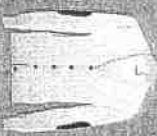


HOW TO TEACH



Anyone can feed sweets to sharks ...

Primary Maths

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EDITED BY PHIL BEADLE

Chapter 2

THE FOUR OPERATIONS

"Manners are not taught in lessons," said Alice. "Lessons teach you to do sums, and things of that sort."

"And you do Addition?" the White Queen asked. "What's one and one and one and one and one and one and one and one and one?"

"I don't know," said Alice. "I lost count."

"She can't do Addition," the Red Queen interrupted. "Can you do Subtraction? Take nine from eight."

"Nine from eight I can't, you know," Alice replied very readily; "but ..."

"She can't do Subtraction," said the White Queen. "Can you do Division? Divide a loaf by a knife – what's the answer to that?"

"I suppose –" Alice was beginning, but the Red Queen answered for her. "Bread-and-butter, of course. Try another Subtraction sum. Take a bone from a dog: what remains?"

Alice considered. "The bone wouldn't remain, of course, if I took it – and the dog wouldn't remain; it would come to bite me – and I'm sure I shouldn't remain!"

"Then you think nothing would remain?" said the Red Queen.

"I think that's the answer."

"Wrong, as usual," said the Red Queen: "the dog's temper would remain."

"But I don't see how –"

Lewis Carroll, *Through the Looking-Glass* (1871)

I love this little section from *Through the Looking-Glass*, and think it sums up just how confused we can become when solving calculations involving the four operations. Addition, subtraction, multiplication and division are the crux of primary mathematics, and quite rightly occupy a substantial place in the primary curriculum. They are vital for the development of good numeracy skills and have wide ranging application in everyday life. However, in the wrong hands they can bore children to tears. Please don't turn your pupils into young arithmophobics.

Arithmophobia: we've all seen it and many of us have felt it ourselves. The minute the maths lesson starts the heads drop; beaming smiles turn to frowns and eyes drift upwards towards the clock, every second stretching out to what seems an eternity. The only thing worse than the monotony of repeating the same questions over and over again is the fear that at any moment you could be put on the spot. What if I don't know the answer? I never know the answer. I'm not some sort of human calculator! The really concerning thing with arithmophobia is that it is entirely contagious and, if left unchecked, your whole class can come down with it. You could even find yourself succumbing to arithmo-jitters yourself, draining your confidence in how you teach the subject. Thankfully, it is easily treated, requiring nothing more than a little understanding and a healthy dose of creativity. By the end of this chapter, you should be buzzing with plenty of ideas to turn your little ones not just into human calculators but into arithmetical acrobats.

ADDITION

Addition is the easiest of the four operations to grasp, as it is the operation that most naturally relates to straightforward counting upwards. In the early stages of learning addition, children will often use counting up on their fingers as the 'go to' strategy for adding. When working out $3 + 2$, children will usually start with three fingers up and then put two more fingers up before counting them to make five. This makes complete sense: we tend to use our hands and fingers to demonstrate many things, such as giving directions, pointing out something of interest or indicating a preference.

However, this fine strategy starts to become slightly more cumbersome as the maths becomes more complex. It should be just another tool in our pupils' toolkits, not something that governs how they work. As the venerable Mickey Mouse is reputed to have said: "Arithmetic is being able to count up to twenty without taking your shoes off." The last thing you want is a classroom full of children with their shoes and socks off! What happens if they need to count to 21? What about the stench of post-PE cheesy feet? Children need to develop a mental agility around solving any question involving any one of the four operations, and the start of this process is learning simple number facts. Children need to work on their number facts, such as bonds to ten, from the earliest possible age and be able to recall them with both pace and accuracy.

PEL MANISM

One way to work on developing pupils' understanding of number bonds is to play Pelmanism (otherwise known as concentration or pairs). This can be done by taking 11 small cards and writing on them the numbers from 0 to 10. Once you have made your cards, give them a shuffle and lay them face down on a table top. The pupil then has to turn over two cards at a time with the aim of making a number bond that equals ten (e.g. $0 + 10$, $1 + 9$, $2 + 8$). If the pupil picks a correct pair, they leave the cards turned face up and pick again; if they are wrong, they turn them back over and the next pupil gets a go. This is great played in small groups or head to head. Alternatively, try placing a number of small items (e.g. counters, paper clips, Lego bricks, Multi-Link cubes) under plastic cups, so that you have 11 upside down cups covering objects of ascending value from 0 to 10. (You can write the value of the contents on the inside of the cup base for additional support, if necessary.) This has the further benefit of providing concrete examples of what each number actually represents.

One of the reasons that Pelmanism is so useful is that it provides the perfect 'in' for discussing one of the key properties of addition: commutativity. Commutativity means that you can reverse the terms in a sum, left to right, and it will still give you the same answer. For example:

$$a + b = b + a$$

$$1+2=2+1$$

The commutative property of addition is important because, once children grasp the fact that they can change the order of their sum and still get the same answer, they are free to work out the sum in the way that is most effective for them. For instance, $3 + 11$ is more easily solved mentally if viewed as $11 + 3$ (i.e. when we start with the largest number). By understanding this

property, pupils are able to answer sums with greater confidence, pace and accuracy. It also provides an excellent contrast when covering subtraction, as this is the inverse of addition and is *not* commutative: meaning that correct order in subtraction is vital.

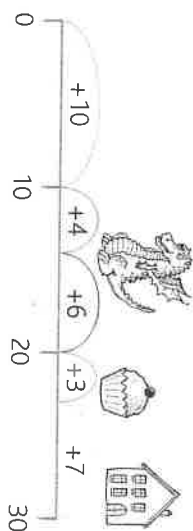
It all seems pretty simple so far, so where does it go wrong? One of the first common difficulties to occur is that of bridging through ten. This is where the sum of two numbers goes over the next ten: $9 + 5 = 14$ or $16 + 7 = 23$.

In both these examples, the sum of the two digits in the ones column goes above ten. Providing pupils have a solid understanding of place value (never assume this), they should know by now that the highest digit you can have in any column or place is 9. Therefore, when the sum of two digits in the ones column goes above ten, we must carry across a ten into the tens column.

Remember: without understanding the rules of the number system, we can't work with numbers!

If your pupils have not grasped the idea of basic place value, then repeat until they are concrete in their understanding. Even with a solid comprehension of how place value works, children may still make arithmetical mistakes when adding together two numbers that bridge through ten. This is often because they try to jump straight to the answer. In which case, the solution here is to slooooooooooooooowwwwww! If your pupils have a tendency to speed through work making careless errors ask them to check their work, check again and ask a friend.

