

Complete, fold and seal
(see note 1 below)

43/0

SURNAME: WILLIAM GROOM

FORENAME(S): LAWRENCE

Candidate's Exam Number:

Book No.

**Total Books
Handed In:**

1. *Journal of the American Medical Association*, 1997; 277: 1033-1038.

100

Programme Title: _____

Year/Level/Stage: _____

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Examination Paper Ref No.: _____ Desk Letter/Number: _____

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Instructions to Candidates:

Summary of Marks

1. Please give the information requested above and enter your name in the space provided at the top right of this page. If your answers are to be marked under a scheme whereby the examiners will not know your identity, fold over and seal down the top right-hand corner of the page (using the gummed label provided) in order to conceal your name.
2. Commence each answer on a fresh page.
3. Do all rough work in the answer book and cross through it.
4. No part of this booklet is to be torn out.
5. Supplementary answer books if used must:-
 - (a) bear the candidate's name (concealed, where appropriate), reference number and paper title;
 - (b) be numbered consecutively;
 - (c) be fastened together by the tags available from the invigilator.
6. Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the exam room.
7. Your answers must be legibly written **IN INK**.
The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.

Question Numbers Answered	1st Examiner	2nd Examiner (optional)	3rd Examiner (optional)
1	24		
3	10		
	34		
TOTAL			
RESULT			



COMMONWEALTH OF MASSACHUSETTS

Department of Transportation

Department of Transportation

Form No. 1
Revised 10/1/80

Form No. 1

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Summary of Data

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The following information was obtained from the records of the Department of Transportation for the period of January 1, 1980, to December 31, 1980. The data was compiled by the Department of Transportation and is presented for your information.

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1

a b c

$$a=1 \quad b=-12 \quad c=37$$

ai) Solve $x^2 - 12x + 37 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{+12 \pm \sqrt{(-12)^2 - 4(1 \times 37)}}{2(1)} \quad \begin{array}{l} 148 \\ -144 \end{array} \quad \begin{array}{l} 7 \\ -4 \end{array}$$

$$\frac{+12 \pm \sqrt{292}}{2} \rightarrow \frac{12 + 17.0880}{2} \quad \frac{12 - 17.0880}{2}$$

$$= 14.544$$

$$x = 14.544 \quad \text{and} \quad x = -2.544$$

aii) quadratic roots

$$4 - 5j \quad \text{and} \quad \underline{4 + 5j}$$

b) $p = -4 + 8j$ $q = 2 + 6j$ $r = -1 - 3j$

i) $\frac{p}{2} + \frac{q}{2}$

$$\frac{-4 + 8j}{2} + \frac{2 + 6j}{2}$$

$$\downarrow \quad \downarrow$$

$$-2 + 4j + 1 + 3j$$

$$= \underline{-1 + 7j}$$

ii) $(p+r)(p-r) \cdot (-4 + 8j + -1 - 3j)(-4 + 8j - -1 - 3j)$

$$(-5 + 5j)(-3 + 11j)$$

$$j^2 = -1$$

$$15 + 55j - 15j + 55j^2$$

$$15 + 55j - 15j - 55$$

$$-40 + 40j$$

$$p = -4 + 8j \quad q = 2 + 6j \quad r = -1 - 3j$$

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iii) $p^2 + q^2 + r^2$

$$j^2 = -1$$

$$-j^2 = +1$$

$$(-4 + 8j)(-4 + 8j) + (2 + 6j)(2 + 6j) + (-1 - 3j)(-1 - 3j)$$

$$16 - 32j - 32j + 64j^2 + 4 + 12j + 12j + 36j^2 + 1 + 3j + 3j + 9j^2$$

top

$$16 - 64 = 0$$

$$-48$$

$$+ -32$$

$$+ -8$$

$$= -88$$

$$-34j$$

iv) $\frac{q}{r} = \frac{(2 + 6j) \times (-1 + 3j)}{(-1 - 3j) \times (-1 + 3j)}$

Complex conjugate = $-1 + 3j$

Top

$$-2 + 6j - 6j + 18j^2$$

$$16 + 12j$$

bottom

$$1 - 3j - 3j - 9j^2$$

$$1 + 9j^2$$

$$10$$

$$j^2 = -1 \quad -j^2 = +1$$

$$\frac{16 + 12j}{10}$$

$$\frac{16}{10}$$

$$\frac{12j}{10}$$

v) $2p - 3r \rightarrow 2(-4 + 8j) - 3(-1 - 3j)$

$$4q + p$$

$$4(2 + 6j) + (-4 + 8j)$$

$$-8 + 16j + 3 + 9j$$

$$8 + 24j + -4 + 8j$$

$$-5 + 25j$$

$$4 + 32j$$

Complex

Conjugate =

$$4 - 32j$$

$$(-5 + 25j) \times (4 - 32j) = -20 - 800j^2 + 100j$$

$$(4 + 32j) \times (4 - 32j)$$

$$16$$

$$-20 + 160j + 100j - 800j^2$$

$$-20 + 100$$

$$160 + 100$$

$$280 + 260j$$

$$1040$$

$$280$$

$$1040$$

$$260j$$

$$1040$$

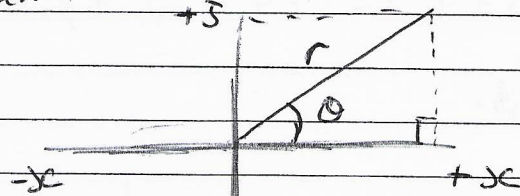
$$16 - 128j + 128j - 1024j^2$$

$$16 + 1024$$

c) $r(\cos \theta + j \sin \theta)$ explain r and θ

r is the line drawn on a graph between the cartesian complex number - known as the argument

θ is the angle between the argument and the positive x axis as shown on an Argand diagram



ii) $r = 5$ $\theta = 120^\circ$

$$180 - 120 = 60^\circ$$

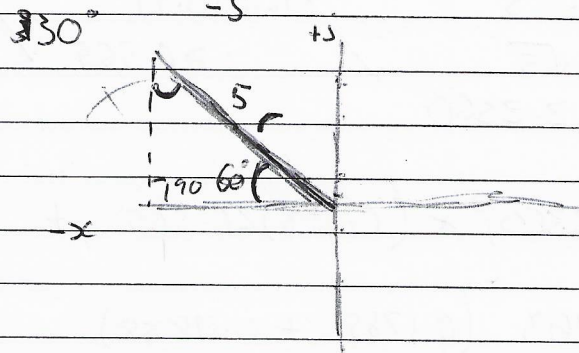
$$\theta = 60^\circ$$

$$r(\cos \theta + j \sin \theta)$$

$$5(-0.5 + j0.8660)$$

$$-2.5 + j4.33$$

$$-2.5 + j4.33j$$



iii) $-3 - 5j$ in polar

$$r^2 = -3^2 + -5^2$$

$$= -34$$

$$r = \sqrt{34}$$

$$= 5.8309$$

$$\tan^{-1} \frac{-5}{-3}$$

$$= 1.6$$

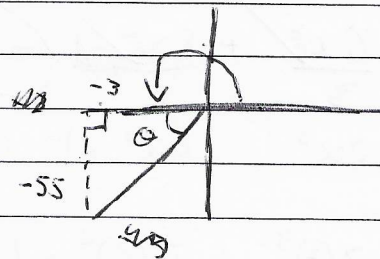
$$\tan^{-1}(1.6)$$

$$= 59.0362$$

$$270 - 59.0362$$

$$= 210.9638$$

$$5.831 \angle 210.9638^\circ$$



d)

$$r^n (\cos n\theta + j \sin n\theta)$$

$$(2+j)^3$$

$$(2+j)(2+j)(2+j)$$

$$(4+2j+2j+j^2)(2+j)$$

$$8+4j+4j+2j^2+4j+2j^2+2j^2+j^3$$

$$8+j^3$$

X

ii

$$r^2 = 2^2 + 1^2$$

$$= 5$$

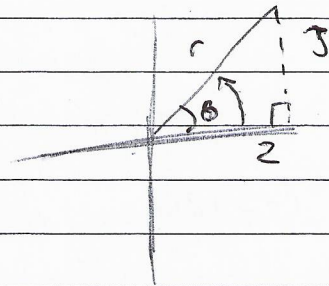
$$r = \sqrt{5}$$

$$= 2.2360$$

$$\tan \theta = \frac{1}{2}$$

$$\tan^{-1}(0.5)$$

$$= 26.565^\circ$$



$$(2.2360)^3 \times (\cos(3)26.565 + j \sin(3)26.565)$$

$$79.695$$

$$11.1793 (0.1788 + 0.9838j)$$

$$1.9988 + 10.9918j$$

3

ai)

$$\int_0^2 (4x^3 - 6x^2 + 8x - 1) dx$$

$$\frac{4x^4}{4} - \frac{6x^3}{3} + \frac{8x^2}{2} - 1$$

$$(2)^4 - 2(2)^3 + 4(2)^2 - 1$$

$$16 - 16 + 16 - 1$$

$$15$$

$$2$$

$$15+1$$

$$16$$

$$(0)^4 - 2(0)^3 + 4(0)^2 - 1$$

$$0 - 2 + 4 - 1$$

X

(24)

3

3a ii) $\int_0^1 (e^{2x} - e^x + 3) dx$

rule 8
 $\frac{1}{a} e^{ax} + C$

$\frac{1}{2} e^{2x} - \frac{1}{1} e^x + 3x$

$\frac{1}{2} e^{2(1)} - e^{(1)} + 3$

$\frac{1}{2} e^{2(0)} - e^0 + 3$

2

iii) $\int (4 \sin 2x - 3 \cos 4x) dx$

$4 \left(-\frac{1}{2} \cos 2x \right) + 3 \frac{1}{4} \sin 4x + C$

2

Integration by parts

b) $\int x \sin 4x$

$u = x$ $\frac{du}{dx} = 1$ $v = \frac{1}{4} \sin 4x$ $\frac{dv}{dx} = \cos 4x$

$\frac{dv}{dx} = \frac{x^2}{2}$ $v = 4 \cos 4x$

$\int x \sin 4x dx = x \frac{1}{4} \sin 4x - \int \frac{1}{4} \sin 4x \cdot x^2 dx$

$x \cos 4x - \sin 4x \frac{-2x^2}{3} + C$

2

1

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Question No.

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