

Advanced Physics of OME exam.

- * 1.) a.) From steady state parameters either side of shock front:

ρ_2	ρ_1
u_2	u_1
H_2	H_1

2

$$\rho_1 u_1 = \rho_2 u_2$$

as $\rho = \text{kg m}^{-3}$ mass is conserved.

- b.) specific volume:

$$\rho_1 D = \rho_2 (D - u_2)$$

$$\therefore \rho_2 = \frac{\rho_1 D}{D - u_2}$$

$$D = a + bu$$

$$\therefore D = 1450 + (1.99 \times \frac{500}{1000})$$

$$= 2445$$

$$\rho_2 = \frac{1000 \times 2445}{2445 - 500}$$

$$= 1257.069 \dots$$

$$\text{Spec. volume} = \frac{1}{\rho}$$

$$\therefore = \frac{1}{1255.06 \dots}$$

$$= 7.96 \times 10^{-4} \text{ m}^3 \text{ kg}^{-1}$$

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Pressure: $(p_2 - p_1) = \rho_1 D \times u_2.$

initial pressure = 0.

$\therefore (p_2 - 0) = 1000 \times 2445 \times 500$

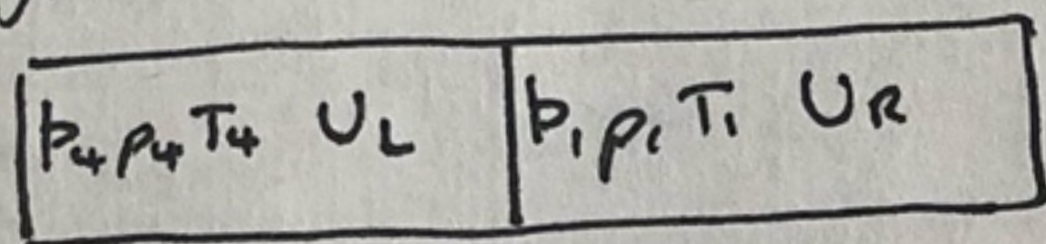
$p_2 = 122250000$

$= \underline{\underline{1.22 \text{ GPa}}}$ to 2 d.p.

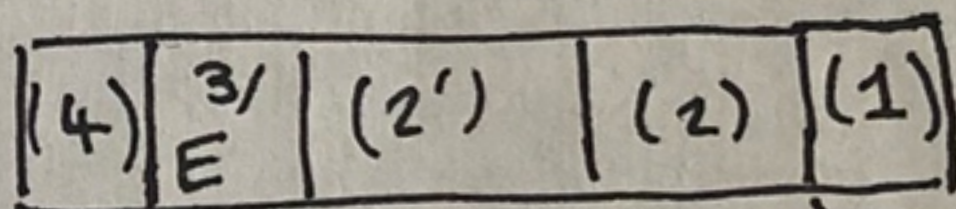
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c.) When driven section and working section diaphragm is removed, a shock front moves through the ~~medium~~ fluid. This is supersonic for detonation and subsonic for deflagration. Shock front is followed by fluid separated with contact discontinuity boundary. (Areas with the same pressure, but different density). This is followed by rarefaction region travelling back into driven section.