Once pupils have the understanding that they can use their knowledge of basic number facts (such as bonds to ten), they will be more successful. Children need to see each set of ten as a numerical milestone. The way I approach this is to talk to the children about journeys. I tell them about trips I've been on, where I've had to make more than one stop on my route. At each stop we calculate how far I have travelled; first, adding to the next ten and then adding on the remaining distance for that leg.



This number shows a journey where, initially, I had to drive 14 miles to drop off my pet dragon, Derrick, at the Dragon Day Care; followed by another nine miles to the bakery to pick up some cake for the teachers; then another seven miles to arrive at work. Once the pupils have grasped the idea, they can create little journeys of their own, mapping out a linear route using small plastic sports cones that are equally spaced apart, with different colour cones to signify tens.

Splitting the numbers into their separate parts is another successful strategy, providing that pupils have a solid understanding of the number system. Pupils must know that the 7 in 79 stands for seven tens and that the 9 stands for nine ones. Once the numbers have been split into tens and ones (possibly hundreds but unlikely at this stage) they can be added together as follows:

$$\begin{array}{r}
 40 + 30 = 70 \\
 7 + 9 = 16 \\
 70 + 16 = 86
 \end{array}$$

Once we are at this stage, we then recombine the numbers by adding the two totals together:

$$70 + 16 = 86$$

It is worth reminding pupils, once again, that the reason this works is because of the commutative property of addition: any sum can be calculated in any order.

Another trick worth mentioning is that of adding 'near multiples'. This is where you round one of the ones to ten and then readjust by subtracting or adding the amount you rounded up by at the end:

$$42 + 19 \text{ becomes } 42 + 20 - 1$$

This is a really useful trick, but the term can be a little confusing because it uses the word 'multiple'. Children need to have this method clearly explained to them, preferably with the use of a hundred square and a number line. Ensure that the pupils can explain why you might use this method and what potential errors they could make (such as forgetting to subtract the amount rounded up by at the end).

Once children have achieved a good understanding of the basics of mental methods for addition, written methods can then be introduced. This should be done carefully using a step-by-step approach to every detail. Checking of understanding must be done at every stage, because this is when misconceptions can become apparent as there are a greater number of factors in play. With mental methods, we were just relying on the ability to manipulate numbers internally with the only external output being the answer. Once we progress on to written methods, we are asking pupils to show us, on paper, exactly what is going on inside their heads. I have seen bright, switched on youngsters completely lose their spark as soon as they

are required to write down their workings because they are unsure of how exactly to set out their work. As teachers, we often neglect this part of the process because putting a digit in a box is not the difficult bit, is it? Well, actually, it can be really bloody hard for some children, especially if you want the digit in the correct box.

In my experience, the formal written method set out below is the most effective, so I generally teach this compact method in the first instance:

	H	T	O	
	4	3	2	
+	1	4	5	
	5	7	7	

Take your time and explain things thoroughly. I like to get pupils to jot down a step-by-step guide to how to set out their work, like this:

Step 1:

Set out your working:

	H	T	O	
	4	3	2	
+	1	4	5	

Step 2:
Add your 'ones' column:

	H	T	O	
	4	3	2	
+	1	4	5	
			7	

$2 + 5 = 7$

Step 3:
Add your 'tens':

	H	T	O	
	4	3	2	
+	1	4	5	
		7	7	

$3 + 4 = 7$

One potential area of frustration for the pupils is forgetting to add on the amount that has been carried over or regrouped into the next column:

	4	6	
+	3	8	
	~~7~~	4	X

	4	6	
+	3	8	
	8	4	✓
	1		

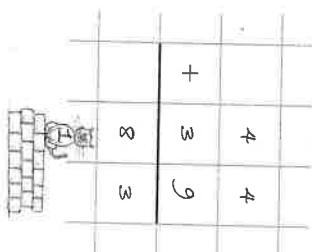
	1		
	4	6	
+	3	8	✓
	8	4	

Remembering to add on what you have just carried under is not a difficult skill. But it isn't particularly exciting or enthralling either, so pupils hardly feel compelled to remember to do it. It is our job, therefore, to make it as enjoyable and interesting as possible so they don't forget. When I first started teaching, there were quite a few pupils in my class who struggled to remember to carry over, and the method that I came up with to demonstrate this concept was that of the 'yo-yo digit'. I start by showing the pupils how a yo-yo works, casting it down and watching it return back to my hand. I explain that, when adding using the column method, if the answer in any given column goes over 9, then a yo-yo number appears! It is cast down underneath the answer into the next column along. At this stage, I ask students whether yo-yos ever stay put? "No, they don't," comes the reply. "They fly back upwards!" I then explain that it is for this reason that the students must remember to add the value of the yo-yo digit to the total of that column (before the yo-yo disappears again). Pupils can even draw little yo-yo calculations themselves by putting the digit that is being regrouped or carried inside a picture of a yo-yo.

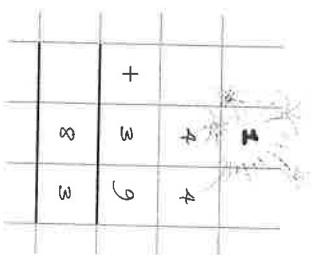
	4	4	
+	3	9	
	8	3	
		1	

Another approach to explain this method was suggested to me by a colleague who had stumbled on a local news story about a cat that waited on

the wall outside its house whenever its owners were out. Using the details of the story, my colleague would draw a little brick wall underneath the addition sums and, whenever there was something to carry over, she would draw a cartoon cat with the value carried over written on its chest. She said that the cat was so important to the owners that they would never forget it and would pick it up whenever they returned home.



For those who prefer to regroup above the calculation, I have in the past explained the regrouping by talking about fireworks, human cannonballs or flying fish, as all these things fly up, but, sadly, and as we know, they always come back down again.



Another approach that works for numbers being written above or below the main body of the calculation is using the good old plane/train ticket upgrade

approach. When you get above 9 in any given column (class) you have run out of room, and so number 1 is given an upgrade to a seat in the next class (column) up! The added bonus of this approach is that you can make a link to the fact that, as you progress from left to right through the columns, the place value of the digits (tickets) increases; more specifically, it increases by ten times the amount. This means that one ticket in the business class (tens) column is worth ten tickets in the economy class column, and a ticket in first class (hundreds) is worth 100 tickets in economy class.

