

UWTSD

Wales Institute of Science and Art



Prifysgol Cymru
Y Drindod Dewi Sant
University of Wales
Trinity Saint David

Analytical Methods Coursework

Analytical Methods ACHA4000

Rhodri Jones

2305734

Question 1:

Question Qno:

i/ $(x+5)^2 + 2x = x^2 - 5$

$$x^2 + 12x + 35 + 2x = x^2 - 5$$

$$14x = -41$$

$$x = \frac{-41}{14}$$

ii/ $2^{x+5} = 6^x$

Take log of both sides

$$\log(2^{x+5}) = \log(6^x)$$

$$(x+5) \times \log(2) = (x) \times \log(6)$$

$$\log 2 \cdot x + 5 \log 2 = \log 6 \cdot x$$

$$5 \log 2 = (\log 6 - \log 2) x$$

$$\log 2^5 = \log \frac{6}{2} x$$

$$x = \frac{\log 2^5}{\log 3}$$

$$x = 3.15$$

$$\text{iii./ } 2 \log(x+3) = \log(x+5) + \log(x+8)$$

$$\log(x+3)^2 = \log((x+5) \times (x+8))$$

$$(x+3)^2 = (x+5) \times (x+8)$$

$$x^2 + 6x + 9 = x^2 + 13x + 40$$

$$-31 = 7x$$

$$x = \frac{-31}{7}$$

$$\text{iv./ } x - \frac{1 - \frac{6x}{5}}{3} = \frac{4 - \frac{x}{3}}{12} + \frac{3}{4}$$

$$x = \frac{4(1 - \frac{6x}{5})}{12} - \frac{4 - \frac{x}{3}}{12} + \frac{15}{12}$$

Multiply throughout by 12:

$$12x = 4(1 - \frac{6x}{5}) - (4 - \frac{x}{3}) + 15$$

$$12x - (4 - \frac{24x}{5}) = (4 - \frac{x}{3}) + 15$$

$$12x + \frac{24x}{5} + \frac{x}{3} = 23$$

(x15)

$$180x + 72x + 5x = 345$$

$$257x = 345$$

$$x = \frac{345}{257} = 1.34$$

Question 2:

Question two:

$$2x + 4y - 3z = 120 \quad (1)$$

$$5x + 2y + z = 140 \quad (2)$$

$$x - 3y + 5z = 100 \quad (3)$$

$$(1) \times 3: 6x + 12y - 9z = 360 \quad (4)$$

$$(2) \times 6: 30x + 12y + 6z = 840 \quad (5)$$

$$(3) \times 4: 4x - 12y + 20z = 400 \quad (6)$$

$$(5) - (4): 24x + 15z = 480 \quad (7)$$

$$(5) + (6): 34x + 26z = 1240 \quad (8)$$

$$(7) \times 26: 624x + 390z = 12480$$

$$(8) \times 15: 510x + 390z = 18600$$

$$(7) - (8): 114x + 0z = -6120$$

$$x = \frac{-6120}{114}$$

$$x = -53.68$$

Question 2:

Sub x into ①

$$2 + (-53.68) + 15Z = 480$$

$$-1288.32 + 15Z = 480$$

$$15Z = 1768.32$$

$$Z = \underline{117.89}$$

Sub x and z into ②

$$5(-53.68) + 2y + 117.89 = 140$$

$$-150.51 + 2y = 140$$

$$2y = 290.51$$

$$y = \underline{145.26}$$

Question 3:

Question Three:

$$0.6x^2 + 3.4x = 7.4$$

$$a = 0.6 \quad b = 3.4 \quad c = -7.4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3.4 \pm \sqrt{3.4^2 - (4 \times 0.6 \times -7.4)}}{2 \times 0.6}$$

$$\underline{x = 1.679} \quad \text{or} \quad \underline{x = -7.346}$$

Question 4:

Question Four:

i./ $y = 5x^3 - 7x^2 + 3x - 1$

$$\frac{dy}{dx} = 15x^2 - 14x + 3$$

ii./ $y = -\frac{1}{x^5} = -x^{-5}$

$$\frac{dy}{dx} = 5x^{-6}$$

iii./ $y = e^{\frac{x}{2}} + \cos \frac{x}{6} + \sin 2x$

$$\frac{dy}{dx} = \frac{1}{2} e^{\frac{x}{2}} - \frac{1}{6} \sin \frac{x}{6} + 2 \cos 2x$$

iv./ $y = (x^7 + 4)^5 \times (3x^2 - 1)^4$

$$y = u \times v$$

$$u = (x^7 + 4)^5 \quad v = (3x^2 - 1)^4$$

$$\frac{du}{dx} = 5(x^7 + 4)^4 \times (7x^6)$$

$$\frac{dv}{dx} = 4(3x^2 - 1)^3 \times (6x)$$

$$\frac{dy}{dx} = (x^7 + 4)^5 \times 4(3x^2 - 1)^3 \times (6x) + (3x^2 - 1)^4 \times 5(x^7 + 4)^4 \times (7x^6)$$