fluid $t=0$

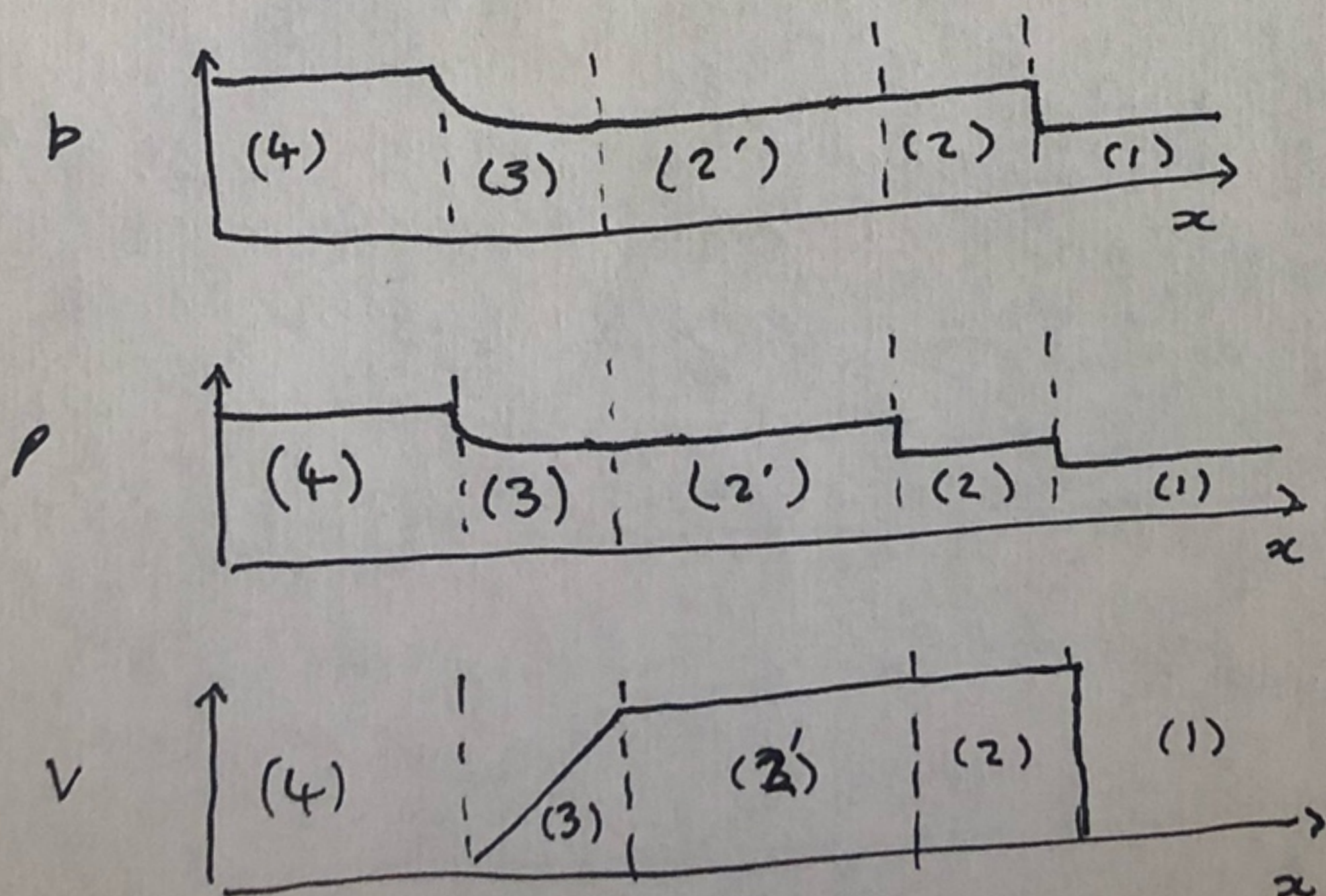


fluid $t > 0$.