SUBTRACTION

Subtraction is tough. For a start, at even its most basic level, it requires pupils to count backwards, which is far more difficult than counting forwards.

Before doing the even the simplest subtraction calculations with children, the most important thing to get nailed is the issue of commutativity (which we've covered above). As a reminder, commutativity is the ability to change the order of numbers in a calculation. Put simply, you can change the order of numbers in an addition calculation, but you can't with subtraction:

$$3 + 7 = 10 \quad \text{and} \quad 7 + 3 = 10$$

However:

$$10 - 3 = 7 \quad \text{but} \quad 3 - 10 = -7$$

If pupils aren't able to grasp this concept, they will be absolutely stumped when it comes to using formal written methods or when attempting to

solve more complicated problems at a later stage. So, don't just tell them about it, get them to do it!

Give the pupils ten sweets (or counters, which are just as good, if nowhere near as tasty) and get them to explore the addition and subtraction calculations above. Try to resist the temptation of telling the pupils that the bigger number always goes on top, or first, as this is a lie and an unnecessary short-cut. Instead, explain that unlike addition calculations, which are actually pretty easy going and don't mind in which order they line up, subtraction calculations are a bit fussy and, as such, insist on being put in exactly the correct and appropriate order. Explain that most of the time this is done with the biggest number going first, but it isn't always the case. To reinforce this idea, ask the pupils to take a certain number of objects away which is a greater number than they currently possess – for example, take four pencils when they only have two, or take off three shoes when they are only wearing two, or put six pieces of scrap paper in the bin when they only have three. Repeat this regularly. Repeat this regularly. Repeat this regularly.

Once the pupils are getting used to this concept, introduce the idea of relating addition to subtraction. Explain how it is the inverse, or opposite, calculation and that this relationship can be super-useful when checking our work. Break out the number tabards at this point: these are mini whiteboard type things with a piece of string attached to the top two corners with sticky tape to create half a sandwich board. Group the children, ideally into fives. Each child gets a tabard and a board marker. I then write three numbers on the board that can be used to create addition and subtraction calculations:

$$12 \quad 8 \quad 20$$

I tend to set out place-markers using hoops or coloured card (or even chairs if you're feeling brave), to mark each part of the calculation. The aim is to see

how quickly they can make all the possible addition and subtraction calculations using the three numbers on the board:

$$12 + 8 = 20$$

$$8 + 12 = 20$$

$$20 - 8 = 12$$

$$20 - 12 = 8$$

Each child takes the role of either one of the numbers or one of the symbols (though make sure that the same child is not putting themselves forward to be a symbol each time in the hope of an easy ride). Make sure that you take the time to address any mistakes and to point out the inverse relationship between addition and subtraction after each set of numbers. Explain how they can use this inverse relationship to check their work:

$$13 - 7 = 6$$

Therefore:

$$6 + 7 = 13$$

Also, pay particular attention to the fact that, although you can create two subtraction calculations out of the three numbers, the answers will be different. (With two different addition calculations with the same numbers the answers are the same, of course.) This method works because it is a damn sight more fun wearing a mini whiteboard and shuffling your group into the correct places than just writing the calculations out and holding it

up in the vague hope that your hard work will be rewarded with a pleasing word from teacher.

Once the children are comfortable with subtracting numbers in the correct order, and are able to check their work with an inverse calculation, it is time to move on to more formal methods. As with addition, the 2014 national curriculum places a great deal of importance on the use of formal written methods. The curriculum asks for these to be introduced at a younger age, with pupils in Year 3 being required to carry out column subtraction involving decomposition (or breaking down numbers into their constituent elements). My advice would be to take a good long look at your pupils and gauge their readiness. Expose them to formal written methods too early and you risk devaluing the progress they have made developing mental methods of working: too late and you risk not ticking all of the national curriculum's many boxes. (I'll leave it to you to work out which one is more important and act accordingly.)

Once the lovelies are ready for written methods involving decomposition, I tell them the story of Robin Hood, introducing the moral idea of stealing from the rich to give to the poor. Robin Hood lends himself incredibly well to being the central character in my explanation of subtraction involving decomposition. We start with a calculation:

	5	2	
-	1	7	

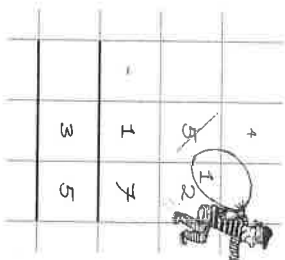
In this example, 7 cannot be subtracted from 2 because we cannot take 7 away from 2 without going into negative numbers. This is a good opportunity to take a brief foray into explaining why we do not want to go into negative numbers by introducing the character of the Sheriff of Nottingham as the dastardly villain who would chop off your mother's fingers for falling behind with her tax payments! Robin Hood comes to our rescue here. Adorned in his customary green tights, Robin unleashes an arrow right through the heart of the rich, taking only what he needs to help the poor settle their debts, and thereby avoiding us from going into negative numbers:

		4	
	5	12	
-	1	7	
	3	5	

It is important to reiterate the fact that Robin is stealing not borrowing. Decomposition is often explained as borrowing – for example, with 1 being 'borrowed' from a next-door neighbour. I used to do this when I first trained as a teacher using a raft of awful accents to show how we could borrow 1 from whoever was next door (a bit like borrowing a cup of sugar). However, borrowing implies that you are going to return it at some point, and this is not the case with decomposition: it is flagrant and unapologetic theft.

Continuing this theme, another character you can call on to weave a little bit of storytelling into subtraction is Burglar Bill. Many children encounter this story in Key Stage 1 and will already be familiar with Bill. His catchphrase of, "I'm having that", provides a super prompt for reminding the pupils when

to 'steal 1' from the next-door number digit that lives in the column to the left. When working through examples in groups or as a whole class, the cacophony of pupils chirping, "I'm having that", in their very best cockney accents, as they move 1 across is joyous to behold. (I like to draw Burglar Bill exiting the scene of the crime with his swag bag bulging with the numbers he has stolen.)



