

Control System Design

CLOSED LOOP CONTROL SYSTEM ANALYSIS OF A PHALANX
CLOSE IN WEAPON SYSTEM (CIWS)
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Abstract

Using MATLAB and Simulink a closed loop system was designed and tested to represent an angular positioning system attached to an automatic CIWS turret.

The system included a comparator to produce a feedback signal, a PID controller to produce the desired transient response and an angular positioning system given as a transfer function.

Additionally, the designed system was mathematically represented as a transfer function. This allowed stability testing of the system, as well as comparison of the output from the transfer function and overall system.

It was found that the transfer function accurately replicated the response of the designed system when comparing the outputs in the time domain. Additionally, Nyquist plots and Pole-Zero plots both indicated the transfer function to be stable in the complex frequency domain; with bode diagrams showing good phase and gain margins in standard frequency domain increasing the stability of the overall system.

Introduction

Background of the Phalanx Close in Weapon System (CIWS)



Figure No.1 – Phalanx CIWS System [1]

The Phalanx Close in Weapon System is a computer controlled, radar guided weapon system that is used to engage and defend against multiple threats, [2] such as:

- Anti-Ship Missiles,
- Surface Craft,
- Helicopters and drones,
- Aircraft.

The weapon system is equipped with an M61A1 20mm cannon capable of firing 3000 - 4500 rounds per minute. [1] The system has been designed to be independent of the ship and can therefore easily be installed and integrated into any system.

The system itself can conduct several roles such as autonomous searching, detection, evaluation, tracking and engagement and kill assessment functions. To achieve engagement the turret has a pneumatic drive to rotate the canon into position towards the direction of the target. [1]

For this report there is a requirement to design a closed loop control system capable of controlling the angular position system attached to the phalanx CIWS turret.

Transfer Function

Before describing a transfer function, the definition of a control system will be first established to help understand what is being represented by the transfer system.

- **Control System:** Is an interconnection of components forming a system that will provide a desired system response. This assumes the system will have a cause-effect relationship for the components of the system using linear system theory. [3]

This means when a singular input is applied to the control system, it is expected for the control system to achieve the desired output (system response).

An example related the Phalanx CIWS is: When there is an input applied to the angular positioning control system, the Phalanx will manoeuvre itself to the desired angle to be able to engage the target and defend the vessel.

This angular positioning control system can be mathematically modelled as a **transfer function**. Which is a mathematical representation of the control system, that models the systems output for each possible input. [4]

For example the transfer function of an amplifier can be calculated by finding the ratio between the input and output.

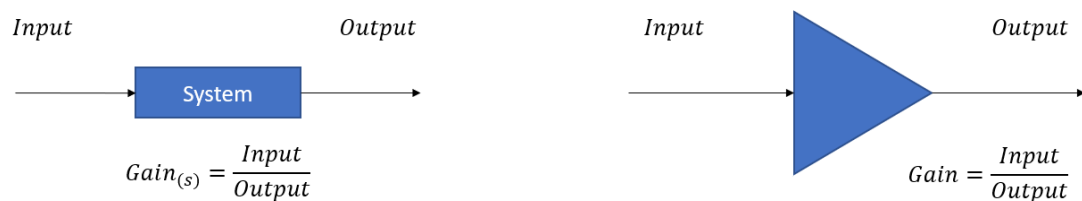


Figure No.2 – Transfer Function with an Amplifier Example

By being able to mathematically describe a system with a transfer function, further analysis can be made to determine the stability of the system. This can be useful if analysis of a live control system would be too costly due to components or operating restrictions.

However, the transfer function changes depending on if the circuit is open or closed loop.

Open Looped Systems

An open loop system is a system that contains a controller and an actuator (sometimes represented as process). [3] There is no feedback signal within the control system, and therefore can be affected by external disturbances without the user knowing.

An example of an open looped system is a motor attached to a potentiometer. The greater the input from the potentiometer, the greater the velocity of the motor.

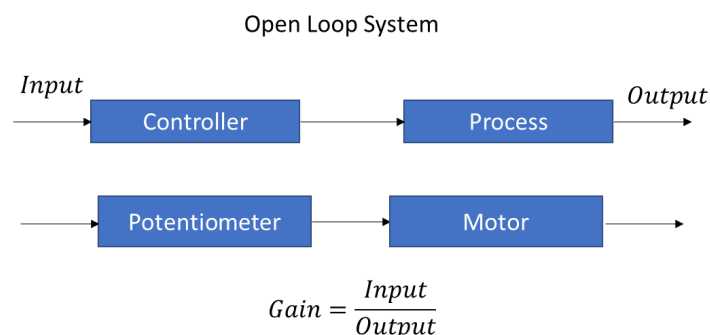


Figure No.3 – Open Loop System Example with Equation [5]

Due to its simplicity the gain of this control system is the ratio between the input over the output.

Closed Looped Systems

A closed looped system utilises an additional component to compare the output with the desired output response. Creating a feedback signal.

The feedback signal maintains the prescribed relationship of the system variable to another by comparing functions of these variables by measuring the difference as a means of control. [3]

This means that the feedback signal compares the desired output with measured output, and any difference between the two, this component will remove.

For example, if our turret had the input to move to position X, the circuit will take the current measurement of the position of the turret Y to calculate Z. This will then allow the system to understand how far it needs to rotate the turret.

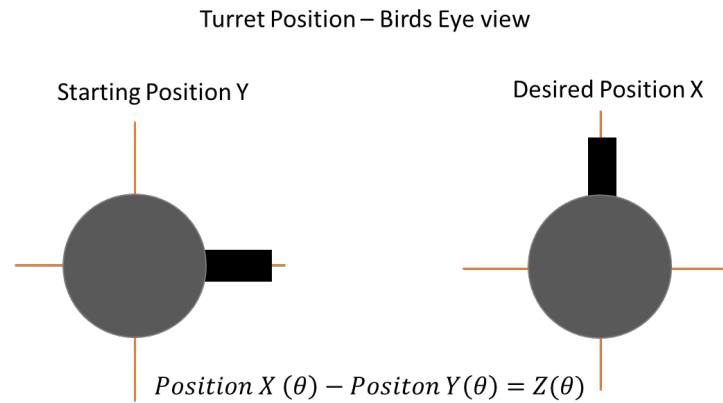


Figure No.5 – Comparator process

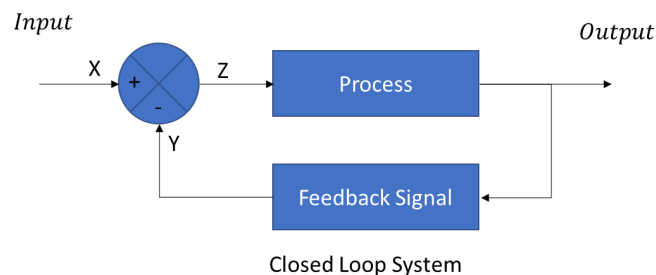


Figure No.6 – Closed Loop System Example

$$G = \frac{1}{GH + 1}$$

Equation No. 1 – Transfer Function of Closed Loop [5]

The benefit of a closed loop system is that the closed system can reject external disturbances. However, the system can be affected by lag and overshooting. Therefore, a controller is required.

