

SPC for Short Production Runs

Traditional average and range control charts require long production runs to set up and check control. This is not possible where small batch sizes are being produced to satisfy specific customers' orders or to implement the Just in Time (JIT) philosophy.

The Short Run $\bar{X}$, R chart has been developed to overcome these restrictions. It enables all parts produced to be plotted on one chart, with no requirement for large amounts of data to set it up. This important development is obtained by an elegant mathematical development of the existing control limits.

Consider the RANGE chart, it is in control when a plotted range falls between the upper and lower control limits.

$$\begin{aligned} & LCL_R < R < UCL_R \\ \text{i.e. } & D_3 \bar{R} < R < D_4 \bar{R} \end{aligned}$$

Dividing through by $\bar{R}$

$$\begin{aligned} \frac{D_3 \bar{R}}{\bar{R}} & < \frac{R}{\bar{R}} < \frac{D_4 \bar{R}}{\bar{R}} \\ D_3 & < \frac{R}{\bar{R}} < D_4 \end{aligned}$$

$$\frac{R}{\bar{R}} = \text{plot point}$$

This means that the short run range chart can be set up before the process is even run, and any part number can be controlled using the chart D_3 is the lower control limit, D_4 the upper control limit and 1 is the centre line.

As far as the AVERAGE chart is concerned the concept is similar, but slightly more complex. The chart is in control if: -

$$\begin{aligned} & LCL_{\bar{X}} < \bar{X} < UCL_{\bar{X}} \\ \text{i.e. } & \bar{\bar{X}} - A_2 \bar{R} < \bar{X} < \bar{\bar{X}} + A_2 \bar{R} \end{aligned}$$

by subtracting $\bar{X}$ throughout, this becomes

$$\begin{aligned} \bar{\bar{X}} - A_2 \bar{R} - \bar{X} & < \bar{X} - \bar{X} < \bar{\bar{X}} + A_2 \bar{R} - \bar{X} \\ -A_2 \bar{R} & < \bar{X} - \bar{\bar{X}} < +A_2 \bar{R} \end{aligned}$$

Dividing through by $\bar{R}$

$$-A_2 < \frac{\bar{X} - \bar{\bar{X}}}{\bar{R}} < +A_2$$

The control limits are $+A_2$ and $-A_2$ and its centre line is at zero.

The key constraint is that the sample size must be kept constant from batch to batch and from one part number to another.

The difficulty is in determining target values of $\bar{R}$ and $\bar{\bar{X}}$ for each part number, ideally from past control charts, other historical data, prior experience of similar parts or by calculating back from the specification limits.

Exercise

Two types of Brake linings (Asbestos and Metallic) are run over the same piece of equipment where they are ground to the proper width. The difference in material causes their R-bar values to be quite different.

Target Values

	Lining	$\bar{X}$	$\bar{R}$
	Asbestos (A)	50	20
	Metallic (M)	10	4
Day	Mon	Mon	Mon
Time	8.00	8.30	9.00
Lining	A	A	M
	48	38	6
	55	55	11
	<u>53</u>	<u>27</u>	<u>10</u>
	$\Sigma X = 156$	120	27
	$\bar{X} = 52$	40	9
	Target $\bar{X} = 50$	50	10
	$\bar{X} - \text{Target } \bar{X} = 2$	-10	-1
	Target $\bar{R} = 20$	20	4
	$(\bar{X} - \text{Target } \bar{X}) / \text{Tar } \bar{R} = 0.1$	-0.5	-0.25
	$R = 7$	29	5
	$R / \text{Target } \bar{R} = 0.35$	1.4	1.25

Continue plotting the subgroups from this process on the chart using the data below.

9.30	10.00	10.30	11.00	11.30	12.00	12.30	1.00	1.30
M	M	M	A	A	A	M	M	M
14	8	14	35	35	55	6	9	11
12	6	13	13	57	62	11	9	11
13	10	6	24	67	63	7	21	14

Use a broken line when changing from one part to another.