QUESTION FOUR:

$$\text{v. } y = (5x^2 + 3x + 1)^5 \times e^x$$

$$y = \frac{u}{v}$$

$$u = (5x^2 + 3x + 1)^5$$

$$v = e^x$$

$$\frac{du}{dx} = 5(5x^2 + 3x + 1)^4 \times (10x + 3)$$

$$v = e^x$$

$$\frac{dv}{dx} = e^x$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

~~$$\frac{dy}{dx} = \frac{e^x \times 5(5x^2 + 3x + 1)^4 - ((5x^2 + 3x + 1)^5 \times e^x)}{(e^x)^2}$$~~

$$\frac{dy}{dx} = \frac{(e^x \times 5(5x^2 + 3x + 1)^4 \times (10x + 3)) - ((5x^2 + 3x + 1)^5 \times e^x)}{(e^x)^2}$$

$$\text{vi} / y = \frac{\sqrt{5x+1}}{\sqrt{8x+5}}$$

$$y = \sqrt{w} = w^{1/2}$$

$$\frac{dy}{dw} = \frac{1}{2} w^{-1/2}$$

$$w = \frac{5x+1}{8x+5}$$

$$w = \frac{u}{v}$$

$$u = 5x+1$$

$$v = 8x+5$$

$$\frac{du}{dx} = 5$$

$$\frac{dv}{dx} = 8$$

$$\text{vii} / \frac{dw}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$= \frac{5(8x+5) - 8(5x+1)}{(8x+5)^2}$$

$$= \frac{17}{(8x+5)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dw} \times \frac{dw}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{5x+1}{8x+5} \right)^{-1/2} \times \frac{17}{(8x+5)^2}$$

$$\text{vii. / } y = (10x + \sqrt[3]{x^5 + 1})^5$$

$$y = w^5 \text{ where } w = 10x + \sqrt[3]{x^5 + 1}$$

$$\frac{dy}{dw} = 5w^4$$

$$\frac{dw}{dx} = 10 + (x^5 + 1)^{-\frac{2}{3}}$$

$$\frac{dw}{dx} = 10 + \frac{1}{x^{\frac{2}{3}} \sqrt[3]{5x^4}}$$

$$\frac{dw}{dx} = 10 + \frac{1}{3} (x^5 + 1)^{-\frac{2}{3}} \times 5x^4$$

$$\frac{dy}{dx} = \frac{dy}{dw} \times \frac{dw}{dx}$$

$$\frac{dy}{dx} = 5(10x + \sqrt[3]{x^5 + 1})^4 \times (10 + \frac{1}{3} (x^5 + 1)^{-\frac{2}{3}} \times 5x^4)$$

Question 5:

Question Five:

$$\frac{dx}{dt} = \text{Velocity}$$

$$\frac{d^2x}{dt^2} = \text{Acceleration}$$

$$\frac{d^3x}{dt^3} = \text{Jerk}$$

$$\frac{d^4x}{dt^4} = \text{Snap}$$

$$\frac{d^5x}{dt^5} = \text{Crackle}$$

$$\frac{d^6x}{dt^6} = \text{Pop}$$

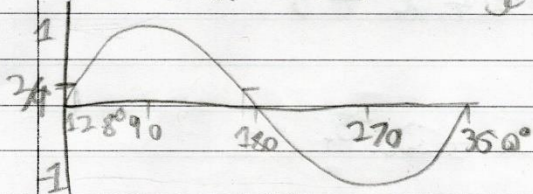
Question 6:

Question Six:

i. $\sin x = \frac{2}{9}$

$\sin x = \frac{2}{9}$ $x = \sin^{-1}\left(\frac{2}{9}\right)$

sin curve:



$x = 180^\circ - 12.8$

$x = 167.2^\circ$

ii. $5\sin^2 x - 0.3\cos x = 2$

$5(1 - \cos^2 x) - 0.3\cos x = 2$

$5 - 5\cos^2 x - 0.3\cos x = 2$

$3 - 5\cos^2 x - 0.3\cos x = 0$

$y = \cos x$

$3 - 5y^2 - 0.3y = 0$ $a = 5$ $b = 0.3$ $c = -3$

$5y^2 + 0.3y - 3 = 0$

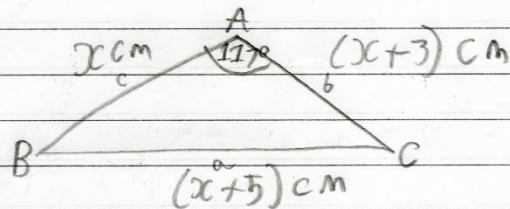
$a = 5$ $b = 0.3$ $c = -3$

$\cos x = \frac{-0.3 \pm \sqrt{0.3^2 - (4 \times 5 \times -3)}}{2 \times 5}$

$\cos x = 0.745$ or $\cos x = -0.805$

Question 7:

question seven:



$$a^2 = b^2 + c^2 - (2bc \times \cos A)$$

$$(x+5)^2 = (x+3)^2 + x^2 - (2 \times (x+3) \times x \times \cos 117^\circ)$$

$$\cos 117^\circ = -0.454$$

$$x^2 + 10x + 25 = x^2 + 6x + 9 + x^2 - (2 \times (x^2 + 3x) \times \cos 117^\circ)$$

$$x^2 + 10x + 25 = x^2 + 6x + 9 + x^2 - (-0.908)(x^2 + 3x)$$

$$10x + 25 = x^2 + 6x + 9 + 0.908(x^2 + 3x)$$

$$0 = 1.908x^2 - 1.276x - 16$$

$$a = 1.908 \quad b = -1.276 \quad c = -16$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{1.276 \pm \sqrt{(-1.276)^2 - (4 \times 1.908 \times -16)}}{2 \times 1.908}$$

$$x = 3.249 \quad \text{or} \quad x = -9.848$$

$$x \text{ cannot be -ve, } \therefore x = 3.249$$

$$x = 3.249 \text{ cm}$$

$$AB = 3.249 \text{ cm}$$

$$BC = 8.249 \text{ cm}$$

$$AC = 6.249 \text{ cm}$$

$$\text{ii./ area} = \frac{1}{2} bc \sin(A)$$

$$= \frac{1}{2} \times 6.249 \times 3.249 \times \sin 117^\circ$$

$$= 9.045 \text{ cm}^2$$