Discrete On – Off Controller

This is the simplest form of control, and has two states which the actuator can be in. Either on or off. An example of a discrete control system is in a central heating system.

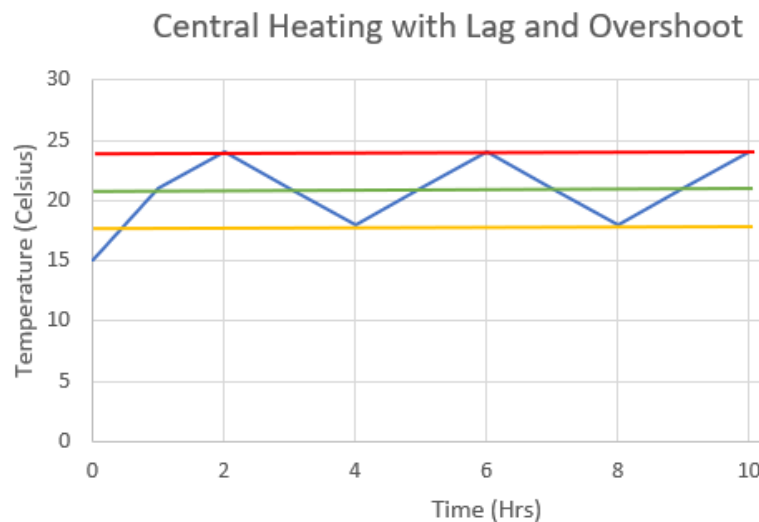


Table No.1 – Discrete on – off controller with an example central heating system

In this example the desired temperature for the room is 21 Celsius. As temperature is below, the heating turns on and temperature rises. However, **measurement lag** can lead to overshooting.

When the thermostat takes a measurement of the room the temperature measured can be different to the true temperature of the room, due to location of the thermostat. Therefore, the heating will remain on until the measured result reaches the desired result.

However, because the measured result is not the same as the true result this can lead to overshooting. In which the real temperature becomes greater than the desired result because of the measurement lag.

Therefore, for the Phalanx CIWS control system a controller is required which can dampen the deviation of the overshooting/undershooting and has a real time response to reduce measurement lag.

Proportional Integral Derivative (PID) Controller

A PID controller is more complex than the discrete controller type and therefore requires time tuning the controller to the system to achieve the correct transient response. [6]

It works by using applying integration, derivative, and differentiation to the difference between the desired and measured output of a controller. A summary of each component.

- **Proportional Control** – This part of the controller measures the difference between the desired output the measured output and applies an output proportional to the difference. Therefore, the greater the difference it will apply a greater input to correct the control system.

However, the disadvantage is that this type of controller is prone to failure from offset. In which a load cannot be added as this would result in a greater deviation between the measured and desired output. Therefore, requiring manual reset.

- **Integral Control** - is used to fight offset as it measures the time sum of the deviation. Therefore, over time the integral action will apply a greater action to the control system to correct for load changes.

However, integral control is not effective for systems that require multiple start-ups.

- **Derivative Control** – Applies a correction that is proportional to the rate of change of the deviation. Therefore, when the difference is greater this will increase the rate of the actuator. This system is useful for control systems that require quick response times.

However, the disadvantage to this type of controller is that if there is no change, then the actuator does not react.

Overall, using a PID controller the desired transient response of the controller can be tuned for the Phalanx CIWS turret. This would include a fast rise time, with a quick settling time with little to no overshoot.

Stability of a system

Due to the operational requirements for the Phalanx CIWS, the control system should be stable to reduce any transient response errors. To validate the stability of the control system, the transfer function can be used to determine Pole – Zero (P-Z) Plots and Nyquist plots.

- **P-Z plots** are diagrams plotted on the complex frequency domain to represent how the system behaves at various frequencies. If the frequency approaches a zero, the magnitude of gain approaches zero. While if the frequency approaches a pole, then the magnitude of gain will be infinite. [7]

Depending on the location of the Poles and Zeros the following equation can be used which describes the time response of the control system.

$$C_i e^{\sigma t}$$

Equation No.2 – Exponentially decaying component [8]

Using this equation, it can be used in the following to calculate how the system would respond to an applied input. This will determine if the system will either dampen or oscillate the control system over time.

$$A_0 e^{-\sigma t} \sin(+\phi)$$

Equation No.3 – Response Component [8]

However, it should be noted. If there is no imaginary part, then this would result in the gain of the system remaining constant.

- **Nyquist Plots** investigate stability in the frequency domain of linear control systems using Cauchy's theory. [3] In which the Poles and Zero's from the P-Z plot are used to draw a contour in the $F(s)$ – plane.

From the Nyquist plot, the Nyquist stability criterion can be applied with the following equation:

$$P = N + Z$$

Equation No.4 – Nyquist Stability Criterion [3]

This determines the stability of the system by determining if there are any Poles or Zero's in the right-hand side of the S-plane. Additionally, from the Nyquist plot, gain margin and phase margin can be determined.

- **Phase Margin** – The amount of phase shift on the $L(j\omega)$ at unity magnitude that will result in a marginally stable system with intersection of the $-1 + j0$ point. [3]
- **Gain Margin** – The increase in system gain when phase = -180 degree that will result in a marginal stable system with intersection of the $-1 + j0$ point. [3]

Simulation Method

System Requirements

Below are the following control system requirements for the design of the closed loop system.

- System does not overshoot by more than 5%
- System brings the process under control within 2 seconds.
- The control system follows a unit step

Design of Circuit

With this information the following control system was designed.