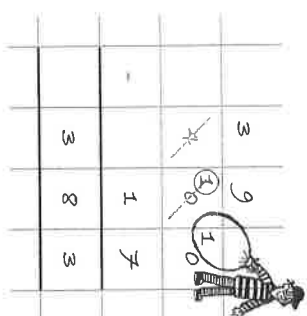
4	1	3
0	7	5
0	7	5

The Burglar Bill link also helps when confronted with a problem like this one:

4	0	0	
	1	7	
3	8	3	

This idea of having to take 1, not only from the immediate neighbouring column but also from the next one along too, is notoriously difficult for children to grasp. However, explain that it is jolly hard work heaving a wide-screen TV or a three-piece-suite over not one but two neighbours' fences,

so it's a good idea to have a rest after each fence before carrying the 1 over the final obstacle:



3	9	10
1	7	3
3	8	3

So, there you have it: subtraction. It's not as bad as everyone thinks and with a little effort can even become vaguely enjoyable(ish).

MULTIPLICATION

Multiplication relies on sound recall of multiplication facts, or times tables as they are commonly known. The general order for approaching the learning of tables is to start with multiples of 2 and 10 before moving on to multiples of 5, then multiples of 3 and 4. Pupils then progress on to learning multiples of 6, 7, 8 and, finally, 9. The 2014 national curriculum now states that pupils should learn up to the 12 times table by the age of 9, and there has been some significant grumbling over the feasibility of this. A common complaint from teachers is that they already have too much to cover and don't have the time to spend lessons drilling pupils in their times tables. However, times tables are a necessary, though not insignificant, pain. A firm grasp of times tables pays off in the long term, and is vital for providing a solid arithmetical foundation for future problem-solving.

When it comes to teaching times tables, a nice idea or principle is to SMASH them to pieces! The five principles behind SMASHing times tables are as follows:

- 1 Short and sweet
- 2 Mix it up!
- 3 Arrangement
- 4 Stir and stimulate
- 5 Hammer away

SHORT AND SWEET

Times tables are like cabbage or broccoli to children: they are hardly any child's preferred choice of diet, but they are full of the vital nourishment they need in order to function well. With this gustatory metaphor in mind, let's reconsider times tables and compare them to plain rice: filling but devoid of flavour. Rice can be delicious when accompanied by something exotic, but consumed on its own in large quantities, it is uninspiring at best. The message here is that a lesson where the sole focus is on the learning of times tables is a bland one.

However, rice goes well with so many things, and so do times tables. I try to incorporate a little times tables work into every single primary mathematics lesson. For the last couple of years, I've spent at least seven minutes of every lesson on them and on related mental arithmetic work. I've found this to be the optimal portion of times tables sustenance in order to provide the diner with enough to sate their hunger, while still having them wanting to come

back for seconds. In some lessons, I will give the pupils more input while exploring particular times tables in greater detail, but never to a point where all they have covered in a lesson are times tables.

The best way I have found to incorporate daily times tables practice into every lesson has been through the use of multiplication grids (see the Appendix for some examples). These are relatively hassle free and fit well with the other principles of SMASH: they ensure that times tables are not presented in a linear fashion, they encourage pupils to embrace the commutative aspect of multiplication and they stir up a sense of competition, regardless of ability level, allowing pupils to hammer away at their tables work both at school and at home.

While on the notion of keeping it short and sweet, it is also important that the pupils aim for this in their answering of times tables questions. Mastery of times tables should not be judged solely on 'knowing' the multiplication facts, but also on the speed at which they can be recalled. If a pupil takes more than a few seconds to answer a times tables question, then they need further work on that times table.

MIX IT UP!

The most common mistake we make when teaching pupils times tables is to assume that a child has grasped a times table because they can repeat it parrot fashion in ascending order. Put simply, if a child has to start from $1 \times$ each time they want to work out an answer, then they don't know that times table well enough. They need to be able to recall multiplication facts going forwards, backwards or out of order, so that they can solve related division calculations, and, most importantly, they need to be able to do all of this with pace. This can be a tall order for many children, but we are not

doing them any favours by lowering our expectations. We should be aspiring to more than them simply counting upwards in multiples.

I've found it useful to look at pupils' progress in times tables through a graduated approach. Pupils who can recall multiples in ascending order would be classed as having an *emerging understanding*; if they can recall facts out of order then they can be considered to have a *firm understanding*; if they can use their multiplication facts quickly to solve related division calculations or 'fill in the missing digit' style calculations then I'd consider them to have an *advanced understanding*. Viewing the learning of times tables in this way gives greater importance to mastery of a key area of numeracy development. It also allows pupils to work on a particular times table to a certain level, and then move on to other times tables, revisiting the initial one at a later stage to extend their understanding to the next level.

Once again, the grid approach (see Appendix) is perfect for ensuring that pupils become used to working on times tables out of sequential order. As the level of difficulty increases, new times tables are introduced to the grid, but the previously learned times tables remain. This provides pupils with further drill in the areas they have worked on previously, sharpening their recall with every grid completed.

ARRANGEMENT

Taking a look at the way that the numbers are arranged within times tables gives pupils a deeper understanding of what times tables actually are and, crucially, how they interact with the number system. Furthermore, it becomes vital that we reinforce the fact that multiplication has a commutative property, and that swapping the digits around can dramatically reduce the difficulty of a given multiplication question. We sometimes take

it for granted that children will recognise this aspect automatically, but they will often overlook this as a strategy for solving a multiplication calculation. Arrays can work wonders for explaining this concept:

- 1 Invite the pupils to work out a given multiplication calculation, such as 3×5 , by placing dots (or counters, sweets, smiley faces, kittens, etc.) in three rows of five, like so:

$$\begin{array}{ccccccccc} \bullet & & & & & & & & & \\ \bullet & & & & & & & & & \\ \bullet & & & & & & & & & \end{array} = 15$$

- 2 Ask them to count up the total number of dots and write the total next to their dots.
- 3 Ask them to do the same for 5×3 alongside their previous working:

$$\begin{array}{ccccccccc} \bullet & & & & & & & & & \\ \bullet & & & & & & & & & \\ \bullet & & & & & & & & & \\ \bullet & & & & & & & & & \\ \bullet & & & & & & & & & \end{array} = 15$$

- 4 Discuss how three columns of five give us the same number of dots as five columns of three.

Repeat this process with several examples and it shouldn't be long before they have cracked it. By teaching this simple rule, you are dramatically reducing the number of multiplication facts that kids have to memorise. We've already explored the importance of looking for patterns in developing sound numerical understanding. Times tables are no different, so exploring the patterns that exist in them is time well spent.

