

University of Wales

Trinity Saint David

School of Engineering

Calculation on Piston, Rod, Crankshaft

Assembles

Xiaotong Jiang

1904747

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Advanced Engine Design(M6X00410)

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1. Introduction

The report mainly put emphasis on three areas of the engine – piston, rod and crankshaft.

Throughout the tasks, fatigue safety factor plays an important role to estimate the durability of the parts of the engine. Also, the skills of calculating the mean stress and ranges of stress to plot the Goodman diagram can be cultivated to calculate and analyze the FSF.

The first part of the task is to analyze the gudgeon pin and rod. During the strokes of the engine movements, bending stress and shear stress appearing on it needed to be calculated for the Goodman diagram. And the concentration factors were applied in three sectional area of the con-rod and also worked out the safety factors.

In addition, for the area of rod-cap and bolt, the stiffness of them, maximum inertia load on the bolt, bolt tightening torque, FSF of the critical parts of rod bolt were calculated. Furthermore, bolt needed to be redesigned to be suitable for 9000rpm.

The final task about fillet sections of the crankshaft may deepen the knowledge of the link area between rod and crankshaft.

2.Engine Data

The standard engine components data, which the report need to base calculations on, are showed on the table 3.1 below.

Table 2.1 Engine components data from standard engine

Type	Producton Engine
Con-rod Mass	0.2674kg
Conrod Cap Mass	0.1038kg
Conrod Bolts Mass	0.03519kg
Conrod Assembly Mass	0.4064kg
Gudgeon Pin Mass	0.08022kg
Piston Mass	0.2577kg
Piston Ring Mass	0.00549kg
Circlip Mass	0.00077kg
Con-rod Length	0.134m
Crank Radius	0.0407m
UTS Con-rod, gudgeon pin	750 N/mm ²
Young's Modulus of rod cap	170000N/mm ²
Young's Modulus of Bolt	210000 N/mm ²

3. Gudgeon Pin

3.1 Mean Stress

From the previous report, the maximum net down force and net tensile force on rod for the standard engine were worked out.

$$\text{highest net compressive force on rod} = 40733.85082N @ 5700rpm$$

$$\text{highest net tensile force on rod} = 6019.789N @ 6000rpm$$

For gudgeon pin, its middle point is subjected to the opposite force, and there are forces exerting by piston pin on both sides of the gudgeon pin. Each side of the pin has the half value of that of force on rod. In addition, the highest net tensile force on rod indicates the compressive force on gudgeon pin from piston pin.

Hence,

$$\text{tensile force on gudgeon pin from piston pin} = 20366.92541N @ 5700rpm$$

$$\text{compressive force on gudgeon pin from piston pin} = 3005.053N @ 6000rpm$$

Also, there is an inertia force of gudgeon pin sticking out appearing to make the pin bend upwards near TDC.

3.1.1 $F_{inertia\ pin}$

$$F_{inertia\ pin} = \text{Mass pin sticking out} * acc$$

$$acc = R\omega^2 \cos \theta + \frac{R}{L}R\omega^2 \cos 2\theta$$

$$R = \text{Crank Radius} = 0.0407\ m, \omega = \frac{2\pi n}{60}, L = \text{Con - rod length} = 0.134m$$

For tensile force on gudgeon pin (highest net down force),

$$n = 5700\ rpm, \theta = 20^\circ$$

$$acc (tensile) = 17000.588 \text{ m/s}^2$$

For compressive force on gudgeon pin (highest net up force),

$$n = 6000 \text{ rpm}, \theta = -358^\circ$$

$$acc (compressive) = 20926.307 \text{ m/s}^2$$

For mass pin, the diameters of gudgeon pin are shown below:

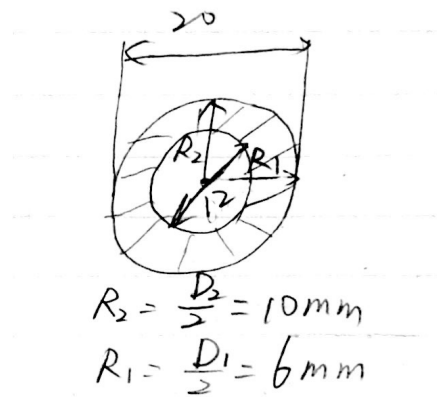


Figure 3.1 Diameters of gudgeon pin

The density of power forged steel is 7600 kg/mm³.

$$\text{Mass pin sticking out} = \text{Length sticking out} * \pi (R_2^2 - R_1^2) * 7600 = 0.3056 \text{ kg}$$

$$F_{\text{inertia pin}} (\text{tensile}) = 519.562 \text{ N}$$

$$F_{\text{inertia pin}} (\text{compressive}) = 639.538 \text{ N}$$

3.1.2 Bending Stress

$$\text{Bending stress at a point} = \frac{M * y}{I}$$

$$I = \text{Second moment of area} = \int_{\text{circle center}}^{\text{surface}} y^2 dA = \frac{\pi(D_2^2 - D_1^2)}{64} = \frac{\pi(20^2 - 12^2)}{64}$$

$$= 6836 \text{ mm}^4$$

1) Tensile bending stress (+VE)

$$\text{Tensile bending moment} = \frac{F_{net}}{2} * L1 - F_{inertia pin} * L2$$

$$= 20366.925 * 15 - 519.562 * 10 = 300308.255 \text{ N} \cdot \text{mm}$$

$$\text{Tensile bending stress} = \sigma_{bending t} = \frac{M * y}{I} = \frac{300308 * 10}{6836} = 439.3 \text{ N/mm}^2$$

2) Compressive bending stress (-VE)

$$\text{Compressive bending moment} = \frac{F_{net}}{2} * L1 - F_{inertia pin} * L2$$

$$= -3005.053 * 15 - 639.538 * 10 = -51471.175 \text{ N} \cdot \text{mm}$$

$$\text{Compressive bending stress} = \sigma_{bending c} = \frac{M * y}{I} = \frac{-51471 * 10}{6836} = -75.3 \text{ N/mm}^2$$

3.1.3 Shear Stress

1) Tensile shear stress (+VE)

$$\text{Net force} = 20366.925 - 519.562 = 19847.36 \text{ N}$$

$$A = \frac{\pi(D2^2 - D1^2)}{4} = 201.06 \text{ mm}^2$$

$$\text{Compressive shear stress} = \tau_t = \frac{19847.36}{201.06} = 98.7 \text{ N/mm}^2$$

2) Compressive shear stress (-VE)

$$\text{Net force} = -3005.053 - 639.538 = -3644.59 \text{ N}$$

$$\text{Tensile shear stress} = \tau_c = \frac{-3644.59}{201.06} = -18.1 \text{ N/mm}^2$$

3.1.4 Combined stress

$$\text{Combined stress} = \sqrt{\sigma_{bending}^2 + (2 * \tau)^2}$$

1) Tensile combined stress

$$\sigma_{max} = \textit{Tensile combined stress} = \sqrt{439.3^2 + (2 * 98.7)^2} = 482 \text{ N/mm}^2$$

2) Compressive combined stress

$$\sigma_{min} = \textit{Compressive combined stress} = -\sqrt{75.3^2 + (2 * 18.1)^2} = -84 \text{ N/mm}^2$$

3.1.5 Mean stress and range of stresses

$$\textit{Mean stress} = \sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} = 199 \text{ N/mm}^2$$

$$\textit{Range of stress} = \sigma_{max} - \sigma_{min} = 566 \text{ N/mm}^2$$

3.2 FSF (Fatigue Safety Factor) by Goodman diagram

The material of the gudgeon pin is power forged steel, and UTS of it is 750 N/mm^2 .

The endurance limit is the max amplitude of stress from a mean stress of zero, which it can go up and down to and last forever.

Considering Steel,

$$\textit{Endurance limit of stress} = 0.45 * \textit{UTS} = 0.45 * 750 = 337.5 \text{ N/mm}^2$$

$$\textit{Allowable range when mean stress is zero} = 2 * 337.5 = 675 \text{ N/mm}^2$$

Goodman graph is shown below,

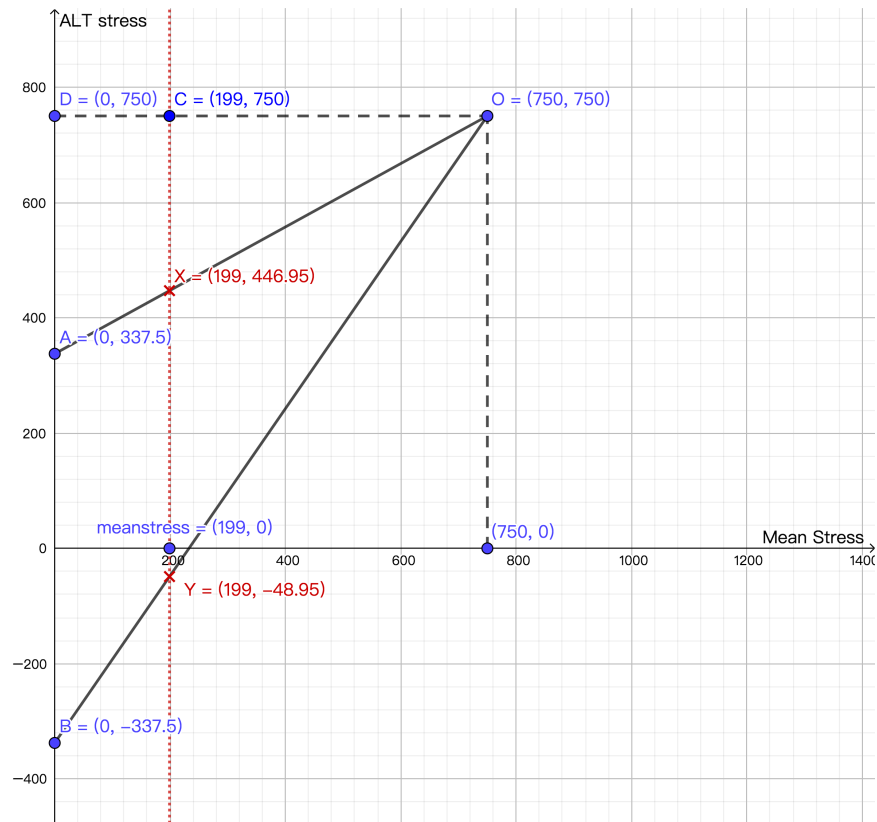


Figure 3.2 Goodman diagram for gudgeon pin

$$\frac{XY}{AB} = \frac{OX}{OA} = \frac{OC}{OD} = \frac{UTS - \text{Mean stress}}{UTS}$$

$$\text{Allowable range at 199 mean stress} = XY = AB * \left(1 - \frac{\text{Mean}}{UTS}\right)$$

$$= 675 * \left(1 - \frac{199}{750}\right) = 495.9 \text{ N/mm}^2$$

$$\text{Actual range} = 566 \text{ N/mm}^2$$

$$FSF = \frac{\text{Allowable range}}{\text{Actual range}} = \frac{495.9}{566} = 0.88 \approx 0.9$$

4. Three critical rod-sections

Three critical rod-sections A, B and C are shown below.

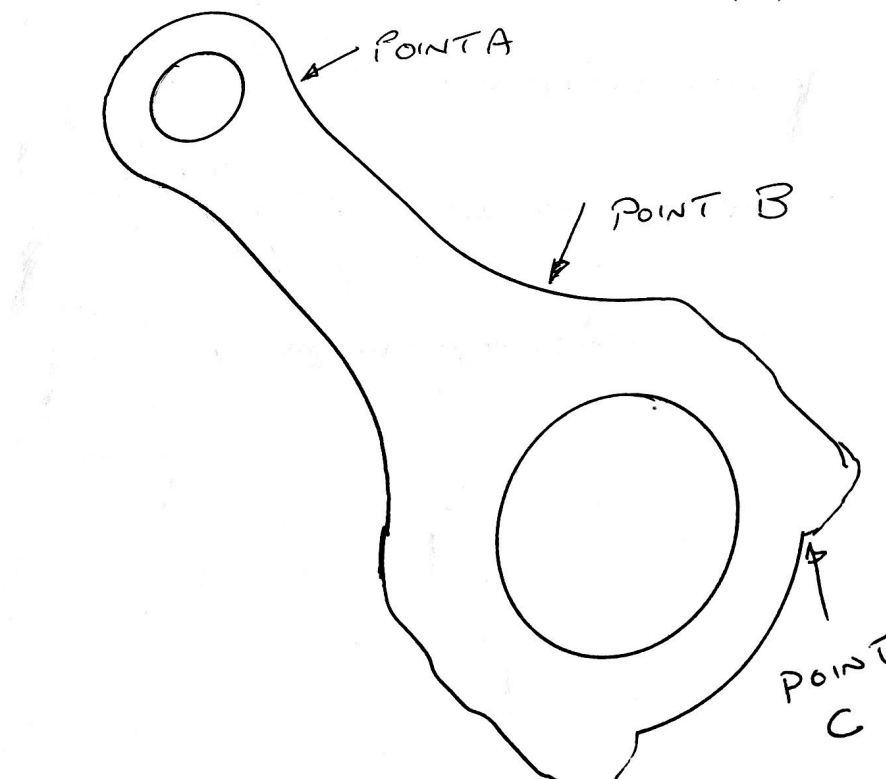


Figure 4.1 Three sectional area on the rod (Mcdonald,2020)

4.1 Point A

4.1.1 Calculation of area



Figure 4.2 Minimum cross sectional area

For the little end of the rod, the minimum cross-sectional area = $13 \times 3 = 39 \text{ mm}^2$.

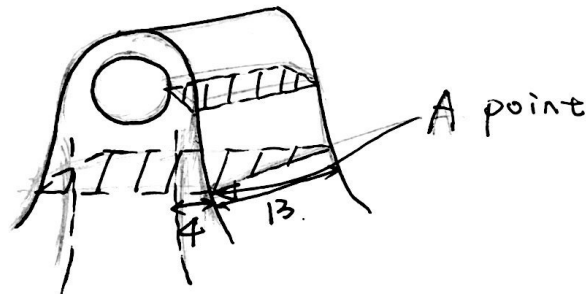


Figure 4.3 Area of point A

Calculating outside area of per side = $4 \times 13 = 52 \text{ mm}^2$.

Total area of point A per side = Middle area per side + Outside area per side

For middle area, 'I' beam area was assumed from previous report and calculating the middle area which is recessed relative to the edge per side:

Middle area per side = 30 mm^2

Hence, the total area of point A = $2 \times \text{per side} = 2 \times (30 + 52) = 164 \text{ mm}^2$

4.1.2 Mean stress

1) Tensile stress

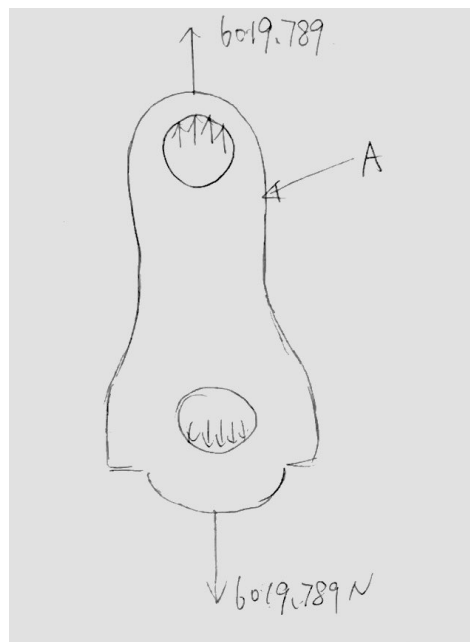


Figure 4.4 Tensile stress

$$\text{Nominal tensile stress at A} = \frac{6019.789}{164} = 36.7 \text{ N/mm}^2$$

2) Compressive stress

$$\text{Similarly, Nominal compressive stress} = -\frac{40733.85082}{164} = -248.4 \text{ N/mm}^2$$

$$\sigma_{\max \text{ nominal}} = 36.7 \text{ N/mm}^2$$

$$\sigma_{\min \text{ nominal}} = -248.4 \text{ N/mm}^2$$

$$\sigma_{\text{mean nominal}} = \frac{\sigma_{\max \text{ nominal}} + \sigma_{\min \text{ nominal}}}{2} = -106 \text{ N/mm}^2$$

4.1.3 Stress concentration factor

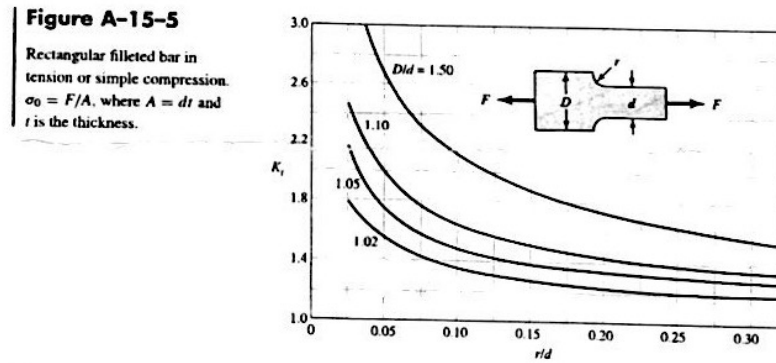


Figure 4.5 Charts of theoretical stress-concentration factors

Considering the stress concentration on point A and from the table 4-1 which shows charts of theoretical stress-concentration factors $-K_t$. This may affect the endurance limit by radius. The author chose $K_t = 1.2$ ($\frac{r}{d} = \frac{12}{24} = 0.5$, $\frac{D}{d} = \frac{33}{24} = 1.4$) as the factor applied in this condition.

4.1.4 Goodman graph

For the con-rod, UTS = 750 N/mm^2

$$\text{Polished endurance limit (no radius or notches)} = 0.45 * 750 = 337.5 \text{ N/mm}^2$$

$$\text{Final endurance limit (with radius)} = \frac{337.5}{1.2} = 281 \text{ N/mm}^2$$

Plotting the Goodman diagram at point A,

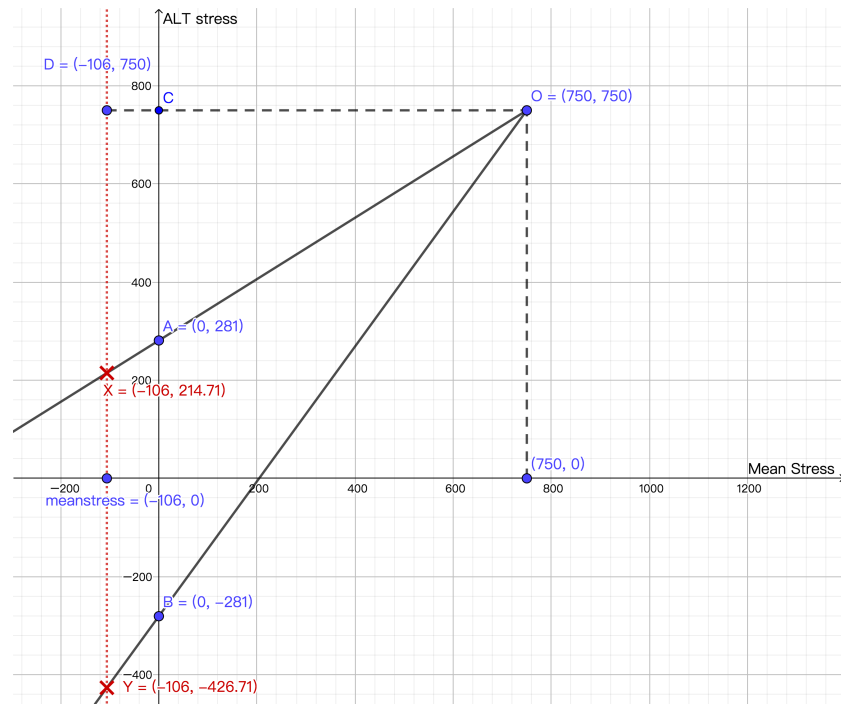


Figure 4.6 Goodman diagram at A

From Goodman diagram,

$$\frac{XY}{AB} = \frac{OX}{OA} = \frac{OD}{OC} = \frac{UTS + \text{Mean stress}}{UTS}$$

$$\text{Allowable range at } -106 \text{ N/mm}^2 \text{ mean stress} = XY = AB * \left(1 + \frac{\text{Mean}}{UTS}\right)$$

$$= 562 * \left(1 + \frac{106}{750}\right) = 641.4 \text{ N/mm}^2$$

$$\sigma_{\text{max nominal}} = 36.7 \text{ N/mm}^2$$

$$\sigma_{\text{min nominal}} = -248.4 \text{ N/mm}^2$$

$$\text{Actual range} = 36.7 + 248.4 = 285.1 \text{ N/mm}^2$$

$$FSF = \frac{\text{Allowable range}}{\text{Actual range}} = \frac{641.4}{285.1} = 2.25 \approx 2.3$$

4.2 Point B

4.2.1 Calculation of area

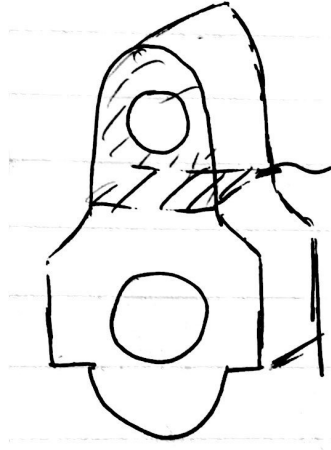


Figure 4.7 Area of B

Calculating outside area of per side = $(39-24)/2 \times 13 = 97.5 \text{ mm}^2$.

Total area of point B per side = Middle area per side + Outside area per side

Hence, the total area of point B $\approx 279 \text{ mm}^2$

4.2.2 Mean stress

1) Tensile stress

At point B, assuming reciprocating mass = Piston pin + 1/3 con-rod

The total tensile force above point B = $6019.789 + 2000 = 8019.789 \text{ N/mm}^2$

$$\text{Nominal tensile stress at point B} = \frac{8019.789}{279} = 28.7 \text{ N/mm}^2$$

2) Compressive stress

The author assumed compressive force was reduced by mostly similar amount considering the inertia force. In fact, maximum net down force appears at 5700rpm which is lower than net up force at 6000rpm, the reduction may be less than that of tensile force.

Maximum compressive force at point B $\approx 40733.85082 - 2000 = 38733.9 \text{ N/mm}^2$

$$\text{Nominal compressive stress} = -\frac{38733.9}{279} = -138.83 \text{ N/mm}^2$$

$$\sigma_{\text{max nominal}} = 28.7 \text{ N/mm}^2$$

$$\sigma_{\text{min nominal}} = -138.83 \text{ N/mm}^2$$

$$\sigma_{\text{mean nominal}} = \frac{\sigma_{\text{max nominal}} + \sigma_{\text{min nominal}}}{2} = -55 \text{ N/mm}^2$$

4.2.3 Stress concentration factor

Figure A-15-5

Rectangular filleted bar in tension or simple compression.
 $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

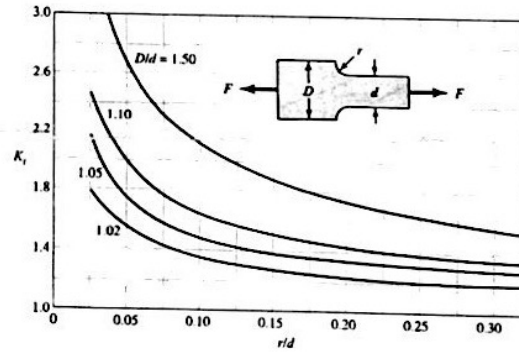


Figure 4.8 stress-concentration factors

Considering the stress concentration on point B and from the table above K_t at point B could be assumed with radius concentration. The author chose $K_t = 1.5$ (r has big radius, $\frac{D}{d} = \frac{70}{24} = 2.9$) as the factor applied in this condition.

4.1.4 Goodman graph

For the con-rod, UTS = 750 N/mm^2

Polished endurance limit(no radius or notches) = $0.45 * 750 = 337.5 \text{ N/mm}^2$

$$\text{Final endurance limit (with radius)} = \frac{337.5}{1.5} = 225 \text{ N/mm}^2$$

Plotting the Goodman diagram at point B,

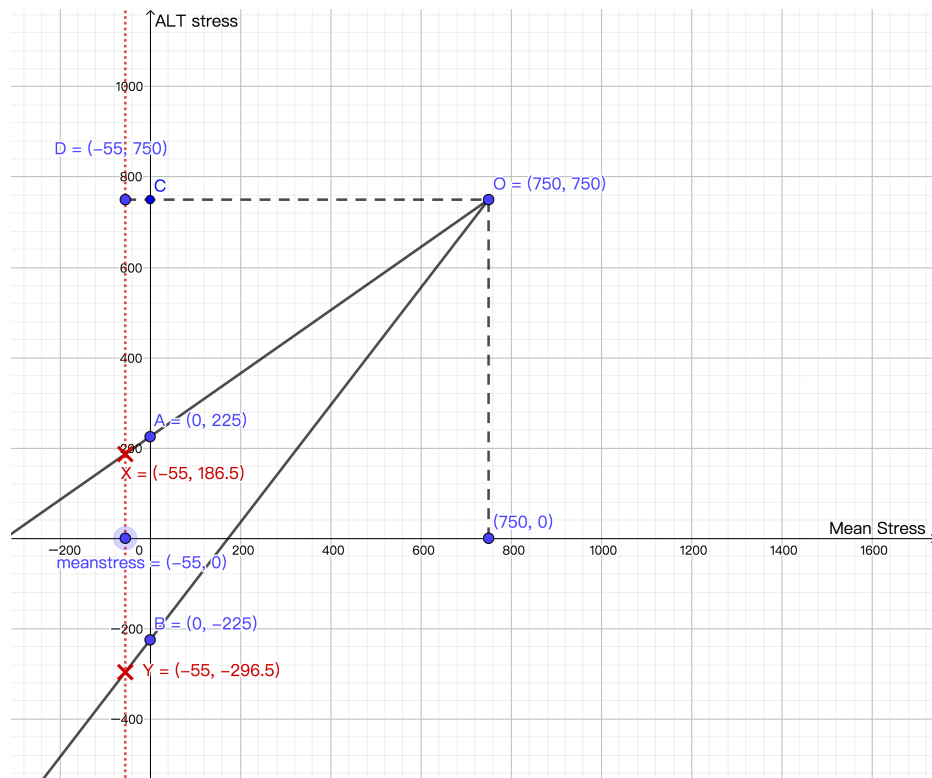


Figure 4.9 Goodman diagram at B

From Goodman diagram,

$$\frac{XY}{AB} = \frac{OX}{OA} = \frac{OD}{OC} = \frac{UTS + \text{Mean stress}}{UTS}$$

$$\text{Allowable range at } -55 \text{ N/mm}^2 \text{ mean stress} = XY = AB * \left(1 + \frac{\text{Mean}}{UTS}\right)$$

$$= 450 * \left(1 + \frac{55}{750}\right) = 483 \text{ N/mm}^2$$

$$\text{Actual range} = 28.7 + 138.83 = 167.5 \text{ N/mm}^2$$

$$FSF = \frac{\text{Allowable range}}{\text{Actual range}} = \frac{483}{167.5} = 2.88 \approx 2.9$$

4.3 Point C

4.3.1 Calculation of area

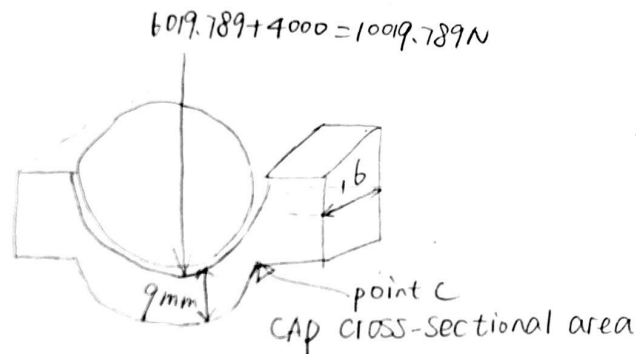


Figure 4.10 Area of C

For point C, the area to calculate the stress is the cap cross-sectional area which shows above.

$$\text{Cap cross - sectional area for point C per side} = 9 * 16 = 144 \text{ mm}^2$$

Hence,

$$\text{the total area of point C} = 2 * \text{Cap cross - sectional area per side} = 288 \text{ mm}^2$$

4.3.2 Mean stress

1) Tensile stress

At point C, more metal could be added and taken into account for assuming the reciprocating mass.

$$\text{The total tensile force above point C} = 6019.789 + 4000 = 10019.789 \text{ N/mm}^2$$

$$\text{Nominal tensile stress at point B} = \frac{10019.789}{288} = 34.8 \text{ N/mm}^2$$

2) Compressive stress

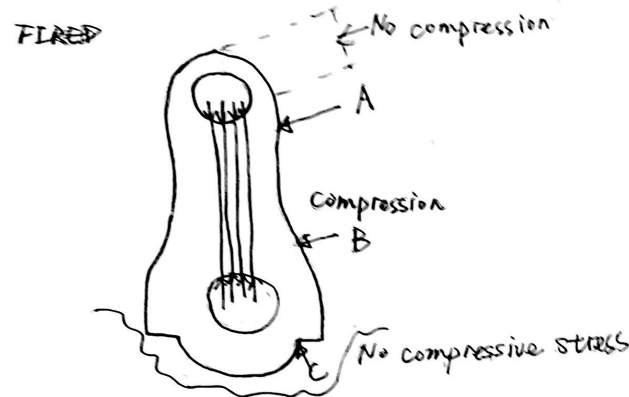


Figure 4.11 Compressive stress

From the graph above, compressive force is only reacted between big & little end rings of con-rod, and the top and bottom of the con-rod do not have compressive stress.