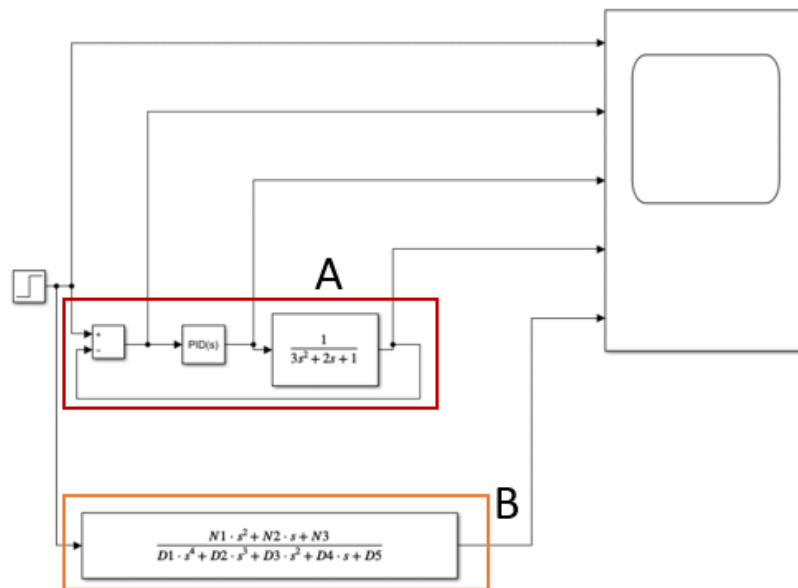


Figure No.7 – The Design of the Control System

It comprised of two individual control systems in parallel to an oscilloscope and both in series of a unit step.

- **Control System A** – Is the designed closed loop system to work alongside the angular position control system. This system comprises of the following components:
 - A comparator to allow a feedback signal to compare the measured results against the desired response. For this scenario negative feedback has been used to maintain system stability. If positive feedback were to be used instead this could cause the system to self-oscillate where the feedback signal is added to input signal.
 - A PID controller is then used to produce the desired response time and curve.

Below is the configuration of the PID block.

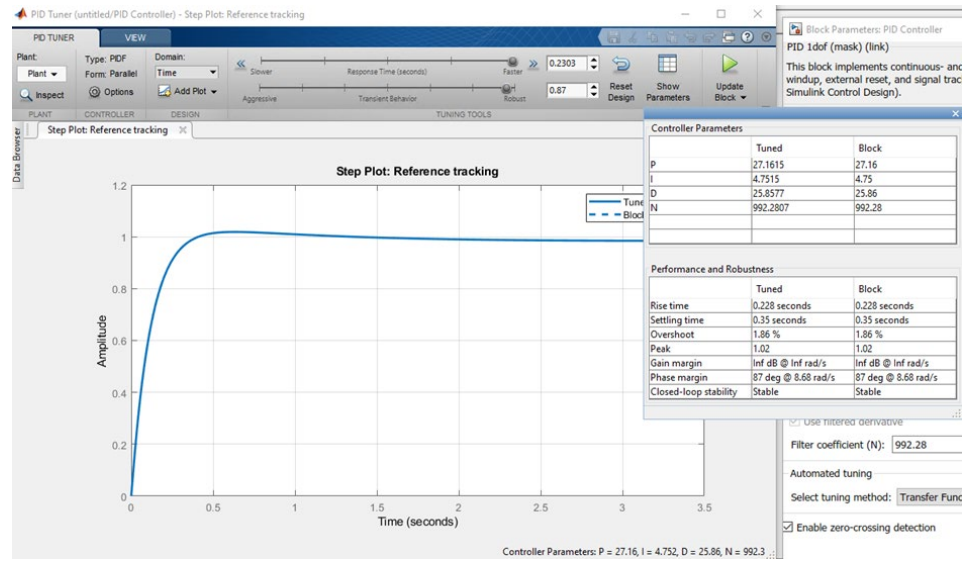


Figure No.8 – PID Controller Response and Parameters

P = 27.16
I = 4.75
D = 25.86
N = 992.28

From Figure X the PID controller rises to the desired output within 0.5 seconds and settles within 1sec. However, the system overshoots by 1.86%.

This shows the advantage of using a PID controller, as it allows quick loading from the derivative and proportional parts of the controller. While the Integral and proportional parts reduce external disturbances and dampens any oscillations or overshooting.

- Finally, the angular position transfer function which represent the process of the system rotating. This part represents the process part of a control system
- **Control System B** – Is a transfer function off all the components in control system combined.

Transfer Function Configuration

To achieve the transfer function the following mathematics were applied to the circuit. The calculations for these equations can be found in Annex A and Annex B.

Algebraic Equations:

$$\left(\frac{(Kp + NKd)s^2 + (NKp + Ki)s + NKi}{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs} \right)$$

Equation No. 5 – Forward Transfer Function Algebraic

$$TF = \left(\frac{(Kp + NKd)s^2 + (NKp + Ki)s + NKi}{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs} \right) \left(\frac{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs}{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs + (Kp + NKd)s^2 + (NKp + Ki)s + NKi} \right)$$

Equation No 6. – Closed System Transfer Function Algebraic

Numerical:

$$\frac{25687.52s^2 + 26955.07s + 4713.33}{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s}$$

Equation No.7 - Forward Transfer Function Numerical

TF

$$= \frac{77062.56s^6 + 76599877.08s^5 + 131313092.09s^4 + 93050376.29s^3 + 36105576.36s^2 + 4676943.09s}{9s^8 + 17873.04s^7 + 8962463.67s^6 + 88436162s^5 + 141167228.1s^4 + 96990840.08 + 37090195.98s^2 + 4676943.09s}$$

Equation No.8 – Closed System Transfer Function Numerical

Dependant Variables

- For each circuit the amplitude of the output will be measured. This is to analyse behaviour of both circuits and to compare.
- Stability Testing. From the transfer function further analysis will be conducted to determine the stability of the circuit. This is to determine if any external disturbances can affect the transfer function, and thus the system and to understand the magnitude of the external disturbance.

Results

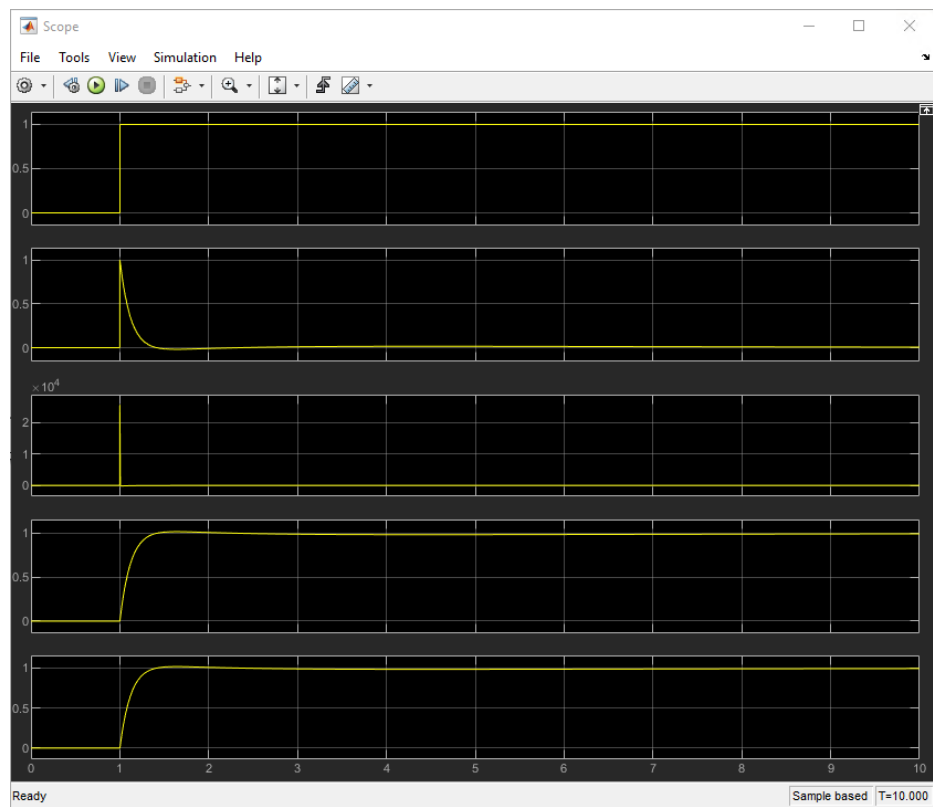


Table No.2 – Oscilloscope measurements of control system A and transfer Function Output

