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- Please give the information requested above and enter your name in the space provided at the top right of this page. If your answers are to be marked under a scheme whereby the examiners will not know your identity, fold over and seal down the top right-hand corner of the page (using the gummed label provided) in order to conceal your name.
- Commence each answer on a fresh page.
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- Your answers must be legibly written **IN INK**. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.

Summary of Marks

Question Numbers Answered	1st Examiner	2nd Examiner (optional)	3rd Examiner (optional)
1	24		
3	29		
	53		
TOTAL			
RESULT			

1. a) $x^2 - 12x + 37 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$ $b = -12$ $c = 37$

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(1)(37)}}{2(1)} = \frac{12 \pm \sqrt{144 - 148}}{2}$$

$$\frac{12 \pm \sqrt{-4}}{2} \quad \frac{12 \pm \sqrt{-4}j}{2} \quad \frac{12 \pm 16j^2}{2} \quad \frac{12 \pm -16}{2}$$

$$x = \frac{28}{2} \text{ \& } \frac{-4}{2} \quad x = \underline{\underline{14 \text{ or } -2}}$$

ii) $\underline{\underline{4+5j}}$

b) $p = -4+8j$ $q = 2+6j$ $r = -1-3j$

i) $\frac{p}{2} + \frac{q}{2} \quad \frac{-4+8j}{2} + \frac{2+6j}{2} \quad -2+4j + 1+3j$

$\underline{\underline{-1+7j}}$

ii) $(p+r)(p-r) \quad (-4+8j)(-4-8j)$

$16 + 32j - 32j - 64j^2 \quad 16 - 64j^2 \quad 16 - 64$

$\underline{\underline{-48}}$

iii) $p^2 + q^2 + r^2 \quad (-4+8j)^2 + (2+6j)^2 + (-1-3j)^2$

$(-4+8j)(-4+8j) + (2+6j)(2+6j) + (-1-3j)(-1-3j)$

$16 - 32j - 32j + 64j^2 + 4 + 12j + 12j + 36j^2 + 1 + 3j + 3j + 9j^2$

$21 - 34j + 109j^2 \quad 21 - 109 - 34j \quad \underline{\underline{-88 - 34j}}$

iv) $p = -4 + 8j$ $q = 2 + 6j$ $r = -1 - 3j$

$$\frac{q}{r} = \frac{2+6j}{-1-3j} \cdot \frac{(2+6j)(-1+3j)}{(-1-3j)(-1+3j)}$$

$$\frac{-2 + 6j - 6j + 18j^2}{1 - 3j + 3j + 9j^2} = \frac{-2 + 18j^2}{1 + 9j^2} = \frac{-2 - 18}{-8} = \frac{-20}{-8}$$

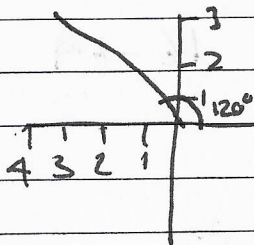
$$\underline{\underline{-2.5}}$$

v) $\frac{2p - 3r}{4q + p} = \frac{2(-4 + 8j) - 3(-1 - 3j)}{4(2 + 6j) + (-4 + 8j)}$

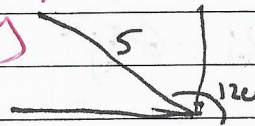
$$\frac{-8 + 16j + 3 + 9j}{8 + 24j - 4 + 8j} = \frac{-5 + 25j}{4 + 32j}$$

- c) i) r = the modulus. The size of the number (magnitude) ~~of the~~
 θ = the argument. The angle at which the modulus is angled

ii) $r = 5$ $\theta = 120^\circ$



labelling?



