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Analytical methods
13/05/2021

2004475
exam 1

Question one

a)

$$y = 2 \sqrt{\frac{3}{x^3}}$$

$$\frac{dy}{dx} = \sqrt{3/x^2}$$

$$\frac{y^2}{4} = \frac{3}{x^2}$$

$$\frac{12}{y^2} = x^2$$

$$x = \sqrt{\frac{12}{y^2}} = \frac{\sqrt{12}}{y}$$

$$x = \frac{\sqrt{12}}{y}$$

b

i

$$\frac{2x+3}{4} = \frac{5x+1}{-3}$$

$$(-3)2x+3 = (4)5x+1$$

$$-6x + -4 = 20x + 1$$

$$-4 - 1 = 20 + 6x$$

$$-13 = 26x$$

$$26x = -13$$

$$x = \frac{-13}{26}$$

$$x = \underline{\underline{-0.5}}$$

$$\text{ii) } \frac{(x-1)}{2} - \frac{5x}{4} = \frac{(x+2)}{2}$$

$$\text{C.P.} = 4$$

$$\frac{4(x-1)}{2} - \frac{4(5x)}{4} = \frac{4(x+2)}{2}$$

$$2(x-1) - 1(5x) = 2(x+2)$$

$$2x - 1 - 5x = 2x + 4$$

$$2x - 5x + 2x = 4 + 1$$

$$-1x = 5$$

$$-x = 5$$

$$\text{iii) } 2 \log(x+3) = \log(x+5) + \log(x+7)$$

3rd law

$$\log(x+3)^2 = \log(x+5)(x+7)$$

$$(x+3)^2 = (x+5)(x+7)$$

$$(x+3)(x+3) = (x+5)(x+7)$$

$$x^2 + 3x + 3x + 9 = x^2 + 7x + 5x + 35$$

$$6x + 9 = 12x + 35$$

$$6x - 12x = 35 - 9$$

$$-6x = 26$$

$$x = \frac{26}{-6}$$

$$x = -4.3$$

$$iv) 4^x \times 3^{x+5} = 2^x$$
~~$$2^{2x} \times 3^{x+5} = 2^x$$~~
~~$$2^{2x} = 2^x$$~~

$$x \log 4 + (x+5) \log 3 = x \log 2$$

$$(x+5) \log 3 = \frac{x \log 2}{x \log 4}$$

$$(x+5) \log 3 = \frac{1}{2}$$

$$x+5 = \frac{1}{2(\log 3)}$$

~~xxxxxx~~

$$x = \frac{1}{2 \log 3} - 5$$

$$x = -3.9$$

c)

i

$$\frac{T^2 + 11T + 30}{(T+6)(T+5)}$$

ii)

$$\frac{4x^2 - 100y^2}{(2x)^2 - (10y)^2}$$

iii)

$$\frac{Tr - Ty + ur - uy}{(r+u)(r-y)}$$

d)

$$\begin{aligned} \text{i)} \quad & x^2 + 14x + 10 \\ & (x+10)(x+4) = 0 \\ & x = -10 \quad x = -4 \end{aligned}$$

$$\begin{aligned} \text{ii)} \quad & x^2 - 5x + 6 \\ & (x-3)(x-2) = 0 \\ & x = 3 \quad x = 2 \end{aligned}$$

$$\begin{aligned} \text{iii)} \quad & x^2 + 8x = 4 \\ & x^2 + 8x - 4 = 0 \\ & (x+4)(x-1) \\ & x = -4 \quad x = 1 \end{aligned}$$

~~iii)~~

$$\begin{aligned} \text{iv)} \quad & x^2 - 225 = 0 \\ & (x-15)(x+15) \\ & x = 15 \quad x = -15 \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & (x+2)^2 - x^2 = (x+5)(x+1) - x^2 \\ & (x+2)(x+2) - x^2 - 2 = x^2 + x + 5x + 5 - x^2 \\ & x^2 + 2x + 2x + 4 - x^2 - 2 = 6x + 5 \\ & 4x + 2 = 6x + 5 \\ & 0 = 2x + 3 \\ & -3/2 = x \end{aligned}$$