For example, if we write out the 6 times table we get the following:

$$\begin{aligned} 6 \times 1 &= 6 \\ 6 \times 2 &= 12 \\ 6 \times 3 &= 18 \\ 6 \times 4 &= 24 \\ 6 \times 5 &= 30 \\ 6 \times 6 &= 36 \\ 6 \times 7 &= 42 \\ 6 \times 8 &= 48 \\ 6 \times 9 &= 54 \\ 6 \times 10 &= 60 \\ 6 \times 11 &= 66 \\ 6 \times 12 &= 72 \end{aligned}$$

Can they spot the pattern?

How about now?

$$\begin{aligned} 6 \times 1 &= 6 \\ 6 \times 2 &= 12 \\ 6 \times 3 &= 18 \\ 6 \times 4 &= 24 \\ 6 \times 5 &= 30 \\ 6 \times 6 &= 36 \\ 6 \times 7 &= 42 \\ 6 \times 8 &= 48 \\ 6 \times 9 &= 54 \\ 6 \times 10 &= 60 \\ 6 \times 11 &= 66 \\ 6 \times 12 &= 72 \end{aligned}$$

We can see that whenever we multiply an even number by 6 the digit in the ones column matches the number that we are multiplying by:

$$\begin{aligned} 2 \times 6 &= 12 \\ 4 \times 6 &= 24 \\ 6 \times 6 &= 36 \end{aligned}$$

What about if we take a look at the 9 times table more closely?

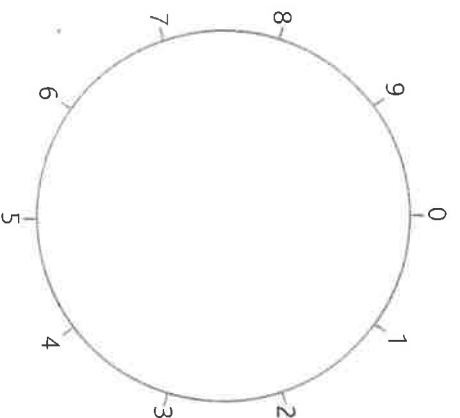
$$\begin{aligned} 1 \times 9 &= 09 \\ 2 \times 9 &= 18 \\ 3 \times 9 &= 27 \\ 4 \times 9 &= 36 \\ 5 \times 9 &= 45 \\ 6 \times 9 &= 54 \\ 7 \times 9 &= 63 \\ 8 \times 9 &= 72 \\ 9 \times 9 &= 81 \\ 10 \times 9 &= 90 \end{aligned}$$

The relationship between the tens and ones columns is the interesting bit, obviously. The ones column descends from 9 down to 0, while the tens column ascends from 0 to 9. Not only that, but if you add the digits together, then the answer will always be 9.

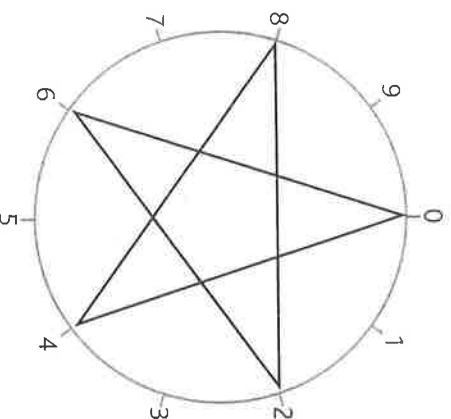
As well as spotting numerical patterns in times tables, visual patterns can be extremely useful too. I first stumbled across using visual patterns with times tables in the work of Tony Biesanz. He has some useful videos on YouTube explaining, among other ideas, how to use visual patterns to explore times tables. In one example he uses the four times table to draw a star.¹

First, draw a circle and place points for 0 through to 9 around the circumference of the circle at equidistant intervals.

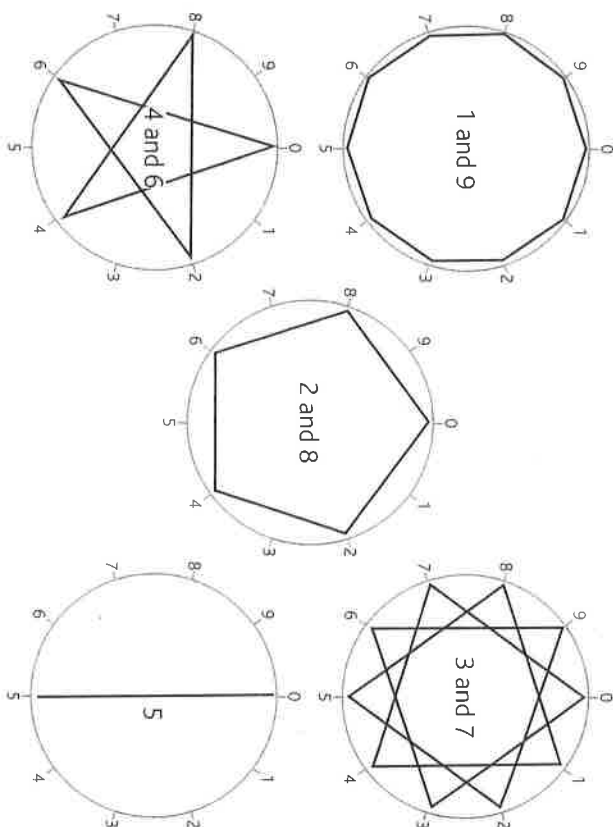
¹ See <https://www.youtube.com/watch?v=N1ALx5q6Q04>. For more videos, check out his MisterNumbers channel at: <https://www.youtube.com/channel/UCNab1XwWR1KnF53DUmTtP6w>.



Then join the dots by going through the four times table in ascending order and drawing a line between the points based on the digit in the ones column, so 0 joins to 4, 4 joins to 8 and 8 then joins to 2 (because 2 is in the ones column for 12). If we continue to repeat the pattern, what we get is:



The really incredible thing about these patterns is that each times table, from 1 to 10, has a corresponding partner that will create the same pattern in reverse. The partners with corresponding patterns are the number bonds to 10: 1 and 9, 2 and 8, 3 and 7, 4 and 6, and 5.



There is a fun property, which can be useful for checking your answer, that lurks within the 3 times table. If you take any multiple of 3, and add the digits from the multiple, they will always equal another multiple of three:

$$36: 3 + 6 = 9 \text{ or } 3 \times 3$$

$$33: 3 + 3 = 6 \text{ or } 3 \times 2$$

$$30: 3 + 0 = 3 \text{ or } 3 \times 1$$

$$27: 2 + 7 = 9 \text{ or } 3 \times 3$$

Or even:

$$2997: 2 + 9 + 9 + 7 = 27 \text{ or } 3 \times 9$$

Ask the children to use a calculator to multiply any number they like by 3 and see if the property remains true.

