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Mathematics
Test 2 / Year one
ACM A4006

$$AB = \begin{pmatrix} 4 & 5 & 1 \\ 2 & 0 & 3 \\ 2 & 1 & 2 \end{pmatrix} \begin{matrix} 1^{st} & 2^{nd} & 3^{rd} \end{matrix} \begin{pmatrix} 3 & 8 & 1 \\ 12 & 2 & 3 \\ 0 & 4 & 3 \end{pmatrix}$$

~~$$\begin{pmatrix} 12 & 60 & 0 \\ 6 & 0 & 0 \\ 6 & 12 & 0 \end{pmatrix} \begin{pmatrix} 32 & 10 & 4 \\ 16 & 0 & 12 \\ 8 & 2 & 8 \end{pmatrix} \begin{pmatrix} 4 & 15 & 3 \\ 2 & 0 & 9 \\ 2 & 3 & 6 \end{pmatrix}$$~~

$$\begin{pmatrix} 12+60+0 & 32+10+4 & 4+15+3 \\ 6+0+0 & 16+0+12 & 2+0+9 \\ 6+12+0 & 8+2+8 & 2+3+6 \end{pmatrix}$$

$$= \begin{bmatrix} 72 & 46 & 22 \\ 6 & 28 & 11 \\ 18 & 26 & 11 \end{bmatrix}$$

$$ii) AC = \begin{bmatrix} 4 & 5 & 1 \\ 2 & 0 & 3 \\ 2 & 1 & 2 \end{bmatrix} \begin{matrix} 1^{st} & 2^{nd} & 3^{rd} \end{matrix} \begin{bmatrix} 0 & 1 \\ 1 & 4 \\ 4 & 5 \end{bmatrix}$$

$$AC = \begin{pmatrix} 0+5+4 & 4+20+5 \\ 0+0+12 & 2+0+5 \\ 0+1+8 & 2+4+10 \end{pmatrix}$$

$$\begin{bmatrix} 9 & 29 \end{bmatrix}$$

iii)

$$A+B = \begin{bmatrix} 4 & 5 & 1 \\ 2 & 0 & 3 \\ 2 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 8 & 1 \\ 12 & 2 & 3 \\ 0 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 5+8 & 1+1 \\ 2+12 & 0+2 & 3+3 \\ 2+0 & 1+4 & 2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 13 & 2 \\ 14 & 2 & 6 \\ 2 & 5 & 5 \end{bmatrix}$$

iv) $A+D$ is not possible since they don't have the correct values to this
(e.g. A is 3×3 & D is 2×2 matrix)

v) BD is not possible since they don't have the correct values to this as ~~A is~~ B is 3×3 & D is 2×2 matrix. So as B columns are 3 and D rows are only 2.

$$v:) \quad 4D = 4 \begin{bmatrix} 5 & 1 \\ 12 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \times 5 & 4 \times 1 \\ 4 \times 12 & 4 \times 5 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 4 \\ 48 & 20 \end{bmatrix}$$

$$b) \quad 2x + y + 4z = 11$$

$$x - y + z = 2$$

$$3x + 2y + 2z = 9$$

$$\begin{bmatrix} 2 & 1 & 4 \\ 1 & -1 & 1 \\ 3 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \\ 9 \end{bmatrix}$$

$$\begin{aligned} |A| &= 2(-2-2) - 1(2-3) + 4(2+3) \\ &= 2(-4) - 1(-1) + 4(5) \\ &= -8 + 1 + 20 \end{aligned}$$

$$D = 13$$

$$\begin{aligned} D_x &= \begin{vmatrix} 11 & 1 & 4 \\ 2 & -1 & 1 \\ 9 & 2 & 2 \end{vmatrix} = 11(-2-2) - 1(4-9) + 4(4-9) \\ &= 11(-4) - 1(-5) + 4(-5) \\ &= -44 + 5 - 20 \\ D_x &= 13 \end{aligned}$$

$$D_y = \begin{vmatrix} 2 & 11 & 4 \\ 1 & 2 & 1 \\ 3 & 9 & 2 \end{vmatrix} = 2(4-9) - 11(2-3) + 4(9-6) \\ = 2(-5) - 11(-1) + 4(3) \\ = -10 + 11 + 12 \\ \Rightarrow D_y = 13$$

$$D_z = \begin{vmatrix} 2 & 1 & 11 \\ 1 & -1 & 2 \\ 3 & 2 & 9 \end{vmatrix} = 2(-9-4) - 1(9-6) + 11(2+3) \\ = 2(-13) - 1(3) + 11(5) \\ = -26 - 3 + 55 \\ D_z = 26$$

$$x = \frac{13}{13} = 1 \quad y = \frac{13}{13} = 1 \quad z = \frac{26}{13} = 2$$

$$2a) \quad z_1 = 2 + 5j \quad z_2 = 1 - 8j \quad z_3 = -3 + 2j$$

$$i) \quad z_1 + z_2 = 2 + 5j + 1 - 8j \\ 3 - 3j$$

$$ii) \quad z_1 - z_3 \\ 2 + 5j - (-3 + 2j) \\ 2 + 5j + 3 - 2j \\ 5 + 3j$$

$$iii) \quad \frac{z_1}{z_2} = \frac{2 + 5j}{1 - 8j} \\ = \frac{(2 + 5j)(1 + 8j)}{(1 + 8j)(1 - 8j)} \\ = \frac{2 + 16j + 5j - 40j^2}{1 - 8j + 8j - 64j^2} \\ = \frac{2 - 11j - 40j^2}{1 - 64j^2}$$

$$2b) 10x^2 + 3x + 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(10)(2)}}{2(10)}$$

$$x = \frac{-3 \pm \sqrt{911j}}{20}$$

$$\sqrt{911j} = 30.18j$$

$$x = \frac{-3 \pm 30.18j}{20}$$

$$x = -0.15 + 30.18j$$

$$x = -0.15 - 30.18j$$

$$2c) (4 + 5j)$$

$$r^2 = 4 + 5^2$$

$$r = 6.4$$

$$\tan^{-1}\left(\frac{4}{5}\right) = 0.67^\circ$$

$$r^n (\cos n\theta + j \sin n\theta)$$

$$(6.4)^5 (\cos 5 \times 0.67 + j \sin 5 \times 0.67)$$

$$10737.42 (\cos 3.35 + j \sin 3.35)$$

$$10737.42 (-0.98 - 0.21j)$$

$$= -10522.67 - 2254.86j$$

Q3)

$$(i) \int 4x^4 - 3x^3 + 3x - 10 \, dx$$

$$= \frac{4x^5}{5} - \frac{3x^4}{4} + \frac{3x^2}{2} - 10x + C$$

$$(ii) \int (x-6)(x-4) \, dx$$

$$= x^2 - 4x - 6x + 24$$

$$= x^2 - 10x + 24$$

$$= \frac{x^3}{3} - \frac{10x^2}{2} + 24x + C$$

$$(iii) \int 3 \cos 5x - 1/x \, dx$$

$$= \frac{3}{5} \sin 5x - \ln x + C$$

$$(iv) \int_0^4 (3x+5)(x+2) \, dx$$

$$3b) \int 5x \cos 2x \, dx$$
$$= 5x \left(\frac{1}{2} \sin 2x \right) - \int \frac{1}{2} \sin 2x \, dx + C$$