

Design and Control of a Mass-Spring-Damper System

using a PID Controller

Dynamics Control and Instrumentation

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Introduction

Understanding the conduct of various daily processes involving human beings is now crucial. The perception of control significantly influences the satisfaction derived from various amenities, such as hot water during bathing, the internal combustion engine in public transport, the motor control of electric trains along with their suspension, and the cooling mechanisms of refrigerators, among others. A range of control methods is employed across several processes, typically utilising the best suitable approach for each, resulting in a comprehensive and thorough system behaviour for effective plan implementation. A primary model utilised to comprehend these dynamics is the mass-spring-damper system, which demonstrates the interaction of mechanical components under diverse forces.

This experiment centres on the development of a control system for a mass-spring-damper system utilising a PID (Proportional-Integral-Derivative) controller. The simulation of this mathematical model plays a very important role because we generate a perspective of the theoretical behavior of the system, giving a comparison between the real and the theoretical which in most cases is a little different, but it does not stop being useful. The controller is also at one point a simulation, as it is by means of a program called MATLAB® where it is made and modified, passing through an interface between the computer and the system. This system was created to exemplify control knowledge and is subject to subsequent modifications for improvement and optimisation.

Objectives

1. Understand the dynamic behaviour of a mass-spring-damper system and its significance in engineering applications.
2. Create a mathematical model for the system and simulate its response using MATLAB.
3. Design a PID controller to manage the position of the mass effectively and analyse its performance.
4. Compare the open-loop and closed loop system responses, focusing on metrics such as overshoot, settling time, and steady state-error.

Mass-Spring-Damper System

As the systems considered are of a dynamic nature, the equations descriptors are usually differential equations. In practice, because of the systems and ignorance of all relevant factors, is It is necessary to introduce a hypothesis about the operation of the system (Chen et al., 2011). Therefore, a times it will be useful to consider the physical system, outline some necessary hypotheses and linearize the system. Then, using physical laws that describe the system linear equivalent, a set of differential equations can be obtained linear. Most control systems contain both mechanical components and as electrical, although some systems also have pneumatic elements and hydraulics. From a mathematical point of view, the description of elements mechanical and electrical is analogous. The movement of mechanical elements can describe in various dimensions such as translation, rotation or their combinations. The mass-spring-damper is type of system and its associated second order differential equation occur in many areas of engineering; in particular, in various mechanical systems and electrical circuits(Ejiroghene Kelly Orhorhoro, 2016).

- Mass (m): Represents the inertia of the system, determining how much force is needed to change its velocity.
- Spring (k): Provides restoring force proportional to the displacement from its equilibrium position, following Hooke's law ($F = -kx$).
- Damper (c): Opposes the motion, providing resistance proportional to the velocity, which serves to dissipate energy and reduce oscillations (Nyathi, 2025).

An ideal mass spring-damper system is represented in Figure 1:

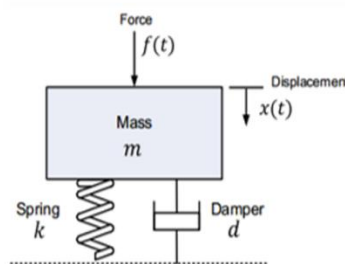


Figure 1. Mass Spring Damper Representation

2nd Newton Law

An important part of physics is the study of motion, including accelerations, which are changes in velocities. But physics also studies which can cause an object to accelerate. That cause is a force, which in general terms is a push or pull exerted on an object; it is said that a force acts on an object to change its speed (Samuel J. Ling, 2020). To Describe this is important to mention the one of motion laws of Newton. Isaac Newton was the first understood the relationship between a force and acceleration. The study of that relationship as Newton presented it, is called Newton's mechanics. But the Newton's mechanics does not apply to all situations, it is a special case very important because it applies to the movement of objects that go in size from very small (Samuel J. Ling, 2020). Newton's second law is a fundamental law of nature, the relationship basic between force and movement. The governing differential equation of motion for the mass-spring-damper system can be derived using Newton's second law (Nyathi, 2025):