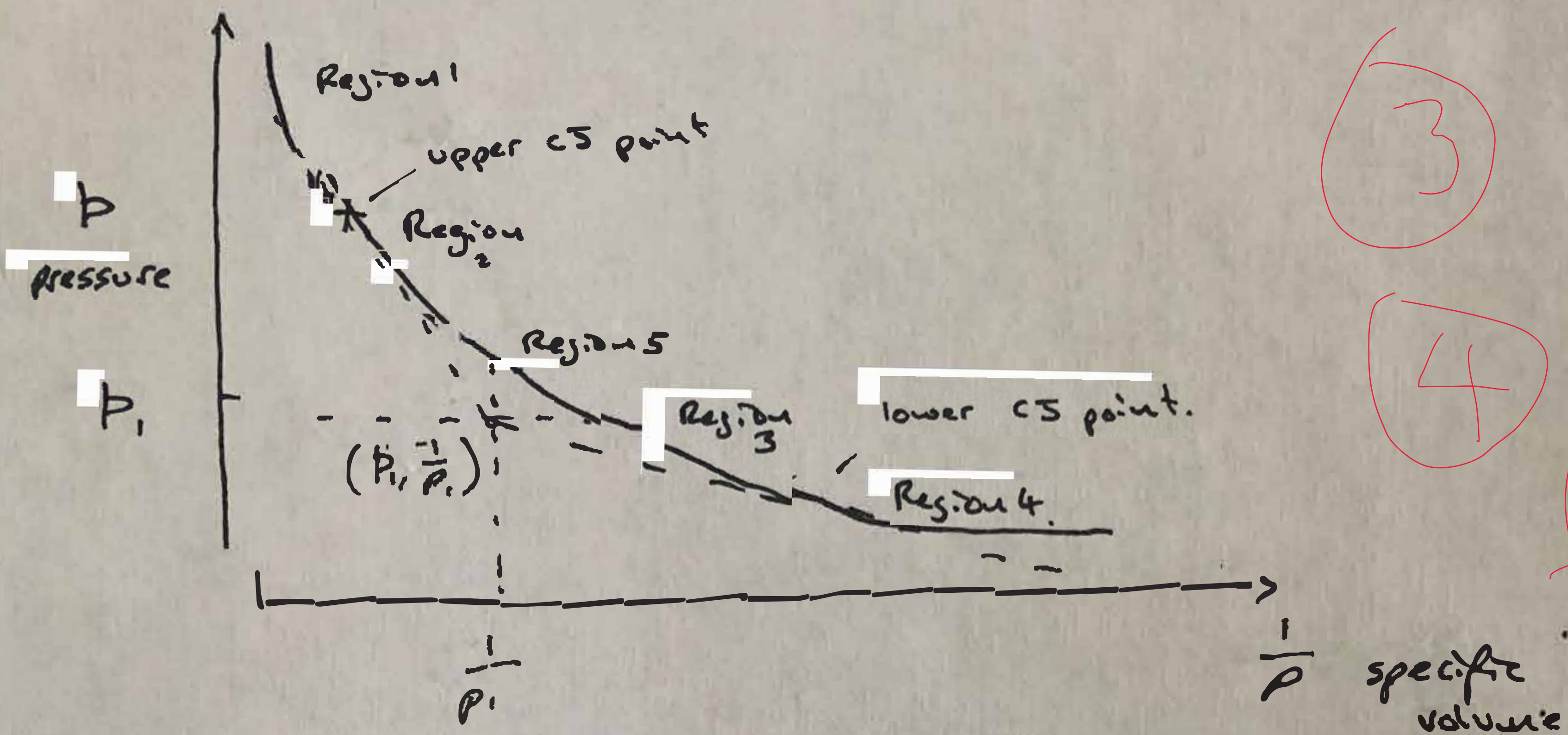
contact discontinuity / shock front.

Pressure, density and velocity profiles:



Around contact discontinuity no fluid flows. 2' and 2 are at the same pressure but have different densities and temperatures.

d.) Hugoniot curve with Rayleigh lines:



Detonation is a supersonic combustion wave with high velocities $\sim 2000 \text{ ms}^{-1}$. Pressure increases across the shock front whilst specific volume decreases.

This is represented by region 1 (strong detonation) and region 2 (weak detonation) on the Hugoniot curve.

Deflagration is a subsonic detonation wave, with low velocities of $\sim 1-100 \text{ ms}^{-1}$. Pressure decreases across shock front whilst specific volume increases. This is represented by regions 3 and 4 on the Hugoniot curve. Which show weak and strong deflagrations respectively.

Region 5 is impossible as there are no real solutions that would satisfy points in that area of the curve.

2.) a.) Cauchy stated that the 6 stress components are linearly related to the 6 strain components.
Expressed mathematically:

$$\{\sigma\} = \{C\} \cdot \{\epsilon\}$$

where C is the stiffness matrix.

Essentially each individual component of stress depends on all 6 strain components.

b.) i.)

$$\text{stiffness matrix} = \frac{300 \times 10^9}{(1 + 0.3)(1 - 2 \times 0.3)}$$

$$\begin{bmatrix} 1-0.3 & 0.3 & 0.3 & 0 & 0 & 0 \\ 0.3 & 1-0.3 & 0.3 & 0 & 0 & 0 \\ 0.3 & 0.3 & 1-0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-0.6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-0.6 \end{bmatrix}$$

$$= (5.769230769 \times 10^{11})$$

$$\therefore \{C\} = \begin{bmatrix} 4.04 \times 10^{11} & 1.73 \times 10^{11} & 1.73 \times 10^{11} & 0 & 0 & 0 \\ 1.73 \times 10^{11} & 4.04 \times 10^{11} & 1.73 \times 10^{11} & 0 & 0 & 0 \\ 1.73 \times 10^{11} & 1.73 \times 10^{11} & 4.04 \times 10^{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.31 \times 10^{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.31 \times 10^{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.31 \times 10^{11} \end{bmatrix}$$

