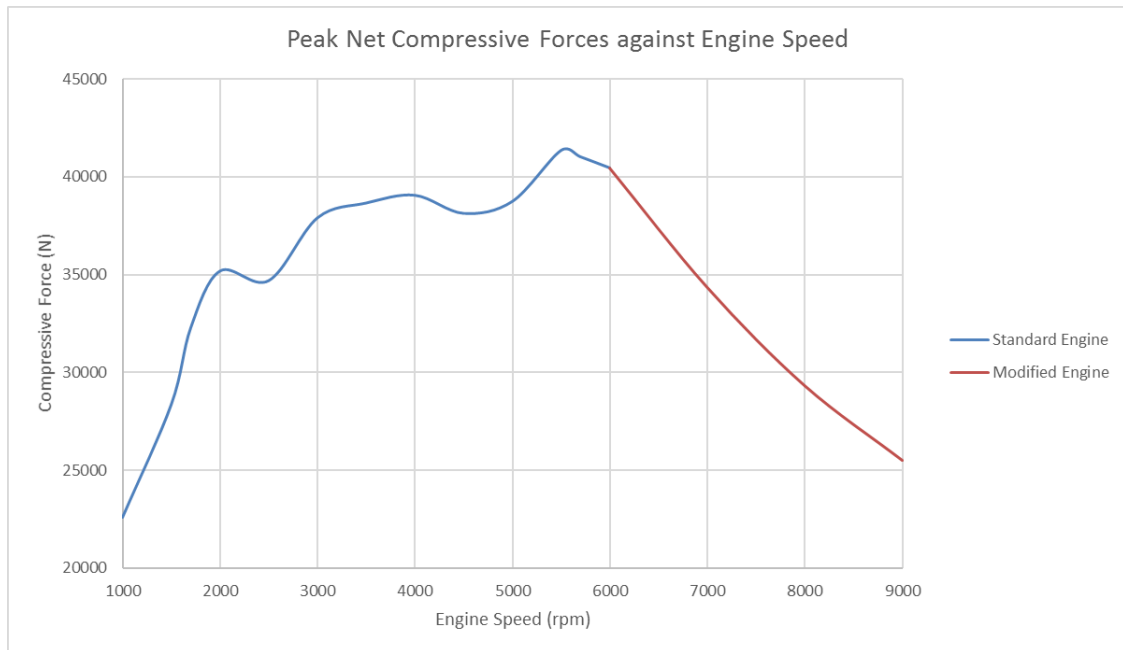


Figure 14 - Peak Net Compressive Force Position (Due to Ignition Advance)

Figures 14 and 15 show the overall Peak Net Compressive Force is **41326.64N @ 5500rpm** (21° ATDC). The peak net compressive forces in the modified engine decrease as expected due to having no cylinder pressure compensation.

