

Worked example

Show that the resonant frequency is given by $f = 1 / 2\pi\sqrt{LC}$?

$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

↑ at resonance this equals zero

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$(2\pi f)^2 = \frac{1}{LC}$$

$$4\pi^2 f^2 = \frac{1}{LC}$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Worked Example

$$R = 200\Omega, \quad X_C = 150\Omega \quad X_L = 80\Omega \quad V = 120V \quad f = 60\text{Hz}$$

How much capacitance must be added to maximize the power in the circuit?

maximum power occurs at resonance

at resonance $X_C = X_L$

$$X_C = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi f X_C}$$

$$C = \frac{1}{2\pi \times 60 \times 150}$$

$$C = 1.768 \times 10^{-5}$$

$$C = 17\mu\text{f}$$

$$X_C = X_L = 80\Omega$$

$$80 = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi \times 60 \times 80}$$

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$$C = \frac{1}{2\pi \times 60 \times 80}$$

$$C = 3.31 \times 10^{-5}$$

$$C = 33\mu\text{f}$$

therefore we must add 16 μf capacitance to maximize power

Worked example

A sinusoidal voltage $= 100\text{V}\sin(\omega t)$ is applied to a series RLC circuit with $L = 20\text{mH}$, $C = 100\text{nF}$, $R = 20\Omega$, calculate

- the resonant frequency
- the amplitude of the current at resonance
- the Q factor
- the amplitude of the voltage across the inductor at resonance

using

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{20 \times 10^{-3} \times 100 \times 10^{-9}}}$$

$$f = \frac{1}{2.8 \times 10^{-6}}$$

$$f = 3558.8 \text{ Hz}$$

at resonance

$$I = \frac{V}{R} = \frac{100}{20}$$

$$I = 5 \text{ A}$$

$$\text{the } Q \text{ factor} = \frac{\omega L}{R}$$

$$= \frac{2\pi \times 3558.8 \times 20 \times 10^{-3}}{20}$$

$$= 22.3$$

$$V_L = I X_L$$

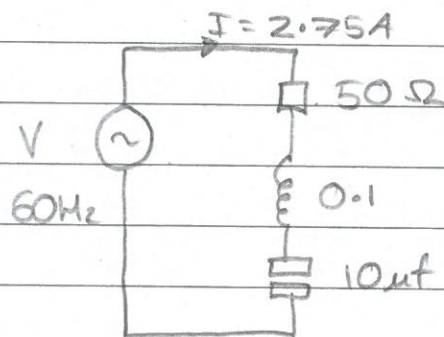
$$V_L = 5 \times 2\pi \times 3558.8 \times 20 \times 10^{-3}$$

$$V_L = 2236 \text{ V}$$

Worked example

A 50Ω resistor, a 0.1H inductor, and a $10\mu\text{f}$ capacitor are connected in series to a 60Hz source. The current in the circuit is 2.75A , find the voltages across

- a) the resistor
- b) the inductor
- c) the capacitor
- d) the RLC network
- e) sketch the phasor diagram



$$\begin{aligned}V_R &= I R \\&= 2.75 \times 50 \\&= 138\text{V}\end{aligned}$$

$$\begin{aligned}X_L &= 2\pi f L \\&= 2\pi 60 \times 0.1 \\&= 37.7\Omega\end{aligned}$$

$$\begin{aligned}V_L &= I X_L \\&= 2.75 \times 37.7 \\&= 103.67\text{V}\end{aligned}$$

$$X_c = \frac{1}{2\pi f C}$$

$$X_c = \frac{1}{2\pi \times 60 \times 10 \times 10^{-6}}$$

$$X_c = 265 \Omega$$

$$V_c = I X_c$$

$$V_c = 2.75 \times 265$$

$$V_c = 729V$$

use either

$$V = I \times Z$$

$$V = I \times \sqrt{R^2 + (X_L - X_c)^2}$$

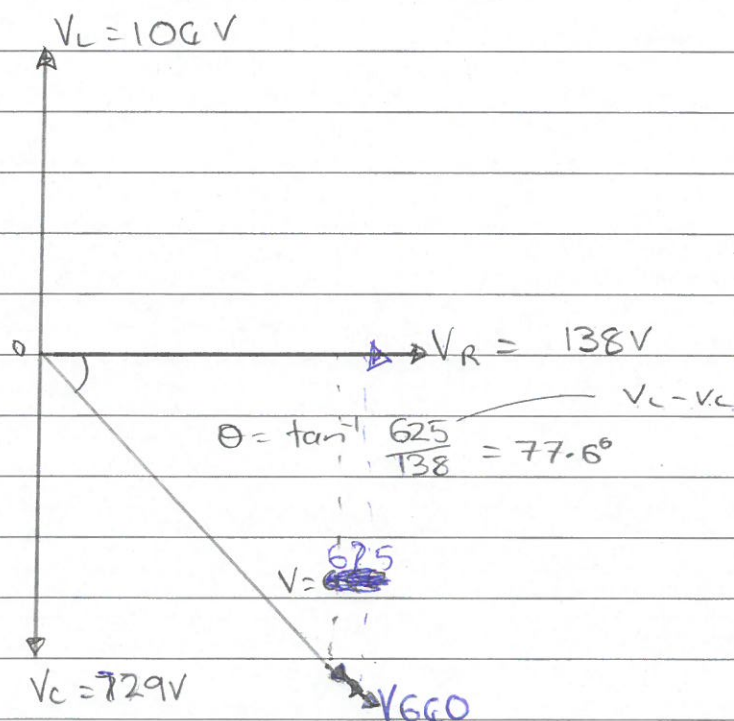
$$V = I \times \sqrt{50^2 + (37.7 - 265)^2}$$

