

ACMA 5011: Thermo-Fluid Mechanics

1. Base units

2 Concept of an Ideal gas

(i) Boyles Law (ii) Charles' Law

(iii) Pressure Law (iv) The Combined equation

(v) Ideal gas equation for any amount of gas

(vi) Real gases? Van Der Waals equation

3 Kinetic Theory: The Derivation of the Pressure in a Container for an Ideal gas. The Link between KE and temperature

4 Adiabatic changes:

5 The Zeroth and 1st Law of Thermodynamics

6 The Concept of Ideal heat engine

7 Concept of Entropy a Measure of disorder of a system.

Base units and Derived units

The Seven Base (fundamental) units.

Mass — kilogram — kg

length — metre — m

time — Second — s

Temperature — kelvin — K

Current — Ampere — A

Amount of — Mole — mol
Substance

Luminous — Candela — Cd.
Intensity

Derived units

Area = length $\times$ length

$$\Rightarrow m \times m$$

$$= m^2$$

Volume = Area $\times$ length

$$\Rightarrow m^2 \times m$$

$$= m^3$$

$$\text{Density} = \frac{\text{mass}}{\text{Volume}}$$

$$\Rightarrow \text{Density} = \frac{\text{kg}}{\text{m}^3} \text{ or } \text{kg m}^{-3}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\Rightarrow \frac{\text{m}}{\text{s}} \\ = \text{ms}^{-1}$$

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$\Rightarrow \frac{\text{m}}{\text{s}} \\ = \text{ms}^{-1}$$

$$\text{Acceleration} = \frac{\text{Change in Velocity}}{\text{Time}}$$

$$\Rightarrow \frac{\text{ms}^{-1}}{\text{s}} \\ = \text{ms}^{-2}$$

$$\text{Force} = \text{mass} \times \text{acceleration} \\ \Rightarrow \underline{\text{kg ms}^{-2}} \quad (\text{or Newton} - \text{N})$$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \\ \Rightarrow \frac{\text{kg ms}^{-2}}{\text{m}^2} \\ = \underline{\text{kg m}^{-1} \text{s}^{-2}}$$

$$\text{Work} = \text{Force} \times \text{Distance} \\ \Rightarrow \text{kg ms}^{-2} \times \text{m} \\ = \underline{\text{kg m}^2 \text{s}^{-2}} \quad [\text{or Joule} - \text{J}]$$

$$\text{GPE} = mgh \\ \Rightarrow \text{kg ms}^{-2} \cdot \text{m} \\ = \underline{\text{kg m}^2 \text{s}^{-2}} \quad (\text{Joule}) \\ (\text{Note same units as Work})$$

$$\text{Charge} = \text{Current} \times \text{time}$$

$$\Rightarrow \text{A} \cdot \text{s}$$

Ex. Definition of "The volt"

$$W = Q \cdot V$$

$W = \text{Work Done}$ $Q = \text{Charge}$ $V = \text{volt}$

$$\Rightarrow \text{kg m}^2 \text{s}^{-2} = \text{AS} \times 'V'$$

$$\frac{\text{kg m}^2 \text{s}^{-2}}{\text{AS}} = 'V'$$

$$\underline{\text{kg m}^2 \text{A}^{-1} \text{s}^{-3} = 'V'}$$

Ex. If $F = \frac{G m_1 m_2}{r^2}$ Base units of 'G'

$$\Rightarrow \text{kg m s}^{-2} = G \cdot \frac{\text{kg} \cdot \text{kg}}{\text{m}^2}$$

$$\Rightarrow \frac{\text{kg m}^3 \text{s}^{-2}}{\text{kg}^2} = G$$

$$\underline{\text{kg}^{-1} \text{m}^3 \text{s}^{-2} = G}$$

$$\text{Number of Moles} = \frac{\text{Mass}}{\text{Molar Mass}}$$

Example Find the number of moles and the number of particles/Molecules contained within 8g of O_2 gas.

$$\text{Molar Mass of } O_2 = 32 \text{ g.}$$

$$\begin{aligned} \text{i) No Moles of } O_2 &= \frac{8}{32} \\ \text{gas} & \\ &= 0.25 \text{ moles.} \end{aligned}$$

1 mole of any substance has 6.023×10^{23} particles/molecules.

$$\begin{aligned} \Rightarrow 0.25 \text{ moles} & \dots \dots 0.25 \times 6.023 \times 10^{23} \text{ molecules} \\ & = 1.50575 \times 10^{23} \text{ molecules.} \end{aligned}$$

Temperature (T)

Temperature is a measure of how "hot" something is.