Hence,

$$\text{Nominal compressive stress at point C} = 0 \text{ N/mm}^2$$

$$\sigma_{\text{max nominal}} = 34.8 \text{ N/mm}^2$$

$$\sigma_{\text{min nominal}} = 0 \text{ N/mm}^2$$

$$\sigma_{\text{mean nominal}} = \frac{\sigma_{\text{max nominal}} + \sigma_{\text{min nominal}}}{2} = 17 \text{ N/mm}^2$$

4.3.3 Stress concentration factor

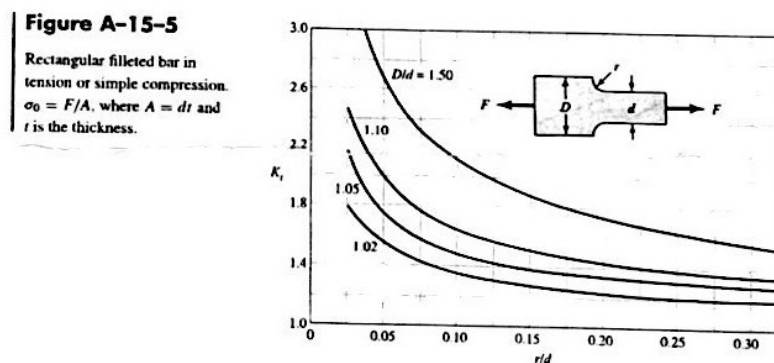


Figure 4.12 stress-concentration factors

Considering the stress concentration on point B and from the table above K_t at point B could be assumed with radius concentration. The author chose $K_t = 3.0$

(r has a small radius, $\frac{D}{d} = \frac{64}{40} = 1.6$) as the factor applied in this condition.

4.1.4 Goodman graph

For the con-rod, UTS = 750 N/mm^2

$$\text{Polished endurance limit (no radius or notches)} = 0.45 * 750 = 337.5 \text{ N/mm}^2$$

$$\text{Final endurance limit (with radius)} = \frac{337.5}{3} = 113 \text{ N/mm}^2$$

Plotting the Goodman diagram at point C,

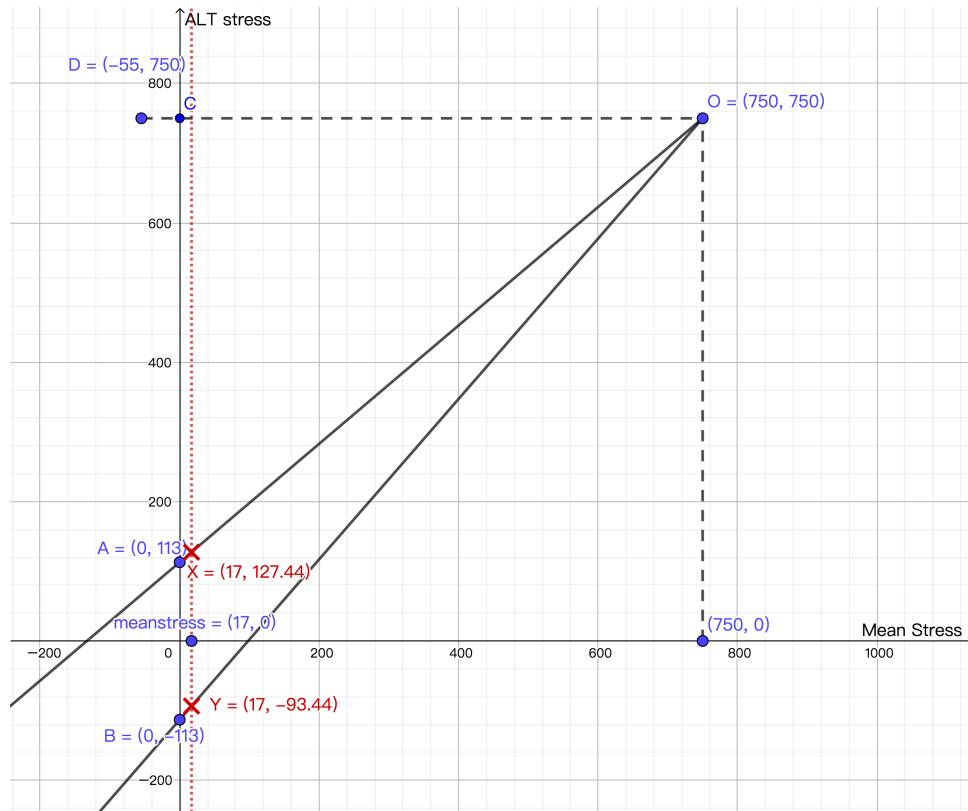


Figure 4.13 Goodman diagram at C

From Goodman diagram,

$$\frac{XY}{AB} = \frac{OX}{OA} = \frac{OD}{OC} = \frac{UTS - \text{Mean stress}}{UTS}$$

$$\text{Allowable range at } 17 \text{ N/mm}^2 \text{ mean stress} = XY = AB * \left(1 - \frac{\text{Mean}}{UTS}\right)$$

$$= 226 * \left(1 - \frac{17}{750}\right) = 221 \text{ N/mm}^2$$

$$\text{Actual range} = 34.8 \text{ N/mm}^2$$

$$FSF = \frac{\text{Allowable range}}{\text{Actual range}} = \frac{221}{34.8} = 6.34 \approx 6.3$$

Position	A	B	C
K_t	1.2	1.5	3.0
FSF	2.3	2.9	6.3

5. Bolt Design

5.1 Ratio of bolt stiffness to cap-rod stiffness

$$K_{bolt} = \frac{E_{bolt} \cdot A_{bolt}}{L}$$

$$K_{Cap-rod} = \frac{E_{Cap-rod} \cdot A_{Cap-rod}}{L}$$

To calculate the stiffness of both bolt and cap-rod, E_{bolt} and $E_{Cap-rod}$ should be determined.

$$E_{bolt} = 210 \times 10^3 \text{ N/mm}^2$$

$$E_{Cap-rod} = 170 \times 10^3 \text{ N/mm}^2$$

Rod cap interface and cross-sectional area of bolt have been measured and calculated as follows.

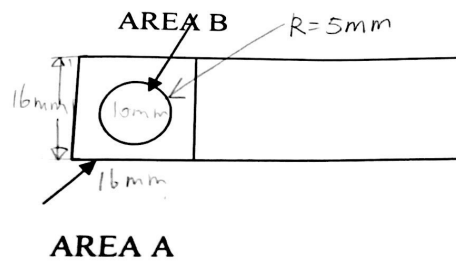


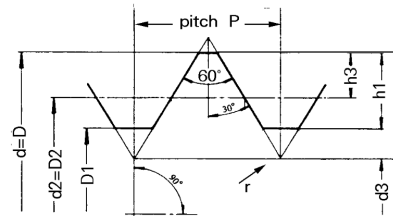
Figure 5.1 Parameters (Mcdonald, 2020)

From the graph above, the outer diameter of the bolt is measured 10mm, and the side length of square in area A is 16mm.

The author searched for the metric fine bolt data for M10 bolt and the relative data shows followed.

THREAD DATA

METRIC THREAD -- FINE PITCH -- M (1 mm - 28 mm)



METRIC THREAD 60° PROFILE

THREAD DATA CHART: Metric Thread -- Fine Pitch										
Nominal Size ISO MF	Thread Form Type	Major Diameter mm d=D	Pitch mm p	Root Radius mm r	Pitch Diameter mm d2=D2	Minor Diameter Male Thd. d3	Minor Diameter Female Thd. D1	Thread Height Male Thd. h3	Thread Height Female Thd. H1	Tap Drill Diameter mm
1.0x0.2	M	1.00	0.20	0.029	0.870	0.755	0.783	0.123	0.108	0.80
1.1x0.2	M	1.10	0.20	0.029	0.970	0.855	0.883	0.123	0.108	0.90
1.2x0.2	M	1.20	0.20	0.029	1.070	0.955	0.983	0.123	0.108	1.00
1.4x0.2	M	1.40	0.20	0.029	1.270	1.155	1.183	0.123	0.108	1.20
1.6x0.2	M	1.60	0.20	0.029	1.470	1.355	1.383	0.123	0.108	1.40
1.8x0.2	M	1.80	0.20	0.029	1.670	1.555	1.583	0.123	0.108	1.60
2x0.25	M	2.00	0.25	0.036	1.838	1.693	1.729	0.153	0.135	1.75
2.2x0.25	M	2.20	0.25	0.036	2.038	1.893	1.929	0.153	0.135	1.95
2.5x0.35	M	2.50	0.35	0.051	2.273	2.071	2.121	0.215	0.189	2.10
3x0.35	M	3.00	0.35	0.051	2.773	2.571	2.621	0.215	0.189	2.60
3.5x0.35	M	3.50	0.35	0.051	3.273	3.071	3.121	0.215	0.189	3.10
4x0.5	M	4.00	0.50	0.072	3.675	3.387	3.459	0.307	0.271	3.50
4.5x0.5	M	4.50	0.50	0.072	4.175	3.887	3.959	0.307	0.271	4.00
5x0.5	M	5.00	0.50	0.072	4.675	4.387	4.459	0.307	0.271	4.50
5.5x0.5	M	5.50	0.50	0.072	5.175	4.887	4.959	0.307	0.271	5.00
6x0.75	M	6.00	0.75	0.108	5.513	5.080	5.188	0.460	0.406	5.20
7x0.75	M	7.00	0.75	0.108	6.513	6.080	6.188	0.460	0.406	6.20
8x0.75	M	8.00	0.75	0.108	7.513	7.080	7.188	0.460	0.406	7.20
8x1.0	M	8.00	1.00	0.144	7.350	6.773	6.917	0.613	0.541	7.00
9x0.75	M	9.00	0.75	0.108	8.513	8.080	8.188	0.460	0.406	8.20
9x1	M	9.00	1.00	0.144	8.350	7.773	7.917	0.613	0.541	8.00
10x0.75	M	10.00	0.75	0.108	9.513	9.080	9.188	0.460	0.406	9.20
10x1	M	10.00	1.00	0.144	9.350	8.773	8.917	0.613	0.541	9.00

Figure 5.3 Metric fine thread data

Nominal size/ISO MF	Major diameter/mm	Pitch/mm	Minor diameter/mm	L/mm
10 x 1	10.00	1	8.773	40

$$\text{Average diameter of 10mm bolt} = \frac{10 + 8.773}{2} = 9.39 \approx 9.4\text{mm}$$

$$A_{\text{bolt}} = \frac{\pi * 9.4^2}{4} = 69.4\text{mm}^2$$

$$A_{\text{cap-rod}} = 16 * 16 - A_{\text{bolt}} = 186.6\text{mm}^2$$

$$\text{The ratio of bolt stiffness to cap - rod stiffness} = \frac{K_{\text{bolt}}}{K_{\text{cap-rod}}} = \frac{E_{\text{Bolt}} * A_{\text{bolt}}}{E_{\text{cap-rod}} * A_{\text{cap-rod}}}$$

$$= \frac{210 \times 10^3 \times 69.4}{170 \times 10^3 \times 186.6} = 0.46$$

5.2 Bolt stiffness when stretch reach to the elastic limit

The stretch region for the standard bolt is the length that is from the head of the bolt to the 2nd engaged thread.

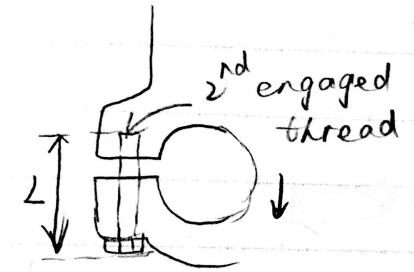


Figure 5.4 Bolt & cap area

$$K_{bolt} = \frac{E_{bolt} \cdot A_{bolt}}{L}, L = 40\text{mm}$$

Hence, the stiffness of the standard bolt = $K_{bolt} = \frac{210 \times 10^3 \times 69.4}{40} = 364350 \text{ N/mm}$

4.3 Maximum loads on the bolt

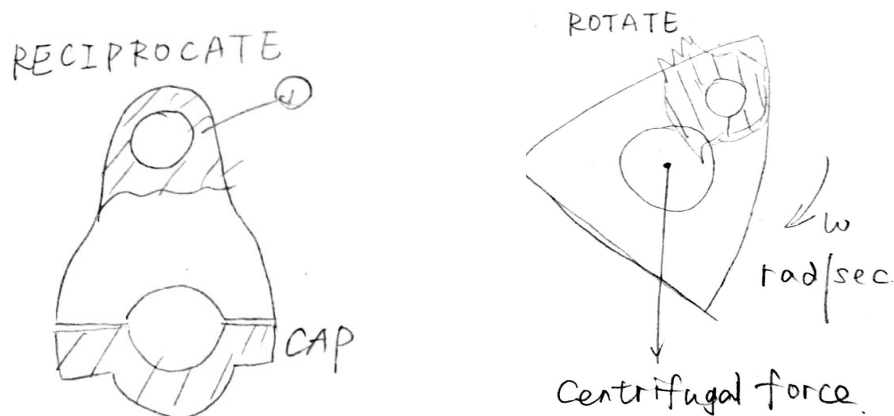


Figure 5.5 Reciprocating and rotating mass and forces

From the graph, to calculate the maximum force on bolts, both reciprocating and rotating inertia force need to be taken into account.

$$\text{Total reciprocating inertia} = M_{rec}(R\omega^2 \cos \theta + \frac{R}{L} * R\omega^2 \cos 2\theta)$$

$$\text{Total rotating inertia} = M_{rot}R\omega^2$$

$$\text{Total upward inertia force on bolt} = M_{rec}\left(R\omega^2 \cos \theta + \frac{R}{L} * R\omega^2 \cos 2\theta\right) + M_{rot}R\omega^2$$

The equations show the alternating load on the bolt, and the highest load on the bolt should happen at TDC ($\theta = 0$ and both reciprocating and rotating are at vertical).

$$M_{rec} = \text{Total reciprocating mass} = \text{Piston} + \text{gudgeon pin} + \frac{1}{3}\text{Rod} \text{ (①)}$$

$$= 0.2702 + 0.08022 + \frac{1}{3} \times 0.4064 = 0.4859 \text{ kg}$$

For total rotating mass, because of the crankpin rod cap cannot pull up and rotate.

$$M_{rot} = \text{Total rotating mass} = \frac{2}{3}\text{Rod} - \text{Cap} = \frac{2}{3} \times 0.4064 - 0.1038 = 0.1671 \text{ kg}$$

Hence,

$$\text{Maximum load on bolt} = M_{rec} \left(R\omega^2 + \frac{R}{L} * R\omega^2 \right) + M_{rot} R\omega^2$$

$$\omega = \frac{2\pi n}{60}, R = 0.0407 \text{ m}, n = 6000 \text{ rpm}$$

$$\text{Maximum load on bolt} = 12864 \text{ N}$$

4.4 Bolt tightening torque

When inertia force acts on the bolt and rod cap, bolt is stretched and cap is compressed. When the bolt is tightened to 90% yield, it may have axial stress and torsional stress. For a typical thread, if it is tightened to 90% yield, axial stress may have 60% yield and the remaining is torsional stress.

1) Clamping force

The M10 x 1 bolt is made of forged steel and its UTS is 1100 N/mm^2 . For most of the engine bolts, they have high tensile and are not plated.

$$\text{yield} = 0.9 * \text{UTS} = 0.9 * 1100 = 990 \text{ N/mm}^2$$

$$\text{Axial stress} = 60\% * \text{yield} = 0.6 * 990 = 594 \text{ N/mm}^2$$

Then the clamping force at vertical can be calculated as follows:

*Clamping force = Average cross – sectional area of bolt * Axial stress*

$$= \frac{\pi * 9.4^2}{4} * 594 = 41222 \text{ N/mm}^2$$

2) Torque to overcome frictions

The tightening torque should be calculated considering it need to overcome 2 frictions (thread friction and head friction).

For the friction force of thread, it is generated by the clamping force and opposite with tightened force.

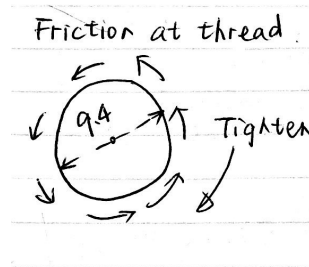


Figure 5.6 Friction at thread

*The circumferential force of thread friction = μ_1 * Clamping force*

$$= 0.18 * 41222 = 7420 \text{ N}$$

(μ = coefecient used in the thread friction ≈ 0.18)

Then the tightening torque at threads can be calculated.

$$\text{Moment at threads} = \text{Average } R * \text{circumferential force} = \frac{9.4}{2} * 7420 = 34.9 \text{ N} \cdot \text{m}$$

For the friction beneath bolt head, the friction happens on the seating face of the bolt when tightened.

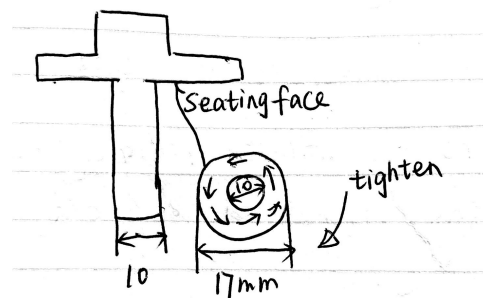


Figure 5.7 Friction at seating face

Also, *The circumferential force = μ_2 * Clamping force = $0.12 * 41222 = 4947 \text{ N}$*

(μ_2 = coefecient of bolt head friction ≈ 0.12)

*Moment at threads = Average $R_{seating\ face}$ * circumferential force*

$$= \frac{10 + 17}{2 * 2} * 4947 = 33.4 N \cdot m$$

Hence,

Total bolt(M10) tightening torque = 34.9 + 33.4 = 68.3 N · m

5.5 Critical parts of bolt and Goodman diagram

From previous calculation, the inertia force on cap at TDC is 12864N and the bolt for each side just has 6432N.

$\delta_x = \text{stretched length of bolt or compressive length of cap}$

$$F_{\text{bolt static}} + K_{\text{bolt}} * \delta_x = 6432 + F_{\text{cap static}} - K_{\text{cap rod}} * \delta_x$$

$$\delta_x = \frac{6432}{K_{\text{bolt}} + K_{\text{cap rod}}}$$

$$\text{Extra force on bolt} = K_{\text{bolt}} * \delta_x = \frac{2504 * K_{\text{bolt}}}{K_{\text{bolt}} + K_{\text{cap rod}}} = \frac{6432}{1 + \frac{K_{\text{cap rod}}}{K_{\text{bolt}}}} = \frac{6432}{1 + 2} = 2144N$$

$$\left(\frac{K_{\text{cap rod}}}{K_{\text{bolt}}} = 2.17 \approx 2 \right)$$

1) Bolt thread

$$\text{Extra nominal stress at root of thread} = \frac{\text{Extra force}}{\frac{\pi * 9.4^2}{4}} = 31N/mm^2$$

Hence, the **range of stress on bolt thread = 31 N/mm²**.

$\sigma_{\min}$ of the bolt = highest bolt stress when tightened to 90% = 90% of yield

$$= 0.9 * 990 = 891N/mm^2$$

$$\sigma_{\max} \text{ of the bolt} = \sigma_{\min} \text{ of the bolt} + \text{range} = 891 + 31 = 922N/mm^2$$

$$\sigma_{\text{mean}} = \frac{\sigma_{\max} + \sigma_{\min}}{2} = 907N/mm^2$$

For the stress concentration,

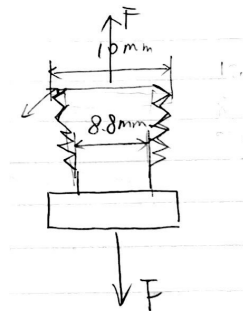
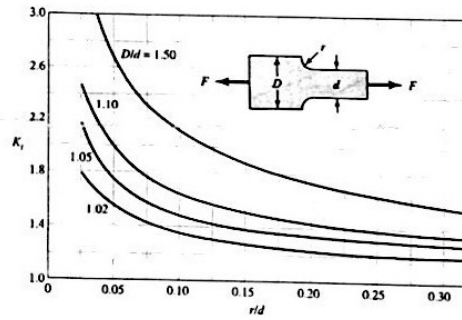


Figure 5.8 Graph of bolt

Figure A-15-5

Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.



The author chose $K_T = 2.4$ at root of the thread ($\frac{D}{d} = 1.1, r$ has a small radius).

$$\text{Polished endurance limit} = 0.45 * UTS = 0.45 * 1100 = 495 \text{ N/mm}^2$$

$$\text{Final endurance limit} = \frac{495}{2.4} = 206 \text{ N/mm}^2$$

Plotting the Goodman diagram of bolt at thread root,

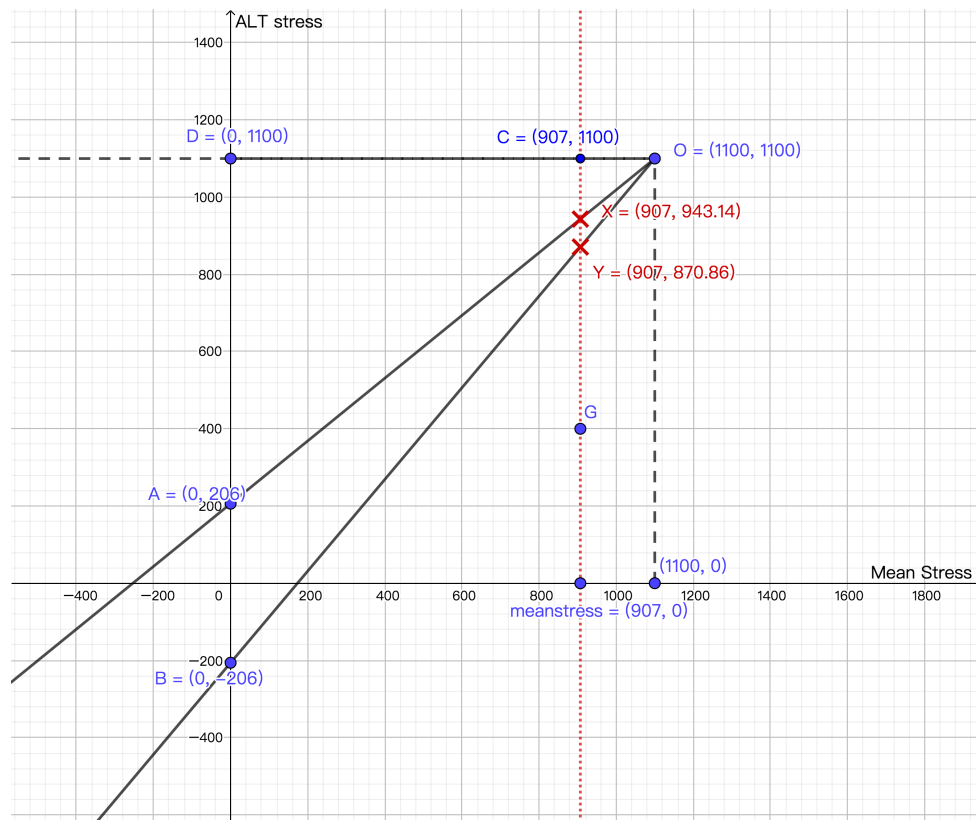


Figure 5.9 Goodman diagram at thread

From Goodman diagram,

$$\frac{XY}{AB} = \frac{OX}{OA} = \frac{OC}{OD} = \frac{UTS - \text{Mean stress}}{UTS}$$

$$\text{Allowable range at } 907 \text{ N/mm}^2 \text{ mean stress} = XY = AB * \left(1 - \frac{\text{Mean}}{UTS}\right)$$

$$= 412 * \left(1 - \frac{907}{1100}\right) = 72.3 \text{ N/mm}^2$$

$$\sigma_{max} = 922 \text{ N/mm}^2$$

$$\sigma_{min} = 891 \text{ N/mm}^2$$

$$\text{Actual range} = 31 \text{ N/mm}^2$$

$$FSF = \frac{\text{Allowable range}}{\text{Actual range}} = \frac{72.3}{31} = 2.33 \approx 2.3$$

2) Bolt head (seating face)

$$\text{Extra nominal stress at head} = \frac{\text{Extra force}}{\text{Seating face area}} = \frac{2144}{\frac{\pi(17^2 - 10^2)}{4}} = 12 \text{ N/mm}^2$$

Hence, the **range of stress on bolt head** = 14 N/mm^2 .

$$\sigma_{min} \text{ of the bolt} = 90\% \text{ of yield} = 0.9 * 990 = 891 \text{ N/mm}^2$$

$$\sigma_{max} \text{ of the bolt} = \sigma_{min} \text{ of the bolt} + \text{range} = 891 + 12 = 903 \text{ N/mm}^2$$

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} = 897 \text{ N/mm}^2$$

For the stress concentration,

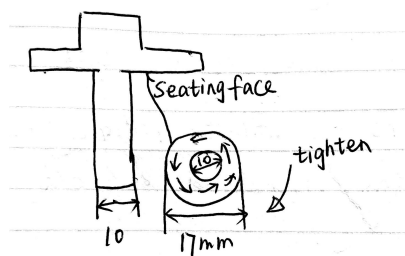


Figure A-15-5

Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

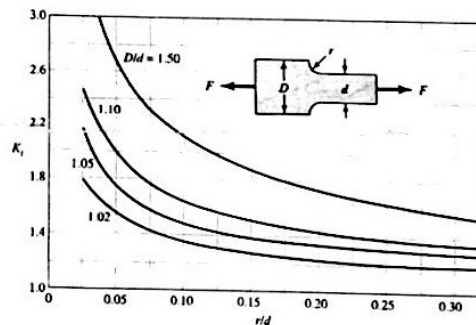


Figure 5.10 Stress concentration at seating face

The author chose $K_T = 2.6$ at root of the thread ($\frac{D}{d} = 1.7$).

$$\text{Polished endurance limit} = 0.45 * UTS = 0.45 * 1100 = 495 \text{ N/mm}^2$$

$$\text{Final endurance limit} = \frac{495}{2.6} = 190 \text{ N/mm}^2$$

Plotting the Goodman diagram of bolt at head,

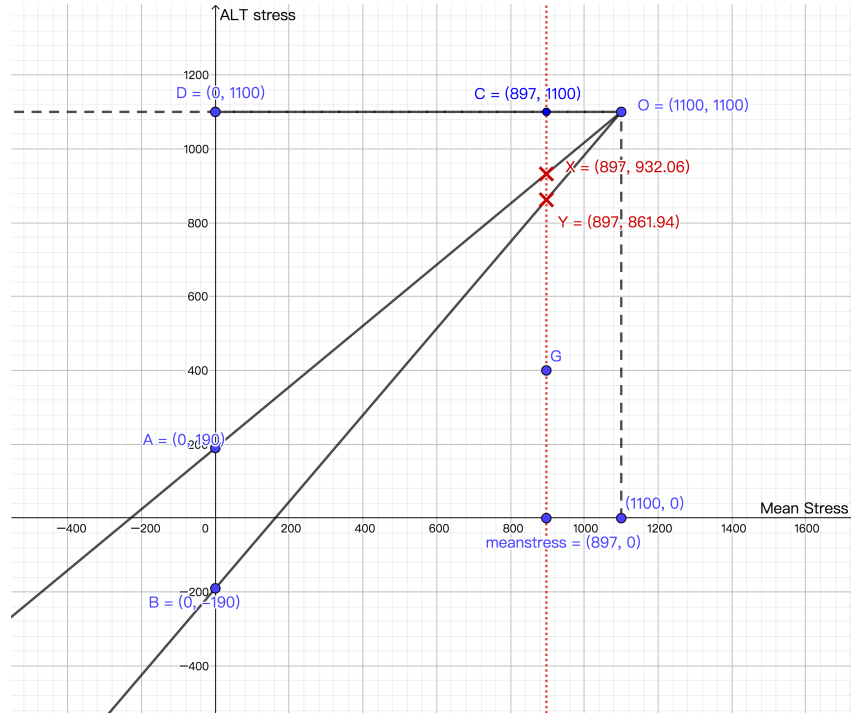


Figure 5.11 Goodman diagram at head

From Goodman diagram,

$$\text{Allowable range at } 897 \text{ N/mm}^2 \text{ mean stress} = XY = AB * \left(1 - \frac{\text{Mean}}{UTS}\right)$$

$$= 380 * \left(1 - \frac{897}{1100}\right) = 70.1 \text{ N/mm}^2$$

$$\sigma_{\max} = 903 \text{ N/mm}^2$$

$$\sigma_{\min} = 891 \text{ N/mm}^2$$

$$\text{Actual range} = 12 \text{ N/mm}^2$$

$$FSF = \frac{\text{Allowable range}}{\text{Actual range}} = \frac{70.1}{12} = 5.84 \approx 5.8$$

Position	Thread Root	Head (seating area)
K_t	2.4	2.6
FSF	2.3	5.8

5.6 Redesign of bolt

The author modified engine to 9000rpm and the inertia load reacted on the bolt changed.

$$\omega_2 = \frac{2\pi * 9000}{60} = 942.5 \text{ rad/sec}$$

$$\text{Maximum load on uprated bolt} = M_{rec} \left(R\omega^2 + \frac{R}{L} * R\omega^2 \right) + M_{rot} R\omega^2 = 28944N$$

The author chose to use the same rod, size of bolts thread could not be changed.

For the previous bolt,

$$\delta_x = \frac{14472}{K_{bolt} + K_{cap rod}}$$

$$\text{Extra force on bolt} = K_{bolt} * \delta_x = \frac{14472}{1 + 2} = 4824N$$

Considering FSF of stress at bolt thread is smaller than that at bolt head,

$$\text{Extra nominal stress at root of thread} = \text{range} = \frac{\text{Extra force}}{\frac{\pi * 9.4^2}{4}} = 70N/mm^2$$

$$\sigma_{min} = 90\% \text{ of yield} = 0.9 * 990 = 891N/mm^2$$

$$\sigma_{max} = \sigma_{min} + \text{range} = 891 + 70 = 961N/mm^2$$

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} = 926N/mm^2$$

$$\text{Allowable range} = 2 * EL * \left(1 - \frac{\text{Mean}}{UTS} \right)$$

$$= 412 * \left(1 - \frac{926}{1100} \right) = 65.2 N/mm^2$$

$$FSF = \frac{\text{Allowable range}}{\text{Actual range}} = \frac{65.2}{70} = 0.93 \approx 0.9 < 1.0$$

Hence, when engine is at 9000 rpm, FSF < 1. The bolt needs to be redesigned to keep the acceptable FSF to improve its durability or realize infinite loop.

In order to increase the FSF, actual range needs to be controlled smaller.

$$Actual\ range = \frac{Extra\ force}{\frac{\pi * 9.4^2}{4}} = \frac{\frac{14472 * K_{bolt}}{K_{bolt} + K_{cap\ rod}}}{\frac{\pi * 9.4^2}{4}}$$

To reduce actual range is to reduce extra force.

$$Extra\ force = \frac{14472}{1 + \frac{K_{cap\ rod}}{K_{bolt}}}$$

$K_{cap\ rod}$ will not change.

Hence, reducing the K_{bolt} is the efficient way to reduce stretch to improve FSF.

The author changed standard bolt into super bolt which might increase the UTS of bolt ($1400\ N/mm^2$).

The high-performance engines must make the con-rod crankshaft big end tunnels resized to ensure that the size con-rod bearing tunnel is still round (Hammill, 2007). Hence, when redesigning the bolt, if the clamping force changes, the big-end tunnel should be resized.

In order to keep the same minimum stress of $891\ N/mm^2$, less than 90 percent yield should be applied to the new bolt.

$$x\% \text{ yield of new bolt} = x\% * 0.9 * 1400 = 891\ N/mm^2$$

$$x\% = 71\%$$

$$Allowable\ range\ uprated = 2 * EL * \left(1 - \frac{Mean}{UTS}\right)$$

$$EL = \frac{1400 * 0.45}{2.4} = 262.5\ N/mm^2$$

Super bolt has three different sections stiffness, and the total stiffness which is lower than the weakest spring can be calculated as followed.

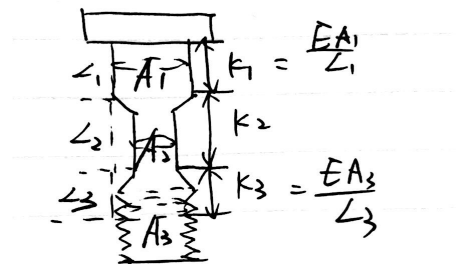


Figure 5.12 Super bolt

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$$

The author designed the super bolt as the graph bellow.

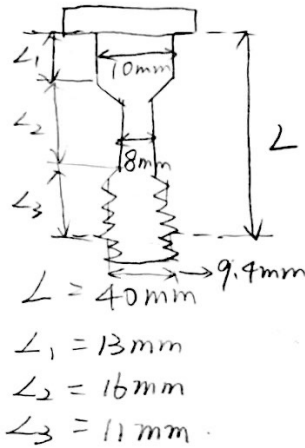


Figure 5.13 parameters of redesigned bolt

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} = 326930 \text{ N/mm}$$

$$\frac{K_{cap \text{ rod}}}{K_{bolt}} = 2.46 \approx 2.5$$

$$Actual \text{ range} = \frac{Extra \text{ force}}{\frac{\pi * 9.4^2}{4}} = \frac{\frac{14472 * K_{bolt}}{K_{bolt} + K_{cap \text{ rod}}}}{\frac{\pi * 9.4^2}{4}} = 59.58 = 60 \text{ N/mm}^2$$

$$Mean = 891 + 30 = 921 \text{ N/mm}^2$$

$$Allowable \text{ range uprated} = 2 * 262.5 * \left(1 - \frac{Mean}{UTS}\right) = 178 \text{ N/mm}^2$$

$$FSF = \frac{Allowable \text{ range}}{Actual \text{ range}} = \frac{178}{60} \approx 2.9$$

Hence,

$$FSF \text{ standard} = 0.9 (@9000 \text{ rpm})$$

$$FSF \text{ uprated} = 2.8 (@9000 \text{ rpm})$$

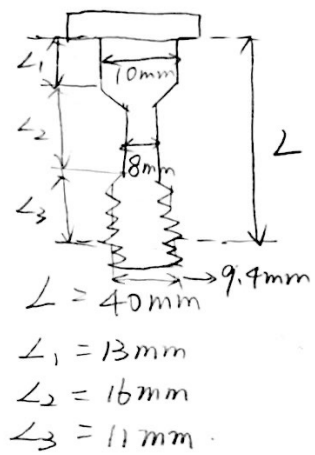
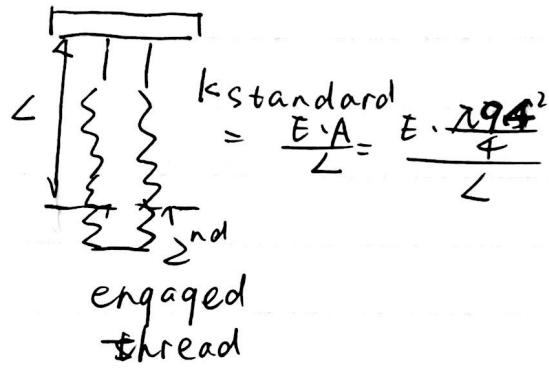


Figure 5.14 The standard and uprated bolt design

6. Fillet sections of crankshaft

The graph below shows the relative parameters of the main bearing cap.

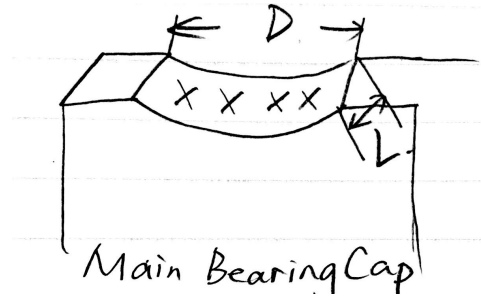


Figure 6.1 Main bearing cap

$$D \approx 45\text{mm}, L \approx 20\text{mm}$$

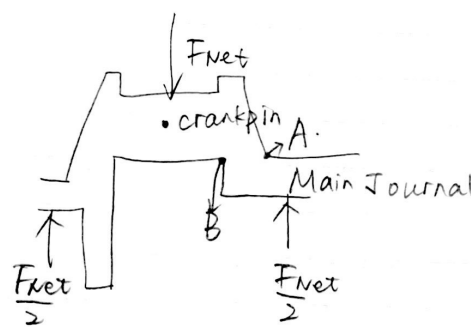


Figure 6.2 Forces on crankpin

As figure 6-2 shows, during the period of firing, forces down of con-rod are reacted on the crankshaft and bending stress was generated. Section A and B are fillets on the crankshaft which are weakest area even though stress concentrations are applied to them (Mcdonald, 2020).

For the firing process, $F_{net} = F_{gas} - F_{inertia}$. In order to calculate σ_{max} , the net down force = 40010.89N (@6000 rpm).

For σ_{min} , it happens when engine pushes down at the center of main journal.

$$F_{net \text{ on crankpin}} = F_{inertia} - F_{gas} \text{ (up is + ve)}$$

$$F_{inertia} = M_{rec} \left(R\omega^2 \cos \theta + \frac{R}{L} * R\omega^2 \cos 2\theta \right) + M_{rot} R\omega^2$$

$$M_{rec} = \text{Piston} + \text{gudgeon pin} + \frac{1}{3} \text{Rod} = 0.4859 \text{kg}$$

$$M_{rot} = \frac{2}{3} \text{Rod} = \frac{2}{3} \times 0.4064 = 0.2709 \text{kg}$$

If assuming the net force is at TDC, $F_{net} = 13200.6 \text{N} (@ \text{TDC} @ 6000 \text{rpm})$

$$\text{Maximum } F_{net} = 13207.3 \text{N} (@ - 358^\circ @ 6000 \text{rpm})$$

5.1 Bending stress

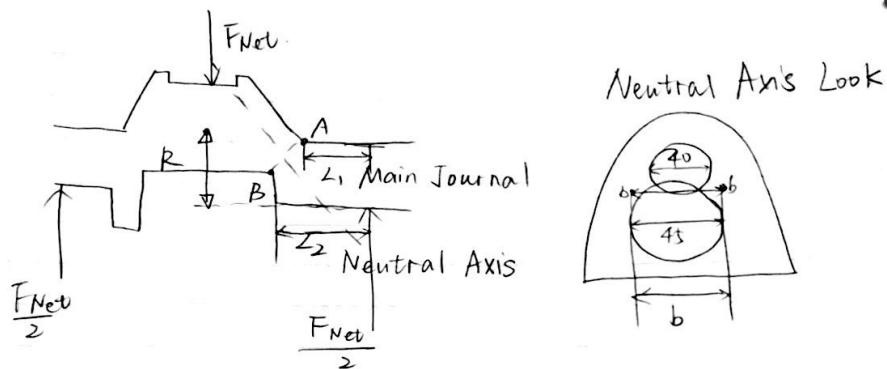


Figure 6.3 Forces on crankpin and diameters

From the graph, when looking along the neutral axis, the effective width of critical beam could

be calculated as $\frac{40+45}{2} = 42.5 \text{mm}$.

$$L1 \approx 12 \text{mm}$$

$$L2 \approx 40 \text{mm}$$

For the effective beam,

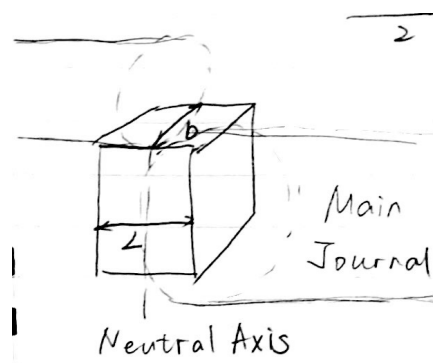


Figure 6.4 effective beam

$$I = \text{second moment of area} = \frac{bL^3}{12} = \frac{42.5 * 20^3}{12} = 28333 \text{ mm}^4$$

For the bending moment, there is one force up (20005.45 N), and A goes into compression while B is at tensile condition.

$$\text{Bending moment at A} = \frac{F_{net}(\text{net down})}{2} * l_1 = \frac{40010.89}{2} * 12 = 240060 \text{ N} \cdot \text{mm}$$

$$\text{Bending moment at B} = \frac{F_{net}(\text{net down})}{2} * l_2 = 20005.45 * 40 = 800218 \text{ N} \cdot \text{mm}$$

For the bending stress,

$$\sigma = \frac{M * y}{I}$$

The value of y is calculated relative to the neutral axis.

$$y = \frac{L}{2} = 10 \text{ mm}$$

$$\sigma_{A \text{ bending}} = \frac{M_A * y}{I} = \frac{240060 * 10}{28333} = 85 \text{ N/mm}^2$$

$$\sigma_{B \text{ bending}} = \frac{M_B * y}{I} = \frac{800218 * 10}{28333} = 282 \text{ N/mm}^2$$

5.2 Shear stress

$$\text{Shear force} = 20005.45 \text{ N}$$

$$\sigma_{A \text{ shear}} = \frac{\text{Shear force}}{\text{Area of A}} = \frac{20005.45}{\frac{\pi * 45^2}{4}} = 13 \text{ N/mm}^2$$

$$\sigma_{B \text{ shear}} = \frac{\text{Shear force}}{\text{Area of B}} = \frac{20005.45}{\frac{\pi * 40^2}{4}} = 16 \text{ N/mm}^2$$

5.3 Mean stress and Goodman diagrams

$$\text{Resultant stress at A} = \sigma_{A \text{ max}} = \sqrt{85^2 + (2 * 13)^2} = 88.9 \text{ N/mm}^2$$

$$\text{Resultant stress at B} = \sigma_{B \text{ max}} = \sqrt{282^2 + (2 * 16)^2} = 283.8 \text{ N/mm}^2$$

$$\sigma_{A \min} = -\frac{13207.3}{40010.89} * 85 = -28 \text{ N/mm}^2$$

$$\sigma_{B \min} = -\frac{13207.3}{40010.89} * 282 = -93 \text{ N/mm}^2$$

For fillet A,

$$\sigma_{A \min} = -28 \text{ N/mm}^2$$

$$\sigma_{A \max} = 89 \text{ N/mm}^2$$

$$\sigma_{A \text{ mean}} = 31 \text{ N/mm}^2$$

$$\text{Ranges of stress at A} = 117 \text{ N/mm}^2$$

$$\text{Polished endurance limit for crank at A} = 0.45 * \text{UTS} = 0.45 * 750 = 337.5 \text{ N/mm}^2$$

$$\text{Final endurance limit for crank at A} = \frac{337.5}{K_t} = 169 \text{ N/mm}^2 (K_t = 2)$$

Plotting the Goodman diagram of fillet A,

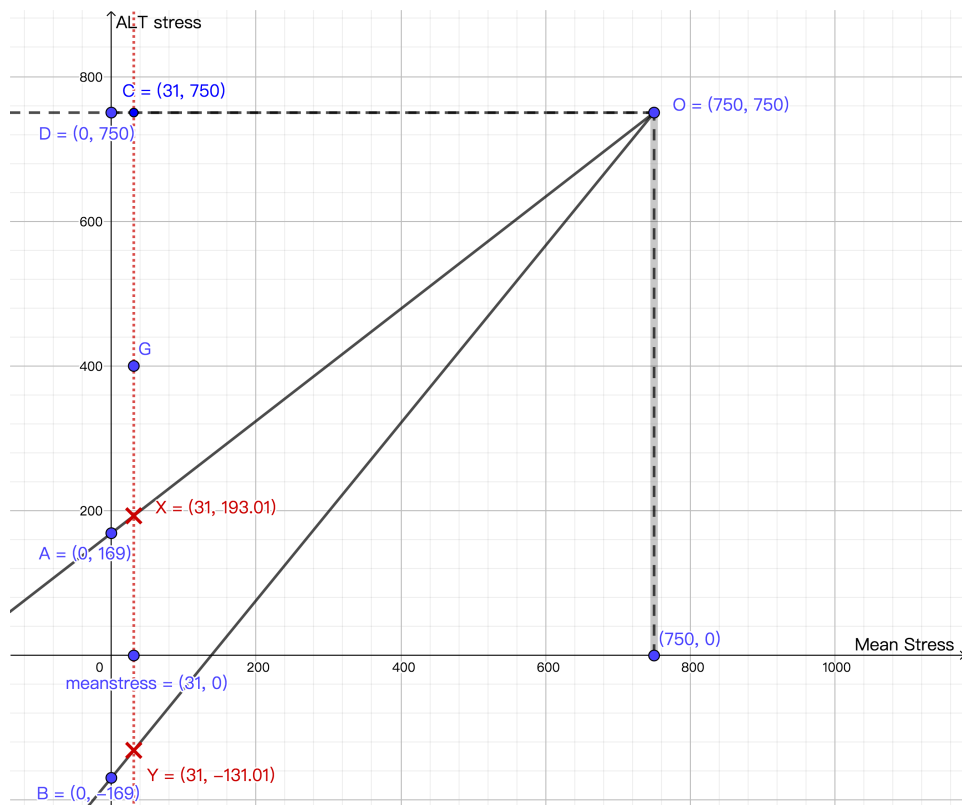


Figure 6.5 Goodman diagram of fillet A

From Goodman diagram,

$$\text{Allowable range at } 31 \text{ N/mm}^2 \text{ mean stress} = XY = AB * \left(1 - \frac{\text{Mean}}{UTS}\right)$$

$$= 338 * \left(1 - \frac{31}{750}\right) = 324 \text{ N/mm}^2$$

$$\text{Actual range} = 117 \text{ N/mm}^2$$

$$FSF = \frac{\text{Allowable range}}{\text{Actual range}} = \frac{324}{117} = 2.76 \approx 2.8$$

For fillet B,

$$\sigma_{B \text{ min}} = -93 \text{ N/mm}^2$$

$$\sigma_{B \max} = 284 \text{ N/mm}^2$$

$$\sigma_{B\ mean} = 96\ N/mm^2$$

Ranges of stress at B = 377 N/mm²

$$\text{Final endurance limit for crank at B} = \frac{337.5}{K_t} = 169 \text{ N/mm}^2 (K_t = 2)$$

Plotting the Goodman diagram of fillet B,

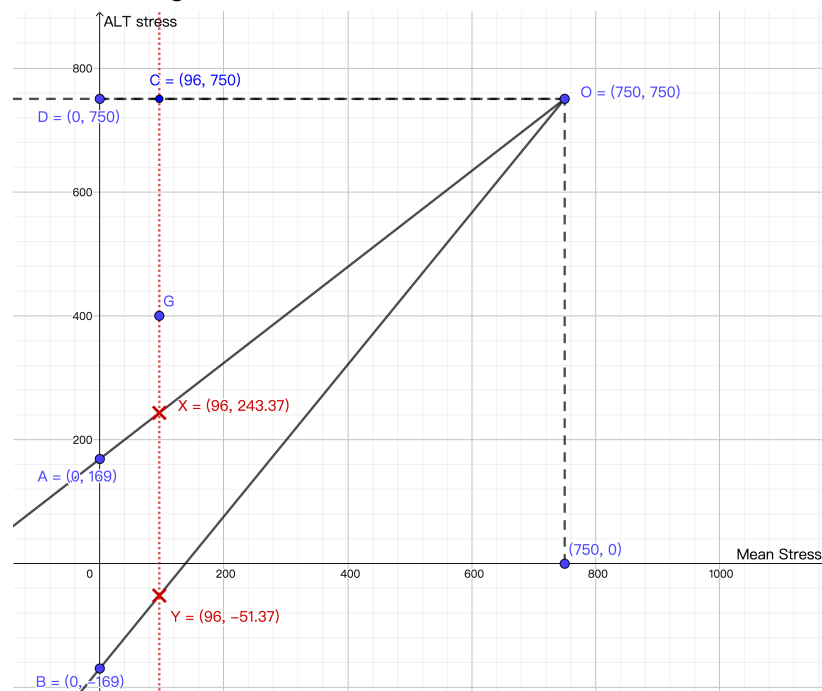


Figure 6.6 Goodman diagram of fillet B

From Goodman diagram,

$$\text{Allowable range at } 96 \frac{N}{mm^2} \text{ mean stress at } B = XY = AB * \left(1 - \frac{Mean}{UTS}\right)$$

$$= 338 * \left(1 - \frac{96}{750}\right) = 295 N/mm^2$$

$$\text{Actual range} = 377 N/mm^2$$

$$FSF = \frac{\text{Allowable range}}{\text{Actual range}} = \frac{295}{377} = 0.78 \approx 0.8$$

6.Conclusion

Throughout the report, many fatigue safety factors were calculated and Goodman diagrams were plotted which may be beneficial to learn the relative stress and fatigue life for the duty cycle of the engine. Also, some conclusions can be drawn during the process.

Stress concentration factors includes notches and radius. Generally, for the radius fillet point, choice of the value may take the diameter of both sides and the radius itself. Different concentration factors may exert effects on the FSF. If choosing smaller value, the author found that the value of FSF could drop a little. Hence, a reasonable choice of the concentration stress is important.

From the bolts redesigning, many parameters influence the FSF of the bolt. But in order to obtain longer fatigue life even infinite life, the stiffness of bolt should not be too big and in this report the author changed the type of bolt and divided the whole length into 3 to obtain 3 stiffness to reduce the stiffness of bolt while controlling the same minimum stress, bolt thread diameter.

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