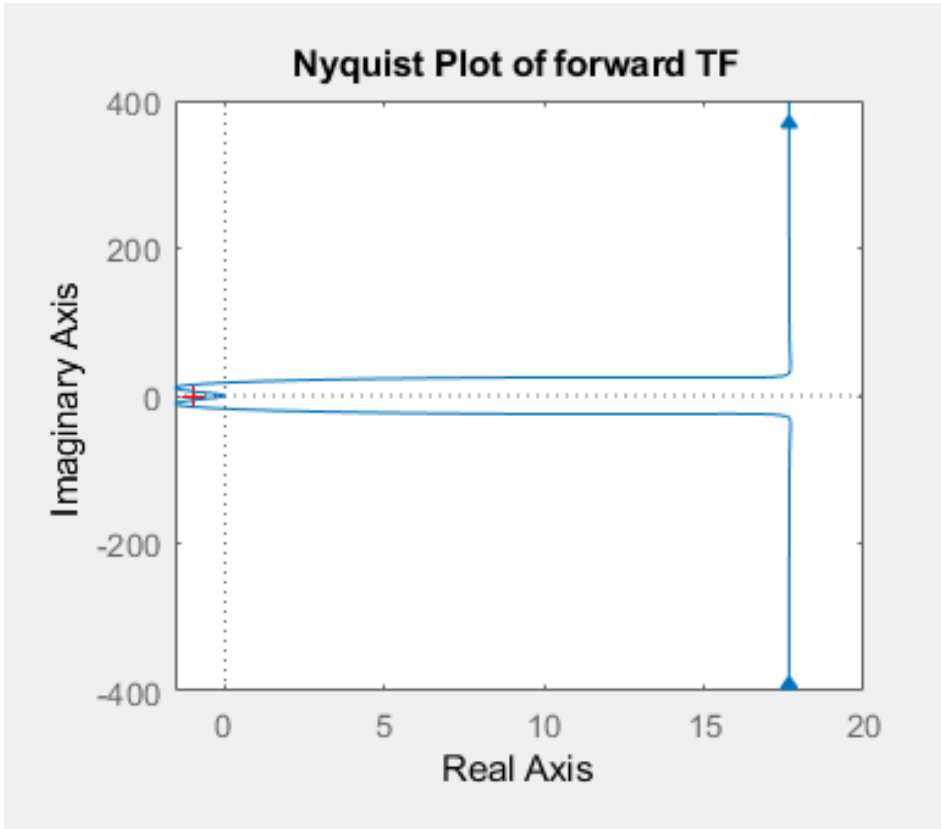


Table No.3 – Nyquist Plot of Forward Transfer Function

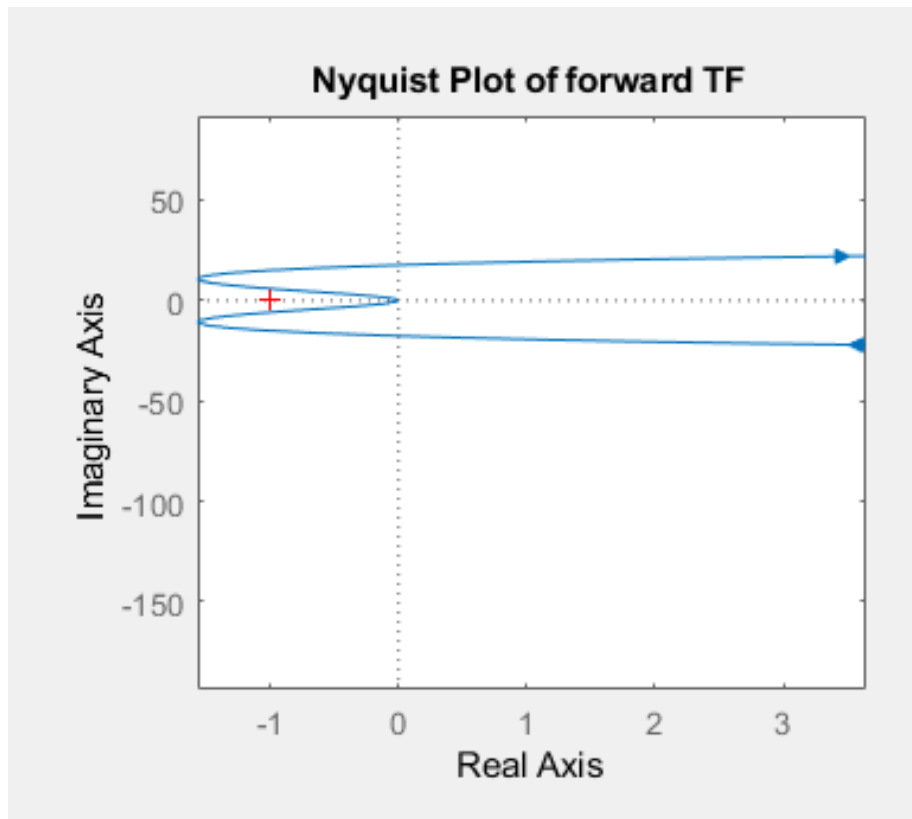


Table No.4 – Nyquist Plot with focus around the unit circle and $(-1, 0j)$

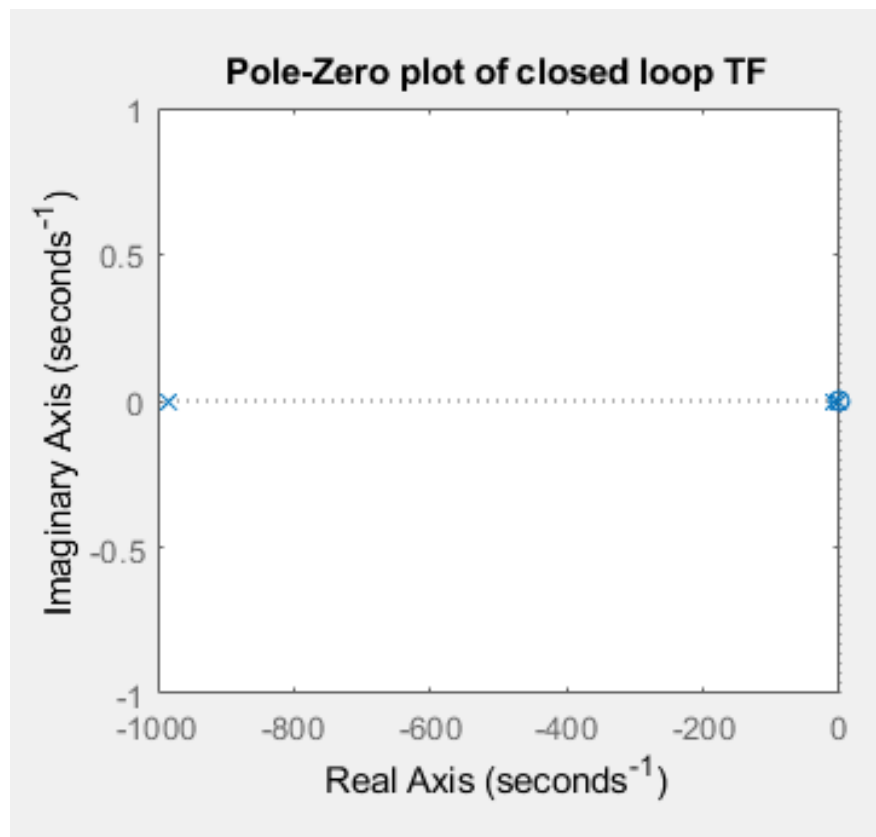


Table No.5 – Pole-Zero Plot of Closed Loop Transfer Function



Table No.6 – Pole-Zero Plot of Closed Loop Transfer Function

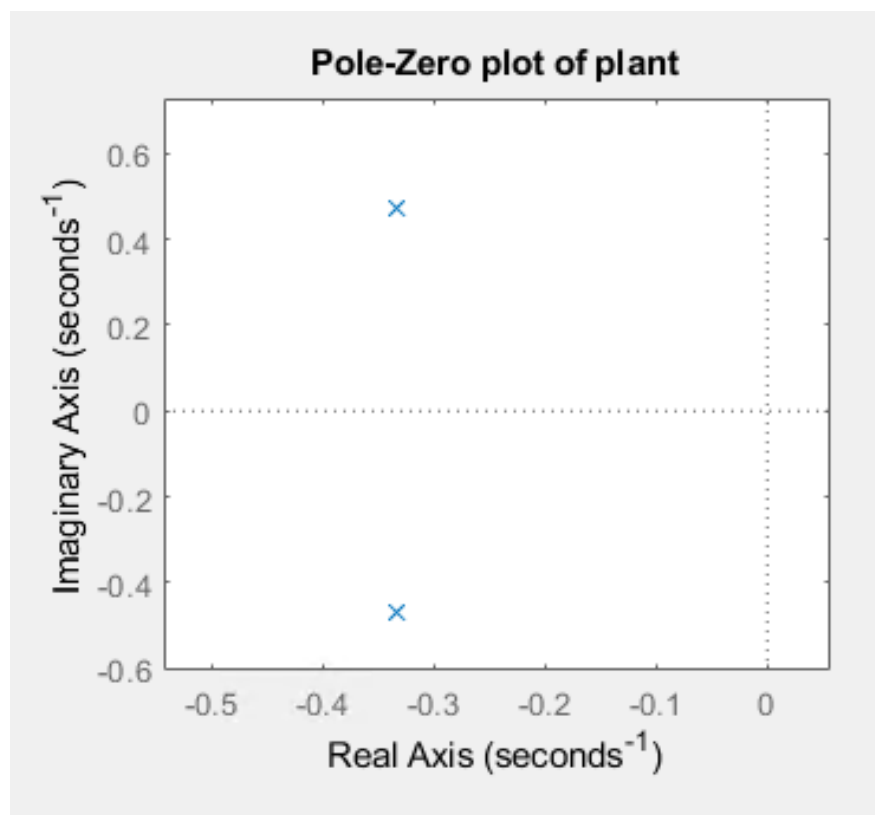


Table No.7 – Pole-Zero Plot of plant

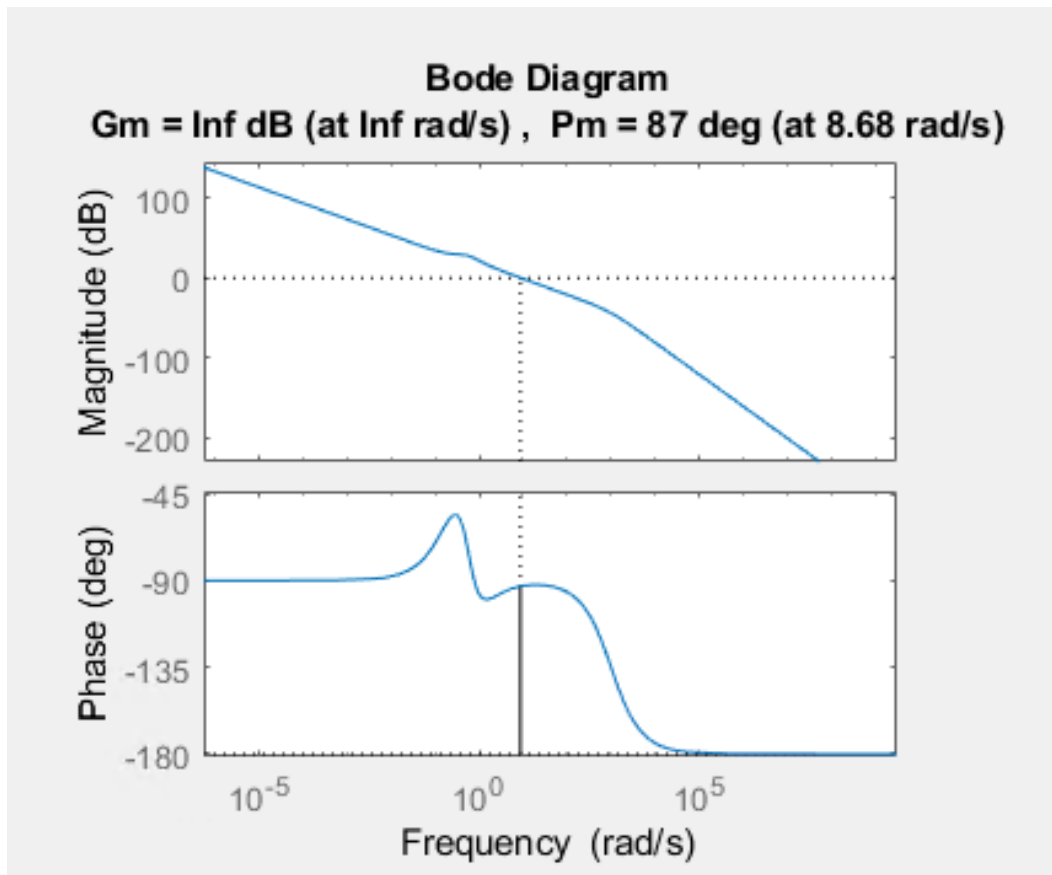


Table No.8 – Bode Diagram

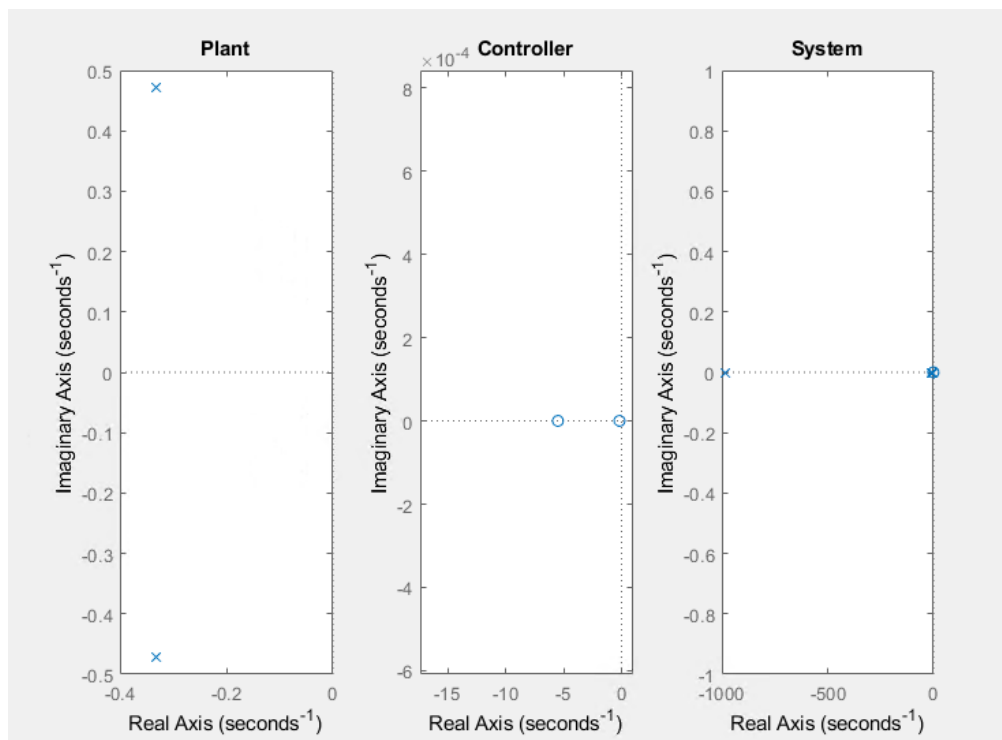


Table No.9 – Pole-Zero Plots of Plant, Controller and System

Discussion

Oscilloscope Measurements

From the Oscilloscope measurements in Table 2 both the Control System A and the transfer function behave similarly to the same input, producing the same output. They both respond to the input within a second and almost instantaneous (reducing lag), while producing little to no overshoot and have quick settling times.

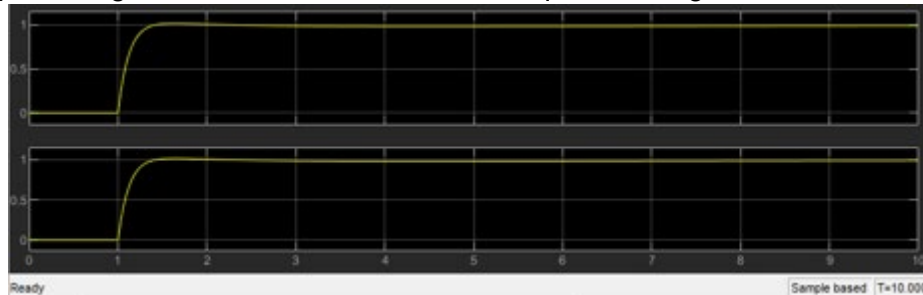


Figure No.8 – Control System Response Above – Transfer Function Response Below

From the requirements both systems in the time domain show the desired results and desired transient responses in the last two graphs. This indicates that the transfer function is a correct mathematical representation of the system. However, it does not show how stable the transfer function is against frequency, or gain.

Pole-Zero Analysis

Pole-Zero Plots of the Plant: The plant is the transfer function given for the angular rotational positing system is:

$$G(s) = \frac{1}{3s^2 + 2s + 1}$$

Equation No.9

When the denominator value is solved to equal zero, by using a quadratic equation, this produces a conjugate pair of poles in the left hand plane on the imaginary axis of the P-Z plot.

From these points, it can be determined that this system by itself is stable, however will slowly dampen over time. This can be proven by entering the value into equation:

$$C_i e^{\sigma t}$$

Equation No.10

$$A_0 e^{-\sigma t} \sin(+\phi)$$

Equation No.11

From here it can be seen that over time slowly the amplitude of the output will slowly be dampened. Depending on the deviation of the real part is from zero, will affect the rate of dampening.

Because there are no co-efficients on the top part of equation No.7 this highlights there should be no poles. From the graph this is confirmed. Therefore, the plant itself cannot reach zero at any frequency.

Pole-Zero Plots of the System: When comparing the plant P-Z plot to the P-Z Plot of the transfer function system it can be seen there are no conjugate pairs. All Poles and Zeros remain on the real axis. As a result, this shows that the system does not dampen over time, but instead remains at a constant amplitude.

It could then be argued that due to conjugate pairs being removed, this has improved the system by reducing the oscillations in the output, resulting in a faster settling time.

It should be noted that in the P-Z plot of the system there are in total 4 poles and 2 zeros. This would highlight that the order of the numerator polynomial is smaller than the denominator polynomial and be confirmed using by looking at the final transfer function from the simulation method.

Nyquist Analysis

The Nyquist plot can be used to determine the frequency response and the phase response against of real values of frequency.

From Figure No.4 the Nyquist plot shows a clockwise rotation and intercepts through a real value at 0 and does not encircle the real value: (-1, 0j). Since this position is not encircled this indicates there are no zero plots on the right-hand side. Which agrees with the P-Z plots from Table No.5 - 6.

However, using the equation:

$$Z = N + P$$

Equation No.12

As $Z = 0$ and there are no counterclockwise encirclements of the real value: (-1, 0j) therefore P must be 0 meaning there are no Poles on the right-hand side either within the contour. This means that the feedback system is stable as the contour does not pass through (-1,0) and there are no poles on the right-hand s-plan. [3] As also shown in the P-Z plots.

Bode Diagrams

From the bode diagrams these show the gain margin and the phase margin of the transfer function. These are used to represent external disturbances on the transfer function and the required quantity of each to reach system instability.

From the graphs it shows the transfer function can resist any quantity of gain. This is most likely due to an integral part of the controller; and the fact it this part can use integration. This prevents errors of load bearing errors from adding gain or removing overall amplitude. However, while being an infinite amount, this is very unlikely. As with a large amount of gain added to the circuit, this attenuates any noise and could most likely make the system unstable. All this does is confirm that small disturbances of gain will not negatively affect the circuit.

The phase margin has been measured from the graph as 87 degrees at a frequency of 8.66 rads/sec. This shows us that the phase shifts this system can still accurately work up to.

Conclusion

In conclusion the input and output of both control system A and the transfer function have similar input / output responses. This is because the transfer function is a mathematical replication of control system A.

Additionally, the responses of both control systems are within the desired remit of:

- Not overshooting more than 5%
- System reaction and settling within 2 seconds.

This is a result of the PID being tuned to ensure the desired transient response of quick rise time, little to no overshoot and fast settling time.

From the stability testing of the transfer function, it was determined the Nyquist plot and P-Z plots showed signs of a stable system. These were because the P-Z plots were on the left-hand side of the F-plane. While remaining on the real axis. This would result in a constant amplitude and would not lead to the system oscillating or dampening over time.

Additionally, the Nyquist plot was determined to fulfil the Nyquist criterion theory due to no encirclement of point $(-1, j0)$ with a clockwise rotation of the contour. Indicating a stable system.

The system itself also had good gain margin, showing it could withstand large magnitudes of gain. Additionally, for the phase to become unstable an almost 90-degree phase shift would be required. Indicating a good sign of the system resisting external disturbances.

Finally, further research and analysis could be to consider the use of root locus and Routh-Hurwitz for further evidence of stability. However, this requires an understanding of mathematics to be applied and is difficult to graphically represent.

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Annex A – Algebraic Calculations for Forward and Closed Transfer Function

$$G(s) = \frac{1}{as^2 + bs + c}$$

Equation No.13

$$Gc(s) = Kp + Ki \frac{1}{s} + kd \frac{s}{\frac{1}{N}s + 1}$$

Equation No.14

$$(Kp + Ki \frac{1}{s} + kd \frac{s}{\frac{1}{N}s + 1}) (\frac{1}{as^2 + bs + c})$$

Equation No.15

$$(Kp + Ki \frac{1}{s} + kd (\frac{sN}{s+N})) (\frac{1}{as^2 + bs + c})$$

Equation No.16

$$(\frac{s(s+N)Kp + (s+N)Ki + s(KdNs)}{s(s+N)}) (\frac{1}{as^2 + bs + c})$$

Equation No.17

$$(\frac{s^2Kp + sNKp + sKi + NKi + NKds^2}{s^2 + Ns}) (\frac{1}{as^2 + bs + c})$$

Equation No.18

$$(\frac{(Kp + NKd)s^2 + (NKp + Ki)s + NKi}{s^2 + Ns}) (\frac{1}{as^2 + bs + c})$$

Equation No.19

$$(\frac{(Kp + NKd)s^2 + (NKp + Ki)s + NKi}{(s^2 + Ns)(as^2 + bs + c)})$$

Equation No.20

$$\left(\frac{(Kp + NKd)s^2 + (NKp + Ki)s + NKi}{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs} \right)$$

Equation No.21

$$TF = \frac{G}{1 + GH}$$

Equation No. 22

$$\frac{\left(\frac{(Kp + NKd)s^2 + (NKp + Ki)s + NKi}{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs} \right)}{1 + \left(\frac{(Kp + NKd)s^2 + (NKp + Ki)s + NKi}{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs} \right)} (1)$$

Equation No.23

$$\frac{\left(\frac{(Kp + NKd)s^2 + (NKp + Ki)s + NKi}{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs} \right)}{\left(\frac{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs}{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs} \right) + \left(\frac{(Kp + NKd)s^2 + (NKp + Ki)s + NKi}{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs} \right)}$$

Equation No.24

$$\frac{\left(\frac{(Kp + NKd)s^2 + (NKp + Ki)s + NKi}{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs} \right)}{\left(\frac{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs + (Kp + NKd)s^2 + (NKp + Ki)s + NKi}{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs} \right)}$$

