

Q1 a) Simplify: $G(s) = \frac{k}{(s+3)(s+2)(s+5)}$ (1 mark)

Then with unity feedback and gain of 100, the open loop TF becomes:

$$GH(s) = \frac{100}{(s+3)(s+2)(s+5)}$$

Let s approach $j\omega$:

$$GH(j\omega) = \frac{100}{(j\omega+3)(j\omega+2)(j\omega+5)} \quad (1 \text{ mark})$$

$$\therefore |GH(j\omega)| = \frac{100}{(\sqrt{\omega^2+3^2})(\sqrt{\omega^2+2^2})(\sqrt{\omega^2+5^2})} \quad (1 \text{ mark})$$

$$|GH(j\omega)| = \frac{100}{(\sqrt{\omega^2+9})(\sqrt{\omega^2+4})(\sqrt{\omega^2+25})} \quad (1 \text{ mark})$$

$$\therefore |GH(j\omega)|_{dB} = 20 \log_{10} \left(\frac{100}{(\sqrt{\omega^2+9})(\sqrt{\omega^2+4})(\sqrt{\omega^2+25})} \right) \quad (1 \text{ mark})$$

$$\angle GH(j\omega) = -\tan^{-1}\left(\frac{\omega}{3}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{5}\right) \quad (1 \text{ mark})$$

$$|GH(j\omega)|_{dB} = 20 \log_{10} \left(\frac{100}{(\sqrt{(0.16)^2+9})(\sqrt{(0.16)^2+4})(\sqrt{(0.16)^2+25})} \right) = 10.41 \text{ dB} \quad (1 \text{ mark})$$

$$\angle GH(j\omega) = -\tan^{-1}\left(\frac{0.16}{3}\right) - \tan^{-1}\left(\frac{0.16}{2}\right) - \tan^{-1}\left(\frac{0.16}{5}\right) = -9.46^\circ \quad (1 \text{ mark})$$

From the figure DC gain = 4.7dB

$$G(\text{dB}) = 20 \log K$$

$$4.7 = 20 \log K$$

$$\frac{4.7}{20} = \log K$$

$$K = 10^{4.7/20} = 1.72$$

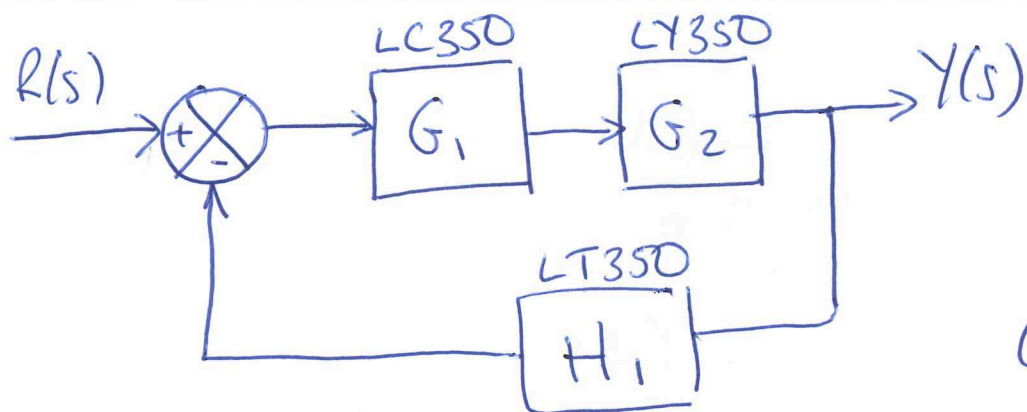
Therefore in standard form:

$$\begin{aligned} G(j\omega) &= \frac{1.72 (1 + j\omega/5)}{(1 + j\omega/15)(1 + j\omega/35)} \\ &= \frac{1.72 \left(\frac{15 \times 35}{5} \right) (j\omega + 5)}{(j\omega + 15)(j\omega + 35)} \\ &= \frac{180 (j\omega + 5)}{(j\omega + 15)(j\omega + 35)} \\ &= \frac{180 (s + 5)}{(s + 15)(s + 35)} \end{aligned}$$

- c) Annotate Bode and show phase never reaches -180° therefore Gain Margin method cannot be used. (2 marks)
- Phase Margin method shows that at 0dB phase is approx -75° (Annotations to show this) (2 marks)
- PM = $-75 - (-180^\circ) = 105^\circ$ (2 marks)
- Since this is greater than 45° , system is stable (1 mark).