Exercise 2

Average and range

A manufacturer makes small batches of nails. An important quality characteristic is length. Three types of nails A, B and C are to be manufactured during a 4-hour cycle. You are given the following information:

Nail	Target	Values
Type	X	R
A	60	4
B	120	10
C	90	8

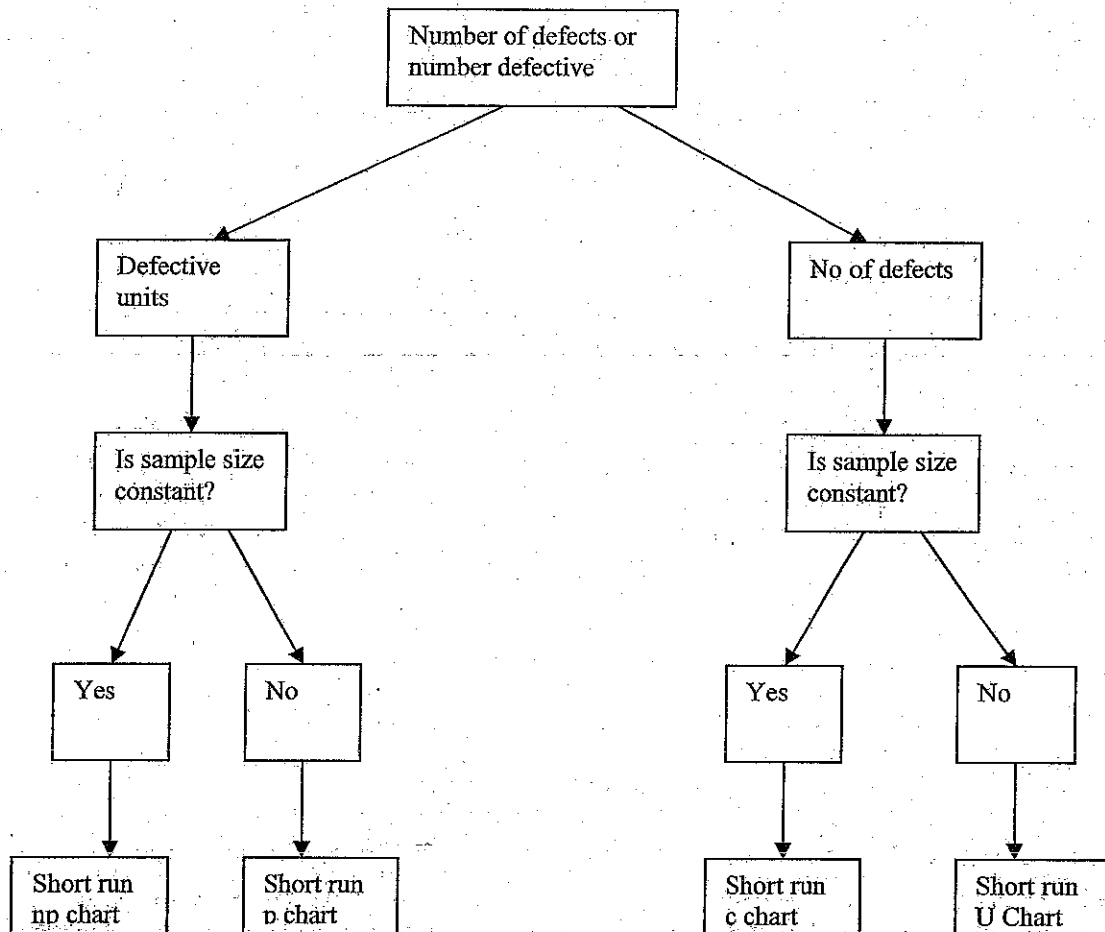
The following sample information has been obtained

Day	Mon							
Time	8.00	8.30	9.00	9.30	10.00	10.30	11.00	11.30
Type	A	A	B	B	B	C	C	C
Item 1	56	58	122	128	114	94	98	102
Item 2	60	60	114	126	126	96	100	106
Item 3	64	56	118	130	130	98	96	98

Determine and plot the appropriate statistics on the enclosed short run control chart. Comment on the state of statistical control of the process.