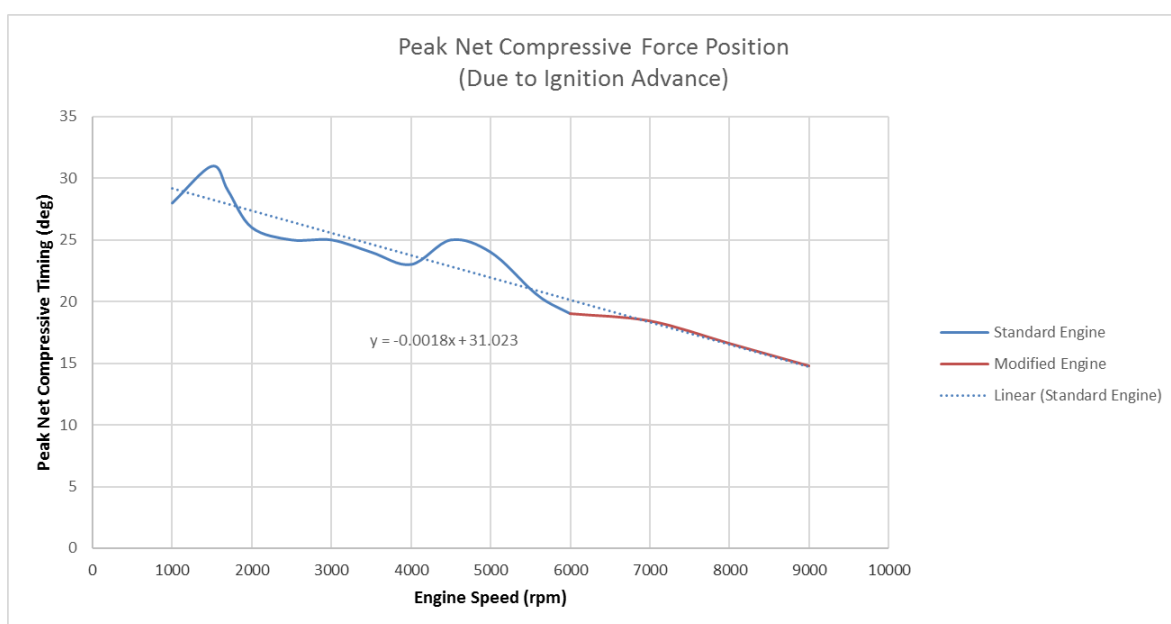


Figure 15 - Peak Net Compressive Forces against Engine Speed

II. Peak Net Tensile Load

The Peak Net Tensile Load acts around TDC on exhaust stroke at every measured engine speed. The point in where the largest inertia load is exerted is at exactly TDC on each cycle. However, due to residual gas forces remaining in the cylinder, the net tensile load can be a product of cyclic pressure variation throughout the engine speed range (Manning, 2012).

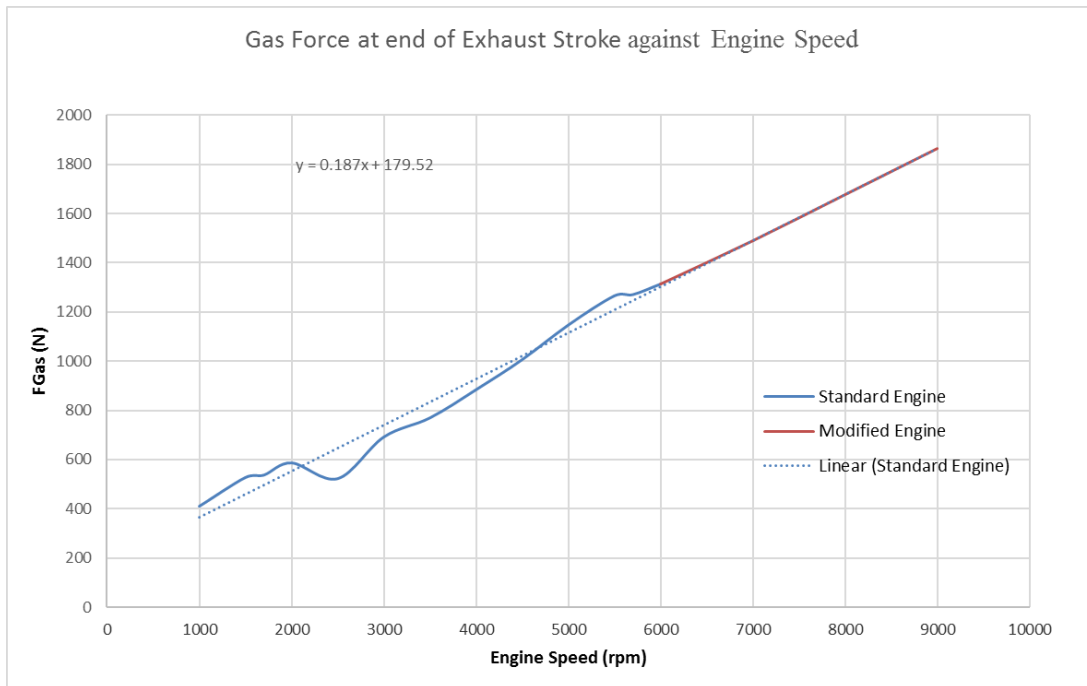


Figure 16 - Gas Force at end of Exhaust Stroke against Engine Speed

The assumption can be made that in the modified engine, peak inertia forces act at TDC on exhaust stroke. This prediction is validated from extrapolating the standard engine gas forces where Peak Net Tensile Loads occur, as shown in Figure 16.