Q2 a)

i)

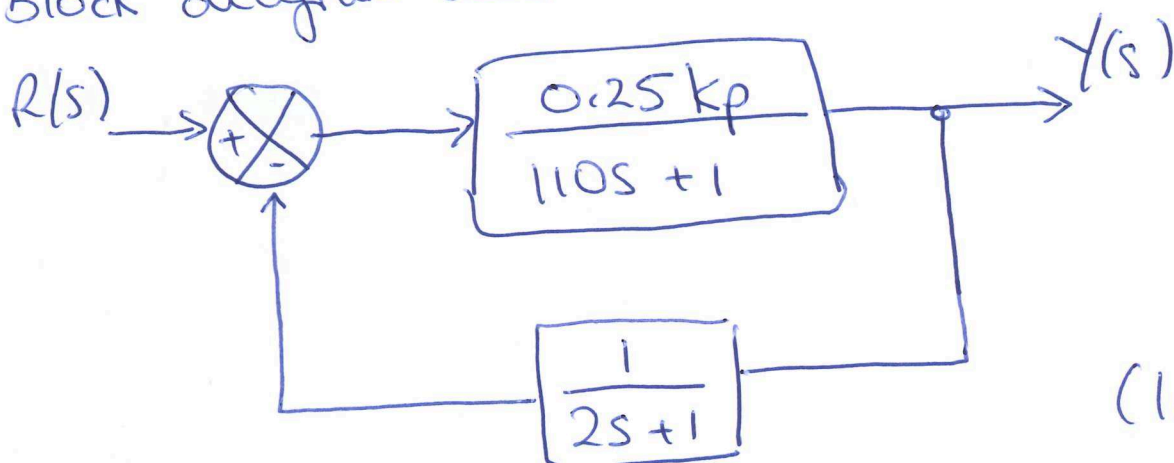


(3 marks)

where $G_2 = \frac{0.25}{110s + 1}$, $G_1 = k_p$

and $H_1 = \frac{1}{2s + 1}$ (2 marks)

ii) Block diagram becomes



(1 mark)

Using the feedback equation

$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

(1 mark)

$$= \frac{0.25 k_p}{110s + 1}$$

$$\frac{1}{1 + \frac{0.25 k_p}{110s + 1} \times \frac{1}{2s + 1}}$$

(1 mark)

$$= \frac{0.25 k_p (2s + 1)}{(110s + 1)(2s + 1) + 0.25 k_p}$$

(1 mark)

$$= \frac{0.5 k_p (s + 0.5)}{220s^2 + 112s + 1 + 0.25 k_p}$$

$$= \frac{0.5 k_p (s + 0.5)}{220 \left(s^2 + 0.51s + \left(\frac{1 + 0.25 k_p}{220} \right) \right)}$$

$$= \frac{0.5 k_p}{220} \frac{(s + 0.5)}{s^2 + 0.51s + \left(\frac{1 + 0.25 k_p}{220} \right)}$$

(4 marks)

when $k_p = ~~0.1~~ 1.1$

$$T(s) = \frac{0.5 \times 1.1}{220} \frac{(s + 0.5)}{s^2 + 0.51s + 0.017} \quad (1 \text{ mark})$$

$$\text{overall gain} = \frac{0.5 \times 1.1}{220} = 0.0025 \quad (1 \text{ mark})$$

$$y_{ss} = \lim_{s \rightarrow 0} \{ s Y(s) \} = \lim_{s \rightarrow 0} \left\{ s \times \frac{1}{s} \times T(s) \right\} \quad (1 \text{ mark})$$