Equation No.25

$$TF = \left(\frac{(Kp + NKd)s^2 + (NKp + Ki)s + NKi}{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs} \right) \left(\frac{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs}{as^4 + (aN + b)s^3 + (bN + c)s^2 + cNs + (Kp + NKd)s^2 + (NKp + Ki)s + NKi} \right)$$

Equation No.26

Annex B – Numerical Calculations for Forward and Closed Transfer Functions

$$G(s) = \frac{1}{as^2 + bs + c}$$

Equation No.27

$$K_p + K_i \frac{1}{s} + K_d \frac{s}{\frac{1}{N}s + 1}$$

Equation No.28

$$(K_p + K_i \frac{1}{s} + K_d \frac{s}{\frac{1}{N}s + 1}) \left(\frac{1}{as^2 + bs + c} \right)$$

Equation No.29

$$\left(27.16 + \frac{4.75}{s} + 25.86 \frac{s}{\frac{s}{992.28} + 1} \right) \times \left(\frac{1}{3s^2 + 2s + 1} \right)$$

Equation No.30

$$\left(27.16 + \frac{4.75}{s} + 25.86 \frac{s}{\frac{s + 992.28}{992.28}} \right) \times \left(\frac{1}{3s^2 + 2s + 1} \right)$$

Equation No.31

$$\left(27.16 + \frac{4.75}{s} + 25.86 \left(s \div \frac{s + 992.28}{992.28} \right) \right) \times \left(\frac{1}{3s^2 + 2s + 1} \right)$$

Equation No.32

$$\left(27.16 + \frac{4.75}{s} + 25.86 \left(s \left(\frac{992.28}{s + 992.28} \right) \right) \right) \times \left(\frac{1}{3s^2 + 2s + 1} \right)$$

Equation No.33

$$\left(27.16 + \frac{4.75}{s} + 25.86 \left(\frac{992.28s}{s + 992.28} \right) \right) \times \left(\frac{1}{3s^2 + 2s + 1} \right)$$

Equation No.34

$$\left(27.16 + \frac{4.75}{s} + \left(\frac{25660.36}{s + 992.28} \right) \right) \times \left(\frac{1}{3s^2 + 2s + 1} \right)$$

Equation No.35

$$\left(\frac{((s(s + 992.28)) \times 27.16) + (s + 992.28)4.75 + s(25660.36s)}{s(s + 992.28)} \right) \times \left(\frac{1}{3s^2 + 2s + 1} \right)$$

Equation No.36

$$\left(\frac{27.16s^2 + 26950.32s + 4.75s + 4713.33 + 25660.36s^2}{s^2 + 992.28s} \right) \times \left(\frac{1}{3s^2 + 2s + 1} \right)$$

Equation No.37

$$\left(\frac{25687.52s^2 + 26955.07s + 4713.33}{s^2 + 992.28s} \right) \times \left(\frac{1}{3s^2 + 2s + 1} \right)$$

Equation No.38

$$\frac{25687.52s^2 + 26955.07s + 4713.33}{(s^2 + 992.28s)(3s^2 + 2s + 1)}$$

Equation No.39

$$\frac{25687.52s^2 + 26955.07s + 4713.33}{3s^4 + 2s^3 + s^2 + 2976.84s^3 + 1984.56s^2 + 992.28s}$$

Equation No.40

$$\frac{25687.52s^2 + 26955.07s + 4713.33}{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s}$$

Equation No.41 – Forward Transfer Function

$$F = \frac{G}{1 + GH}$$

Equation No.42

$$TF = \frac{\frac{25687.52s^2 + 26955.07s + 4713.33}{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s}}{1 + \frac{25687.52s^2 + 26955.07s + 4713.33}{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s}} \times 1$$

Equation No.43

$$TF = \frac{\frac{25687.52s^2 + 26955.07s + 4713.33}{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s}}{\frac{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s}{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s} + \frac{25687.52s^2 + 26955.07s + 4713.33}{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s}}$$

Equation No.44

$$TF = \frac{\frac{25687.52s^2 + 26955.07s + 4713.33}{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s}}{\frac{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s + 25687.52s^2 + 26955.07s + 4713.33}{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s}}$$

Equation No.45

$$TF = \frac{\frac{25687.52s^2 + 26955.07s + 4713.33}{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s}}{\frac{3s^4 + 2978.84s^3 + 27673.08s^2 + 27947.35s + 4713.33}{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s}}$$

Equation No.46

$$TF = \frac{25687.52s^2 + 26955.07s + 4713.33}{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s} \times \frac{3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s}{3s^4 + 2978.84s^3 + 27673.08s^2 + 27947.35s + 4713.33}$$

Equation No.47

$$TF = \frac{(25687.52s^2 + 26955.07s + 4713.33)(3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s)}{(3s^4 + 2978.84s^3 + 1985.56s^2 + 992.28s)(3s^4 + 2978.84s^3 + 27673.08s^2 + 27947.35s + 4713.33)}$$

Equation No.48

$$\begin{aligned}
& TF \\
& = \frac{77062.56s^6 + 76519012.08s^5 + 51004112.21s^4 + 25489212.35s^3 + 80865s^2 + 80294840.72s^4 + 53520908.79s^3 + 26746976.86s^2}{9s^8 + 8936.52s^7 + 83019.24s^6 + 83842.05s^5 + 14139.99s^4 + 8936.52s^7 + 8873487.75s^6 + 82433677.63s^5 + 83250684.07s^4 + 14040255.94s^3} \dots \\
& \quad + 14139.99s^4 + 14040255.94s^3 + 9358599.5s^2 + 4676943.09s \\
& 5956.68s^6 + 5914665.55s^5 + 54946560.72s^4 + 55491140.27s^3 + 9358599.52s^2 + 2976.84s^5 + 2955843.36s^4 + 27459443.82s^3 + 27731596.46s^2 + 4676943.09s
\end{aligned}$$

Equation No.49

$$\begin{aligned}
& TF \\
& = \frac{77062.56s^6 + 76599877.08s^5 + 131313092.09s^4 + 93050376.29s^3 + 36105576.36s^2 + 4676943.09s}{9s^8 + 17873.04s^7 + 8962463.67s^6 + 88436162s^5 + 141167228.1s^4 + 96990840.08 + 37090195.98s^2 + 4676943.09s}
\end{aligned}$$

Equation No.50