$$3^2 + 4^2 = 5^2 \\ \therefore 06A \text{ or } 36A$$

iii) $-3 - 5j$ in Polar

$$r = \sqrt{a^2 + b^2} \quad r = \sqrt{(-3)^2 + (-5)^2} \quad r = \sqrt{9 + 25} = \sqrt{34} = 5.830951$$

$$\theta = \tan^{-1} \frac{b}{a} = \tan^{-1} \frac{-5}{-3} \quad \theta = 59.03624$$

$$r = 5.83 \quad \theta = 59.03^\circ \text{ (to 2 d.p.)}$$

$$\underline{\underline{5.83 \angle 59.03}}$$

d) $(2+j)^3$

$$(2+j)(2+j)(2+j) = (4+2j+2j+j^2)(2+j) = (4+4j+j^2)(2+j)$$

$$8+4j+8j+4j^2+2j^2+j^3 = 8+12j+6j^2+j^3 = 8+12j+6(-1)+j$$

$$8+11j-6 \quad \underline{\underline{2+11j}}$$

2

De Moivre's

$$r = \sqrt{2^2+1^2} = \sqrt{5} \quad r = 2.2361$$

$$\theta = \frac{1}{2} \tan^{-1} \frac{1}{2} \quad \theta = 26.5651$$

$$r^3 < 3\theta \quad 2.2361^3 < 3 \times 26.5651$$

$$11.1803 < 79.6952^\circ \quad 11.18 < 79.70^\circ \text{ (to 2 d.p.)}$$

$$\sqrt{2^2+1^2} = r = 11.1803 \quad \leftarrow \text{From part 1 (2+1j)}$$

5

$$\theta = \tan^{-1} \frac{1}{2} = 79.6952 \quad \therefore 11.18 < 79.70^\circ \text{ (to 2 d.p.)}$$

2H

3. a) $\int_0^2 (4x^3 - 6x^2 + 8x - 1) dx$

$$\int \frac{4x^4}{4} - \frac{6x^3}{3} + \frac{8x^2}{2} - \frac{x}{1}$$

$$\frac{1}{4}(4x^4) - \frac{1}{3}(6x^3) + \frac{1}{2}(8x^2) - \frac{1}{1}x$$

$$[x^4 - 2x^3 + 4x^2 - x]_0^2$$

$$[(2)^4 - 2(2)^3 + 4(2)^2 - 2] - 0$$

$$16 - 16 + 16 - 2 = 14$$

ii) $\int_0^1 (e^{2x} - e^x + 3) dx$

$$\frac{1}{2}e^{2x} - \frac{1}{1}e^x + \frac{3x}{1} \left[\frac{1}{2}e^{2x} - e^x + 3x \right]_0^1$$

$$\left[\frac{1}{2}e^2 - e^1 + 3(1) \right] - \left(\frac{1}{2}e^0 - e^0 + 3(0) \right)$$

$$(3.6945 - 2.7183 + 3) - \left(\frac{1}{2} - 1 + 0 \right)$$

$$3.9762 - \left(-\frac{1}{2} \right) = 3.9762 + \frac{1}{2} = 3.4762$$

$$= \underline{\underline{3.48}} \text{ (2 d.p.)}$$

iii) $\int (4\sin 2x - 3\cos 4x) dx$

$$4\left(-\frac{1}{2}\cos 2x\right) - 3\left(\frac{1}{4}\sin 4x\right) + c$$

$$-2\cos 2x - \frac{3}{4}\sin 4x + c$$

b)

$$\int x \sin 4x \, dx$$

by Parts

$$u = x \quad \frac{du}{dx} = 1$$

$$\frac{dv}{dx} = \sin 4x \quad v = -\frac{1}{4} \cos 4x$$

$$\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$$

$$\int x \sin 4x \, dx = (x)\left(-\frac{1}{4} \cos 4x\right) - \int \left(-\frac{1}{4} \cos 4x\right) \, dx$$

$$= (x)\left(-\frac{1}{4} \cos 4x\right) - (x)\left(-\frac{1}{4} \left(-\frac{1}{4} \sin 4x\right)\right)$$

$$= (x)\left(-\frac{1}{4} \cos 4x\right) - (x)\left(\frac{1}{16} \sin 4x\right)$$

$$= -\frac{1}{4} x \cos 4x - \frac{1}{16} x \sin 4x + C$$

c)

