



EXAMINATION SOLUTIONS

(to accompany the examination paper sent to the external examiner
but NOT to be seen by students)

Programme:	BSc/HND Motorsport/Automotive Engineering	Level:	5
Module:	Mechanical Principles		
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Q	Question	Marks
1 i)	$\epsilon_x = 15 \times 10^{-6}$ $\epsilon_y = -18 \times 10^{-6}$ $E = 169 \times 10^9 \text{ N/m}^2$ $\nu = 0.26$ $\sigma_1 = \frac{E}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2) \checkmark$ $\sigma_1 = \frac{169 \times 10^9}{1 - (0.26^2)} (15 \times 10^{-6} + (0.26 \times -18 \times 10^{-6})) \checkmark$ $\sigma_1 = 1.87 \text{ MPa. (tensile)} \checkmark$ $\sigma_2 = \frac{E}{1-\nu^2} (\epsilon_2 + \nu \epsilon_1)$ $\sigma_2 = \frac{169 \times 10^9}{1 - 0.26^2} (-18 \times 10^{-6} + (0.26 \times 15 \times 10^{-6})) \checkmark$ $\sigma_2 = -2.56 \text{ MPa. (compressive)} \checkmark$	5
ii)	$\sigma_1 = 65 \times 10^6 \text{ N/m}^2$ $\sigma_2 = 10 \times 10^6 \text{ N/m}^2 \checkmark$ $\epsilon_1 = \frac{1}{E} (\sigma_1 - \nu \sigma_2) = \frac{1}{169 \times 10^9} (65 \times 10^6 - (0.26 \times 10 \times 10^6)) = 369.23 \checkmark$ $\epsilon_2 = \frac{1}{E} (\sigma_2 - \nu \sigma_1) = \frac{1}{169 \times 10^9} (10 \times 10^6 - (0.26 \times 65 \times 10^6)) = -40.82 \checkmark$	5



Q	Question	Marks
Q2		
i)	$r = 0.025 \text{ m}$ $P = 20 \times 10^5 \text{ N/m}^2$ $E = 163.2 \times 10^9 \text{ N/m}^2$ $\nu = 0.3$ $K = \frac{E}{3(1-2\nu)} = \frac{163.2 \times 10^9}{3(1-0.6)} = 136 \times 10^9 \text{ N/m}^2$	2
ii)	$K = \frac{\sigma}{E_r}$ $\sigma = -P$ $\therefore K = \frac{-P}{E_r}$ $E_r = \frac{-P}{K} = \frac{-20 \times 10^5}{136 \times 10^9} = -14.71 \times 10^{-6}$	3
iii)	$E_r = \frac{\Delta V}{V} \Rightarrow \Delta V = E_r \times V$ $\Delta V = E_r \times \frac{4}{3}\pi r^3$ $\Delta V = -14.71 \times 10^{-6} \times \frac{4}{3}\pi (0.025)^3$ $\Delta V = -9.63 \times 10^{-3} \text{ m}^3$	3
iv)	An isotropic material has the same material properties in any direction. Examples would be most metals.	2



Q	Question	Marks
Q3	i) Standard case by superposition. $w = 1500 \text{ N/m}$ $EI = 2.2 \times 10^6 \text{ Nm}^2$ $F = 5000 \text{ N}$ $L = 1.3 \text{ m}$ ii) $y_{UDL} = \frac{wL^4}{8EI} = \frac{1500 \times 1.3^4}{8 \times 2.2 \times 10^6} = 2.43 \times 10^{-4} \text{ m}$ iii) $y_{PL} = \frac{FL^3}{3EI} = \frac{5000 \times 1.3^3}{3 \times 2.2 \times 10^6} = 1.66 \times 10^{-3} \text{ m}$ iv) $y = y_{UDL} + y_{PL} = 2.43 \times 10^{-4} + 1.66 \times 10^{-3} = 1.91 \times 10^{-3} \text{ m}$ iv) $y_{PL}' = y - y_{UDL} = 3.5 \times 10^{-3} - 2.43 \times 10^{-4}$ $y_{PL}' = 3.257 \times 10^{-3}$ $y_{PL}' = \frac{F'L^3}{3EI}$ $F' = \frac{y_{PL}' 3EI}{L^3} = \frac{3.257 \times 10^{-3} \times 3 \times 2.2 \times 10^6}{1.3^3}$ $F' = 9784.34 \text{ N}$ $\therefore \Delta F = 9784.34 - 5000$ $\Delta F = 4784.34 \text{ N}$	3 3 1 3



Q	Question	Marks
Q4.	$d = 0.35\text{m}$ $L = 0.8\text{m}$ $\mu_L = 0.85$ ✓ $\mu_C = 0.95$ ✓ $P = 20 \times 10^5 \text{N/m}^2$ $\sigma_{\text{max}} = 110 \times 10^6 \text{N/m}^2$	1
	Longitudinally $\sigma_L = \frac{PD}{4t\mu_C}$ $t = \frac{PD}{4\sigma_L\mu_C}$ $t = \frac{20 \times 10^5 \times 0.35}{4 \times 110 \times 10^6 \times 0.95}$ $t = 1.67 \times 10^{-3} \text{m}$ ✓	2
	Circumferentially $\sigma_C = \frac{PD}{2t\mu_L}$ $t = \frac{PD}{2\sigma_C\mu_L}$ $t = \frac{20 \times 10^5 \times 0.35}{2 \times 110 \times 10^6 \times 0.85}$ $t = 3.74 \times 10^{-3} \text{m}$ ✓	2
	$t_{\text{min}} = 3.74 \times 10^{-3} \text{m}$ ✓	1



Q	Question	Marks
Q5	$d_o = 150\text{mm} = 0.15\text{m} \rightarrow r_o = 0.075\text{m} \checkmark$ $d_i = 130\text{mm} = 0.13\text{m} \rightarrow r_i = 0.065\text{m} \checkmark$ $P = 80 \times 10^5 \text{N/m}^2$ Applying boundary conditions @ $r_o = 0.075\text{m}, \sigma_r = 0 \checkmark$ $r_i = 0.065\text{m}, \sigma_r = -P \checkmark$ $\sigma_r = a - \frac{b}{r^2}$ $\sigma_r = a - \frac{b}{r^2}$ $0 = a - \frac{b}{0.075^2}$ $-80 \times 10^5 = a - \frac{b}{0.065^2}$ $0 = 0.075^2 a - b \checkmark$ $-33800 = 0.065^2 a - b \checkmark$ $0 - -33800 = 0.075^2 a - b + b - 0.065^2 a \checkmark$ $33800 = 0.075^2 a - 0.065^2 a$ $a = \frac{33800}{0.075^2 - 0.065^2} = 24.143 \times 10^6 \checkmark$ $b = 0.075^2 a = 135.8 \times 10^3 \checkmark$ $\sigma_c = a + \frac{b}{r^2} =$ $\sigma_{c_{r_o}} = 24.143 \times 10^6 + \frac{135.8 \times 10^3}{0.075^2} = 48.14 \times 10^6 \text{N/m}^2 \checkmark$ $\sigma_{c_{r_i}} = 24.143 \times 10^6 + \frac{135.8 \times 10^3}{0.065^2} = 56.14 \times 10^6 \text{N/m}^2 \checkmark$	1 2 2 1 2 2