$$= 0.0025 \frac{0.5}{0.017}$$

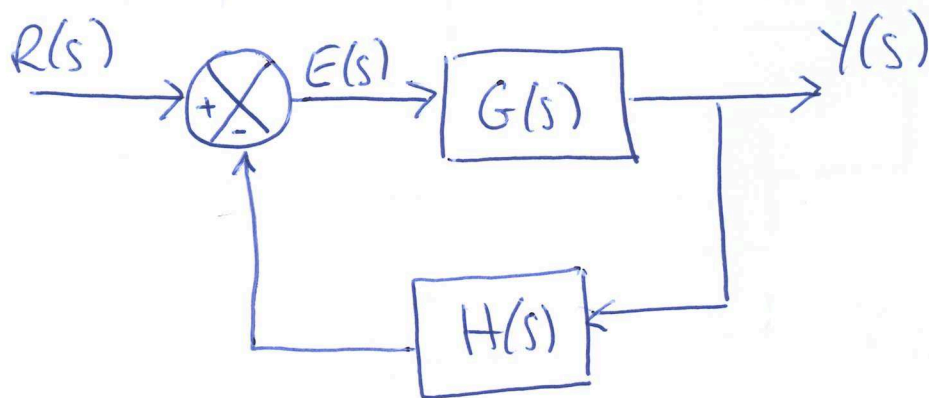
$$= 0.0735 \quad (1 \text{ mark})$$

$$e_{ss} = 1 - 0.0735$$

$$= 0.926 = 92.6\%$$

(2 marks).

Q2 b)



$$Y(s) = G(s)E(s) \quad \text{--- (1)}$$

$$E(s) = R(s) - H(s)Y(s) \quad \text{--- (2)}$$

(1 mark)

Subs (2) into (1)

$$Y(s) = G(s)[R(s) - H(s)Y(s)]$$

(1 mark)

$$Y(s) = G(s)R(s) - G(s)H(s)Y(s)$$

(1 mark)

$$Y(s) + G(s)H(s)Y(s) = G(s)R(s)$$

(1 mark)

$$Y(s)[1 + G(s)H(s)] = G(s)R(s)$$

(1 mark)

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

(1 mark)

Q3 a)

$$G_o = \frac{2}{(s+1)} \times \frac{1}{s(s+3)}$$

$$= \frac{2}{s(s+1)(s+3)}$$

$$= \frac{2}{s(s^2 + 4s + 3)}$$

$$= \frac{2}{s^3 + 4s^2 + 3s}$$

(2 marks)

$$G(s) = \frac{G_o(s)}{1 + G_o(s)H(s)}$$

$$H(s) = k$$

$$= \frac{\frac{2}{s^3 + 4s^2 + 3s}}{1 + \frac{2}{s^3 + 4s^2 + 3s} \times k}$$

$$= \frac{2}{s^3 + 4s^2 + 3s + 2k}$$

(2 marks)

From denominator

$$a_n = 1 \quad a_{n-1} = 4 \quad a_{n-2} = 3 \quad a_{n-3} = 2k$$

(1 mark)

$$\begin{array}{c|cc} s^3 & 1 & 3 \\ s^2 & 4 & 2k \\ s^1 & 3 - \frac{1}{2}k & 0 \\ s^0 & 2k & \end{array}$$

$$b_1 = \frac{4 \times 3 - 1 \times 2k}{4}$$

$$= 3 - \frac{1}{2}k$$

$$C_1 = \frac{(3 - \frac{1}{2}k)2k - 4 \times 0}{3 - \frac{1}{2}k}$$

$$= 2k$$

8(marks)

For a stable system there must be no sign changes in column 1.