SPC for Short Run Production Runs Attribute Data

Short run SPC charts have been developed for dealing with attribute type data. The decision chart given below is useful in selecting the appropriate chart to use.



Short Run np chart

This chart is used when batches of different items are manufactured on the same process. Samples of items are selected and inspected periodically to determine if the items are 'good' or 'bad' 'acceptable' or 'unacceptable' (Conforming or non-conforming).

The sample sizes for each item must be constant. However, the sample sizes may be different for different parts e.g. if the sample size for item A is 30, then whenever item A is being manufactured a sample of 30 items is selected. If item A is followed by manufacturing a batch of item B, the sample size selected for B must all be the same, but may be different from the sample sizes for item A.

The plot point formula is obtained in a similar way to those $\bar{X}$, R charts discussed previously.

Derivation of Plot Point Formula

The chart is in control when the plot point falls between the upper and lower statistical control limits.

$$UCL = \bar{np} + 3 \sqrt{\frac{\bar{np}(1 - \bar{np})}{n}}$$

$$LCL = \bar{np} - 3 \sqrt{\frac{\bar{np}(1 - \bar{np})}{n}}$$

Hence plot point np must fall between these limits

$$LCL < np < UCL$$

$$\bar{np} - 3 \sqrt{\frac{\bar{np}(1 - \bar{np})}{n}} < np < \bar{np} + 3 \sqrt{\frac{\bar{np}(1 - \bar{np})}{n}}$$

Subtract $\bar{np}$

$$-3 \sqrt{\frac{\bar{np}(1 - \bar{np})}{n}} < np - \bar{np} < 3 \sqrt{\frac{\bar{np}(1 - \bar{np})}{n}}$$

$$-3 < \frac{np - \bar{np}}{\sqrt{\frac{\bar{np}(1 - \bar{np})}{n}}} < +3$$

Hence Plot Point is

$$\frac{np - \text{Tar } \bar{np}}{\sqrt{\frac{\text{Tar } \bar{np}(1 - \text{Tar } \bar{np})}{n}}}$$

(Where $\text{Tar } \bar{np}$ is the Target $\bar{np}$ value) and control limits are ± 3

These limits represent the $\pm 3\sigma$ limits of the traditional np chart

Therefore $UCL_{np} = +3$ $LCL_{np} = -3$

Different items with different values of $\text{Tar } \bar{np}$ can all be plotted on the same control chart because they have the same dimensionless control limits of ± 3 . Hence these limits can be inserted on the chart before the first plot point is recorded.

Short Run SPC – Example 1: np chart

Four parts ABC and D are produced in small batches on a tapping machine. The parts are subjected to visual examination and classified as acceptable or non-conforming. The sample sizes vary for each part and are as follows: -

Item	Sample size
A	20
B	35
C	25
D	40

The results of 12 hours of production together with the inspection data is as follows.

Date 26/3/04

Time	6	7	8	9	10	11	12	13	14	15	16	17
Part	A	A	A	B	B	B	C	C	C	D	D	D
No of non conformances	7	3	4	2	5	3	4	1	6	1	0	2

The target $\bar{np}$ values for the parts are: -

Item	Target $\bar{np}$
A	4.1
B	2.3
C	3.8
D	1.9

Plot the relevant information on a short run np chart and comment on the state of statistical control of the process.

A trial run was then conducted on new item E. The sample size used was 35 and a target value of 2.1 established. The sample was found to contain 7 non-conforming items. Determine and record the appropriate plot point on the chart. Comment on the capability of the process to manufacture this new item.