With all of these patterns, children gain a real sense of achievement as they unlock the beautiful world of numbers for themselves. Either that or they'll be chuffed that they have fewer times tables to memorise.

STIR AND STIMULATE

The real key to times tables success is supplementing the hard work and repetitive approach with activities from which the children will get a genuine buzz. Along with encouraging pupils to spot patterns, a sure fire way to stir some interest in the learning of times tables is to encourage some competition. There are tons of way to do this: from having a class league table or a points system to full blown approaches like Times Table Rock Stars.² Contrary to what some touchy-feely types might have you believe, children thrive when challenged and relish the opportunity to duel mathematically with their fellow peers. It just has to be dressed up in the appropriate clothing.

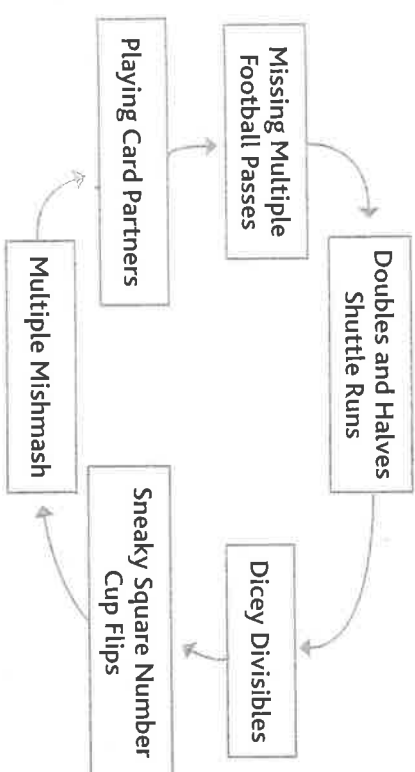
I am a passionate believer in the power of movement during mathematics lessons. A fantastic way of making times tables more exciting and relevant, especially for those who prefer to be on their feet than on their backsides, is to carry out some times tables circuit training every so often. This approach ensures that not only do pupils' brains hurt from challenging times table work, but they also get a chance to exercise the nine-tenths of their bodies below their necks too.

This method works best if you can do it in the playground or the school hall, but it can be adapted to work in a classroom. The idea is that you make several 'stations' where pupils have to use times tables to solve questions, but also that each station requires some form of physical activity. A simple example of this would a series of 'missing gap' times tables questions where pupils are required to pass a football as many times as their answer before moving on to the next question. The key is to make the stations simple to set up, simple to follow and simple to score, but with an appropriate level of challenge to stir a sense of competition. A great way to do this is to link in questions that involve division or square numbers that pupils must use

² See <https://ttrockstars.com/>.

THE FOUR OPERATIONS

their multiplication facts to answer. Have a look at this sample circuit with a selection of stations:



- 1 Missing Multiple Football Passes. Sticky notes with 'missing gap' multiplications on them are placed face down on the plastic sports cone in the middle. The first pupil runs to the cone, flips over the first sticky note and shouts it out to the rest of the group. Pupils work out the missing multiple, then complete as many passes with a football as the answer, before the next pupil runs to the central cone and repeats the process.
- 2 Doubles and Halves Shuttle Runs. Place a set of number cards on either of the two start cones. Pupils take it in turns to flip over a number card from alternating piles. When a card in the first pile is flipped over, the pupils have to double the number and run that many shuttle runs between their group. When they flip over a card from the second pile, they halve the number and run that many shuttle runs.

3 Dacey Divisibles. Prepare a set of sticky notes with two-digit products of times tables that the pupils know on each one. Once you have a decent set (30 should do) scrunch up each note individually into a ball and spread them out. The pupils take it in turns to roll a dice (large foam dice work best) and then find a sticky note with a number on it that is divisible by their dice number.

4 Sneaky Square Number Cup Flips. Prepare a set of cards with the square numbers to 12×12 on them. Shuffle the cards and place them face down on a table. Place 12 paper cups on the table. The first pupil flips a card over and works out what the square root of the number is (e.g. if the card says 81, the square root is 9). If the square root answer is even, then the pupils have to make sure that the same number of cups are flipped facing upwards; if the square root is an odd number, then they must flip the appropriate number of cups facing downwards.

5 Multiple Mishmash. Place sticky notes with a times table calculation on each of them on one wall (or the floor on one side of the room). Place the answers to the calculations on another set of notes of a different colour on the wall or floor on the opposite side of the room. Place an empty bin in between the sets of questions and answers. The pupils line up behind the bin and take it in turns to sprint out and collect a sticky note from the question side, before finding the corresponding answer on the other side. Once they have found a pair, they stick them together and dash back to chuck them in the bin. You can also ask the pupils to repeat this game in reverse, with them getting the answers first before heading to the questions.

6 Playing Card Partners. Prepare a set of sticky notes containing products from times tables up to 10×10 that the pupils know. Lay out a pack of playing cards with the face cards removed (face up for easy, face down for difficult). Pupils must pick a sticky note, then find a

pair of playing cards which, when multiplied together, will equal the number on the sticky note.

The circuit is just the start. These activities are so easily adapted and altered that the possibilities are endless. Imagine how brilliant a full-blown mathematical assault course could be? Or how about staging your very own Mathstlympics? And why limit these ideas just to times tables? They could be used for drilling any type of four operations arithmetic. The only limitations are your imagination, available space and the school's health and safety policy.

For a simpler way of sprucing up your arithmetic drill, how about creating your very own question balloons? The procedure is simple enough:

Step 1: Blow up balloon.

Step 2: Scrawl relevant questions on said balloon in board marker pen.

Step 3: Go back to step 1 and start all over again, because it is better for the pupils to do step 2 themselves. You may get a little out of breath here.

Step 4: Pupils throw question balloons to each other, answering whichever question is closest to their right thumb.

Step 5: Pupils cross out questions on the balloon with a board marker as they answer them.

Step 6: When they have answered all the questions, they can pop their balloon to signify that they have been victorious!

For an added level of difficulty, a 'killer question' can be inserted into the balloon on a small scrap of paper, which must be answered after the balloon has been popped.

Another cracking way to work with times tables (as well as any other form of arithmetic or number properties) is to play bingo. Instead of 'two little ducks' or 'jump and jive', think '2 x 11' or '7 x 5'. To make it more challenging, chuck in some addition or subtraction first to extend it to a two-step problem – for example, '7 plus 4 then multiply by 2' or '6 minus 9 multiplied by 5'.

Remember, mental agility is king.

HAMMER AWAY

Times tables are ideal conduits for teaching children the value of perseverance and resilience, as they are rarely mastered quickly. Put simply, times tables success does not happen overnight and pupils need to realise and be warned about this. It will take time and plenty of repetition for most pupils to become proficient in their application of times tables. They need to hammer away at them every day without fail. A sure fire way to speed up the acquisition of times tables is to involve parents as much as possible. It doesn't take a genius to figure out that the pupils who practise both at school and at home are more likely to pick up their times tables quickly.

A WORD ON TIMES TABLES 'PRODUCTS'

The market for times tables aids is ever increasing and, with the continual rise in mobile devices and apps, people have more choice than ever as to how they can support their little darlings with their learning at school and at home. The products on the market for teaching times tables are wide ranging and one could write a rather boring book just listing them. However, they vary greatly in quality. By all means explore the possibilities, but don't

be fooled by gimmicks claiming to be the magic solution. Keep in mind the SMASH principle and you won't go far wrong.

WRITTEN METHODS

As with all the four operations, there is a move towards ensuring that pupils are accomplished in their use of written methods for multiplication. There is a plethora of written methods for multiplication. My preferred ones are the grid method (multiplication with partitioning) and the traditional long multiplication method of laying out. The grid method is a solid starting point for venturing into written methods because it builds on mental methods and minimises place-keeping errors due to being set out neatly in a grid. I use the method as follows:

$$22 \times 14 =$$

$$20 \quad 2$$

10	200	20
4	80	8

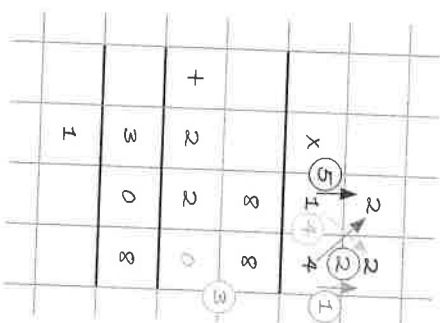
$$200 + 20 + 80 + 8 = 308$$

$$44 \times 6 =$$

	40	4
6	240	24

$$240 + 24 = 244$$

The traditional column method can pose greater difficulty for pupils due to the more intricate nature of knowing the correct order in which to multiply the digits. As explained earlier in this chapter, breaking down the individual steps is vital. The way I tend to approach this is to enlarge the calculation and then draw different coloured arrows that are numbered to show each step:



And that's all I've got to say about written methods of multiplication. Bit short? Maybe. There's a reason for that though. If every primary teacher taught multiplication in the same ratio as this section (about five times more work on mental methods and times tables compared to the time spent on teaching written methods) then pupils would be in a far better position to problem-solve when they hit secondary school. Times tables simply must be the focus – don't marginalise them too early.

DIVISION

Division is, without doubt, the least popular of the four operations. A straw poll on the areas of mathematics that most people associate with confusion and stomach churning dread almost always yields the same result: the dislike of division is as common as the belief that the gruff school dinner lady was not actually a lady at all.

Every time I teach division, I try to think up a new way to present it. As I mentioned previously, "Anyone can feed sweets to sharks", comes from a short piece of in-lesson drama that I use to explain division. The first lesson in the series begins with the teacher (me) entering the room dressed as a deep-sea fisherman (yellow waterproof mac optional but highly advised). I explain that I am a scientist working for the Society of Somalian Sugar Sharks (alliteration works best when trying to introduce an element of absurdity) and that my job is to care for a rare breed of shark: the only vegetarian shark in the world. The pupils are then told that this particular shark feeds only on sweets, and that because of this they need feeding twice a day by me and my team. At this point, I usually ask the pupils to spend a few minutes thinking in groups about what they will need to consider when feeding Somalian sugar sharks. Some of the answers you get here are brilliant, with the best one ever being, "To make sure that you are not dressed in bright colours, otherwise you might look like a sweetie yourself". Following this, I usually split the class into groups to work out how many sweets they would feed to each shark, given a particular number of sweeties and a particular number of sharks.

How many sweets would you give to each shark if you had 11 sweets and three sharks?

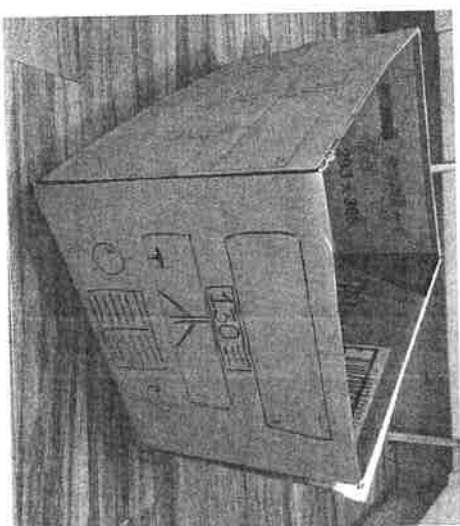
For this part, you can split the pupils up into sharks and scientists and get them to act it out, or you can use a small group to demonstrate to the rest of the class. Once the pupils have worked through several examples, I usually get them to feed back to the class what they have been doing, the process they went through and the answers they got.

For the next step in the sugar shark drama, you will need a small number of pupils who are not afraid to act as sharks (simple shark hats made out of laminated pictures of a shark head help the pupils to get into character). The teacher still plays the role of the scientist who feeds the sharks. The teacher picks a number of sweets that will not share equally between the sharks (pupils), such as 11 sweets for three sharks. The teacher demonstrates sharing the sweets out to the sharks, who greedily chomp on them until the problem occurs of not having enough sweets to give each shark another one (the remainder).

At this point, I ask each of the sharks how they would feel if I gave the other shark another sweet, but didn't give one to them. I also ask the rest of the class to spend a minute or two discussing what I should do next. We play around with the idea until we get to the solution of not feeding the sharks any more unless they can all have one, because all division should be fair and equal. We talk about what we could call the amount of sweets that are left over, before introducing the word 'remainder'. The pupils then work through some examples themselves. I have also asked the pupils to film their own sugar shark drama episode to demonstrate the idea of remainders, which is then shown to the rest of the class.