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F(t)$$

Control Theory

Control theory in engineering and mathematics addresses the behaviour of dynamical systems. The output sought from a system is referred to as the reference. A controller adjusts the inputs of a system to ensure that one or more output variables adhere to a specified reference throughout time. Swift advancements in digital system technology have fundamentally transformed the alternatives for control design. It has become frequently feasible to construct highly complex digital controllers and to do the enormous computations necessary for their creation (Simrock, 2011). It is also important to mention the two types of control systems, A **regulator** sustains a physical variable at a constant value despite disturbances. – A **servo mechanism** refers to a control system designed to ensure that a physical variable adheres to or follows a specified time function, initially utilised for regulating mechanical position or motion.

The primary elements of a control system (**Fig. 1**) are the plant, and a sensor employed to measure the variable intended for regulation (Simrock, 2011). The closed-loop system (**Fig. 2**) necessitates a linkage from the system outputs to the input.

- a) **Sensor:** The system for measurement of a variable (or signal).
- b) **The plant** of a control system is the part of the system to be controlled.

The compensator (or controller or simply filter) provides satisfactory characteristics for the total system.

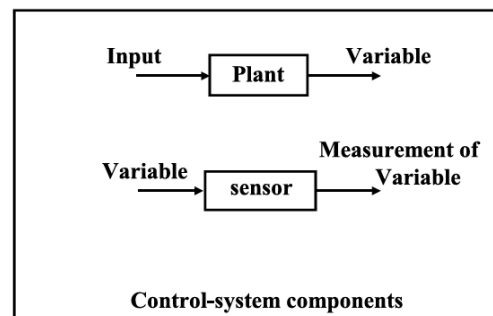


Fig. 1: Components of a control system

A closed-loop system is a system in which certain forces (we call these inputs) are determined, at least in part, by certain responses of the system (we call these outputs) (Simrock, 2011).

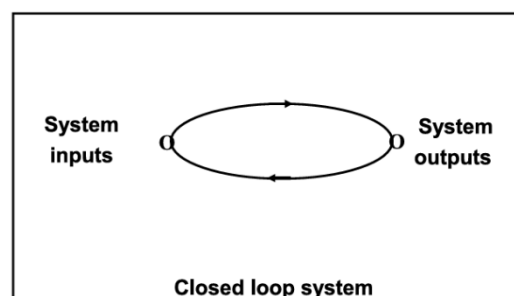


Fig. 2: Schematic of a closed-loop system

PID Controlled System

An automatic controller changes the output value of a plant with the reference input (the desired value), determines the deviation and produces a control signal that will reduce the deviation to zero or a small value (Paz, 2001). The way in which the automatic controller produces the control signal is refers to control action. Many automatic industrial controllers are electronic, hydraulic,

pneumatic or some combination of these Classification of industrial controllers. The acronym PID stands for Proportio-Integro-Differential control. Each of these, the P, the I and the D are terms in a control algorithm, and each has a special purpose. Occasionally, certain terms are omitted as they are unnecessary in the control design. It is possible to implement a PI, PD, or simply a P control (Paz, 2001).

The standard PID control configuration is as shown. It is also sometimes called the “**PID parameter form.**”