$3 - \frac{1}{2}k$ must be positive

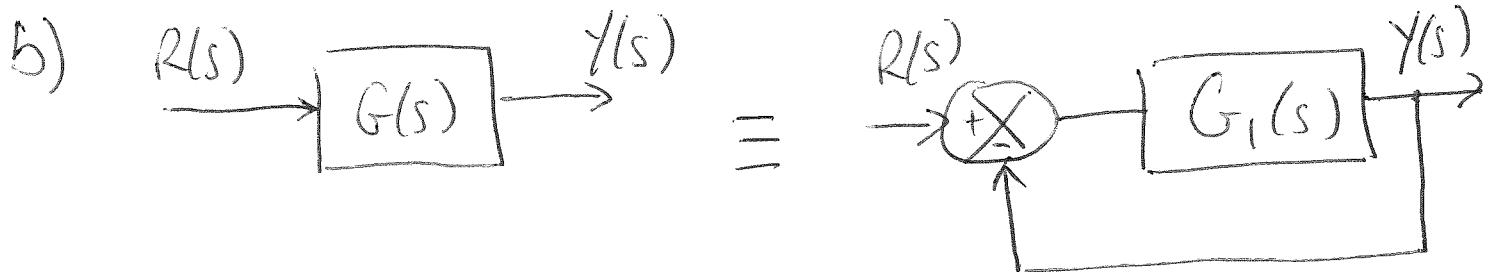
$$3 - \frac{1}{2}k > 0$$

$$3 > \frac{1}{2}k$$

$$3 \times 2 > k$$

$$k < 6 \text{ and } k > 0$$

(3 marks)



$$G(s) = \frac{G_1(s)}{1 + G_1(s)}$$

(2 marks)

$$G(s)[1 + G_1(s)] = G_1(s)$$

$$G(s) + G(s)G_1(s) = G_1(s)$$

$$G(s) = G_1(s) - G(s)G_1(s)$$

$$G(s) = G_1(s)[1 - G(s)]$$

$$G_1(s) = \frac{G(s)}{1 - G(s)}$$

(2 marks)

$$\begin{aligned} G_1(s) &= \frac{2}{s^3 + 4s^2 + 3s + 2k} \\ &= \frac{1}{1 - \frac{2}{s^3 + 4s^2 + 3s + 2k}} \\ &= \frac{2}{s^3 + 4s^2 + 3s + 2k - 2} \\ &= \frac{2}{s^3 + 4s^2 + 3s + 2(k-1)} \end{aligned}$$

Since s cannot be factored out, this system is a type/class 0.

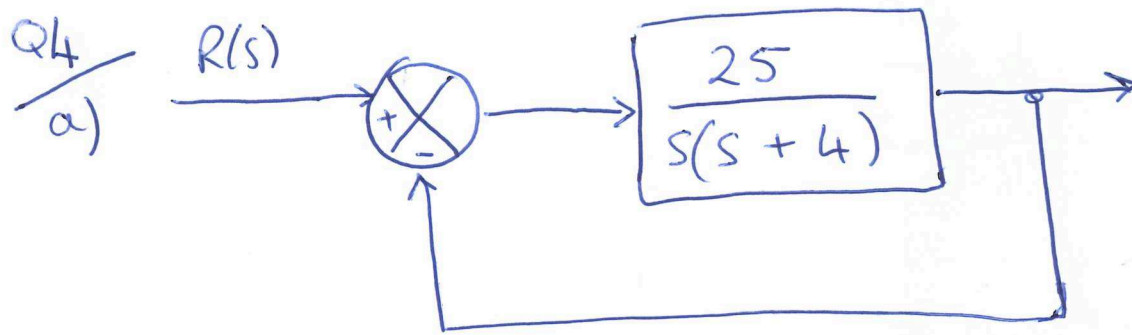
c) i) $s^3 + 10s^2 + 2s + 9$ (3 marks)

Since all coefficients are present and none are negative this system can be stable.

ii) $s^5 + 6s^4 + 9s^3 + 2s^2 - s + 4$

All coefficients are present but one is negative this system is unstable.

iii) $s^4 + 7s^3 + s^2 + 5$ Since one coefficient is missing system is at best critically stable