It is measured on a scale such as $^{\circ}\text{F}$ or $^{\circ}\text{C}$ or on the absolute temperature scale in K (Kelvins)

Heat Heat is thermal energy

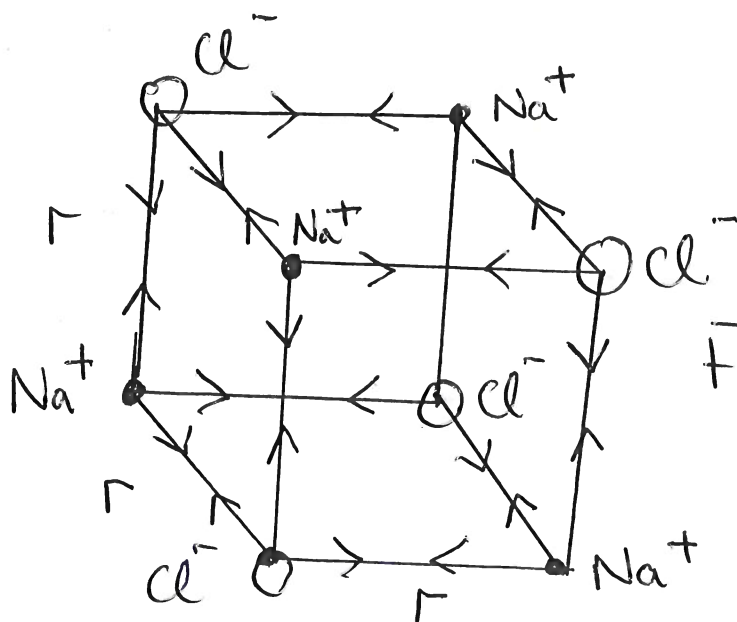
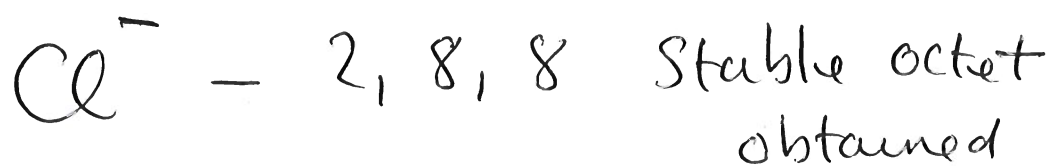
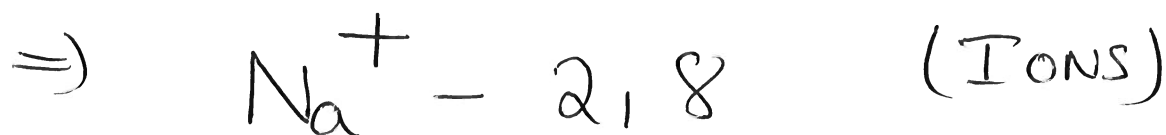
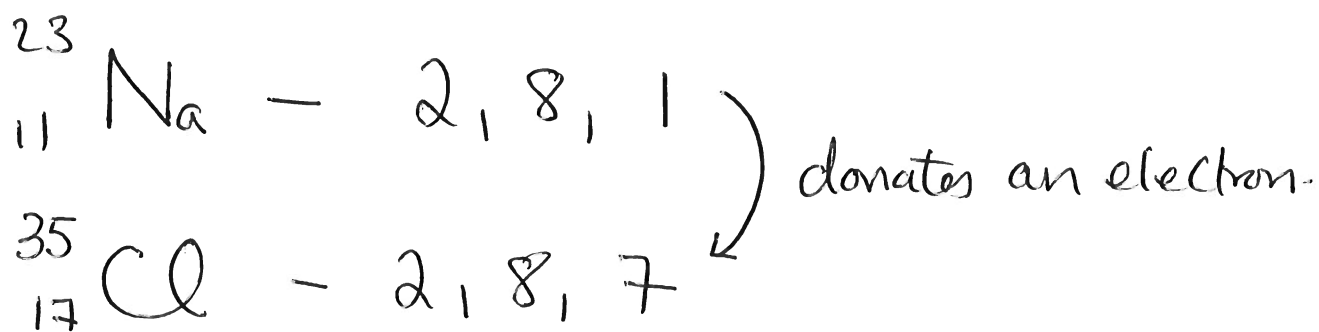
Thermo-Dynamics:

The study of heat energy (Thermo)
in motion (Dynamics).