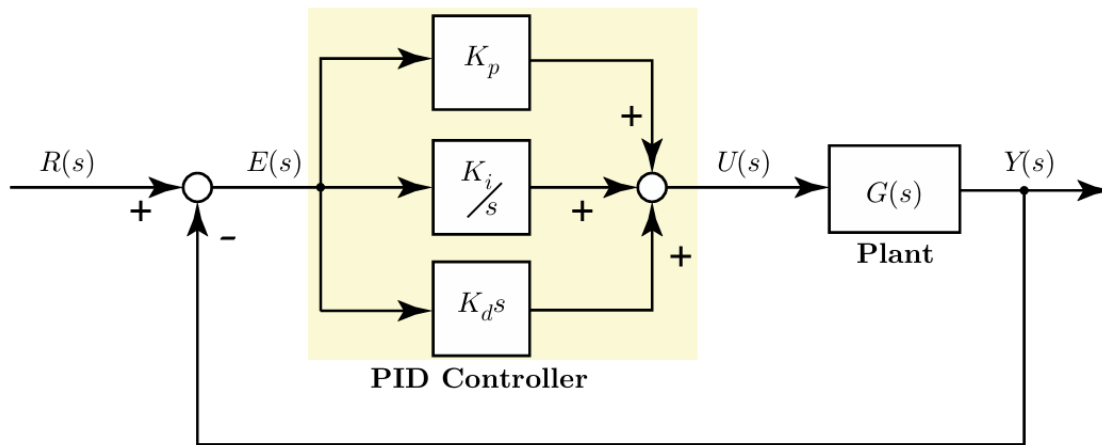


Figure 1: **PID Controlled System**

The overall PID control formula is given by:

$$u(t) = K_p e(t) + K_i \int e(t) dt + K_d \frac{de(t)}{dt}$$

Where $u(t)$ is the control output and $e(t)$ is the error signal (Nyathi, 2025).

The industrial controllers are classified, according to their control actions, as:

- a) Proportional control action. For a controller with proportional the relation between the output of the controller $u(t)$ and the error signal $e(t)$ is:

$$u(t) = K_p e(t)$$

- b) Integral control action. In an integral control action controller, the output value of the controller $u(t)$ is changed to a ratio proportional to the output signal $u(t)$. That is,

$$\frac{du(t)}{dt} = K_i e(t)$$

- c) Derivative Control (K_d): Predicts future errors based on the rate of error change, improving system stability. The proportional derivative action (PD) is defined by:

$$u(t) = K_p e(t) + K_p T_d \frac{de(t)}{dt}$$

This combination enables the PID controller to accurately and effectively minimise error, hence enhancing stability, response speed, and reducing steady-state error (Rodriguez, 2013).

2. Modelling and Simulation

Transfer function of the mass-spring-damper system

✕ Mass-Spring-Damper System Transfer Function

```

1  % Define system parameters
2  m = 1;      % Mass (kg)
3  c = 2;      % Damping coefficient (Ns/m)
4  k = 10;     % Spring constant (N/m)
5
6  % Create numerator and denominator
7  numerator = 1;          % Numerator of G(s)
8  denominator = [m, c, k]; % Coefficients of s^2 + cs + k
9
10 % Create transfer function
11 G = tf(numerator, denominator);
12
13 % Display the transfer function
14 disp('Transfer Function G(s):');
15 G
16

```

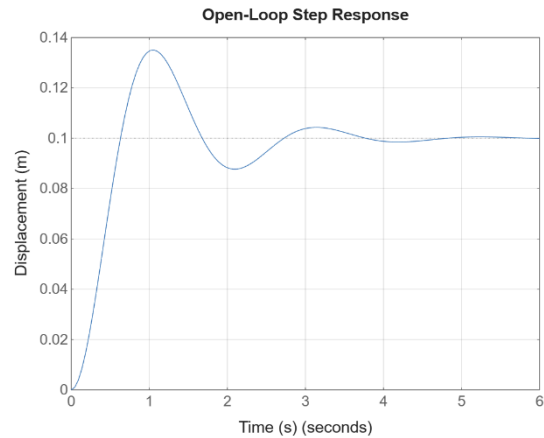
Result:

Transfer Function G(s):