Exercise 2

Small castings are visually inspected for flaws. They are categorised as acceptable and non-conforming. Four different castings are made on the same processes in small batches. The results of 12 hours production and inspection are as follows.

27/3/04

Part	A	A	A	B	B	C	C	C	D	D	D	D
Time	6	7	8	9	10	11	12	13	14	15	16	17
Sample size (n)	45	50	52	25	30	53	50	46	35	30	32	40
Non-conformances	3	2	3	0	1	2	1	4	1	2	1	1

The target values for p are as follows

Part	A	B	C	D
Target p	0.025	0.03	0.01	0.02

The plot point

$$\frac{\bar{p} - \text{Tar } \bar{p}}{\sqrt{\text{Tar } \bar{p} (1 - \text{Tar } \bar{p})}} \cdot \frac{1}{n}$$

Complete a short run SPC chart for attributes from this data and comment on the performance of the process.

Short Run Attribute Chart

When checking components for multiple defects, for example printed circuit boards where a number of defects can occur within each board, it is possible to monitor the total performance of an assembly line producing small batches of different boards using one short run C chart.

Sample sizes must be constant for each part number, the centre line is 0 and the control limits are + 3 and - 3.

$$\text{The C plot point} = \frac{C - \text{target } \bar{c}}{\sqrt{\text{Target } \bar{c}}}$$

Different printed circuit boards with different target C bar values can all be plotted on the same chart, because they all have the same control limits after coding. These limits are constant and can be put on the chart before the first point is plotted.

Printed circuit boards are produced on an assembly line in small batches. The sample sizes are constant for each part. The number and type of defect for 8 consecutive batches were as follows:

Part	J	C	E	B	I	C	H	F
Type of Defect								
Wrong Location		1		1	2	1	1	1
Wrong Value	2	2	1	4	1	1	2	1
Missing Part				2			1	1
Masking Error	1	1		1	1		1	1
Solder Error				1		1	1	1

The target C values and sample sizes for the parts are:

Part	Target C	Sample Size
A	2.2	15
B	6.7	10
C	3.2	5
D	1.8	10
E	2.7	1
F	7.1	5
G	1.9	15
H	4.9	1
I	6.1	5
J	4.2	1

Plot the relevant information on a short run C chart and comment on the state of statistical control of the process.

COMMENTS

Characteristic _____

$$D_n = 0 \text{ for } n = 1 \text{ to } 5$$

DATE
TIME
PART #

P	1
I	2
E	3
C	4
E	5

TARGETS

TOTAL Σx

AVERAGE $\bar{x}$

TARGET $\bar{x}$

$$(\bar{x} - \text{TAR } \bar{x})$$

TARGET R

$$(\bar{x} - \text{TAR} \bar{x}) / \text{TAR} \bar{R}$$

RANGE R

R / TARGET $\bar{R}$
$$UCL = +A_2 = \underline{\hspace{10cm}}$$

CENTER LINE = 0

$$\text{PLOT POINT} = \frac{\bar{x} - \text{TAR } \bar{x}}{\text{TAR } \bar{R}}$$
$$LCL = -A_2 = \underline{\hspace{2cm}}$$

0.

1 2 3 4 5 6 7 8 9 10 11 12

4.0

3.5

3.0

2.5

2.0

1.5

1.0

.5

.0

$$UCL = D_4 = \underline{\hspace{2cm}}$$
$$\text{PLOT POINT} = \frac{R}{\text{TAR} \bar{R}}$$

CENTER LINE = 1

LCL = D₃ = _____

COMMENTS

SH

Chart # _____
 Dept # _____
 Mach # _____
 Characteristic _____

$D_3 = 0$ for $n = 1$ to 5				DATE	
n	D_4	A_2	d_2	TIME	PART #
1	3.27	2.66	1.13		
2	3.27	1.88	1.13		
3	2.57	1.02	1.69		
4	2.28	.73	2.06		
5	2.11	.58	2.33		

