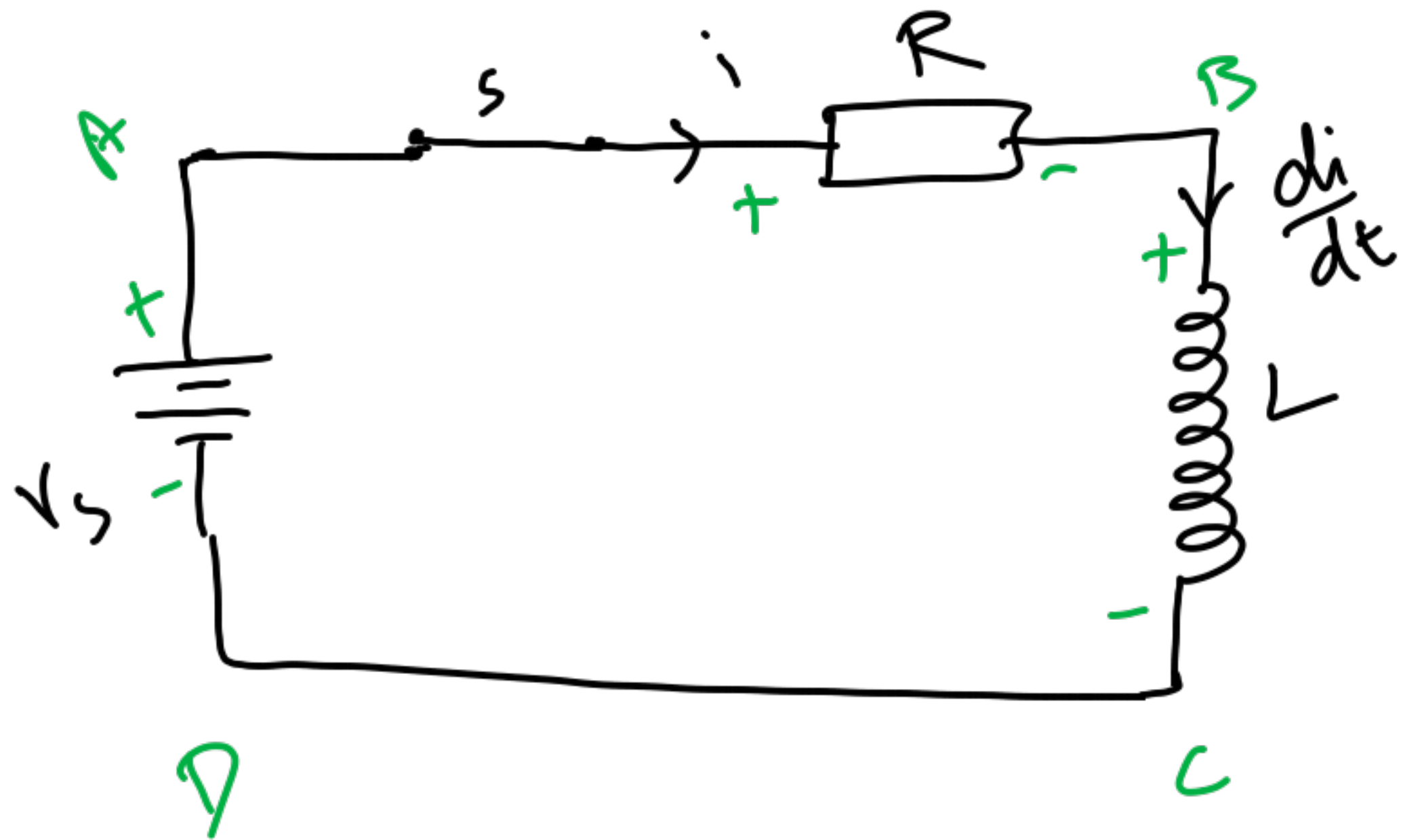


RL transient Circuit



Apply KVL in ABCDA

$$-iR - L \frac{di}{dt} + V_s = 0$$

$$V_s - iR = L \frac{di}{dt}$$

Rearranging,

$$\frac{dt}{L} = \frac{di}{V_s - iR}$$

Apply integration on both the side.

$$\int \frac{dt}{L} = \int \frac{di}{V_s - iR}$$

Limit on, t varies from '0' to 't'
 i varies from '0' to 'i'

Apply the limits

$$\int_0^t \frac{dt}{L} = \int_0^i \frac{di}{V_S - iR}$$

$$\frac{1}{L} \int_0^t dt = \int_0^i \frac{di}{V_S - iR} \quad \text{--- ①}$$

Integration by Substitution.

$$u = V_S - iR$$

Diff. with respect to 'i'

$$\frac{du}{di} = 0 - R$$

$$\frac{du}{di} = -R$$

$$du = -R \cdot di$$

$$-\frac{du}{R} = di$$

Sub. 'u' and 'di' in eq ①.