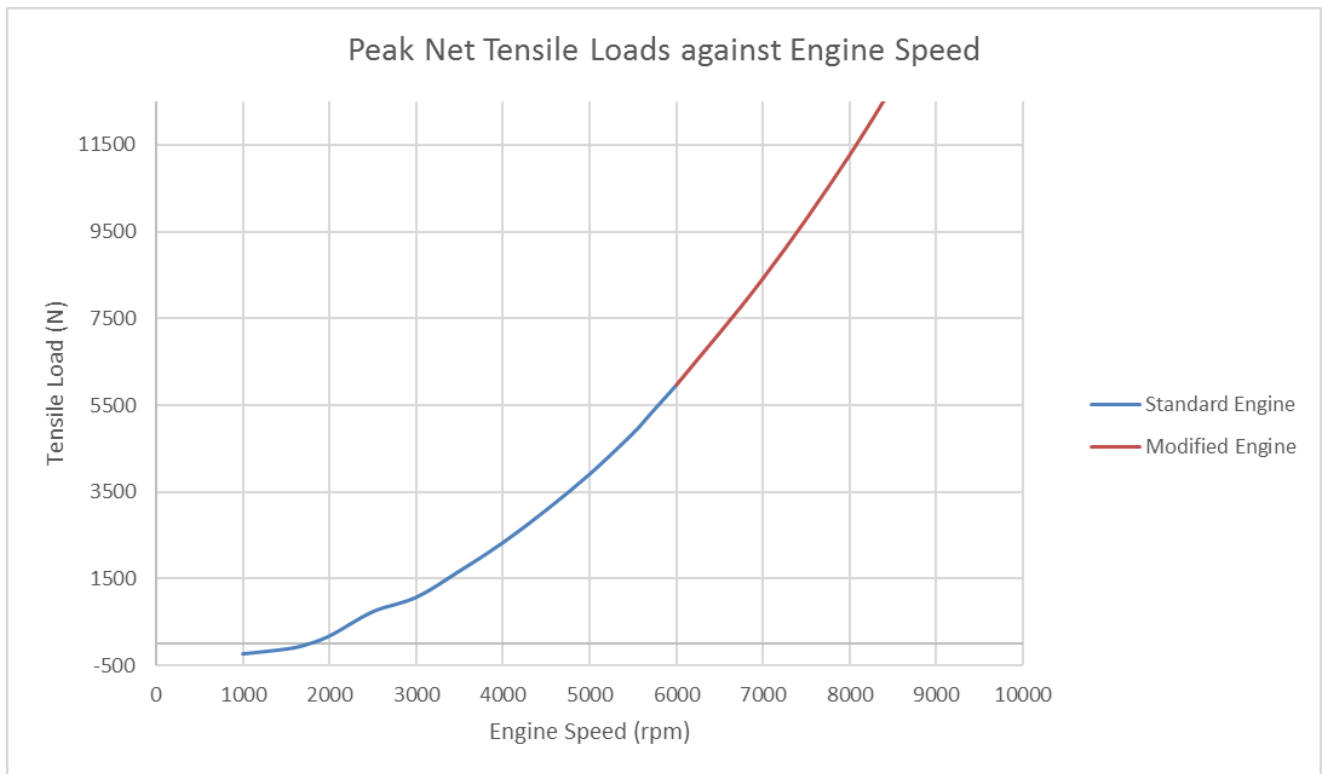


Figure 17 - Peak Net Tensile Loads against Engine Speed

A. Standard Engine

Peak Net Tensile Load is **5963.62N**@ 6000rpm (2° ATDC Exhaust).

B. Modified Engine

Peak Net Tensile Load is **14525.61N** @ 9000rpm (TDC Exhaust). This is anticipated due to the increase in piston acceleration (Pulkrabek, 2004).

7. Stress Analysis

The Solidworks model was used to find the minimum cross-sectional area (**136.26mm**) of the Connecting rod shank, to apply both Compressive and Tensile Loads.

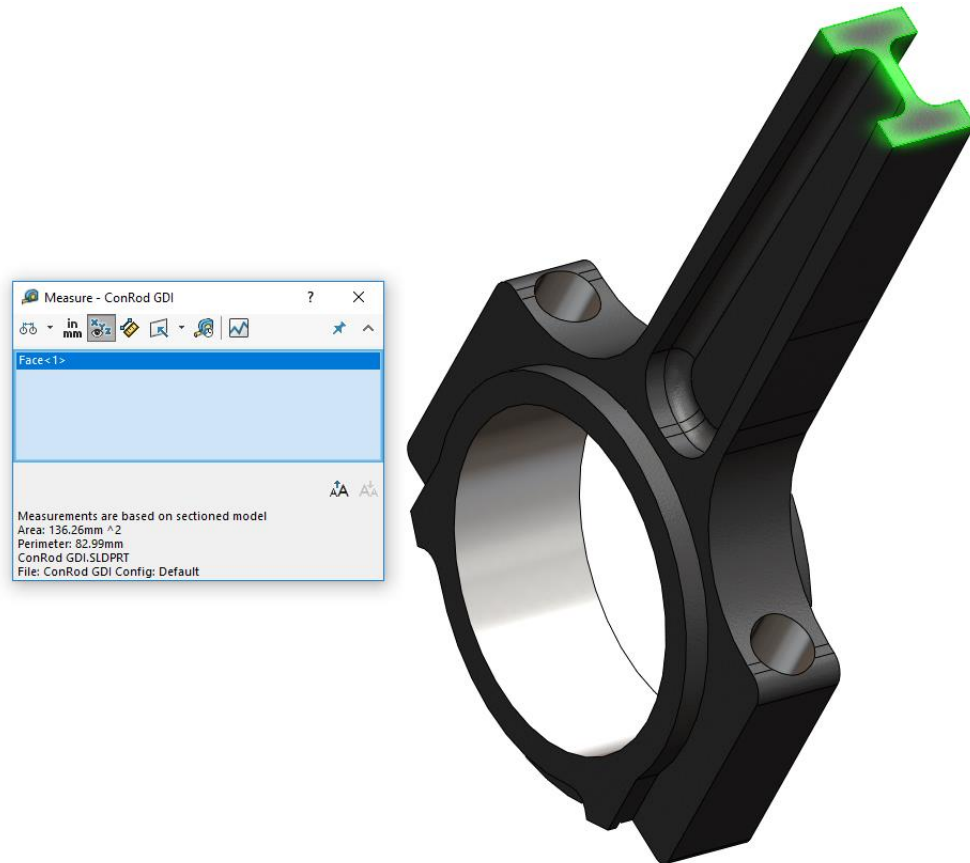


Figure 18 - 3D Model Cross Section

Since,

$$\sigma = \frac{\text{Force}}{\text{Area}}$$

Where σ = Stress (Pascal or N/m^2)

Then,

$$\sigma_c = \frac{F_c}{A_{rod}} \quad \text{and} \quad \sigma_t = \frac{F_t}{A_{rod}}$$

Where F_c = Max Compressive Force (N), F_t = Max Tensile Load (N),

A_{rod} = Minimum Cross Sectional Area of Connecting Rod

I. Safety Factors

The Static safety factor is a design feature used to combat the effect of individual peak cylinder torques, whilst protecting the Connecting Rod in such eventualities as a detonation event. An increased safety factor also increases the life of the product by reducing component fatigue (Leela Krishna Vegi, 2013).

The safety factor (ratio) is calculated from,

$$SSF = \frac{\sigma}{UTS}$$

II. Compressive Stress



Figure 19 - Peak Compressive Stress and Static Safety Factors against Engine Speed

The peak compressive stress exerted on the Connecting Rod is **303.29 N/mm² @ 5500rpm (21° ATDC)**.

The Ford 1.6 GDI has a nominal static safety factor of **2.47** in compression. The Compressive stresses from the modified engine reduce over 6000rpm, as expected, where no additional pressure is acting.