$$V = 660V$$

$$V = \sqrt{V_R^2 + (V_L - V_c)^2}$$

$$V = \sqrt{138^2 + (100 - 729)^2}$$

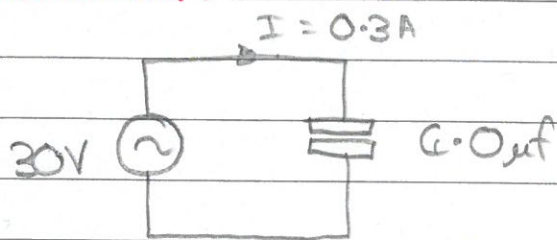
$$V = 660V$$



$$\theta = \frac{\text{opp}}{\text{adj}} \tan^{-1}$$

Worked Example

When a $4.0\mu\text{f}$ capacitor is connected to a generator whose output is 30V , the current in the circuit is observed to be 0.3A . What is the frequency of the source?



$$X_c = \frac{V_c}{I_c}$$

$$X_c = \frac{30}{0.3} \\ = 100\Omega$$

$$X_c = \frac{1}{2\pi fC}$$

$$f = \frac{1}{2\pi \times X_c \times C}$$

$$f = \frac{1}{2\pi \times 100 \times 4 \times 10^{-6}}$$

$$f = 398\text{ Hz}$$

$$\begin{aligned}\text{Current} &= \frac{V}{Z} \\ &= \frac{100}{19.6} \\ &= 5.15 \text{ A}\end{aligned}$$

$$\begin{aligned}V_R &= IR \\ &= 5.15 \times 12 \\ &= 61.7\end{aligned}$$

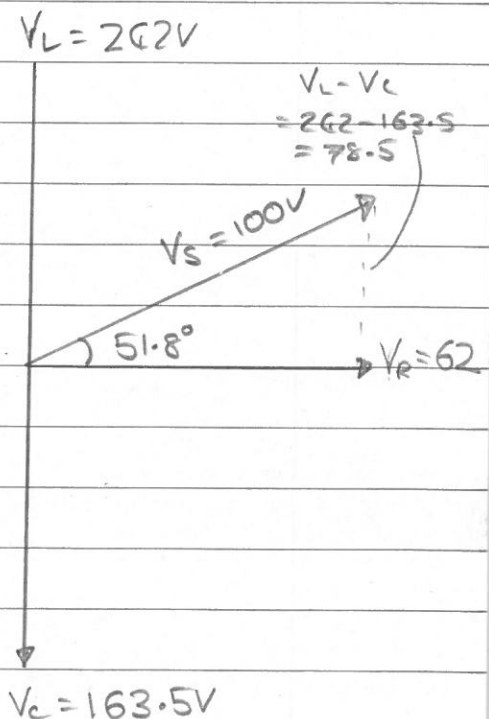
$$\begin{aligned}V_L &= I X_L \\ &= 5.15 \times 47.13 \\ &= 242.2 \text{ Volts}\end{aligned}$$

$$\begin{aligned}V_C &= I X_C \\ &= 5.15 \times 31.8 \\ &= 163.5 \text{ V}\end{aligned}$$

$$\text{Power factor} = \cos \theta = \frac{R}{Z}$$

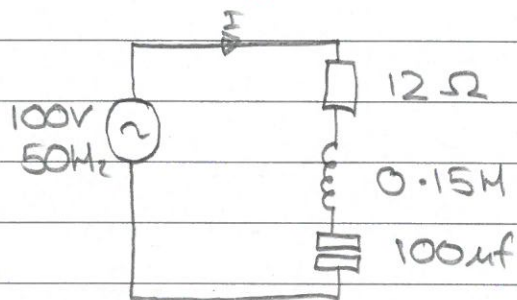
$$\begin{aligned}\text{Power factor} &= \frac{12}{19.6} \\ &= 0.619\end{aligned}$$

$$\therefore \cos^{-1}(0.619) = 51.8^\circ \text{ lagging}$$



Worked Example

A series RLC circuit containing a resistance of $12\ \Omega$, an inductance of 0.15 H and a capacitance of $100\ \mu\text{f}$ are connected in series across 100 V 50 Hz supply. Calculate the total circuit impedance, the circuits power, power factor, and phasor diagram?



$$X_L = 2\pi f L$$

$$X_L = 2\pi \times 50 \times 0.15$$

$$X_L = 47.12\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$$X_C = 31.83\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{12^2 + (47.12 - 31.83)^2}$$

$$Z = \sqrt{144 + 230}$$

$$Z = 19.6\ \Omega$$

Worked examples

what value of C resonates with a
239 μH L at 1000 KHz

using $f = \frac{1}{2\pi\sqrt{LC}}$

C?

$$L = 239 \times 10^{-6} \text{ H}$$

$$f = 1000 \times 10^3 \text{ Hz}$$

$$f = 1 \times 10^6 \text{ Hz}$$

$$C = \frac{1}{4\pi^2 f^2 L}$$

$$C = \frac{1}{4\pi^2 \times (1 \times 10^6)^2 \times 239 \times 10^{-6}}$$

$$C = \frac{1}{39.47 \times (1 \times 10^6)^2 \times 239 \times 10^{-6}}$$

$$C = 1.06 \times 10^{-10}$$

$$C = 10.6 \text{ pF}$$