$$\frac{1}{L} \int_0^t dt = \int_0^i \frac{-du}{R \cdot u}$$

$$\frac{1}{L} \int_0^t dt = -\frac{1}{R} \int_0^i \frac{du}{u}$$

$$\frac{1}{L} \left[t \right]_0^t = -\frac{1}{R} \left[\ln u \right]_0^i$$

$$\left[\int \frac{du}{u} = \ln u \right]$$

Sub. $u = V_s - iR$ in above equatn.

$$\frac{1}{L} \int_0^t = -\frac{1}{R} \left[\ln(V_s - iR) \right]_0^t$$

$$\frac{1}{L} [t - 0] = -\frac{1}{R} \left[\ln(V_s - iR) - \ln(V_s) \right]$$

formula

$$\ln A - \ln B = \ln\left(\frac{A}{B}\right)$$

$$\frac{t}{L} = -\frac{1}{R} \left[\ln\left(\frac{V_s - iR}{V_s}\right) \right]$$

$$-\frac{tR}{L} = \ln\left(\frac{V_s - iR}{V_s}\right)$$

taking exponential on both the side

$$e^{-tR/L} = e^{\ln\left(\frac{V_s - iR}{V_s}\right)}$$

$$e^{-tR/L} = \frac{V_s - iR}{V_s}$$

$$V_s e^{-tR/L} = V_s - iR$$

$$iR = V_s - V_s e^{-tR/L}$$

$$i = \frac{V_s}{R} - \frac{V_s}{R} e^{-tR/L}$$

$$i = \frac{V_s}{R} - \frac{V_s}{R} e^{-tR/L} \quad \text{--- (2)}$$

Steady State Current $\bar{I}_0 = \frac{V_s}{R}$ ✓

time constant $\tau = \frac{L}{R}$ ✓

$$\textcircled{2} \Rightarrow i = \bar{I}_0 - \bar{I}_0 e^{-t/\tau}$$

$$i = \bar{I}_0 (1 - e^{-t/\tau}) \quad \text{--- (2)}$$

When, $t = \tau$

$$i = \bar{I}_0 (1 - e^{-\tau/\tau})$$

$$i = \bar{I}_0 (1 - e^{-1})$$

$$i = \bar{I}_0 (1 - 0.37)$$

$$i = 0.63 \bar{I}_0$$

When $t = 2\tau$

$$i = \bar{I}_0 (1 - e^{-2\tau/\tau})$$

$$i = \bar{I}_0 (1 - e^{-2})$$

$$i = \bar{I}_0 (1 - 0.135)$$

$$i = 0.865 \bar{I}_0$$

$$\bar{I}_0 = 10$$

$$t = \tau \quad i = 6.3$$

$$t = 2\tau \quad i = 8.6$$

When $t = 3\tau$

$$i = I_0(1 - e^{-3\tau/\tau})$$

$$i = I_0(1 - e^{-3})$$

$$i = I_0(1 - 0.05)$$

$$i = 0.95 I_0$$

When $t = 4\tau$

$$i = I_0(1 - e^{-4\tau/\tau})$$

$$i = I_0(1 - e^{-4})$$

$$i = I_0(1 - 0.02)$$

$$i = 0.98 I_0$$

When $t = 5\tau$

$$i = I_0(1 - e^{-5\tau/\tau})$$

$$i = I_0(1 - e^{-5})$$

$$i = I_0(1 - 0.007)$$

$$i = 0.993 I_0$$

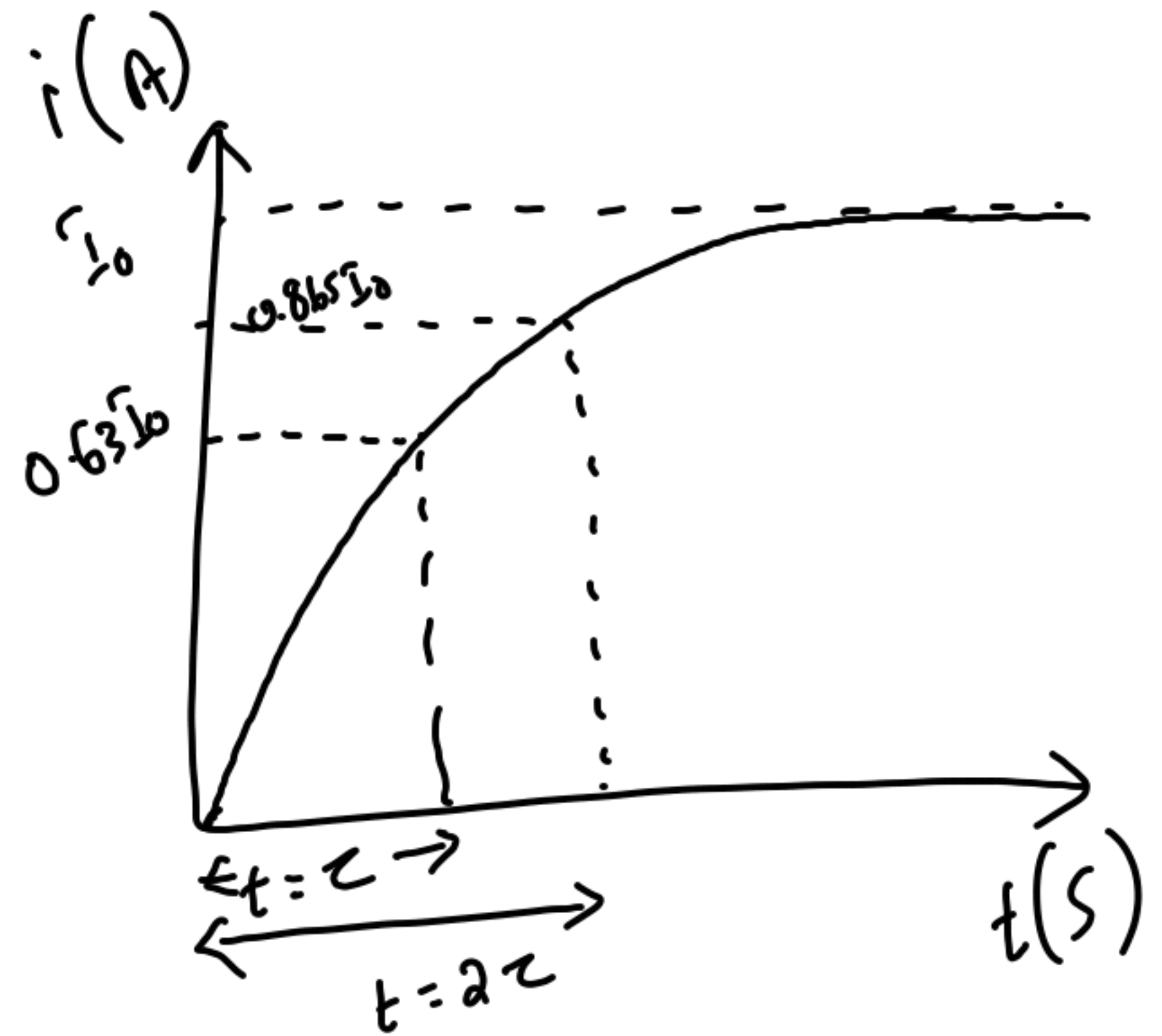
When $t = 6\tau$

$$i = I_0(1 - e^{-6\tau/\tau})$$

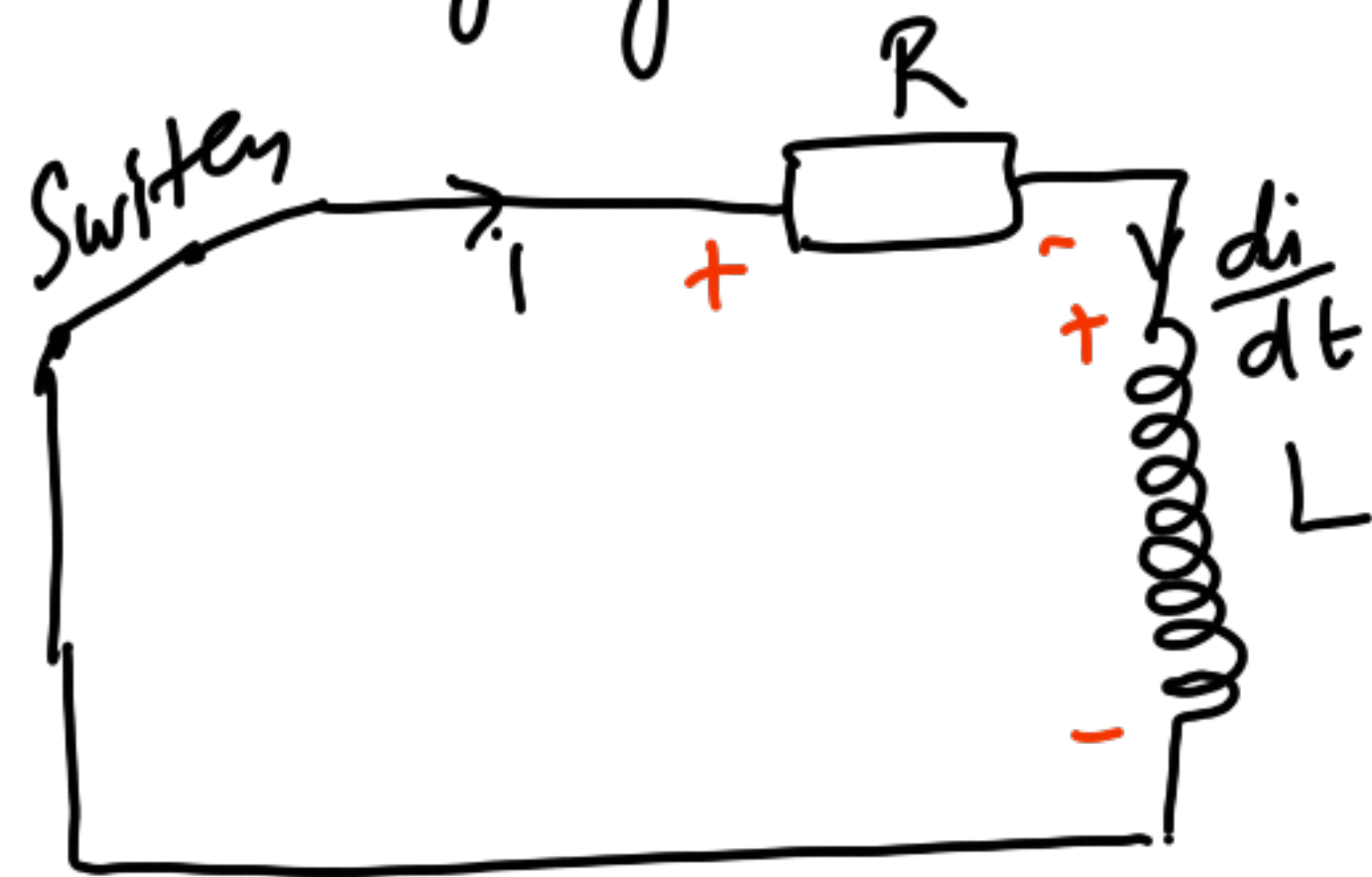
$$i = I_0(1 - e^{-6})$$

$$i = I_0(1 - 0.003)$$

$$i = 0.997 I_0$$



Discharging-RL



Applying KVL in above circuit

$$-iR - L \frac{di}{dt} = 0$$

$$-iR = L \frac{di}{dt}$$

$$-\frac{R \cdot dt}{L} = \frac{di}{i}$$

Integrating both the side.

$$-\int \frac{R}{L} dt = \int \frac{di}{i}$$

$$\text{Limits } t = 0 \text{ to } t$$
$$i = I_0 \text{ to } i$$

$$-\frac{R}{L} \int_0^t dt = \int_{I_0}^i \frac{di}{i}$$

$$-\frac{R}{L} [t]_0^t = [\ln i]_{I_0}^i$$

$$-\frac{R}{L} [t - 0] = \ln(i) - \ln(I_0)$$

$$-\frac{Rt}{L} = \ln\left(\frac{i}{I_0}\right)$$

Taking exponential on both the side.

$$e^{-\frac{Rt}{L}} = \frac{i}{I_0}$$
$$i = I_0 \left(e^{-Rt/L} \right)$$

$$i = I_0 e^{-Rt/L}$$

$$i = I_0 e^{-t \frac{R}{L}}$$

$$\tau = \frac{L}{R}, \text{ Sub in above equatn.}$$

$$i = I_0 e^{-t/\tau}$$

$$\text{When } t = \tau$$

$$i = I_0 e^{-\tau/\tau} = I_0 e^{-1} = 0.37 I_0$$

$$\text{When } t = 2\tau$$

$$i = I_0 e^{-2\tau/\tau} = I_0 e^{-2} = 0.135 I_0$$

$$\text{When } t = 3\tau$$

$$i = I_0 e^{-3\tau/\tau} = I_0 e^{-3} = 0.05 I_0$$

$$\text{When } t = 4\tau$$

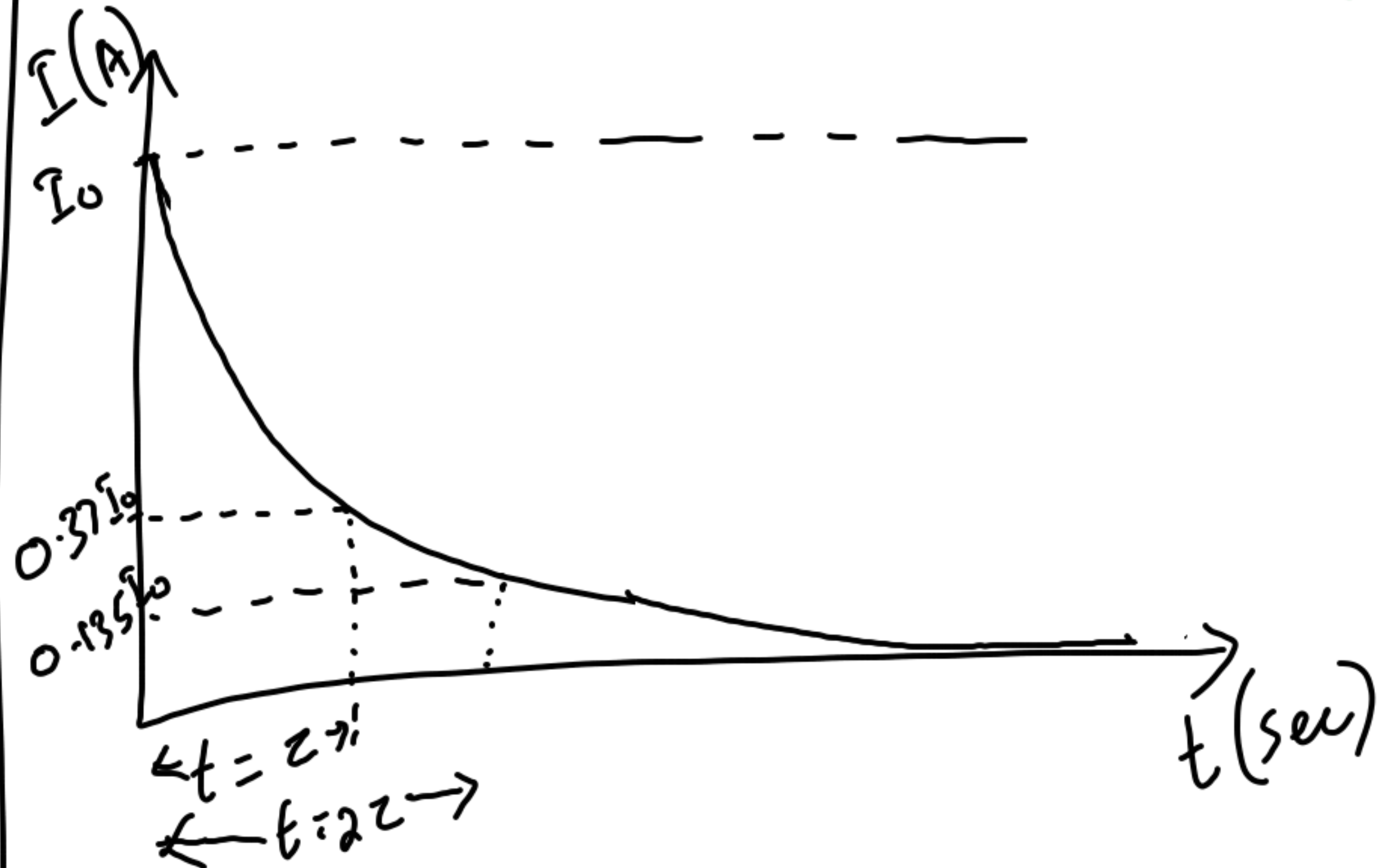
$$i = I_0 e^{-4\tau/\tau} = I_0 e^{-4} = 0.018 I_0$$

$$\text{When } t = 5\tau$$

$$i = I_0 e^{-5\tau/\tau} = I_0 e^{-5} = 0.0067 I_0$$

$$\text{When } t = 6\tau$$

$$i = I_0 e^{-6\tau/\tau} = I_0 e^{-6} = 0.0025 I_0$$



$$I_0 = 10 \text{ A}$$

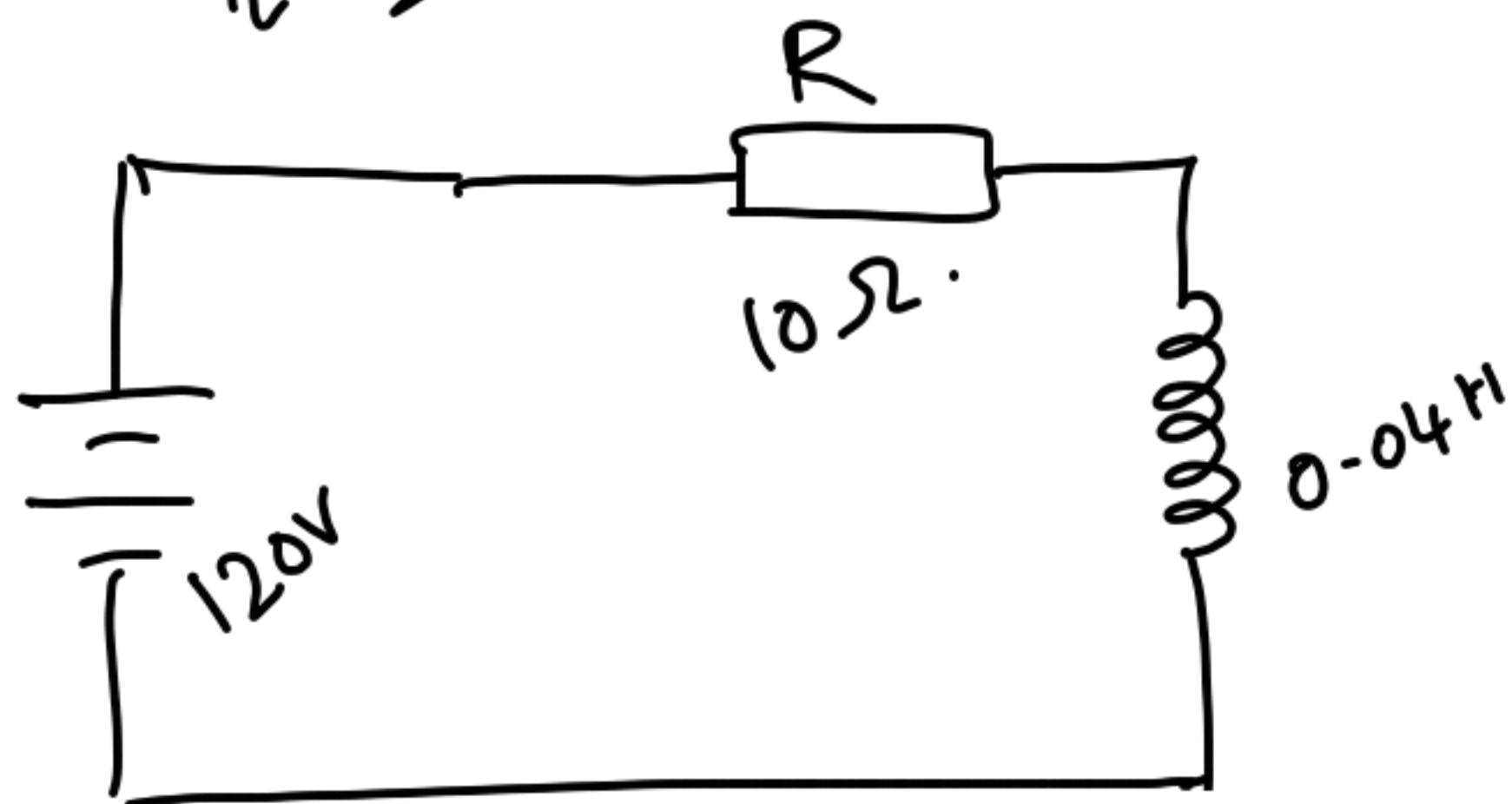
when $t = \tau$, $i = 3.7 \text{ A}$

$$t = 2\tau, i = 1.3 \text{ A}$$

✓
✓
✓

- i) A coil of inductance 0.04 H and Resistance of 10Ω is connected to a 120 V D.C Supply. Determine
- the final value of Current
 - the time constant of the circuit
 - the value of Current after a time equal to the time constant from the instant the Supply voltage is connected.
 - the expected time for the Current to rise to within 1% of its final value.

Soln.
 $t = 0s$
 $t = 0.004s = \tau$



a) $I_0 = \frac{V}{R} = \frac{120}{10} = 12 A$

b) $\tau = \frac{L}{R} = \frac{0.04}{10} = 0.004 s$

c) $I = ?$
 $t = \tau = 0.004 s$

$$I = I_0 (1 - e^{-t/\tau})$$

$$I = 12 (1 - e^{-0.004/0.004})$$

$$= 12 (1 - e^{-1})$$

$$I = 12 (1 - 0.37)$$

$$I = 7.59 A$$

d) $t = ?$

$$I = I_0 (1 - e^{-t/\tau})$$

$$\underline{I} = 12(1 - e^{-t/0.004})$$

$$12 \times 0.99 = 12(1 - e^{-t/0.004})$$

$$11.88 = 12(1 - e^{-t/0.004})$$

$$\frac{11.88}{12} = 1 - e^{-t/0.004}$$

$$0.99 = 1 - e^{-t/0.004}$$

$$e^{-t/0.004} = 1 - 0.99$$

$$e^{-t/0.004} = 0.01$$

taking natural log on both the side.

$$\frac{-t}{0.004} = \ln(0.01)$$

$$\frac{-t}{0.004} = -4.61$$

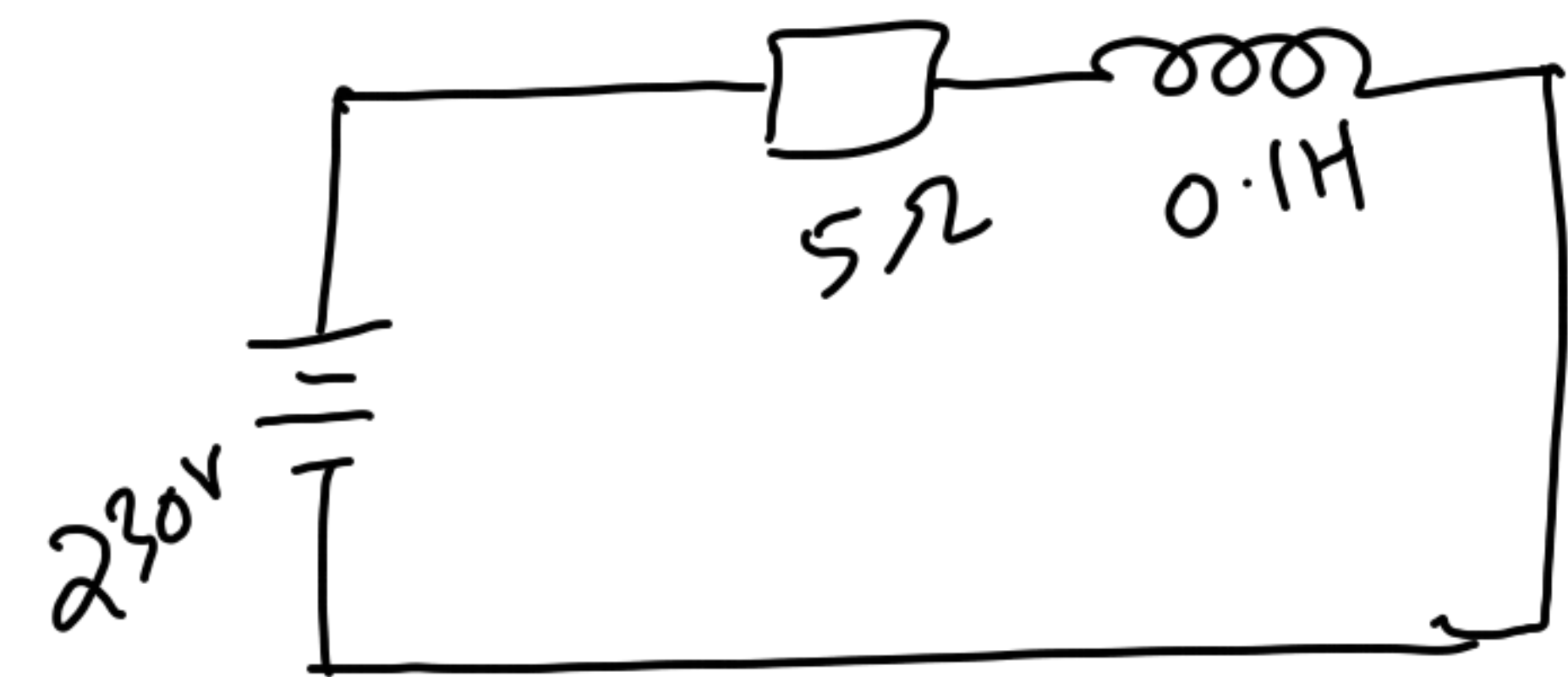
$$t = 4.61 \times 0.004$$

$$t = 0.0185$$

$$t = 0.02 \text{ s}$$

$$t = \underline{\underline{2 \text{ ms}}}$$

2) A coil of Resistance 5Ω and inductance of $0.1H$ is switched onto a $230V$ supply. Calculate the rate of rise of current at $t=0$ and at $t=2\tau$. Also, calculate the Steady State Value of the circuit current.



Soln.

Steady State Current

$$I_0 = \frac{V}{R} = \frac{230}{5} = 46A$$

Rate of Rise of Current:

$$I = I_0 \left(1 - e^{-t/\tau} \right)$$

$$\tau = \frac{L}{R} = \frac{0.1}{5} = 0.02s$$

$$I = 46(1 - e^{-t/0.02})$$

$$I = 46 - \underline{46} e^{-t/0.02}$$

$$x = \underline{\underline{\frac{1}{0.02}}}$$

Differentiate with Respect to 't'

$$\frac{dI}{dt} = 0 - 46 \left(-\frac{1}{0.02} \right) e^{-t/0.02}$$

$$\frac{dI}{dt} = \frac{46}{0.02} e^{-t/0.02}$$

$$\boxed{\frac{dI}{dt} = 2300 e^{-t/0.02}}$$

Diff. of e^{-xt}

$$e^{-xt} = -x e^{-xt}$$

$$\underline{46} e^{-\frac{1}{0.02}(t)} = 46 \left(-\frac{1}{0.02} \right) e^{-\frac{1}{0.02}(t)}$$