III. Tensile Stress

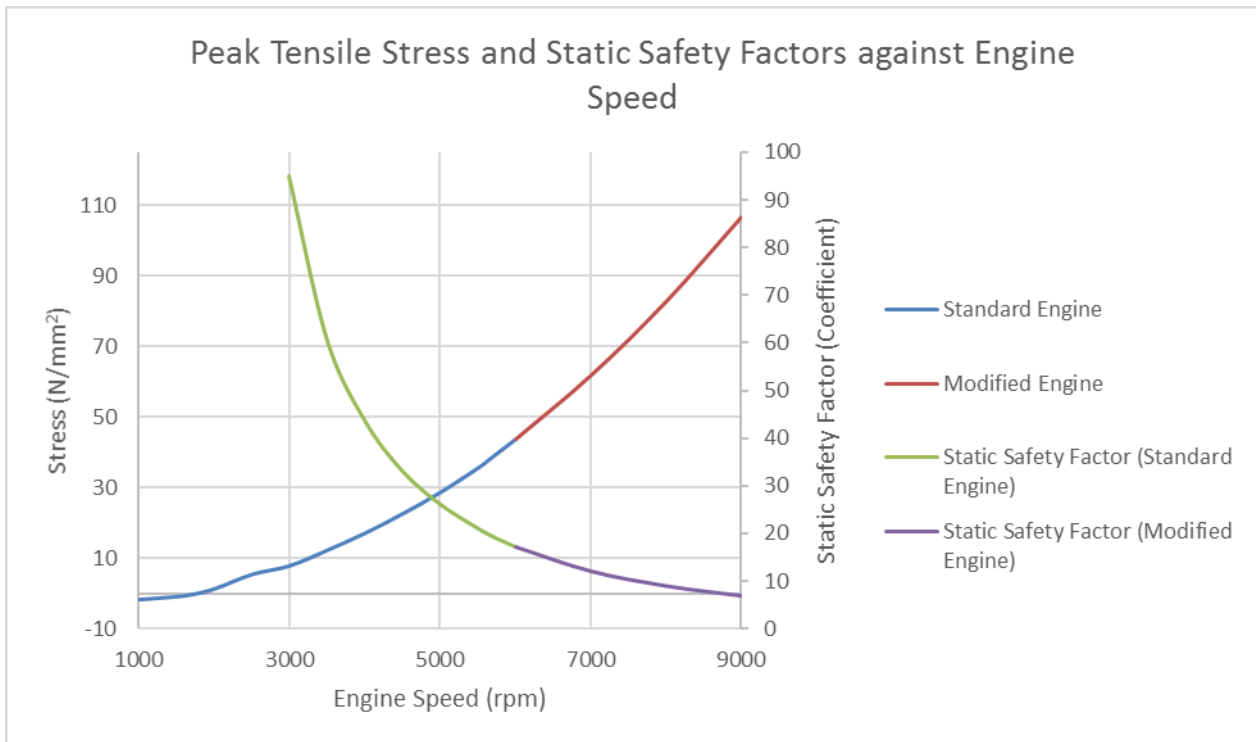


Figure 20 - Peak Tensile Stress and Static Safety Factors against Engine Speed

Due to residual gas forces counteracting the reduced inertial loads at low engine speeds, the stresses seen in the tensile region of the cycle are in fact, acting in compression. These anomalies from the Static Safety Factor have been removed from the graph to show the correct trend in terms of tensile stresses.

The peak tensile stress exerted on the Connecting Rod is **43.77 N/mm² @ 6000rpm** for the standard engine and **106.6 N/mm² @ 9000rpm** for the modified engine. This is a gain of 2.44 times that of the standard engine, through a 3000rpm climb.

The rise in tensile stress comes directly from an increase in inertial loads. This must be considered for Connecting Rod Big End Cap fasteners, as these would have been designed with a relative safety factor for that of the standard tensile stresses and could therefore be subject to excessive stretch/fatigue in such conditions.

The nominal Static Safety Factor for the Connecting Rod in Tensile Stress is **17.14** and **7.04** for the standard and theoretically modified engine, respectively.