$$\int_1^2 \frac{10x}{5x^2-4} \, dx = \ln 16 \quad u = 5x^2-4$$

$$\frac{du}{dx} = 10x \quad \frac{du}{10x} = dx$$

$$\int \frac{10x}{u} \cdot \frac{du}{10x} = \int \frac{1}{u} \, du$$

$$\left[\ln u \right]_1^2$$

$$\ln 20 - \ln 5 = \ln \left(\frac{20}{5} \right) = \ln 4$$

$$\ln(5(2^2)-4) - \ln(5(1^2)-4)$$

$$\ln 16 - \ln 1$$

$$\ln(5 \times 4 - 4) - \ln(5 \times 1 - 4)$$

$$(\ln 20 - 4) - \ln(5 - 4)$$

$$\ln 16 - \ln 1$$

$$\ln 1 = 0$$

$$\ln 16 - 0 = \ln 16$$

d) $\int_5^6 \frac{5x-5}{(x+1)(x-4)} dx$

$$\frac{A}{(x+1)} + \frac{B}{(x-4)} = \frac{5x-5}{(x+1)(x-4)}$$

$$A(x-4) + B(x+1) = 5x-5$$

$x=4$ $B(4+1) = 5(4-5)$

$$5B = 15$$

$$B = 3$$

$x=-1$ $A(-1-4) = 5(-1)-5$

$$A(-5) = -10$$

$$A = 2$$

$$\int \frac{2}{x+1} + \frac{3}{x-4} dx$$

$$\int 2(x+1)^{-1} + 3(x-4)^{-1} dx$$

$$\left[2 \ln(x+1) + 3 \ln(x-4) \right]_5^6$$

$$\left[2 \ln(6+1) + 3 \ln(6-4) \right] - \left[2 \ln(5+1) + 3 \ln(5-4) \right]$$

$$\left[2 \ln 7 + 3 \ln 2 \right] - \left[2 \ln 6 + 3 \ln 1 \right]$$

$$(3.8918 + 2.0794) - (3.5835 + 0)$$

$$1.8124 - 3.5835 = -1.7711$$

$$= -1.77 \text{ (to 2 d.p.)}$$

6

X

$$e) \int \frac{x^2+1}{x^3+3x} dx$$

$$\int (x^2+1)(x^3+3x)^{-1} dx$$

$$u = (x^3+3x)^{-1} \quad \frac{du}{dx} = -(x^3+3x)^{-2} \cdot (3x^2+3)$$

$$v = (x^2+1) \quad \frac{dv}{dx} = 2x$$

$$\int v \frac{dv}{dx} dx = uv - \int u \frac{du}{dx} dx$$

$$\int (x^2+1)(x^3+3x)^{-1} dx = (x^2+1) \ln(x^3+3x) - \int \ln(x^3+3x) (2x) dx$$

$$\int (x^3+3x)^{-1} (x^2+1) dx = (x^3+3x)^{-1} \left(\frac{x^3}{3} + x \right) - \int \left(\frac{x^3}{3} + x \right) (x^3+3x)^{-2} dx$$

$$= (x^3+3x)^{-1} \left(\frac{x^3}{3} + x \right) - \left(\frac{1}{3} \frac{x^4}{4} + \frac{x^2}{2} \right) - \frac{x^4}{4} - \frac{x^2}{2} - (x^3+3x)^{-1}$$

$$\frac{1}{3}x^3 + x - \frac{1}{12}x^4 + \frac{1}{2}x^2 - \frac{1}{4}x^4 - \frac{x^2}{2} - \frac{1}{3}x^3 + x$$

$$\frac{1}{3}x^3 + x - \frac{1}{12}x^4 + \frac{1}{2}x^2 - \frac{1}{4}x^4$$

$$\frac{1}{3}x^3 + x - \frac{1}{6}x^4 + \frac{1}{2}x^2$$

$$-\frac{1}{6}x^4 + \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Question No.

Question No.

Examiner Only

Question No.

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