STATES OF MATTER

SOLIDS Have a fixed shape, usually Crystalline in Structure. The bonds between atoms are usually electrostatic (or Covalent in polymers/plastics etc).

Example : The Structure of Common Salt.

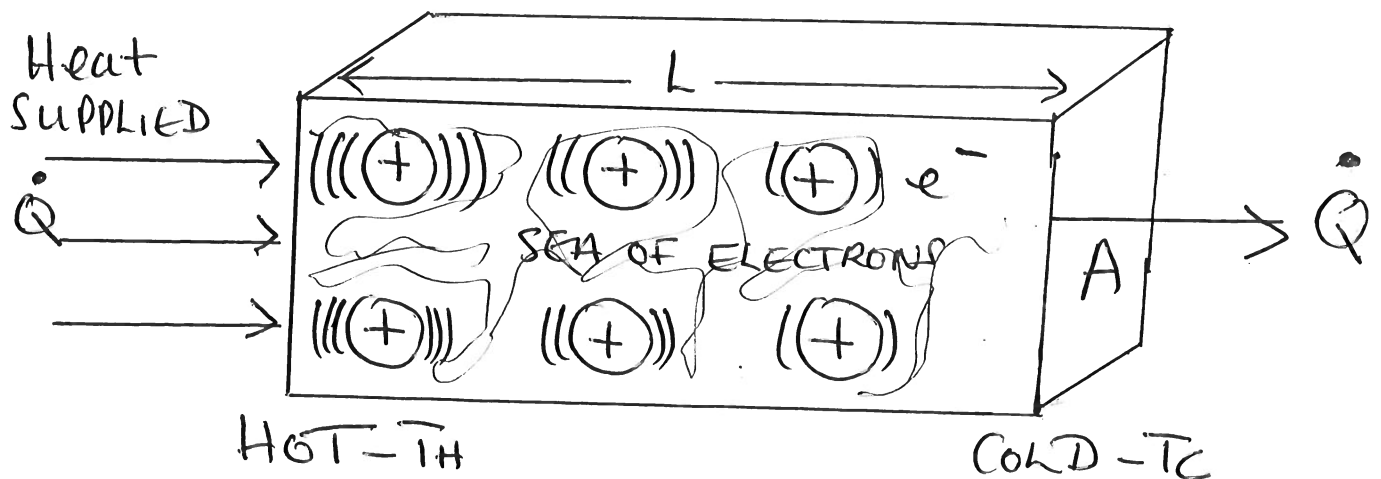


$$F = \frac{k \cdot q_{\text{Na}^+} q_{\text{Cl}^-}}{r^2}$$

The electro-static force $F = \frac{k q_{Na}^+ q_{Cl}^-}{r^2}$

Is very strong and as a consequence solids usually have very high melting and boiling points as a lot of thermal energy needs to be supplied in order to break these bonds.

Heat Conduction through Solids



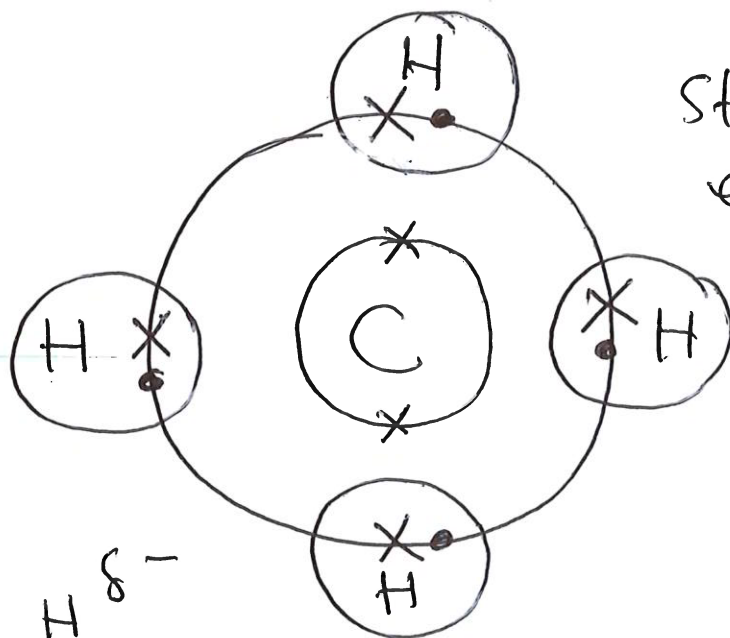
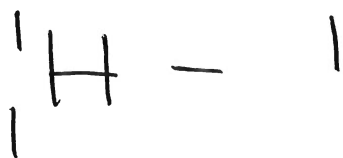
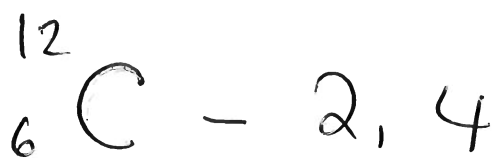
$$\dot{Q} = \frac{k A (T_H - T_C)}{L}$$

Fourier's Law

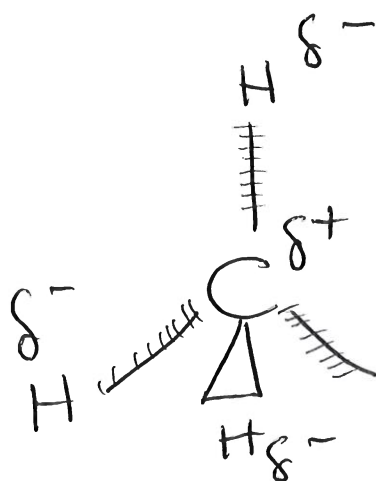
Fluids (Liquids & Gases)

These have no fixed shape, the bonds are covalent and consequently have lower melting and boiling points than solids.

Example: Methane: CH_4



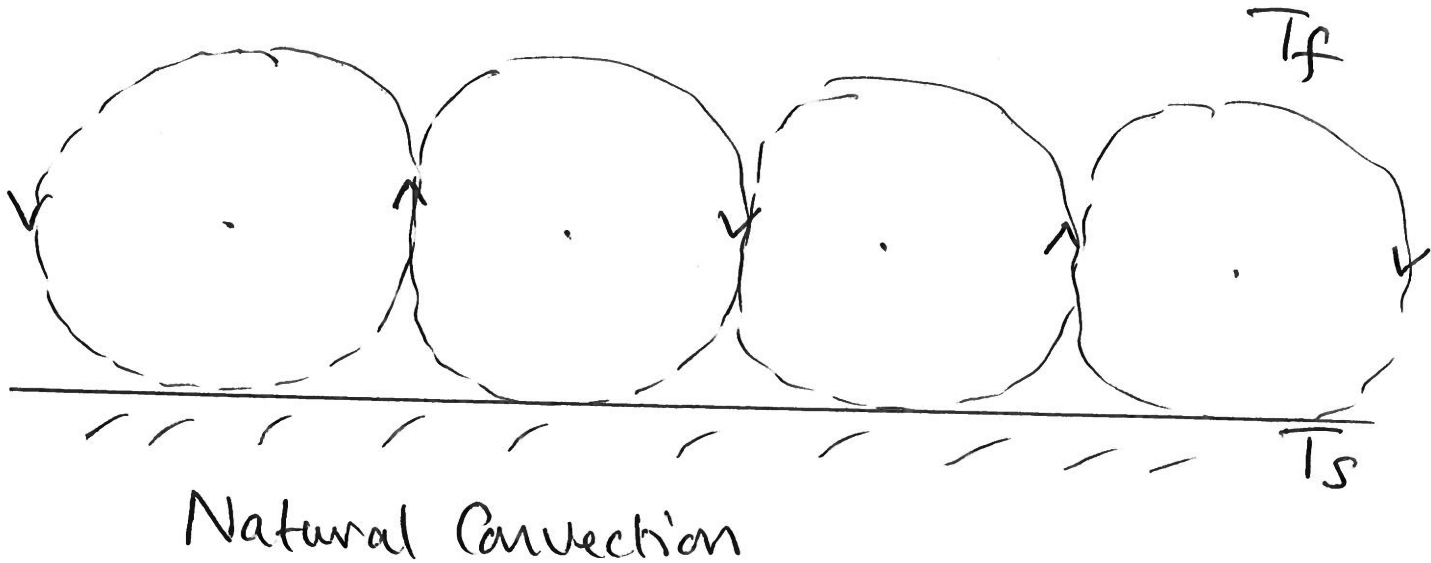
Stable octet of electrons in outer shell.



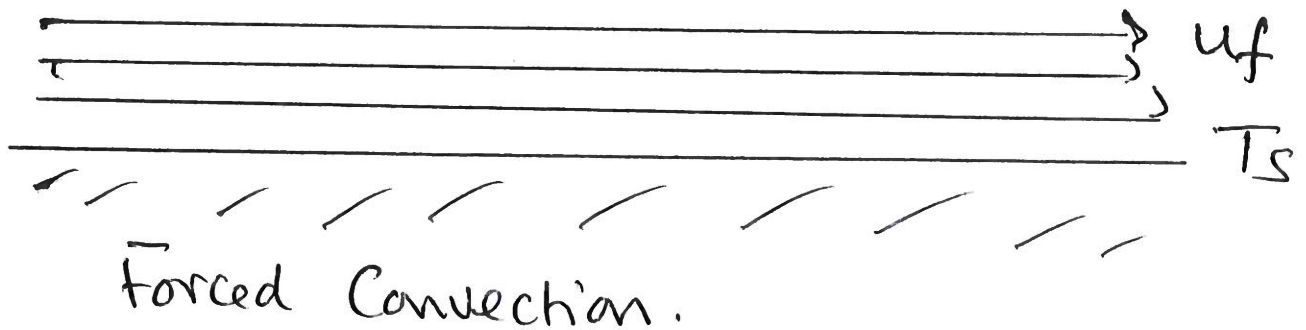
Tetrahedral in shape

Heat Conduction in fluids

This is achieved by means of the Process of Convection. This can be FREE OR NATURAL OR FORCED.



Air blown across a surface.



$$\dot{Q} = h A (T_s - T_f)$$

GASES

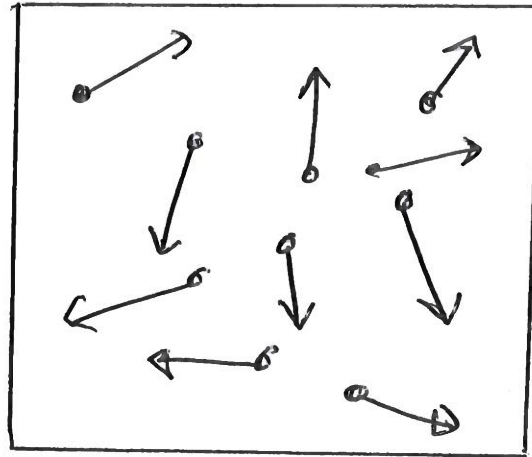
A type of fluid where the atoms or molecules are considered to be very far apart. Compared with solids and liquids, the gas occupies the shape and volume of its container.

In this course we predominantly consider ideal gases.

For an ideal gas we assume that:

- (i) The atoms / molecules can be treated as point masses (zero volume)
- (ii) There are no inter-atomic forces.
- (iii) The collisions between particles and the container walls are perfectly elastic, so no energy is lost

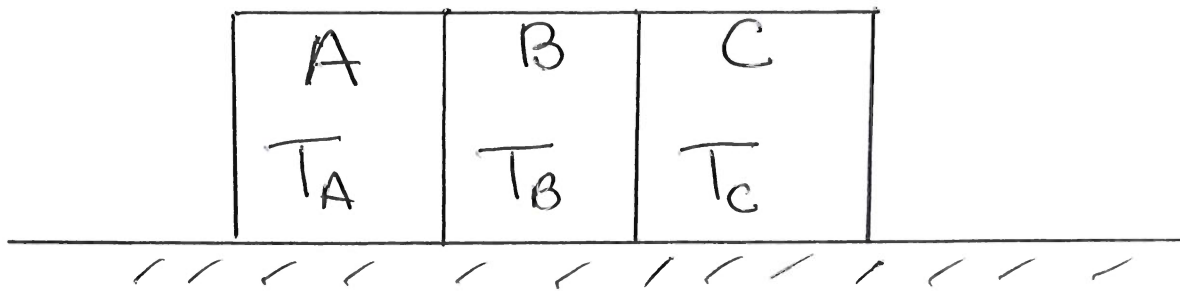
(iv) The Motion of the gas particles is entirely random in nature.



The Zeroth & 1st law of Thermo-dynamics

Zeroth Law

Consider three objects in thermal equilibrium. (No heat transferred)

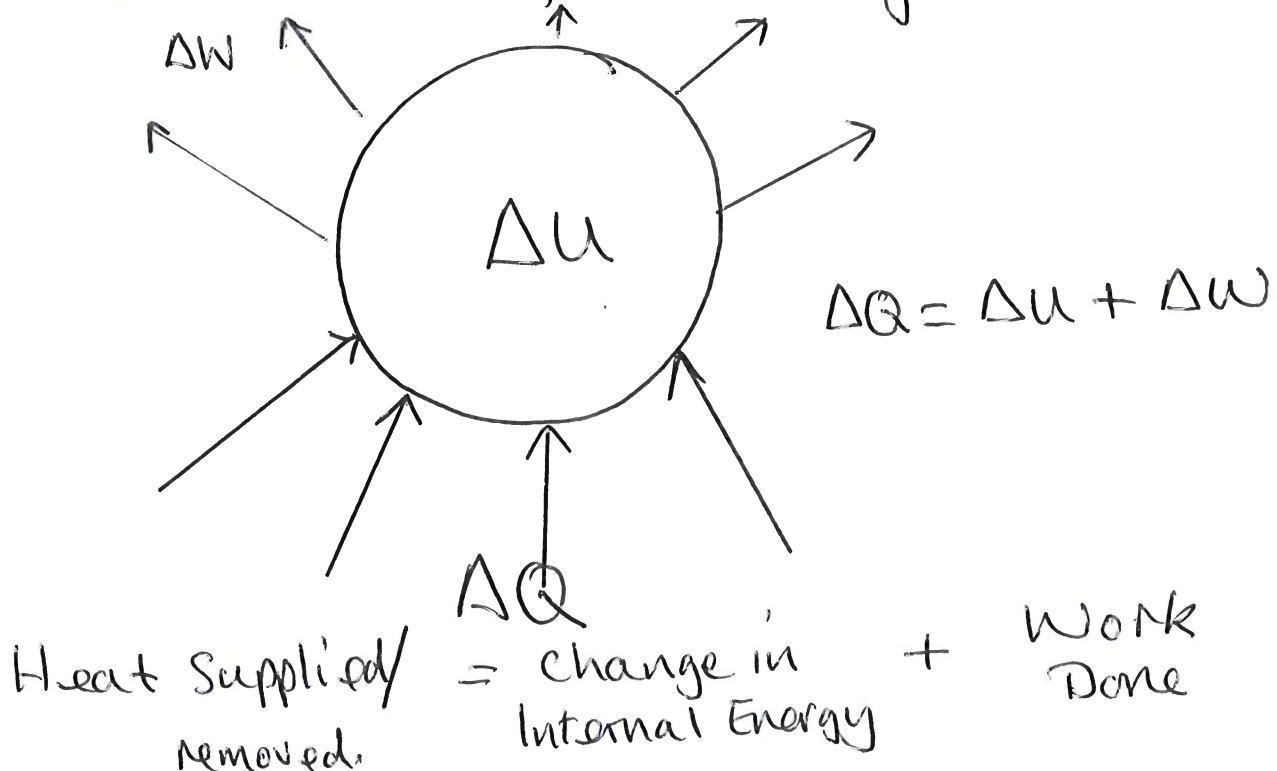


$$\text{If } T_A = T_B$$

$$T_B = T_C$$

$$\text{Then } T_A = T_C$$

The 1st Law of Thermo-dynamics (Closed System)



$$\Delta Q = \Delta U + \Delta W$$

If heat supplied to system ΔQ +ve
 Removed from . ΔQ is -ve

If work is done by system ΔW is +ve
 done on system ΔW is -ve

Ex.

$$\Delta Q = +3000 \text{ J}$$

$$\Delta W = +1000 \text{ J}$$

$$\Delta U = ?$$

$$\Delta Q = \Delta U + \Delta W$$

$$3000 = \Delta U + 1000$$

$$\underline{2000 \text{ J} = \Delta U.}$$

Ex.

$$\Delta Q = -1000 \text{ J}$$

$$\Delta W = -200 \text{ J}$$

$$\Delta U = ?$$

$$\left. \begin{array}{l} \Delta Q = \Delta U + \Delta W \\ -1000 = \Delta U + (-200) \\ -1000 + 200 = \Delta U \\ \underline{-800 \text{ J} = \Delta U} \end{array} \right\}$$

The Ideal Gas Laws

Boyles Law

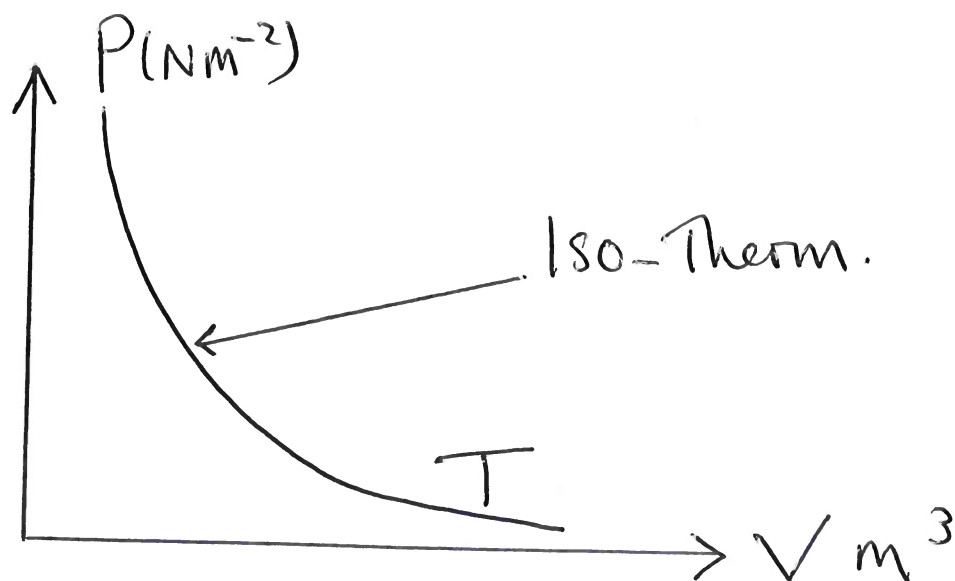
For a fixed mass of gas at a constant temperature, the Pressure is found to be Inversely Proportional to its Volume.

$$\text{i.e. } P \propto \frac{1}{V}$$

$$\Rightarrow P = k \cdot \frac{1}{V}$$

$$\Rightarrow PV = k (\text{constant})$$

$$\Rightarrow P_1 V_1 = P_2 V_2 \text{ etc.}$$



Ex. $P_1 = 2 \times 10^6 \text{ Pa}$ $P_2 = ?$

$$V_1 = 2 \text{ m}^3 \quad V_2 = 3 \text{ m}^3$$

$$P_1 V_1 = P_2 V_2$$

$$2 \times 10^6 \times 2 = P_2 \times 3$$

$$\frac{4 \times 10^6}{3} \text{ Pa} = P_2.$$

Ex. $P_1 = 1 \times 10^5 \text{ Pa}$ $P_2 = 4 \times 10^5 \text{ Pa}$
 $V_1 = ?$ $V_2 = 1 \text{ m}^3$