8. Compression Distance & Tensile Stretch

Since,

$$\varepsilon = \frac{x}{L}$$

Where ε = Strain, x = Displacement (mm)

Then,

$$E = \frac{\sigma}{\varepsilon}$$

Where E = Young's Modulus (GPa)

Therefore,

$$x = \frac{FL}{EA}$$

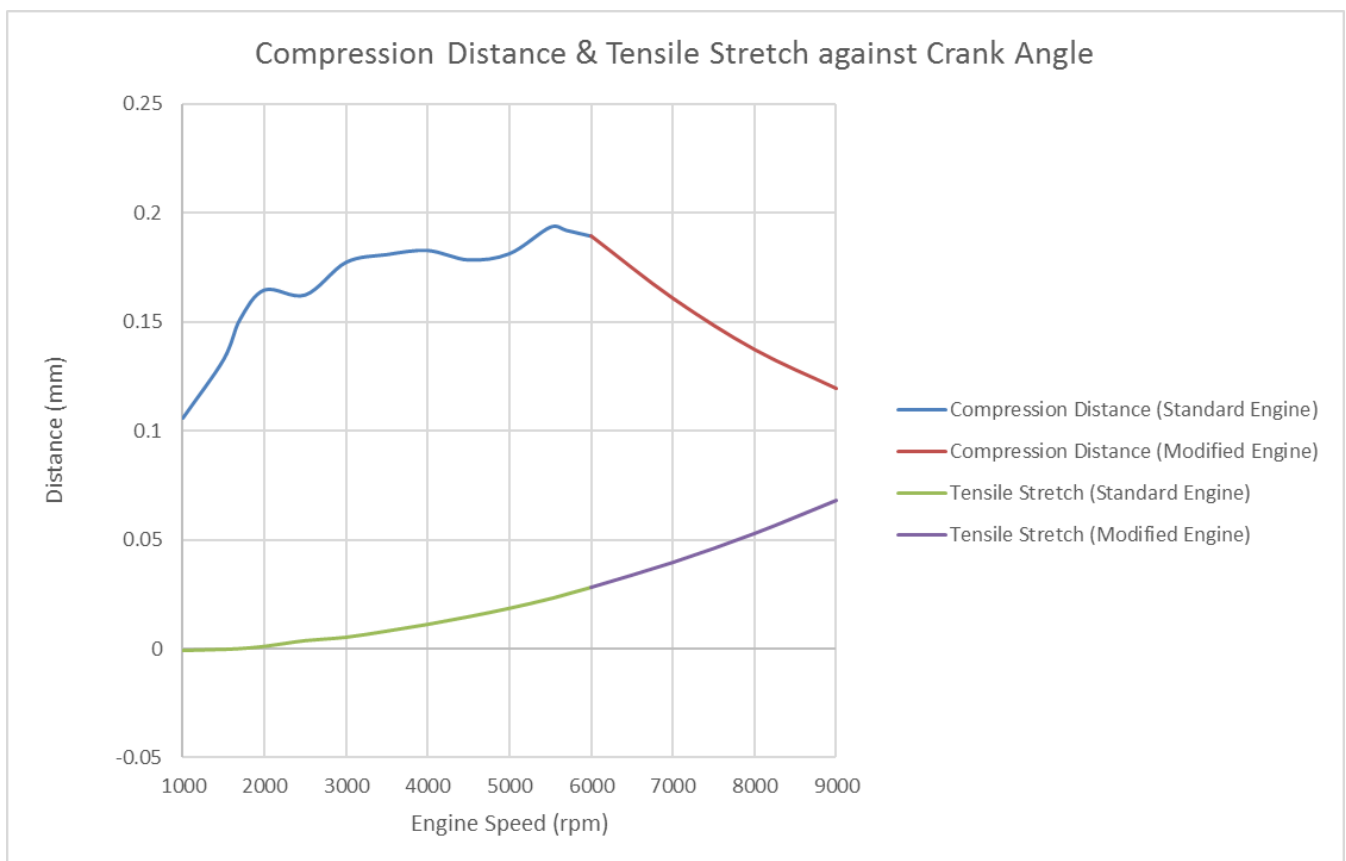


Figure 21 - Compression Distance & Tensile Stretch against Crank Angle

As per section 4 (I, II), the displacement of the Connecting Rod is a function of Peak Net Compressive and Tensile Forces. For further analysis on these calculated distances see section 9.

9. Titanium Connecting Rod Comparison

A theoretical study was then conducted on the effects of changing material from Steel to Titanium (Ti-6Al-4V).