$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \quad \text{And since } H(s) = 1 \quad (1 \text{ mark})$$

$$= \frac{25}{s(s+4)} \quad (1 \text{ mark})$$

$$1 + \frac{25}{s(s+4)}$$

$$= \frac{25}{s(s+4) + 25} \quad (1 \text{ mark})$$

$$= \frac{25}{s^2 + 4s + 25} \quad (1 \text{ mark})$$

b) Compare with $\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (1 \text{ mark})$

i) $\omega_n = \sqrt{25} = 5 \text{ rad/s} \quad (1 \text{ mark})$

ii) $2\zeta\omega_n = 4 \quad \therefore \zeta = \frac{4}{2 \times 5} = 0.4 \quad (2 \text{ marks})$

iii) $\omega = \omega_n \sqrt{1 - \zeta^2} = 5 \sqrt{1 - 0.4^2} = 4.583 \text{ rad/s} \quad (2 \text{ marks})$

iv) $t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1 - \zeta^2}}{\zeta}\right)}{\omega_n \sqrt{1 - \zeta^2}}$

$$= \frac{\pi - \tan^{-1} \left(\frac{\sqrt{1-0.4^2}}{0.4} \right)}{4.583}$$

$$= \frac{\pi - \tan^{-1} 2.29}{4.583} = 0.433 \text{ sec} \quad (2 \text{ marks})$$

$$v) t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{4.583} = 0.685 \text{ sec} \quad (2 \text{ marks})$$

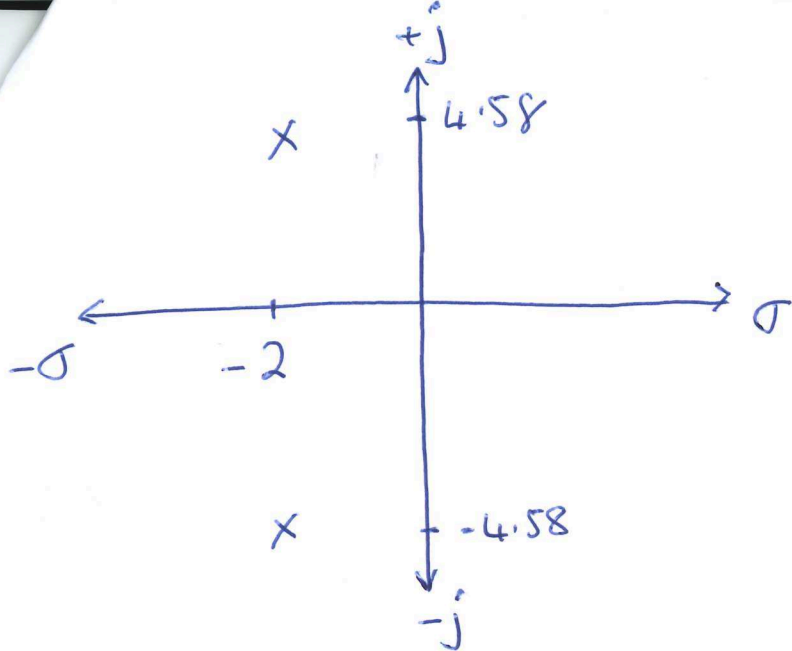
$$\begin{aligned} \text{vi) } \% \text{ OS} &= 100 e^{-\frac{\zeta \pi}{\sqrt{1-\zeta^2}}} \\ &= 100 e^{-\frac{0.4 \pi}{\sqrt{1-0.4^2}}} \\ &= 100 e^{-1.37} = 25.41\% \quad (2 \text{ marks}) \end{aligned}$$

$$\text{vii) } t_s = \frac{4}{\zeta \omega_n} = \frac{4}{0.4 \times 5} = 2 \text{ sec} \quad (2 \text{ marks})$$

$$\begin{aligned} \text{viii) } n &= \frac{2}{\pi} \sqrt{\frac{1}{\zeta^2} - 1} = \frac{2}{\pi} \sqrt{\frac{1}{0.4^2} - 1} \\ &= 1.458 \quad (2 \text{ marks}) \end{aligned}$$

$$\text{c) Poles found by: } s^2 + 4s + 25 = 0 \quad (1 \text{ mark})$$

$$\begin{aligned} s &= \frac{-4 \pm \sqrt{4^2 - 4(1)(25)}}{2(1)} = -2 \pm j \frac{\sqrt{84}}{2} \\ &= -2 \pm j 4.58 \quad (2 \text{ marks}) \end{aligned}$$