$$\begin{aligned} \text{ii) } \sigma_{xx} &= (4.04 \times 10^{11} \times 0.05) + (1.73 \times 10^{11} \times 0.1) + (1.73 \times 10^{11} \times 0.05) \\ &\quad + (0 \times 0.2) + (0 \times 0.1) + (0 \times 0.2) \\ &= (2.02 \times 10^{10}) + (1.73 \times 10^{10}) + (8.65 \times 10^9) \\ &= 4.615 \times 10^{10}. \end{aligned}$$

$$\neq \underline{4.62 \times 10^{10} \text{ Pa}}$$

$$\text{or } 4.62 \times 10^{10} \text{ Pa.}$$

$$= \underline{46.15 \text{ GPa.}}$$

c.) i) The deviatoric stress matrix is the regular stress matrix then with the hydrostatic (or direct stresses) subtracted. 3

ii) The deviatoric matrix essentially represents shear stresses to the material. i.e. stresses that are deforming the material. 2

3.) a) $pV = nRT$.

$$\therefore n = \frac{pV}{RT}$$

$$= \frac{2.5 \times 10^6 \times 0.4}{8.31 \times 313.16} = \frac{2.5 \times 10^6 \times 0.4}{8.31 \times 313.16}$$

~~$$= 410.4821375$$~~
~~$$= 410.48 \text{ moles}$$~~

$$= 384.2666479 \text{ moles}$$

$$= \underline{\underline{384.27 \text{ moles}}}$$

b.) i) As isotherm $p_1 V_1 = p_2 V_2$.

$$\therefore p_2 = \frac{p_1 V_1}{V_2}$$

$$= \frac{5 \times 10^6 \times 0.2}{0.15}$$

$$= 6.666 \dots \times 10^6$$

$$p_2 = \underline{\underline{6.67 \text{ MPa}}}$$

ii). As adiabatic $p_3 V_1^\gamma = p_2 V_2^\gamma$

$$\therefore p_3 = p_2 \left(\frac{V_2}{V_1} \right)^\gamma$$

$$p_3 = 6.67 \times 10^6 \left(\frac{0.15}{0.2} \right)^{\frac{7}{5}}$$

$$= 4458734.398 \text{ Pa.}$$

$$= \underline{\underline{4.46 \text{ MPa.}}}$$

$$\gamma = \frac{919.8}{657.}$$

iii). As gas is ideal $pV = nRT$ still holds.

$$n = \frac{6.67 \times 10^6 \times 0.15}{8.31 \times 300.}$$

$$= 401.3237064 \text{ moles. at } p_2 V_2, T_1.$$

$$p_3 V_1 = n R T_2.$$

$$\therefore T_2 = \frac{p_3 V_1}{n R.}$$

$$= \frac{4.46 \times 10^6 \times 0.2}{401.3237 \times 8.31}$$

$$= 267.4662669$$

$$= \underline{\underline{267.47 \text{ K.}}}$$

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4.) a.) $\Delta Q = \Delta u + \Delta W.$

The energy supplied to a system is equal to the change in internal energy of the system, plus the work done by the ~~the~~ system. E.g. change in internal energy of a gas, plus work done by expanding/pushing a piston etc.

b.) As adiabatic $\Delta Q = 0. \therefore \Delta u = \Delta W.$

$$\Delta u = mc\Delta T.$$

$$m = 0.01 \text{ kg.}$$

$$c = 500.$$

$$\Delta T = \frac{250}{500 \times 0.01}$$

$$= 50$$

$$\therefore \Delta T = 50 \text{ K}$$

3

3

c.) $\Delta Q = \Delta u + \Delta W.$

$$15 = \Delta u + 7.$$

$$\therefore \Delta u = 15 - 7$$

$$= 8.$$

$$\Delta u = \underline{\underline{8.5.}}$$

3

d.) $\Delta S = \int_{T_1}^{T_2} \frac{\delta Q}{T}.$

$$\Delta S = mc \int_{T_1}^{T_2} \frac{1}{T} dt$$

$$\Delta S = mc \left[\ln(T) \right]_{T_1}^{T_2}$$

$$= mc \ln\left(\frac{T_2}{T_1}\right).$$

$$m = 7$$

$$c = 4.2 \times 10^3$$

$$T_2 = 353.16$$

$$T_1 = 293.16.$$

$$\therefore \Delta S = 7 \times 4.2 \times 10^3 \times \ln\left(\frac{353.16}{293.16}\right)$$

$$= \underline{\underline{5474.36 \text{ JK}^{-1}}}$$

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