

Complete, fold and seal
(see note 2 below)

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12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

1	Student Number	2030723
2	Title of Examination Paper	Mathematics Exam 2
3	Module Code	ACDA4008
4	Date and Time of Examination	6/5/22 11:00
5	Examination Venue	FF005
6	Seat Number	N/A
7	Number of Answer Books handed in	

[illegible]

i) $y = 6x - 2$

$\frac{dy}{dx} = 6$

ii) $y = 5x^3 + 17x^{\frac{1}{2}} - \frac{3}{2}$
 $\frac{dy}{dx} = 15x^2 + \frac{17}{2}x^{-\frac{1}{2}}$

iii) $y = \frac{3}{x^4} + 7x^2 \rightarrow 3x^{-4} + 7x^2$
 $\frac{dy}{dx} = -12x^{-5} + 14x \rightarrow 14x - 12x^{-5}$

iv) $y = \sin(6x)$
 $\frac{dy}{dx} = 6\cos(6x)$

v) $y = \cos(x)$
 $\frac{dy}{dx} = -\sin(x)$

vii) $y = 2x^{-3} + 5x^2$
 $\frac{dy}{dx} = -6x^{-4} + 10x \rightarrow 10x - 6x^{-4}$

viii) $y = x + \sin 2x$
 $\frac{dy}{dx} = 1 + 2\cos(2x) \rightarrow 2\cos(2x) + 1$

b) $y = \cos(2x)$
 $\frac{dy}{dx} = -2\sin(2x)$
 $\frac{d^2y}{dx^2} = -2\cos(2x)$

$\frac{d}{dx} \sin$

$\frac{d}{dx} \cos$

$\frac{d}{dx} \sin$

$\frac{dy}{dx} = 2(-\sin(2x))$

$\frac{d^2y}{dx^2} = 4(-\cos(2x))$

$\frac{d^3y}{dx^3} = 8(\sin(2x))$

$\frac{d^4y}{dx^4} = 16(\cos(2x))$

$\frac{d^5y}{dx^5} = 32(-\sin(2x)) \rightarrow -32(\sin(2x))$



1c Quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

$$u = 2x + 1$$

$$\frac{du}{dx} = 2$$

$$v = 3x^2$$

$$\frac{dv}{dx} = 6x$$

$$v^2 = 9x^4$$

$$\frac{3x^2(2) - (2x+1)(6x)}{9x^4} \rightarrow \frac{6x^2 - (12x^2 + 6x)}{9x^4} = \frac{-6x^2 - 6x}{9x^4} = \frac{-2x^2 - 2x}{3x^4}$$

$$-\frac{2}{3}x^{-2} - \frac{2}{3}x^{-3} \rightarrow -\frac{2}{3x^2} - \frac{2}{3x^3} = \frac{dy}{dx}$$

$$\text{If } x = -1, \frac{dy}{dx} = -\frac{2}{3} - \frac{2}{3} = -\frac{2}{3} + \frac{2}{3} = 0$$

$$\text{If } x = -1, \frac{dy}{dx} = 0$$

3a

$$i) \left(\frac{1+i}{1-i}\right)^3 = \left(\frac{1+i}{1-i} \cdot \frac{1+i}{1+i}\right)^3$$

$$\frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1+2i+i^2}{1-i^2} = \frac{1+2i+(-1)}{1-(-1)} = \frac{2i}{2} = i$$

$$(0+i)^3$$

$$(0+i)(0+i) = i+i(0+i) = i^3 = -1i = -i \quad \underline{0-i}$$

$$ii) \frac{3}{1+i} \left(\frac{1-i}{1-i}\right) = \frac{3-3i}{1-i^2} = \frac{3-3i}{2} = \frac{3}{2} - \frac{3}{2}i \rightarrow \underline{1.5 - 1.5i}$$

$$iii) \frac{1}{i^{-17}} = i^{17} \text{ or } i^{15} (i)^2 = i^{16} \quad i^{16} = (i^2)^8 = (-1)^8 = 1$$

$$\frac{1}{i^{-17}} = i^{17} = i \rightarrow \underline{0+1i}$$

$$b) z_1 = \frac{3-2j}{2+j} \quad \text{and} \quad z_2 = \frac{3+2j}{1-2j}$$

$$z_1: \frac{3-2j}{2+j} \times \frac{2-j}{2-j} = \frac{2j^2 - 4j - 3j + 6}{4-j^2} = \frac{2(-1) - 7j + 6}{5} = \frac{4-7j}{5} = \underline{0.8 - 1.4j}$$

$$z_2: \frac{3+2j}{1-2j} \times \frac{1+2j}{1+2j} = \frac{3+6j+2j+4j^2}{1-4j^2} = \frac{3+8j-4}{5} = \frac{8j-1}{5} = \underline{-0.2 + 1.6j}$$

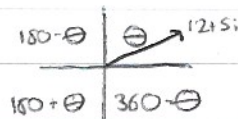
c) Express $(2+3i)(3-2i)$ in polar form.

$$6+9i-4i-6i^2 = 6+5i-6(-1) = 6+5i+6 = 12+5i \text{ (rectangular form)}$$

polar form $r \angle \theta$

$$R = z = \sqrt{x^2 + y^2} = \sqrt{12^2 + 5^2} = 13$$

$$\theta = \arg(z) = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{5}{12}\right) = 22.62^\circ \text{ (2dp)}$$



$$r \angle \theta = \underline{13 \angle 22.62^\circ \text{ (2dp)}} \text{ or } \underline{13(\cos 22.62^\circ + i \sin 22.62^\circ)} \text{ (2dp)}$$