TARGETS			
Part #	$\bar{X}$	$\bar{R}$	CPK
A			
B			
C			
D			
E			
F			
G			
H			
J			

TOTAL Σx	
AVERAGE $\bar{x}$	
TARGET $\bar{x}$	
$(\bar{x} - \text{TAR } \bar{x})$	
TARGET $\bar{R}$	
$(\bar{x} - \text{TAR } \bar{x}) / \text{TAR } \bar{R}$	
RANGE R	
$R / \text{TARGET } \bar{R}$	

SHORT RUN AVERAGE CHART

UCL = $+ A_2 =$ _____

CENTER LINE = 0

PLOT POINT = $\frac{\bar{x} - \text{TAR } \bar{x}}{\text{TAR } \bar{R}}$

LCL = $- A_2 =$ _____

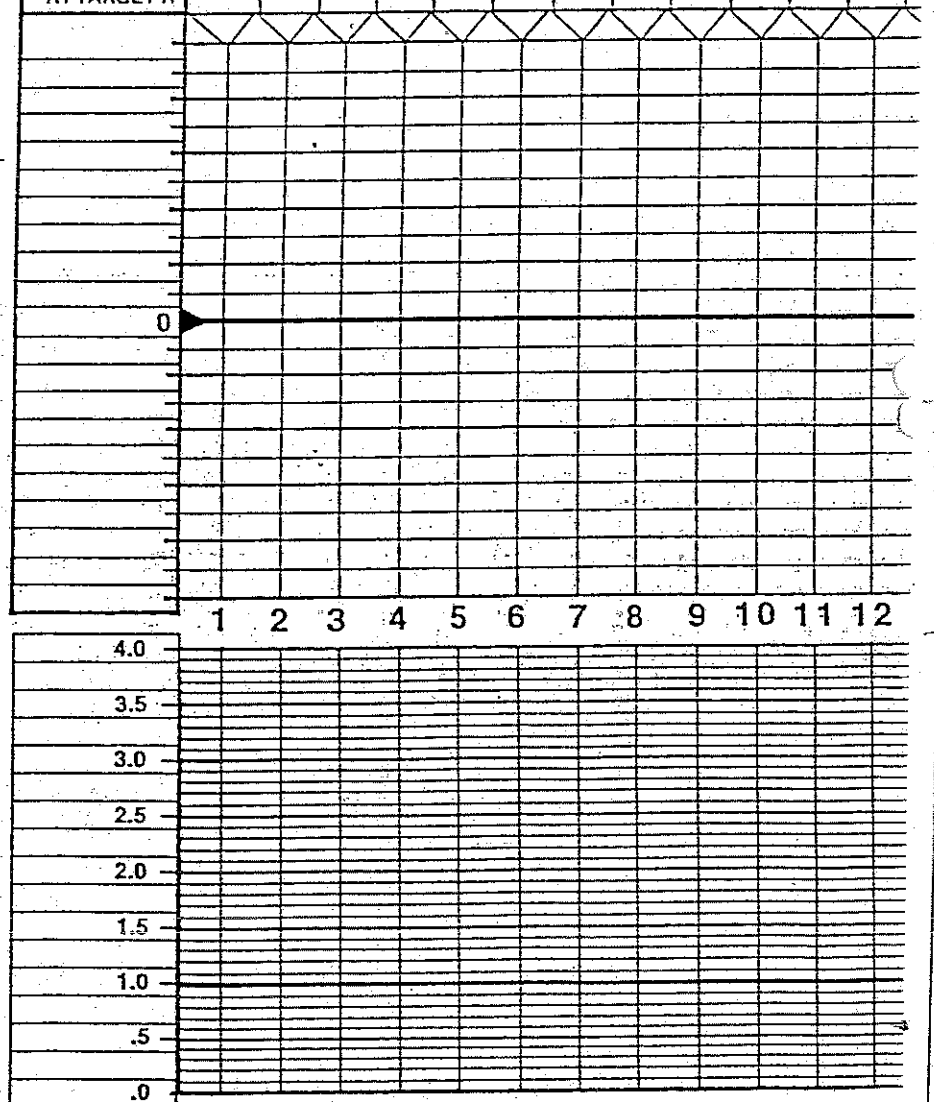
SHORT RUN RANGE CHART

UCL = $D_4 =$ _____

PLOT POINT = $\frac{R}{\text{TAR } \bar{R}}$

CENTER LINE = 1

LCL = $D_3 =$ _____



SHORT RUN ATTRIBUTE CHART

Dept.		Oper. No. & Description				Chart No.	
Average	UCL	LCL	Sample Size	Sample Frequency	Type of Chart		
0	+3	-3					

[illegible][illegible]

NOTES:

**Sample size must
be constant for
c and np charts**

COMMENTS:

SHORT RUN ATTRIBUTE CHART

Dept.	Oper. No. & Description					Chart No.		
Average	0	UCL	+3	LCL	-3	Sample Size	Sample Frequency	Type of Chart

Chart Type	Plot Point
c	$\frac{c - \bar{c}}{\sqrt{\bar{c}}}$
u	$\frac{u - \bar{u}}{\sqrt{\frac{\bar{u}}{n}}}$
np	$\frac{np - n\bar{p}}{\sqrt{n\bar{p}(1-\bar{p})}}$
p	$\frac{p - \bar{p}}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}}$

Part	Avg
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	

TOTAL

[illegible]

NOTES:

Sample size must
be constant for
c and np charts

COMMENTS:

SHORT RUN ATTRIBUTE CHART

Dept.	Oper. No. & Description					Chart No.
Average	0	UCL +3	LCL -3	Sample Size	Sample Frequency	Type of Chart

[illegible][illegible]

NOTES:

**Sample size must
be constant for