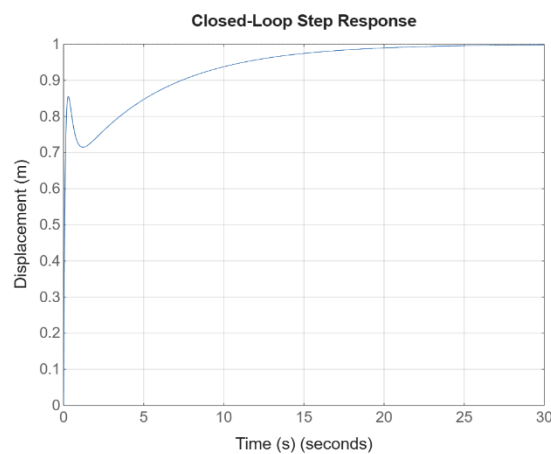
G =

$$\frac{1}{s^2 + 2s + 10}$$

Open-Loop and Close-Loop Response Simulation

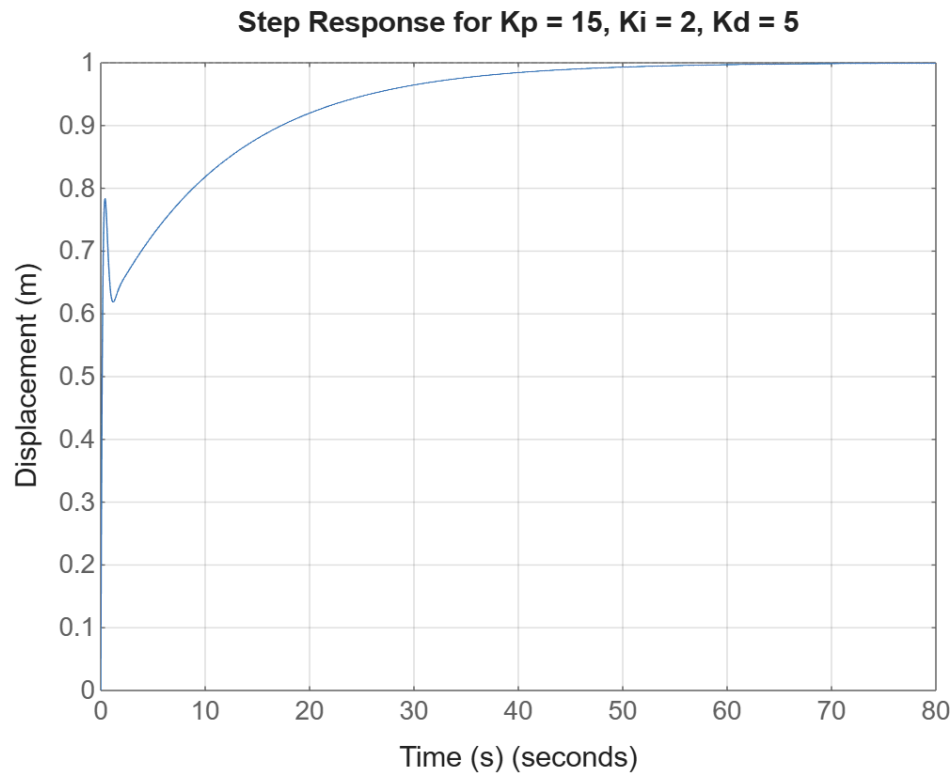


The plot Open-Loop Response demonstrates how the system reacts rapidly, although there is a discernible overshoot of approximately 35% before it levels off. It requires around 5 seconds to completely settle at the final value of 0.1 meters after reaching its peak at about 1 second. It only takes around half a second to a full second for the temperature to climb. In general, the system exhibits some oscillation before reaching a stable state, a property known as underdamped. It gets the job done, but it could be better—particularly if you're after a quicker settling with less overshoot.