I have another tried and tested method for introducing the formal method of written short division. The formal method of written division is very different

to the other four operations: it does not look anything like the other three, which tend to stand in a more orderly manner, in regimented columns. The thing that goes over the top of the number that you are dividing, you know the bit I mean, well, I've always thought it looks a bit like a bus stop. It is for this reason that one day, a couple of years ago, I found myself in the art room (not my natural environment) with a cardboard box and some red paint. My class had just been doing some work on forms of transport and how they travelled to school in their geography lessons. It transpired that several children in the class had more than a passing interest in London buses. So, with this in mind, I set about creating my first Division Bus. More budget *Art Attack* than *Pimp My Ride*. I slathered the box in red paint and crudely drew on the necessary appendages of wheels and headlights. Next, I attached two lengths of yellow string to form shoulder straps. There it was, a thing of beauty, my first Division Bus.



When I unveiled it in front of the class, there were 'oohs' and 'aaahs' from all corners of the room, mainly to do with the burning question of how I had

managed to wedge myself inside such a small box! To use the Division Bus to its full potential you first need to set up your bus stop. I tend to project a background on the whiteboard with a division bracket as the bus shelter. I position a number of chairs under the shelter (division bracket) with a mini whiteboard and pen on the seat, and I pin up some bus timetables either side of the board to set the scene. Pick a number of children to be passengers waiting at the bus stop based on the number of chairs you have laid out. It helps if you have costumes for them to dress up in as every rough, tough boy secretly relishes the opportunity to dress up like a granny. Get the pupils to sit at the bus stop with a single digit written on their mini whiteboard. They must not show the rest of the class their number until the bus arrives in order to keep the suspense rolling. As the bus pulls up to the stop, the driver (pupil) shows the rest of the pupils a mini whiteboard with the divisor written on it. As soon as the divisor has been unveiled, the passengers at the bus stop spin around their mini whiteboards to show the dividends. At this point, every pupil in the class who is not either a driver or at the bus stop dashes to scrawl down the working out and answer to ensure that they might just get picked to be the next bus driver.

When the pupils have a solid understanding of how to use the short division method, the long division method can be introduced. This method enables pupils to divide by numbers that are larger than 12. If you have to teach it, you could do a whole lot worse than visiting this website, which shows a very good demonstration of long division: www.mathsisfun.com/numbers/long-divisionanimation.html.

Despite there being lots of different modified and shortened ways of carrying out this type of calculation, the one demonstrated on the website above is the most common and the easiest to follow. The core idea to reinforce in this method is the 'bringing down' of digits from underneath the division bracket. When I was growing up, one of my favourite computer games was a cracking little number called *lemmings*. The aim of the game was to navigate a certain percentage of your lemmings around a map while ensuring they

didn't plunge off the edge of a cliff to their demise. This idea of lemmings diving is a great way to remind the pupils to bring down the necessary numbers from the division bracket.

A WORD ON CALCULATORS

By time you read this book, the thinking could have changed, but the 2014 national curriculum suggests that calculators should only be introduced towards the end of Key Stage 2, and only if pupils have achieved a solid understanding of the basics. The reasoning is that by introducing calculators at the start of Key Stage 2, pupils will become reliant on them for even basic arithmetic. I'm sure this would actually be the case if pupils were given calculators in every lesson and the teacher allowed the pupils to use them for every question. However, we are professionals and, with a bit of judgement, I am of the belief that calculators can be incredibly useful tools for pupils throughout Key Stage 2 and even in Key Stage 1.

A crucial area of development for any pupil is encouraging them to work with greater independence. In mathematics, one of the most important aspects of this is getting pupils into the habit of checking their own work for mistakes. This is one area where a calculator can come into its own. By providing pupils with calculators, and teaching them how to check their work effectively, they can obtain far more immediate feedback on their work than is possible if they rely solely on the teacher to check it. Of course, there are many other methods for checking work, and I am not suggesting that a calculator is necessarily the best approach for all circumstances, but it can be a very effective tool.

Another use for calculators is to deploy them in contexts where the calculation element of the lesson is playing second fiddle to a more powerful

objective: developing investigative skills. In an investigation, we are often asking pupils to use skills they have already learned to tackle a far wider and more abstract problem. The key aim of such an activity is to get them thinking for themselves, and part of that is working out how they can work most efficiently. Using a calculator to trial and improve working and answers is a valuable learning opportunity for the older primary pupil – for example, asking pupils to find out the four consecutive numbers that add up to 268.

The main reason that calculators can be effective in the primary age phase is simply that they can be great fun. Anything that encourages children to play around with numbers and calculations shouldn't be sniffed at. Here are a few fun ideas to get you started:

- **Beat the calculator.** One person has a calculator, the other has only their brain and they race to answer questions.
- **Messing about writing words.** Try $1 - 0.2266$ or 30218017×2 before turning the calculator upside down to reveal a word.
- **Closest wins.** Pupils have ten seconds to estimate the answer to a difficult calculation; closest answer wins.

For some superb ideas about what can be done with a calculator have a look at the NRICH website,³ which illustrates brilliantly just what can be achieved with a calculator when wielded well in the right hands.

³ See http://nrich.maths.org/public/leg.php?group_id=37&code=5028#results.

IN CONCLUSION

Over the years, I've come up with countless different ways of explaining the four operations and, to be honest, I wouldn't be able to tell you which one I think works the best. Different classes like different approaches. The unifying factor is that it feels a damn sight more interesting learning about yo-yos, fireworks, human cannonballs, burglars, Robin Hood, Somali sugar sharks, buses and lemmings than dredging through some dull textbook, worksheet or PowerPoint that has nothing to excite other than a flashy colour scheme. Not to mention the fact that you can take these ideas as a starting point and present them in all manner of ways through storytelling, drama, artwork and so on.

Making maths memorable by being, erm, zany and far from what the pupils expect has massive benefits. First, when it comes to covering operations such as division again, I can refer to the sugar sharks and the children usually remember the process and approach we went through. I cannot say the same of baldly enquiring, "Remember when we did column multiplication ... " I've spoken to children several years later who tell me that they still think about sharks when solving division problems, or greedy crocodiles when looking at equations that contain inequalities. Second, it serves to reinforce the message that maths can be fun and that it is not simply a subject based on textbooks and the answering of questions which are either right or wrong. Children form early positive memories associated with maths, so if you do it with imagination and verve it becomes less of a Marmite subject. Or it remains a Marmite subject, but a version of Marmite that everyone likes.