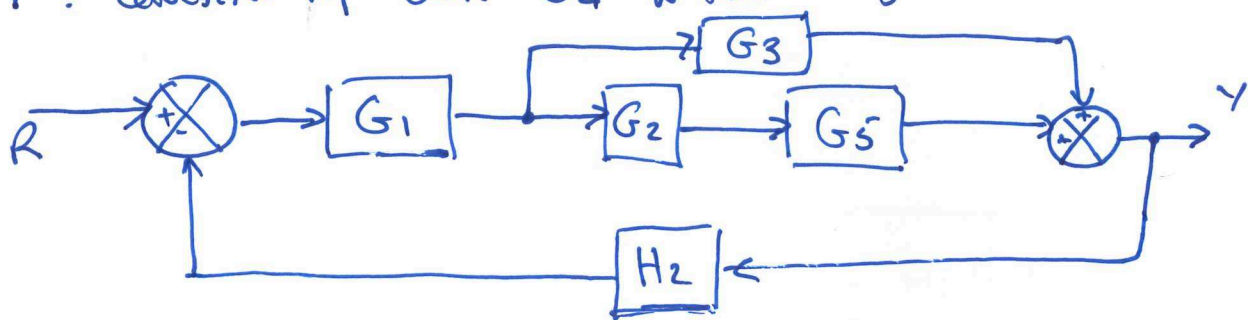


Poles are in LHP $\therefore$ stable

(2 marks)

~~Q5~~

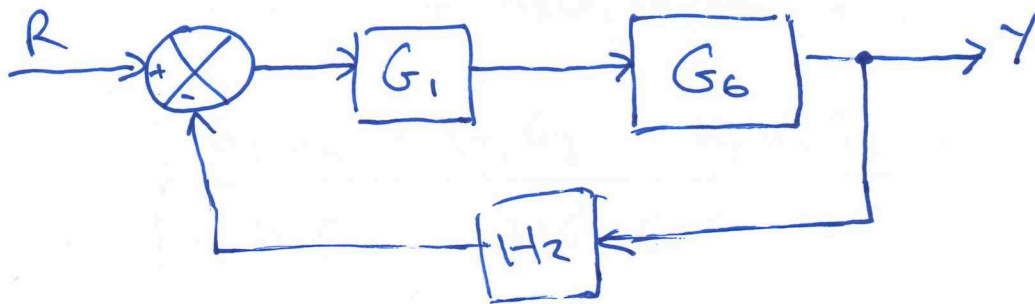
Step 1 : Combine H_1 with G_4 to form G_5



$$\text{where } G_5 = \frac{G_4}{1 + H_1 G_4}$$

(2 marks)

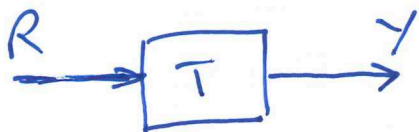
Step 2. Combine G_2 and G_5 in parallel with G_3 to form G_6



$$\text{where } G_6 = G_2 G_5 + G_3$$

(2 marks)

Step 3 Combine G_1 with G_6 with feedback via H_2



$$\text{where } T = \frac{G_1 G_6}{1 + H_2 G_1 G_6}$$

(2 marks)

Subs for G_6

$$\therefore T = \frac{G_1 [G_2 G_5 + G_3]}{1 + H_2 G_1 [G_2 G_5 + G_3]}$$

$$= \frac{G_1 G_2 G_5 + G_1 G_3}{1 + H_2 G_1 G_2 G_5 + H_2 G_1 G_3}$$

(2 marks)

subs for G_5

$$T = \frac{G_1 G_2 \left[\frac{G_4}{1 + H_1 G_4} \right] + G_1 G_3}{1 + H_2 G_1 G_2 \left[\frac{G_4}{1 + G_4 H_1} \right] + H_2 G_1 G_3}$$