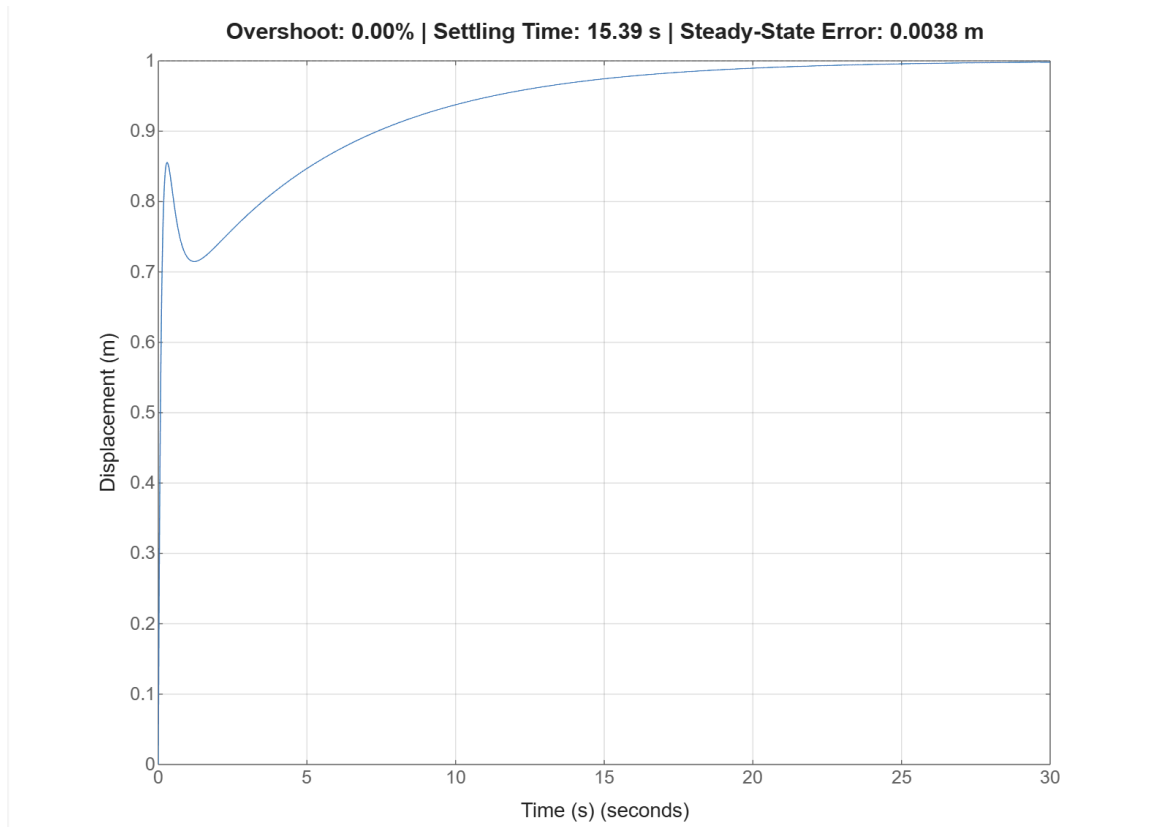
On this Closed-Loop Response, is observed a small miss of about 3% means it briefly goes a little above the goal (1 metre), but not by much. The response settles in about 20 seconds, which means it quickly becomes stable and stays close to the final number. Since there is no steady-state error, the system finishes at the exact value of 1 metre that was wanted.

PID Controller Implementation



The step response illustrates and how the system performs an excellent job of staying within the required range; it reaches the target value of 1 metre without exceeding it. Additionally, it achieves the aim precisely in the end because it does not have any steady-state error. The only real drawback is how long it takes for it to settle. For the reaction to remain stable and near the final value, it takes around 60 seconds. Overall, it's quite precise and steady, however it's a little sluggish. If you would want a quicker response, you could want to consider modest increases to the gains.

Responses for various PID settings



The Illustration represents a carefully tuned closed-loop system with no overshoot, a settling time of 15.39 seconds, and a low steady-state inaccuracy of 0.0038 m. This indicates a smooth, consistent, accurate response showing good PID controller performance.

Conclusion

The development and analysis of the mass-spring-damper system using a PID controller provided the importance of proper control design in the dynamic system performance. The responses of MATLAB providing the overshoot, settling time, and steady-state error as a result of the implementation of a variety of PID configurations in the plots were interesting.

In contrast, the closed loop system demonstrated better stability and precision than the open loop system. This experiment confirmed that PID tuning is necessary for dynamic system behaviour optimising in terms of decisions and overall stability.

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