$$\frac{dI}{dt} = 2300 e^{-t/0.02}$$

When $t = 0$

$$\frac{dI}{dt} = 2300 e^{-0/0.02}$$

$$\frac{dI}{dt} = 2300 \times e^0$$

$$\frac{dI}{dt} = 2300 \times 1 = 2300 \text{ A/s}$$

When $t = 2\tau$

$$\frac{dI}{dt} = 2300 e^{-2\tau/0.02}$$

$$= 2300 e^{-2(0.02)/0.02}$$

$$= 2300 e^{-2}$$

$$= 2300 \times 0.135$$

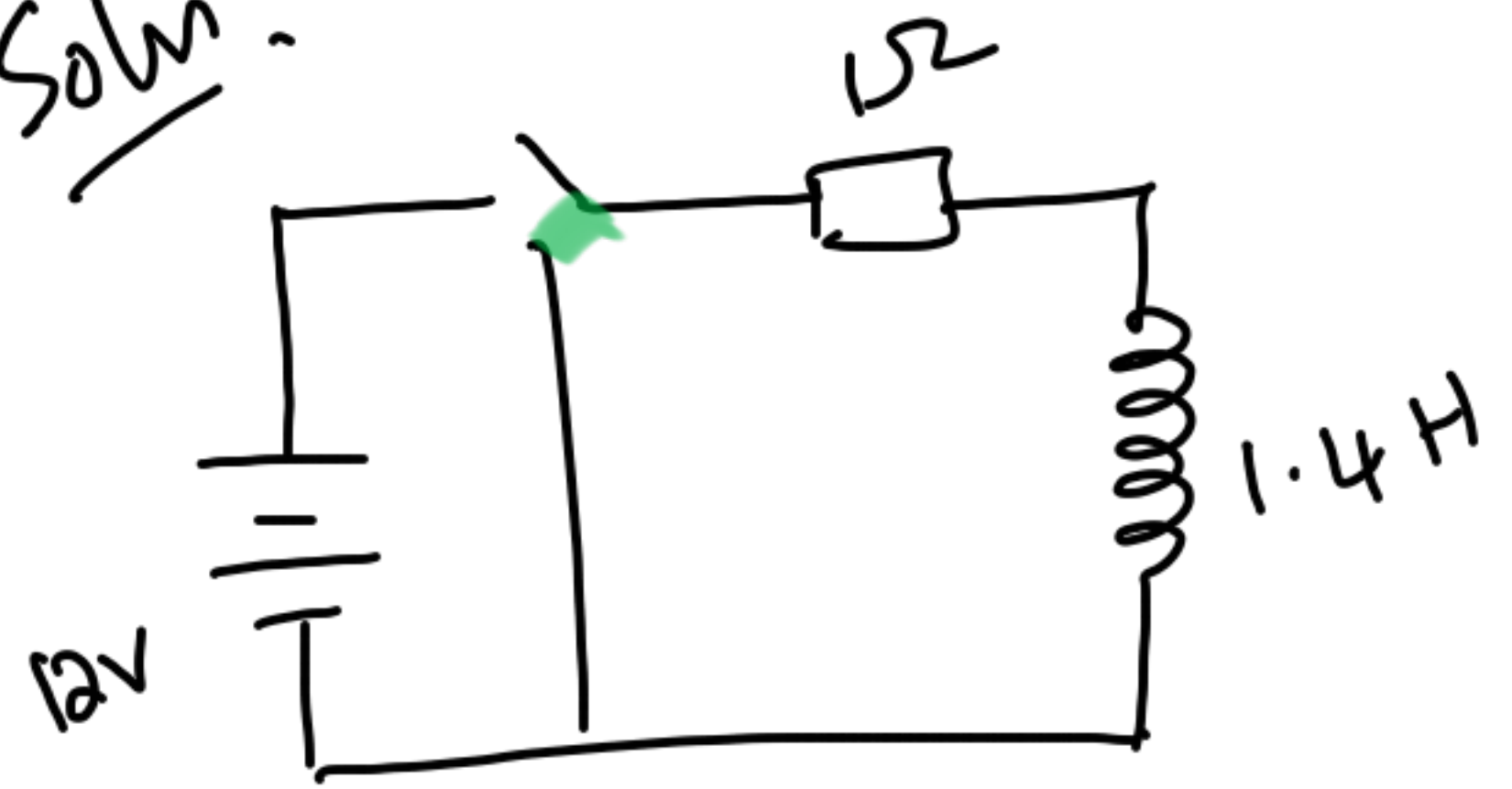
$$\frac{dI}{dt} = 311.27 \text{ A/s}$$

3) A coil having an inductance of 1.4 H and a Resistance 1Ω is connected to a 12 V Supply through a switch.

a) What will be the Value of Current 400 ms after Switching ON the Supply.

b) After the current has reached steady state, the supply is disconnected. How long does it take ^{for} the current to drop to half of its steady-state value.

Soln.



a) $I = ?$, when $t = 400\text{ ms}$

$$I = I_0(1 - e^{-t/\tau})$$

$$I_0 = \frac{V}{R} = \frac{12}{1} = 12\text{ A}$$

$$\tau = \frac{L}{R} = \frac{1.4}{1} = 1.4\text{ s}$$

$$I = 12 \left(1 - e^{-400 \times 10^{-3} / 1.4} \right)$$

$$I = 2.98 \text{ A}$$

b) Discharging.

$$I = I_0 e^{-t/\tau}$$

to calculate time, when $I_0 = \frac{I_0}{2}$

$$I = \frac{I_0}{2} e^{-t/\tau}$$

$$\frac{12}{2} = 12 e^{-t/1.4}$$

$$\frac{6}{12} = e^{-t/1.4}$$

$$0.5 = e^{-t/1.4}$$

taking natural log on both the sides

$$\ln(0.5) = \frac{-t}{1.4}$$

$$-0.69 = \frac{-t}{1.4}$$

$$t = 1.4 \times 0.69$$

$$t = 0.975$$