(2 marks)

x top & btm by $(1 + G_4 H_1)$

$$\begin{aligned} T &= \frac{G_1 G_2 G_4 + G_1 G_3 (1 + H_1 G_4)}{(1 + H_1 G_4) + H_2 G_1 G_2 G_4 + (1 + H_1 G_4) H_2 G_1 G_3} \\ &= \frac{G_1 G_2 G_4 + G_1 G_3 + H_1 G_1 G_3 G_4}{1 + H_1 G_4 + H_2 G_1 G_2 G_4 + H_2 G_1 G_3 + H_1 H_2 G_1 G_3 G_4} \end{aligned}$$

(2 marks)

Q5

$$b) T = \frac{4 \times \frac{1}{(s+1)} \times \frac{1}{s} + 4 \times 2 + 2 \times 4 \times 2 \times \frac{1}{s}}{1 + 2 \times \frac{1}{s} + 1 \times 4 \times \frac{1}{(s+1)} \times \frac{1}{s} + 1 \times 4 \times 2 + 2 \times 1 \times 4 \times 2 \times \frac{1}{s}}$$

(1 mark)

$$= \frac{4}{s(s+1)} + 8 + \frac{16}{s}$$

$$1 + \frac{2}{s} + \frac{4}{s(s+1)} + 8 + \frac{16}{s} \quad (1 \text{ mark})$$

x top & btm by $s(s+1)$

$$T = \frac{4 + 8s(s+1) + 16(s+1)}{s(s+1) + 2(s+1) + 4 + 8s(s+1) + 16(s+1)}$$

(2 marks)

$$= \frac{4 + 8s^2 + 8s + 16s + 16}{s^2 + s + 2s + 2 + 4 + 8s^2 + 8s + 16s + 16}$$

$$= \frac{8s^2 + 24s + 20}{9s^2 + 27s + 22}$$

(2 marks)

c) Zeros occur where $8s^2 + 24s + 20 = 0$

$$s = \frac{-24 \pm \sqrt{24^2 - 4(8)(20)}}{2(8)}$$

$$= \frac{-24}{16} \pm \frac{\sqrt{-64}}{16} = -1.5 \pm j0.5$$

(2 marks)

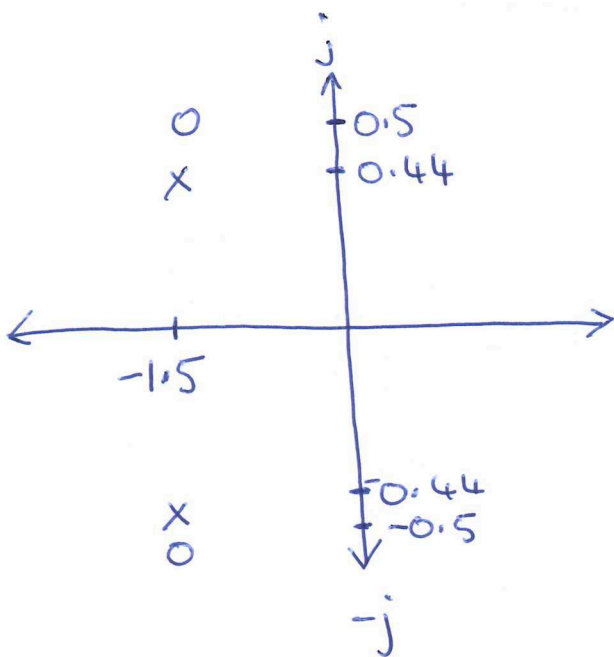
Poles occur where

$$9s^2 + 27s + 22 = 0$$

$$s = \frac{-27 \pm \sqrt{27^2 - 4(9)(22)}}{2(9)}$$
$$= \frac{-27 \pm \sqrt{-63}}{18}$$

$$= -1.5 \pm j 0.44$$

(2 marks)



Since the poles are in the LH plane, the system is stable and since poles have imaginary parts, oscillations will occur before system settles.

(2 marks)