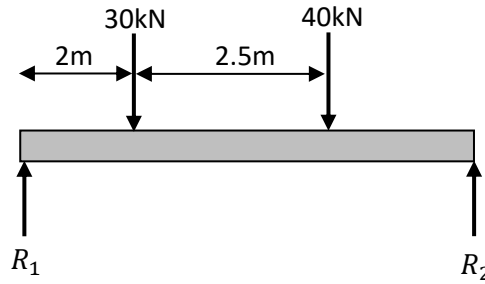


PROBLEM a:

Beam is 7 m long with an $(E \times I)$ value of 200 MN.m^2 . Determine slope and deflection values in the middle.



Implementing the conditions for static equilibrium to determine support reactions R_1 and R_2 :

- Condition 1: the algebraic sum of forces is zero that is,

$$\text{sum of the } \textbf{upward} \text{ component of forces} = \text{sum of the } \textbf{downward} \text{ component of forces}$$

Thus,

$$R_1 + R_2 = 30 + 40$$

- Condition 2: the sum of the anticlockwise moments about a point or axis equals the sum of the clockwise moments about the same point or axis. If we choose left tip of the beam,

$$30 \times 2 + 40 \times 4.5 = 7 \times R_2$$

$$\therefore R_2 = \frac{30 \times 2 + 40 \times 4.5}{7} = 34.29 \text{ kN}$$

And substituting R_2 into condition 1 gives,

$$R_1 + 34.29 = 30 + 40$$

$$\therefore R_1 = 70 - 34.29 = 35.71 \text{ kN}$$

Taking the origin at the left hand end of the beam and writing down the bending moment at a section located at the extreme right hand part of the beam in terms of all loads to the left of that section gives,

$$M_{xx} = EI \frac{d^2y}{dx^2} = 35.71x - 30[x - 2] - 40[x - 4.5]$$

It must be emphasised that all loads in the right-hand side of the above bending moment equation are in units of kN (i.e. newton $\times 10^3$). In subsequent workings, therefore, it is convenient to carry through this factor as a denominator on the left-hand side of the equation in order that the expressions are dimensionally correct.

Integrating gives,

$$\frac{EI}{10^3} \frac{dy}{dx} = \frac{35.71}{2} x^2 - \frac{30}{2} [x - 2]^2 - \frac{40}{2} [x - 4.5]^2 + A$$

$$\therefore \frac{EI}{10^3} \frac{dy}{dx} = \frac{35.71}{2} x^2 - 15[x - 2]^2 - 20[x - 4.5]^2 + A$$

Integrating a second time gives,

$$\frac{EI}{10^3} y = \frac{35.71}{6} x^3 - \frac{15}{3} [x - 2]^3 - \frac{20}{3} [x - 4.5]^3 + Ax + B$$

$$\therefore \frac{EI}{10^3} y = \frac{35.71}{6} x^3 - 5[x - 2]^3 - \frac{20}{3} [x - 4.5]^3 + Ax + B$$

Determination of the constant of integration A and B:

- At $x = 0, y = 0$. Therefore,

$$\frac{EI}{10^3} \times 0 = \frac{35.71}{6} \times 0^3 + A \times 0 + B \quad \therefore \quad 0 = 0 + 0 + B \quad \therefore \quad B = 0$$

- At $x = 7, y = 0$. Therefore,

$$\frac{EI}{10^3} \times 0 = \frac{35.71}{6} \times 7^3 - 5(7 - 2)^3 - \frac{20}{3} (7 - 4.5)^3 + A \times 7$$

$$\therefore \quad 0 = 1312.5 + A \times 7 \quad \therefore \quad A = \frac{-1312.5}{7} = -187.5$$

Therefore the slope equation is,

$$\frac{dy}{dx} = \frac{10^3}{EI} \left(\frac{35.71}{2} x^2 - 15[x - 2]^2 - 20[x - 4.5]^2 - 187.5 \right)$$