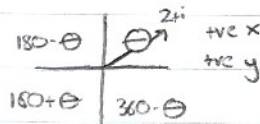
d Quotient of $(1+3i)$ and $(1+i)$ in polar form

$$\frac{1+3i}{1+i} \rightarrow \frac{1+3i}{1+i} \times \frac{1-i}{1-i} \rightarrow \frac{1+3i-i-3i^2}{1-i^2} \rightarrow \frac{1+2i+3}{2} = \frac{4+2i}{2} = 2+i \text{ (rectangular form)}$$

polar form $r \angle \theta$

$$R = z = \sqrt{x^2 + y^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\theta = \arg(z) = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{2}\right) = 26.56^\circ \text{ (2dp)}$$



$$r \angle \theta = \sqrt{5} \angle 26.56^\circ \text{ (2dp)} \text{ or } \sqrt{5} \cos(26.56^\circ) + i \sin(26.56^\circ)$$



4a ~~2.5~~ = 2

$$2^{3x-5} = 4^{5x-6}$$

$$\log_2 2^{3x-5} = \log_2 4^{5x-6} \quad \log_2 (4^{5x-6}) = 3x-5$$

$$(5x-6)\log_2 4 = 3x-5$$

$$(3x-5)\log 2 = (5x-6)\log 4 \quad \log_2 4 = \frac{3x-5}{5x-6}$$

$$(3x)\log 2 - 5\log 2 = 5x\log 4 - 6\log 4 \quad \log_2 4 = 2$$

$$(3x-5)2 = 5x-6$$

$$10x-12 = 5x-6$$

$$7x = 7$$

$$x = 1$$

sub $x=1$ into original expression

$$2^{(3-5)} = 4^{(5-6)}$$

$$\frac{1}{2} = \frac{1}{4}$$

(ii)

$$\log(x^2-4) - \log(x+8) = \log x$$

$$\log\left(\frac{x^2-4}{x+8}\right) = \log x$$

$$\frac{x^2-4}{x+8} = x \rightarrow x^2-4 = x(x+8) \rightarrow x^2-4 = x^2+8x \rightarrow -4 = 8x \rightarrow x = -\frac{1}{2}$$

$$\log\left(\frac{x^2-4}{x+8}\right) = \log x$$

$$\frac{x^2-4}{x+8} = x$$

$$x^2-4 = x(x+8)$$

$$x^2-4 = x^2+8x \rightarrow x^2-x^2-4 = 8x$$

$$-4 = 8x$$

$$x = -\frac{4}{8} = -\frac{1}{2}$$

sub $x = -\frac{1}{2}$ into original expression

$$\log\left(\left(-\frac{1}{2}\right)^2-4\right) - \log\left(-\frac{1}{2}+8\right) =$$

$$\log(2.25)$$

$$x^2-4 \quad x+8 \quad \begin{array}{r} x-8 \\ \sqrt{x^2-0x-4} \\ x^2+8x \\ \hline -8x-4 \\ -8x-64 \\ \hline 0+60 \end{array}$$

b) i) $\log x^2 y$ ~~is~~

$$\therefore \log x^2 + \log y \rightarrow \underline{2\log(x) + \log(y)}$$

q

ii) $\log\left(\frac{x^5 y^3}{z}\right)$

$$\therefore \log(x^5 y^3) - \log z$$

$$\therefore \log x^5 + \log y^3 - \log z$$

$$\therefore \underline{5\log(x) + 3\log(y) - \log(z)}$$



2a

$$i \int (3x^2 + 4x^3) dx = \frac{3x^3}{3} + \frac{4x^4}{4} = \underline{x^3 + x^4 + C}$$

$$ii \int (2x^{3/2} - 3x) dx = \frac{2x^{5/2}}{5/2} - \frac{3x^2}{2} + C$$

$$iii \int \left(\frac{2}{x} + 4\right) dx = \underline{2\ln(x) + 4x + C}$$

$$iv \int (2x^2 + 1)(3x - 4) dx \rightarrow \int (6x^3 + 3x - 8x^2 - 4) dx$$

$$\frac{6x^4}{4} + \frac{3x^2}{2} - \frac{8x^3}{3} - 4x + C \rightarrow \underline{\frac{6x^4}{4} - \frac{8x^3}{3} + \frac{3x^2}{2} - 4x + C}$$

$$v \int_3^6 (3x^2 + x^3) dx = \left. \frac{3x^3}{3} + \frac{x^4}{4} \right|_3^6 \rightarrow x^3 + \frac{1}{4}x^4 \Big|_3^6$$

$$= \frac{(36)^3}{3} \left((6)^3 + \frac{(6)^4}{4} \right) - \left((3)^3 + \frac{(3)^4}{4} \right)$$

$$= \underline{492.75}$$

$$b \quad f(x) = 2\cos x \, dx$$

$$\int 2\cos(x) dx = 2 \int \cos(x) dx$$

$$\therefore \int \cos(x) dx = \sin x + C$$

$$\int 2\cos(x) dx = \underline{2\sin(x) + C}$$

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