The study assumes equivalent mass and shank cross sectional area between both rods to provide a representative comparison.

Material Properties	Material Type	
	Steel (Standard)	Titanium (Ti-6Al-4V)
UTS (MPa)	750	950
Young's Modulus (GPa)	210	113.8
Density (kg/m ³)	7700	4430

Table 2 - Material Properties

I. Compression

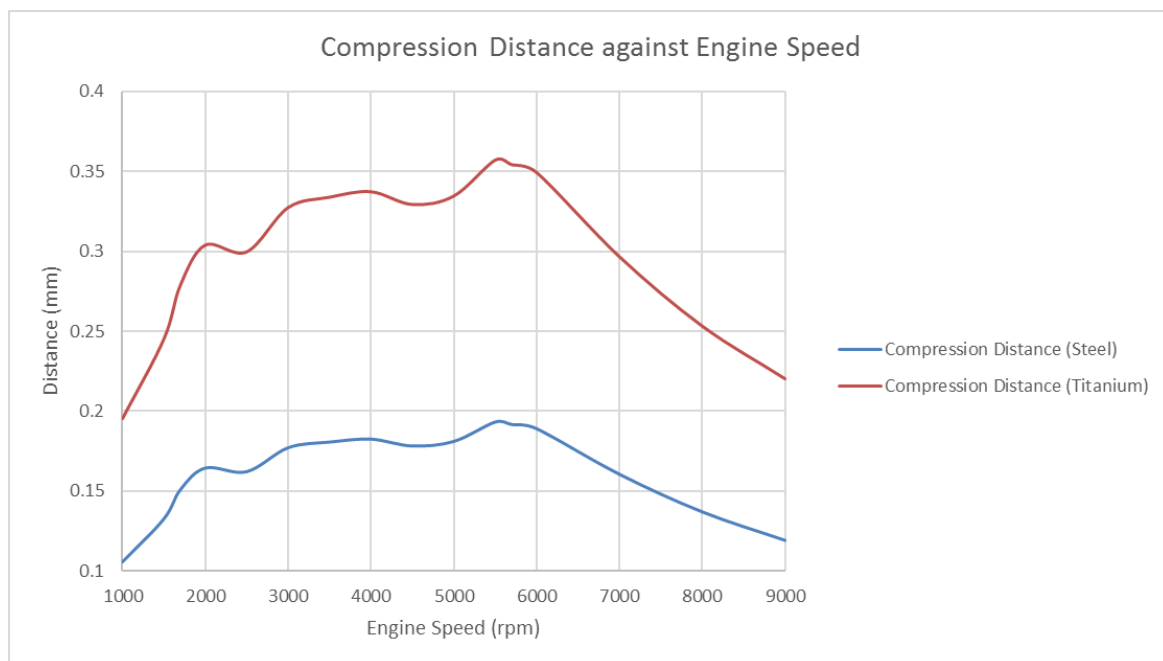


Figure 22 - Compression Distance against Engine Speed

Material Properties	Material Type	
	Steel (Standard)	Titanium (Ti-6Al-4V)
SSF (Peak Compressive)	2.47	3.13
Standard Engine - Compressive Distance (mm)	0.194	0.357
Modified Engine - Compressive Distance (mm)	0.194	0.357

All values recorded @ 5500rpm, 21° ATDC

Table 3 - Compressive Distances

From Table 3, an increase in compressive distance can be seen due to the lower Young's Modulus of Ti-6Al-4V, whereas the safety factor relative to UTS is seen to be increased over that of Steel.

II. Tensile



Figure 23 - Tensile Stretch against Engine Speed

Material Properties	Material Type	
	Steel (Standard)	Titanium (Ti-6Al-4V)
Standard Engine SSF (Peak Tensile)	17.14	21.71
Standard Engine - Tensile Stretch (mm)	0.028	0.068
Modified Engine – Tensile Stretch (mm)	0.052	0.126
Standard Engine @ 6000rpm, 2° BTDC exhaust, Modified Engine @ 9000rpm, TDC exhaust		

Table 4 - Tensile Stretch

Table 4 shows Titanium stretches an additional **58.7%** more in tensile displacement than steel, in the case of the modified engine.

10. Further considerations for Titanium

I. The effects of Titanium Displacements (Modified Engine)

Figures 22 and 23 prove the Elasticity of Titanium is greater than steel. *Tables 3 and 4* indicate the range in displacements are **0.246mm** for steel and **0.483mm** for Titanium. This equates to approximately double the displacement of steel when comparing the peak compressive distance and peak tensile stretch through the full calculated range.

Although doubling the material displacement is significant, the full consequences of such a change is difficult to predict. The only point in which there is an increased likelihood of interference (valve to piston) is around TDC exhaust (maximum tensile stress). At this point, the piston is an additional **0.074mm** closer to the valves. The original production engine design clearance tolerances should be considered in this instance.

In addition, the piston to cylinder wall clearances should be reconsidered as changing the connecting rod length can influence the levels of piston ring friction that occur.

Finally, the cyclic stresses exerted onto the Titanium connecting rod can hold the potential to significantly increase the probability of failure due to fatigue/stress concentration cracks.

II. Simplified Comparison of Volumes & Masses

As previously stated, the mass and cross-sectional area of the Titanium rod was assumed to be equal to the factory steel rod, resulting in higher SSF's and compressive/tensile displacements.

The following calculations provide some perception as to how the titanium rod would compare in terms of its volume and mass, to match the maximum compressive distance that the steel rod experiences @5500rpm.

The new theoretical cross-sectional area of the Titanium Connecting Rod can be found by Transposing the following equation to make A the subject.

$$x = \frac{FL}{EA}$$

Where F is taken as peak compressive force for the standard engine and x is taken as maximum compressive displacement.

From this, the new Titanium minimum shank cross section is estimated to be **251.45mm³**, an 84.5% advance over the steel rod.

Assuming the factory steel rod has a mass of **0.37kg**, the density ratio of **0.58** suggests a Titanium equivalent (in terms of material volume) would have a mass of **0.21kg**.

Designing the Titanium Rod to withstand identical compressive forces and compression displacements to the standard engine would require additional material when compared to the steel rod. This is estimated by taking the cross-section ratio of **1.85** for Titanium and multiplying it by its original mass for the factory equivalent rod (*0.21kg*). This results in a new mass of approximately **0.39kg** (20g heavier than factory steel rod).

However, the static safety factors in Titanium are *1.27 times* greater than that of the steel rod. This means the Titanium rod could withstand a total of **384.6N/mm²** of compressive stress before exceeding the nominal static safety factor found in the factory steel rod. (*This calculates to an additional 81.33N/mm² over the factory steel rod, with an SSF of 2.47*)

11. Conclusion

The connecting rod is subject to immense forces, multiple times per second. These forces have been split up into the directions in which they act and analysed further to comprehend the design processes involved in such a part. The combustion gas forces acting onto the piston assembly have the greatest effect on the resultant stresses through the connecting rod. The inertial loads from the reciprocating masses surpass the effects of the residual gas forces around TDC (exhaust), which is exaggerated further through an increased engine speed.

The Ford 1.6 GDI was designed with an SSF of 2.47 in compression. Being a production engine, a high enough safety factor is required to avoid premature fatigue and costly product recalls. The cyclic displacements exerted through the steel connecting rod has also been factored in. The Titanium (Ti-6Al-4V) theoretical replacement has an increased UTS, but a reduced young's Modulus. This means the safety factor is increased for the same mass and cross section. However, the increase in displacement results in uncertainty in terms of fatigue and clearance penalties.

12. Further Improvements

- Increase the cylinder pressure to observe its effects on compressive SSF
- FEA study of 3D Solidworks model to validate results and observe stress distributions
- Investigate 1st order crankshaft balancing and 2nd order balancing with counterrotating shafts
- Create friction model to investigate the effects on connecting rod forces
- Investigate the effects of changing connecting rod length / crankshaft pin radius
- Investigate the effects of crankshaft offset
- Compare multiple material types to further understand the benefits and limitations of each
- Investigate the effects of firing orders and engine configurations
- Compare Naturally Aspirated with Forced induction forces/stresses
- In depth stress analysis including potential stress risers

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