And the deflection equation is,

$$y = \frac{10^3}{EI} \left(\frac{35.71}{6} x^3 - 5[x - 2]^3 - \frac{20}{3} [x - 4.5]^3 - 187.5x \right)$$

Hence the slope and deflection at mid-span, that is $x = 3.5$, are:

$$\frac{dy}{dx} = \frac{10^3}{200 \times 10^6} \left(\frac{35.71}{2} \times 3.5^2 - 15(3.5 - 2)^2 - 187.5 \right) = -1.25 \times 10^{-5}$$

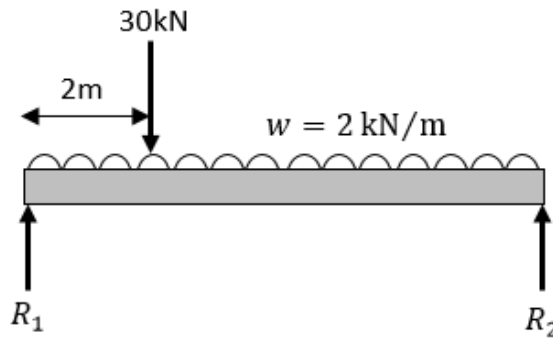
and,

$$y = \frac{10^3}{200 \times 10^6} \left(\frac{35.71}{6} \times 3.5^3 - 5(3.5 - 2)^3 - 187.5 \times 3.5 \right) = -2.0896 \times 10^{-3} \text{ m} = -2.0896 \text{ mm}$$

PROBLEM b:

Beam is 6 m long with an EI value of 300 MN.m².

Determine slope at the left end and deflection in the middle.



Implementing the conditions for static equilibrium to determine support reactions R_1 and R_2 :

- Condition 1: the algebraic sum of forces is zero that is,

$$\text{sum of the **upward** component of forces} = \text{sum of the **downward** component of forces}$$

Thus,

$$R_1 + R_2 = 30 + 2 \times 6$$

- Condition 2: the sum of the anticlockwise moments about a point or axis equals the sum of the clockwise moments about the same point or axis. If we choose left tip of the beam,

$$30 \times 2 + (2 \times 6) \times 3 = 6 \times R_2$$

$$\therefore R_2 = \frac{30 \times 2 + 12 \times 3}{6} = 16 \text{ kN}$$

And substituting R_2 into condition 1 gives,

$$R_1 + 16 = 30 + 12$$

$$\therefore R_1 = 42 - 16 = 26 \text{ kN}$$

Taking the origin at the left hand end of the beam and writing down the bending moment at a section located at the extreme right hand part of the beam in terms of all loads to the left of that section gives,

$$M_{xx} = EI \frac{d^2y}{dx^2} = 26x - 30[x - 2] - 2x \left(\frac{x}{2} \right)$$

It must be emphasised that all loads in the right-hand side of the above bending moment equation are in units of kN (i.e. newton $\times 10^3$). In subsequent workings, therefore, it is convenient to carry through this factor as a denominator on the left-hand side of the equation in order that the expressions are dimensionally correct.

$$\frac{EI}{10^3} \frac{d^2y}{dx^2} = 26x - x^2 - 30[x - 2]$$

Integrating gives,

$$\begin{aligned} \frac{EI}{10^3} \frac{dy}{dx} &= \frac{26}{2}x^2 - \frac{x^3}{3} - \frac{30}{2}[x - 2]^2 + A \\ \therefore \frac{EI}{10^3} \frac{dy}{dx} &= 13x^2 - \frac{1}{3}x^3 - 15[x - 2]^2 + A \end{aligned}$$

Integrating a second time gives,

$$\frac{EI}{10^3} y = \frac{13}{3}x^3 - \frac{1}{12}x^4 - \frac{15}{3}[x - 2]^3 + Ax + B$$

Determination of the constant of integration A and B:

- At $x = 0, y = 0$. Therefore,

$$\frac{EI}{10^3} \times 0 = \frac{13}{3} \times 0^3 - \frac{1}{12} \times 0^4 + A \times 0 + B \quad \therefore \quad 0 = 0 - 0 + 0 + B \quad \therefore \quad B = 0$$

- At $x = 6, y = 0$. Therefore,

$$\frac{EI}{10^3} \times 0 = \frac{13}{3} \times 6^3 - \frac{1}{12} \times 6^4 - \frac{15}{3}(6 - 2)^3 + A \times 6$$

$$\therefore \quad 0 = 936 - 108 - 320 + A \times 6 \quad \therefore \quad A = \frac{-508}{6} = -84.6$$

Therefore the slope equation is,

$$\frac{dy}{dx} = \frac{10^3}{EI} \left(13x^2 - \frac{1}{3}x^3 - 15[x - 2]^2 - 84.6 \right)$$

And the deflection equation is,

$$y = \frac{10^3}{EI} \left(\frac{13}{3}x^3 - \frac{1}{12}x^4 - \frac{15}{3}[x - 2]^3 - 84.6x \right)$$

Hence the slope at the left, that is $x = 0$, is,

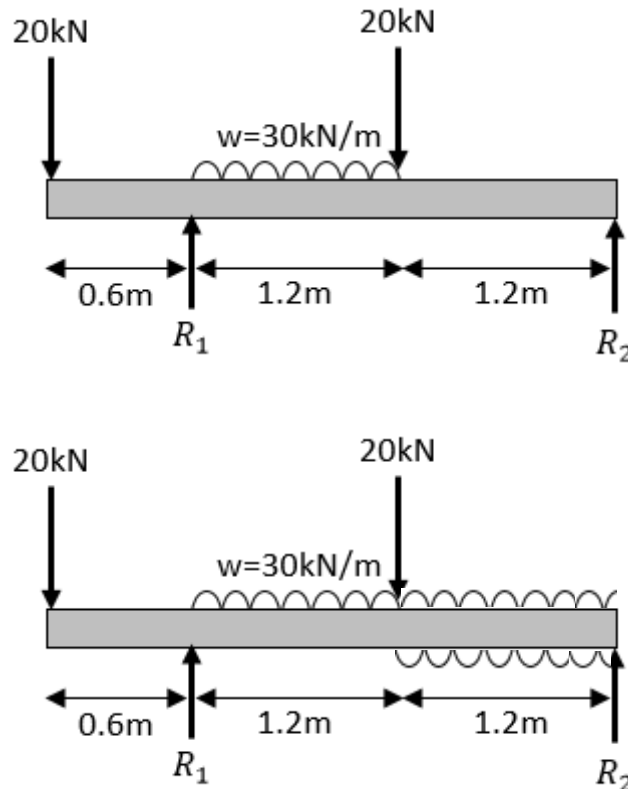
$$\frac{dy}{dx} = \frac{10^3}{300 \times 10^6} \left(13 \times 0 - \frac{1}{3} \times 0 - 84.6 \right) = -2.82 \times 10^{-4}$$

and deflection at mid-span, that is $x = 3$, is:

$$y = \frac{10^3}{300 \times 10^6} \left(\frac{13}{3}3^3 - \frac{1}{12}3^4 - \frac{15}{3}(3 - 2)^3 - 84.6 \times 3 \right) = -0.00049583 \text{ m} \approx -0.5 \text{ mm}$$

PROBLEM c:

Determine the deflection at a point 1 m from the left-hand end of the beam loaded as shown below. Consider an EI value of 0.65 MN.m^2 .