c and np charts**

COMMENTS:

SHORT RUN ATTRIBUTE CHART

Dept.	Oper. No. & Description					Chart No.		
Average	0	UCL	+3	LCL	-3	Sample Size	Sample Frequency	Type of Chart

Chart Type	Plot Point
c	$\frac{c - \bar{c}}{\sqrt{\bar{c}}}$
u	$\frac{u - \bar{u}}{\sqrt{\frac{\bar{u}}{n}}}$
np	$\frac{np - n\bar{p}}{\sqrt{n\bar{p}(1-\bar{p})}}$
p	$\frac{p - \bar{p}}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}}$

Part	Avg
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	
TOTAL	

[illegible]

NOTES:

Sample size must
be constant for
c and np charts

COMMENTS:

SHORT RUN ATTRIBUTE CHART

Dept.	Oper. No. & Description					Chart No.
Average	UCL	LCL	Sample Size	Sample Frequency	Type of Chart	

Chart Type	Plot Point
c	$\frac{c - \bar{c}}{\sqrt{\bar{c}}}$
u	$\frac{u - \bar{u}}{\sqrt{\frac{\bar{u}}{n}}}$
np	$\frac{np - n\bar{p}}{\sqrt{n\bar{p}(1-\bar{p})}}$
p	$\frac{p - \bar{p}}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}}$

Part	Avg
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	

The control chart grid features a vertical axis on the left with major tick marks at -4, -3, -2, -1, 0, 1, 2, 3, and 4. A horizontal center line is drawn at the 0 level. On the right side, there are labels for 'UCL' (Upper Control Limit) at the 3 level and 'LCL' (Lower Control Limit) at the -3 level. The horizontal axis is labeled 'TOTAL' at the bottom right. The grid consists of major lines every 1 unit and minor lines every 0.1 units.

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NOTES:

Sample size must be constant for $\bar{c}$ and np charts

COMMENTS: