

Trinity University

[ACCA4025] - Mathematics

[Examination]

Exam Question Answers

Joshua Piecko

Student Number: 2112389

12th January, 2022.

Question 1

Question 1 (A)

(1) Find Least Common multiplier of 6, 2:

$$2 \times 3 = 6$$

1.1 multiply by LCM = 6

$$= \left(\frac{x-4}{6} \times 6 \right) - \left(\frac{x+1}{2} \times 6 \right) = \left(\frac{x+3}{2} \times 6 \right)$$

1.2 Simplified = $x - 4 - 3(x+1) = 3(x+3)$

1.3 Expand $x - 4 - 3(x+1) = x - 4 - 3x - 3$

1.4 Simplify (1.3) = $-2x - 7$

1.5 Expand $3(x+3) = 3x + 9$

1.6 $-2x - 7 = 3x + 9$

1.7 add 7 to both sides = $-2x = 3x + 16$

1.8 Subtract $3x$ from both sides
 $= -2x - 3x = 3x + 16 - 3x$

1.9 Simplify = $-5x = 16$

Question 1 (A)

1.10 Divide both sides by -5

$$= \frac{-5x}{-5} = \frac{16}{-5}$$

1.11 Simplify $= x = -\frac{16}{5}$

Answers

$$x = -\frac{16}{5}$$

Question 1

(B)

$$5x + 3y = 9, \quad 2x - 3y = 12$$

① Solve $5x + 3y = 9$ for y

$$1.1 \quad 5x + 3y = 9 \Rightarrow 3y = -5x + 9$$

$$1.2 \quad \frac{3y}{3} = \frac{-5x + 9}{3}$$

$$1.3 \quad y = -\frac{5}{3}x + 3$$

② Substitute $-\frac{5}{3}x + 3$ for y in $2x - 3y = 12$, then solve for x .

$$2.1 \quad 2x - 3\left(-\frac{5}{3}x + 3\right) = 12$$

$$2.2 \quad 7x - 9 = 12 \quad (\text{simplified})$$

$$2.3 \quad 7x = 21 \quad (\text{Add } 9 \text{ both sides})$$

$$2.4 \quad x = \frac{7x}{7} = \frac{21}{7} = 3$$

$$2.5 \quad x = 3$$

Question 1 (B)

(3) Substitute 3 for x in $y = -\frac{5}{3}x + 3$

3.1 $y = -\frac{5}{3}x + 3$

3.2 $y = -5 + 3$

3.3 $y = -2$

Answers:

$$x = 3$$

$$y = -2$$

Question 1 (C)

Solve $x^2 - 7x + 12 = 0$ for x .

① Solve with quadratic formula: $ax^2 + bx + c = 0$

$$1.1 \quad x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 1 \times 12}}{2}$$

$$1.2 \quad x = \frac{-(-7) \pm 1}{2}$$

$$1.3 \quad x_1 = \frac{-(-7) + 1}{2}$$

$$x_2 = \frac{-(-7) - 1}{2}$$

Answers

$$x_1 = 4$$

or

$$x_2 = 3$$

Question 2

Question 2

(A)

(A-1)

$$(1.) \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ 5 & 5 & 5 \\ 8 & 8 & 8 \end{bmatrix}$$

Multiply matrix A by matrix B on calculator.

Question 2

(A)

(A-2)

(1) A - B

1.1 Subtract matching positions

$$= \begin{pmatrix} 1-1 & 2-1 & 3-1 \\ 4-(-1) & 5-(-1) & 6-(-1) \\ 7-1 & 8-1 & 9-1 \end{pmatrix}$$

1.2 Simplified

$$= \begin{pmatrix} 0 & 1 & 2 \\ 5 & 6 & 7 \\ 6 & 7 & 8 \end{pmatrix}$$

Answers:

$$A-B = \begin{bmatrix} 0 & 1 & 2 \\ 5 & 6 & 7 \\ 6 & 7 & 8 \end{bmatrix}$$

Question 2

(A)

(A-3)

① $A+B$

1.1 Add matching positions (A+B)

$$= \begin{pmatrix} 1+1 & 2+1 & 3+1 \\ 4+(-1) & 5+(-1) & 6+(-1) \\ 7+1 & 8+1 & 9+1 \end{pmatrix}$$

1.2 Simplified = $\begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 8 & 9 & 10 \end{pmatrix}$

Answers:

$$A+B = \begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 8 & 9 & 10 \end{pmatrix}$$

Question 2

A

A-4

① Transpose A

$$1.1 \quad \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

Answer

$$A^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

Question 2

(B)

① find the determinant of A

1.1 Find matrix determinant according to formula:

$$\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = a \cdot \det \begin{pmatrix} e & f \\ h & i \end{pmatrix} - b \cdot \det \begin{pmatrix} d & f \\ g & i \end{pmatrix} + c \cdot \det \begin{pmatrix} d & e \\ g & h \end{pmatrix}$$

$$= 3 \cdot \det \begin{pmatrix} 9 & -2 \\ 2 & 1 \end{pmatrix} - 1 \cdot \det \begin{pmatrix} 6 & -2 \\ -1 & 1 \end{pmatrix} - 4 \cdot \det \begin{pmatrix} 6 & 9 \\ -1 & 2 \end{pmatrix}$$

$$1.2 \quad \det \begin{pmatrix} 9 & -2 \\ 2 & 1 \end{pmatrix} = 13$$

$$1.3 \quad \det \begin{pmatrix} 6 & -2 \\ -1 & 1 \end{pmatrix} = 4$$

$$1.4 \quad \det \begin{pmatrix} 6 & 9 \\ -1 & 2 \end{pmatrix} = 21$$

$$1.5 \quad \text{Refactor} = 3 \cdot 13 - 1 \cdot 4 - 4 \cdot 21 = -49$$

Answer

$$\det A = -49$$

Question 2 (C)

① Solve system of equations using gaussian elimination

1.1 Multiply first equation by -3 and add result to second equation.

$$\begin{array}{r} x + y - z = 1 \\ -5y + 4z = -8 \\ x + 3y - 2z = 14 \end{array}$$

1.2 Multiply equation 1 by -1 then add to equation 3.

$$\begin{array}{r} x + y - z = 1 \\ -5y + 4z = -8 \\ 2y - z = 13 \end{array}$$

1.3 Multiply equation 2 by $\frac{2}{5}$ and add result to equation 3:

$$\begin{array}{r} x + y - z = 1 \\ -5y + 4z = -8 \\ \frac{3}{5}z = \frac{49}{5} \end{array}$$

Question 2

(C)

1.4 remove fractions by multiplying equation 3 by 5:

$$\begin{aligned} x + y - z &= 1 \\ -5y + 4z &= -8 \\ 3z &= 49 \end{aligned}$$

1.5 Solve for z

$$3z = 49 \quad z = \frac{49}{3}$$

1.6 Solve for y using second equation:

$$-5y + 4z = -8$$

$$-5y + 4 \cdot \frac{49}{3} = -8$$

$$y = \frac{44}{3}$$

1.7 Solve for x by substituting $y = \frac{44}{3}$ and $z = \frac{49}{3}$ into first equation:

$$x = \frac{8}{3}$$

Answers:

$$x = \frac{8}{3}, y = \frac{44}{3}, z = \frac{49}{3}$$

Question 3

Question 3

A

A-1

① Find $|7i + 8j|$

1.1 $\sqrt{(8j)^2 + 7^2}$

1.2 Simplify

$$= \sqrt{64j^2 + 49}$$

Answer:

$$= \sqrt{64j^2 + 49} \quad \text{or } 64j + 7$$

Question 3

(B)

(B-1)

(i) Computing cross product of two vectors =

$$(u_1, u_2, u_3) \times (v_1, v_2, v_3) = (u_2 v_3 - u_3 v_2, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1)$$

$$1.1 = (-4j \cdot 2k - 2k(-6j)) \quad 2k \cdot 9i - 3i \cdot 2k \quad 3i(-6j) - (-4j \cdot 9i)$$

1.2 Simplify each element

$$= (4jk \quad 12ik \quad 18ij)$$

Answer:

$$a \times b = 4jk \quad 12ik \quad 18ij$$

Question 3

(B)

(B-2)

(1) $A \cdot B$

$$\begin{aligned} 1.1 \quad \vec{a} \cdot \vec{b} &= a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z \\ &= 3 \cdot 9 + (-4) \cdot (-6) + 2 \cdot 2 \end{aligned}$$

1.2 Simplify

$$\begin{aligned} &= 27 + 24 + 4 \\ &= 55 \end{aligned}$$

Question 3

(C)

(6-1)

(1) Find $z+2w$ given $z=-2+i$ and $w=3+2i$

1.1 $w(2): 3+2i$

$$= -2+i+2(3+2i)$$

1.2 Expand ~~-2~~ $-2+i+2(3+2i)$

$$= 4+5i$$

$$\text{Answer} = 4+5i$$

Question 3

①

①-2

① Find zw given $z = -2 + i$ and $w = 3 + 2i$

1.1 $w(z)$:

$$= (-2 + i)(3 + 2i)$$

1.2 Expand $(-2 + i)(3 + 2i)$

$$= -8 - i$$

Answer

$$wz = -8 - i$$

Question 3

(C)

(C-3)

(1) Find $\frac{z}{w}$ given $z = -2 + i$ and $w = 3 + 2i$

1.1

$$= \frac{-2+i}{w(-2+i)}$$

1.2 Expand 1.1

$$= -\frac{2}{w(-2+i)} + \frac{i}{w(-2+i)}$$

Answer =

$$= -\frac{2}{w(-2+i)} + \frac{i}{w(-2+i)}$$

Question 4

Question 4

(A)

(A-1)

(1) Derive $9x^2$

$$1.1 \frac{d}{dx} 9x^2 = 18x$$

Answer

$$= 18x$$

Question 4

(A)

(A-2)

(2) Derive: $x^2 + \sin 5x$

$$2.1 \frac{d}{dx} x^2 + \sin 5x = 5\cos(5x) + 2x$$

Answer

$$= 5\cos(5x) + 2x$$

Question 4

A

A-3

(1) Derive $x e^{2x}$

1.1 Apply Product rule

$$= \frac{d}{dx}(x) e^{2x} + \frac{d}{dx}(e^{2x}) x$$

1.2 $\frac{d}{dx}(x) = 1$

1.3 $\frac{d}{dx}(e^{2x}) = e^{2x} \cdot 2$

1.4 $\frac{d}{dx}(x e^{2x}) = 1 \cdot (e^{2x} + e^{2x} \cdot 2x)$

1.5 Simplify:

$$= e^{2x} + 2e^{2x} x$$

Answer

$$= e^{2x} + 2e^{2x} x$$

Question 4

(3)

(3-1)

(1) integrate $\int 3x^2 dx$

1.1 Remove Constant

$$= 3 \cdot \int x^2 dx$$

1.2 Apply power rule

$$= 3 \cdot \frac{x^3}{3}$$

1.4 simplify $= x^3$

1.5 Add constant $= x^3 + C$

Answer:

$$= x^3 + C$$

Question 4

(B)

(B-2)

① integrate $\int x e^{3x} dx$

1.1 Apply u-Substitution

$$= \int \frac{e^u u}{9} du$$

1.2 Remove Constant = $\frac{1}{9} \cdot \int e^u u du$

1.3 Apply integration by parts

$$= \frac{1}{9} (e^u u - \int e^u du)$$

1.4 $\int e^u du = e^u$

$$= \frac{1}{9} (e^u u - e^u)$$

1.5 Substitute back $u=3x = \frac{1}{9} (e^{3x} \cdot 3x - e^{3x})$

1.6 Add constant = $\frac{1}{9} (e^{3x} \cdot 3x - e^{3x}) + C$

Answer:

$$= \frac{1}{9} (e^{3x} \cdot 3x - e^{3x}) + C$$