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Question three

a)

$$i) \quad \underset{a}{15}x^2 = \underset{b}{14}x + \underset{c}{20}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

~~$x = \frac{-15 \pm \sqrt{15^2 - 4(14)(20)}}{2(15)}$~~

$$x = \frac{-14 \pm \sqrt{14^2 - 4(15)(20)}}{2(15)}$$

$$x = \frac{-14 \pm \sqrt{196 - 1200}}{30}$$

$$x = \frac{-14 \pm \sqrt{1004}}{30}$$

$$x = \frac{-14 \pm \sqrt{31.68}}{30}$$

$$x = \frac{-14 \pm 1.056}{30}$$

$$x = \frac{-14 \pm 1.056}{30}$$

$$x = \frac{-14 \pm 1.056}{30}$$

$$x = -14 + 1.056$$

$$x = -14 - 1.056$$

$$x = -12.94$$

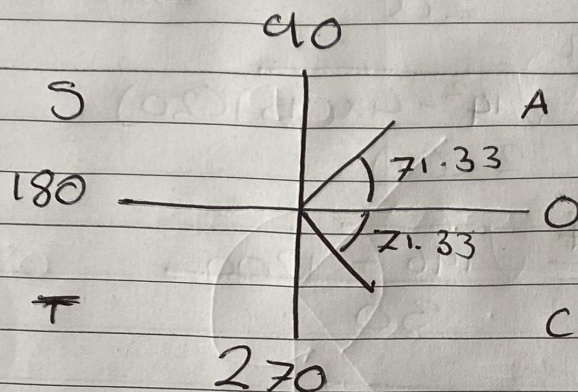
$$x = -15.05$$

b) Trigonometric equations

i) $\cos x = 0.32$

$$x = \cos^{-1} 0.32$$

$$\underline{x = 71.33^\circ}$$



$$360 - 71.33 = 288.67$$

$$\cos(288.67) = 0.32$$

$$x = 0.32$$

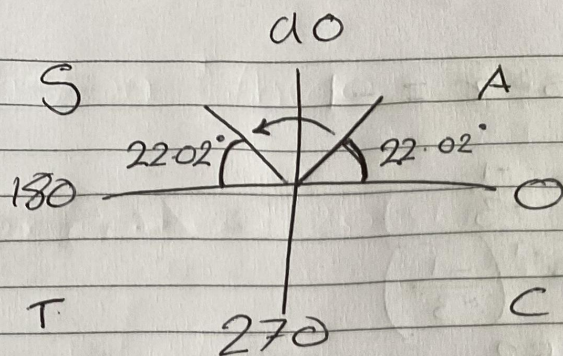
(ii) ~~ii)~~

$$8 \sin x = 3$$

$$\sin x = \frac{3}{8}$$

$$x = \sin^{-1} \left(\frac{3}{8} \right)$$

$$x = 22.02^\circ$$



$$180^\circ - 22.02^\circ = 157.92^\circ$$

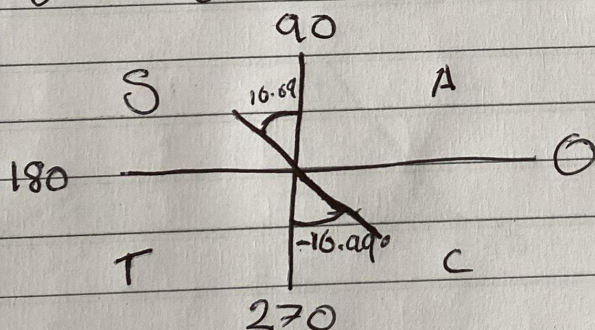
$$x = 157.92$$

$$(iii) -10 \tan x = 3$$

$$\tan x = \frac{3}{-10}$$

$$\tan^{-1} \left(\frac{3}{-10} \right)$$

$$x = -16.69^\circ$$



$$x = 180^\circ + 270^\circ + 16.69^\circ$$

$$x = 286.69$$

$$x = \tan^{-1}(286.69)$$

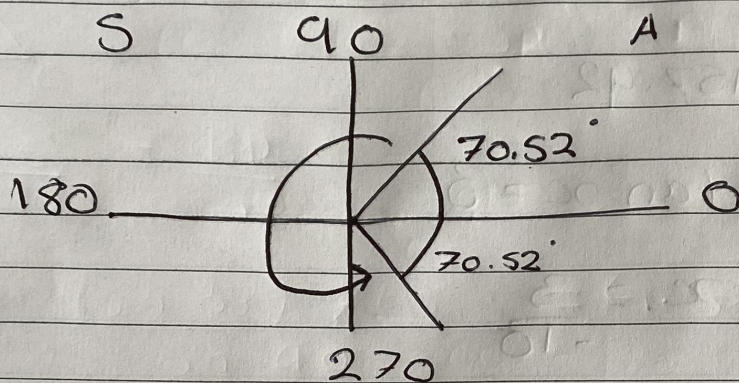
$$x = -3.33$$

(iv) $-3 \cos x = -1$

$$\cos x = \frac{-1}{-3}$$

$$x = \cos^{-1}\left(\frac{-1}{-3}\right)$$

$$x = 70.52$$



$$x = 360 - 70.52^\circ$$

$$x = 289.48$$

$$x = \cos(289.48)$$

$$x = 0.33$$

$$c) \quad x^2 - 7x + 10 \quad 0^\circ \leq x \leq 360$$

↓

$$\begin{array}{cc} (x-5) & (x-2) \\ x=5 & x=2 \end{array}$$

$$\begin{array}{cc} \tan x = 5 & \tan x = 2 \\ x = \tan^{-1}(5) & x = \tan^{-1}(2) \\ x = 78.7 & x = 63.43 \end{array}$$

$$d) \quad y = \cos^2\left(\frac{20}{5}\right)$$

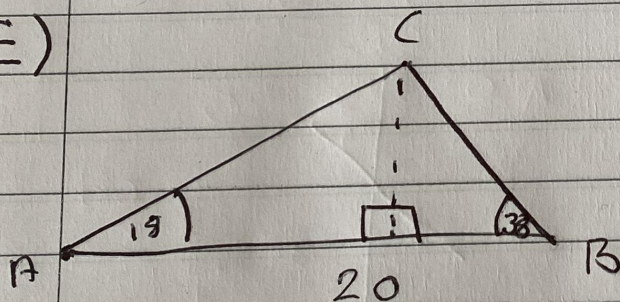
$$\sqrt{y} = \cos\left(\frac{20}{5}\right)$$

$$\cos^{-1}(\sqrt{y}) = \frac{20}{5}$$

$$5 \cdot \cos^{-1}(\sqrt{y}) = 20$$

$$\frac{5}{2} \cdot \cos^{-1}(\sqrt{y}) = 0$$

E)



$$ACB \angle = 124^\circ$$

$$CAB \angle = 18^\circ$$

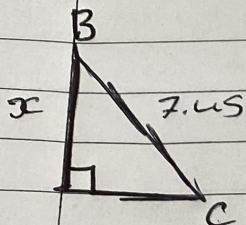
$$CBA \angle = 38^\circ$$

$$\frac{AC}{\sin 38} = \frac{20}{\sin 124}$$

$$AC = \frac{20 (\sin 38)}{\sin 124} \approx \underline{\underline{14.8 \text{ miles}}}$$

$$\frac{BC}{\sin 18} = \frac{20}{\sin 124}$$

$$BC = \frac{20(\sin 18)}{\sin 124} = \underline{7.45 \text{ miles}}$$



$$a = 38$$

$$a_2 = 180 - (38 + 90)$$

$$a_2 = 52$$

$$\frac{7.45}{\sin 90} = \frac{BD}{\sin 38}$$

$$BD = \frac{(\sin 38) 7.45}{\sin 90} = 4.58 \text{ miles}$$