Implementing the conditions for static equilibrium to determine support reactions R_1 and R_2 :

- Condition 1: the algebraic sum of forces is zero that is,

$$\text{sum of the } \textbf{upward} \text{ component of forces} = \text{sum of the } \textbf{downward} \text{ component of forces}$$

Thus,

$$R_1 + R_2 = 20 + 30 \times 1.2 + 20$$

- Condition 2: the sum of the anticlockwise moments about a point or axis equals the sum of the clockwise moments about the same point or axis. If we choose R_1 location,

$$20 \times 1.2 + (30 \times 1.2) \times 0.6 = 20 \times 0.6 + R_2 \times 2.4$$

$$\therefore R_2 = \frac{20 \times 1.2 + 36 \times 0.6 - 20 \times 0.6}{2.4} = 14 \text{ kN}$$

And substituting R_2 into condition 1 gives,

$$R_1 + 14 = 20 + 36 + 20$$

$$\therefore R_1 = 76 - 14 = 62 \text{ kN}$$

Taking the origin at the left hand end of the beam and writing down the bending moment at a section located at the extreme right hand part of the beam in terms of all loads to the left of that section gives,

$$M_{xx} = EI \frac{d^2y}{dx^2} = -20 \times x + 62[x - 0.6] - 30[x - 0.6] \frac{[x - 0.6]}{2} - 20[x - 1.8] + 30[x - 1.8] \frac{[x - 1.8]}{2}$$

It must be emphasised that all loads in the right-hand side of the above bending moment equation are in units of kN (i.e. newton $\times 10^3$). In subsequent workings, therefore, it is convenient to carry through this factor as a denominator on the left-hand side of the equation in order that the expressions are dimensionally correct.

$$\frac{EI}{10^3} \frac{d^2y}{dx^2} = -20x + 62[x - 0.6] - 20[x - 1.8] - 15[x - 0.6]^2 + 15[x - 1.8]^2$$

Integrating gives,

$$\frac{EI}{10^3} \frac{dy}{dx} = -\frac{20}{2}x^2 + \frac{62}{2}[x - 0.6]^2 - \frac{20}{2}[x - 1.8]^2 - \frac{15}{3}[x - 0.6]^3 + \frac{15}{3}[x - 1.8]^3 + A$$

$$\therefore \frac{EI}{10^3} \frac{dy}{dx} = -10x^2 + 31[x - 0.6]^2 - 10[x - 1.8]^2 - 5[x - 0.6]^3 + 5[x - 1.8]^3 + A$$

Integrating a second time gives,

$$\frac{EI}{10^3} y = -\frac{10}{3}x^3 + \frac{31}{3}[x - 0.6]^3 - \frac{10}{3}[x - 1.8]^3 - \frac{5}{4}[x - 0.6]^4 + \frac{5}{4}[x - 1.8]^4 + Ax + B$$

Determination of the constant of integration A and B:

- At $x = 0.6$, $y = 0$. Therefore,

$$\frac{EI}{10^3} \times 0 = -\frac{10}{3} \times 0.6^3 + A \times 0.6 + B \quad \therefore \quad B = 0.72 - 0.6A$$

- At $x = 3$, $y = 0$. Therefore,

$$\frac{EI}{10^3} \times 0 = -\frac{10}{3}3^3 + \frac{31}{3}(3 - 0.6)^3 - \frac{10}{3}(3 - 1.8)^3 - \frac{5}{4}(3 - 0.6)^4 + \frac{5}{4}(3 - 1.8)^4 + A \times 3 + 0.72 - 0.6A$$

$$\therefore 0 = -90 + 142.848 - 5.76 - 41.472 + 2.592 + A \times (3 - 0.6) + 0.72$$

$$\therefore 0 = 8.208 + 2.4 \times A + 0.72 \quad \therefore \quad A = \frac{8.208 - 0.72}{2.4} = 3.12$$

Thus,

$$B = 0.72 - 0.6 \times 3.12 = -1.152$$

Therefore, the deflection at $x = 1$ is,

$$y = \frac{10^3}{0.65 \times 10^6} \left(-\frac{10}{3} \times 1^3 + \frac{31}{3}(1 - 0.6)^3 - \frac{5}{4}(1 - 0.6)^4 + 3.12 \times 1 - 1.152 \right) = -0.00113 \text{ m}$$