$$P_1 V_1 = P_2 V_2$$

$$1 \times 10^5 \times V_1 = 4 \times 10^5 \times 1$$

$$V_1 = \frac{4 \times 10^5}{1 \times 10^5}$$

$$\underline{V_1 = 4 \text{ m}^3}$$

Charles' Law

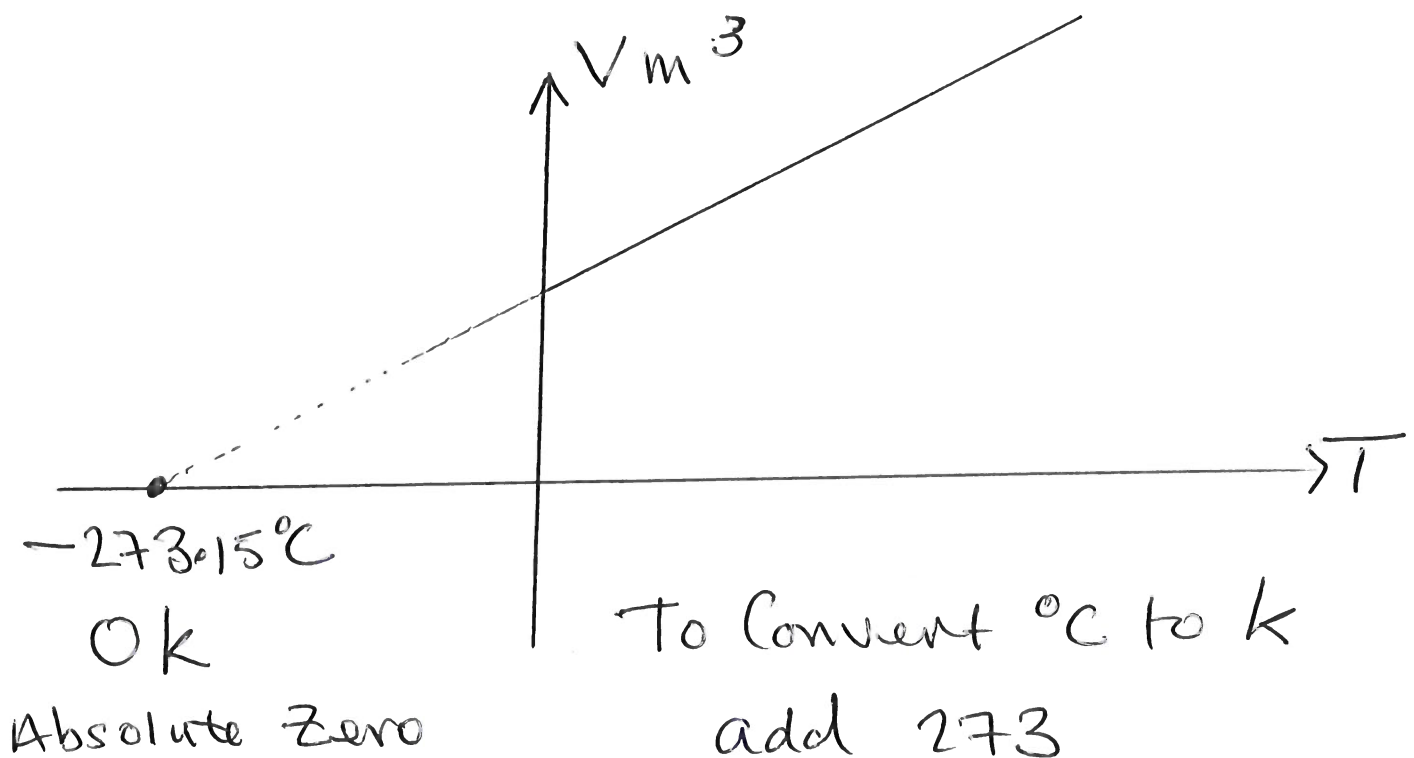
For a fixed mass of gas at Constant Pressure, the volume occupied by the gas is directly proportional to its temperature.

$$\Rightarrow V \propto T$$

$$\Rightarrow V = kT$$

$$\Rightarrow \frac{V}{T} = k \text{ (constant)}$$

$$\Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2} \text{ etc.}$$



Example:

$$V_1 = 2\text{m}^3$$

$$V_2 = 4\text{m}^3$$

$$T_1 = 373\text{k}$$

$$T_2 = ?$$

from Charles' law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\Rightarrow \frac{2}{373} = \frac{4}{T_2}$$

$$\Rightarrow 2T_2 = 4 \times 373$$

$$\Rightarrow T_2 = \frac{4 \times 373}{2}$$

$$\underline{T_2 = 746\text{k}}$$

The Pressure law

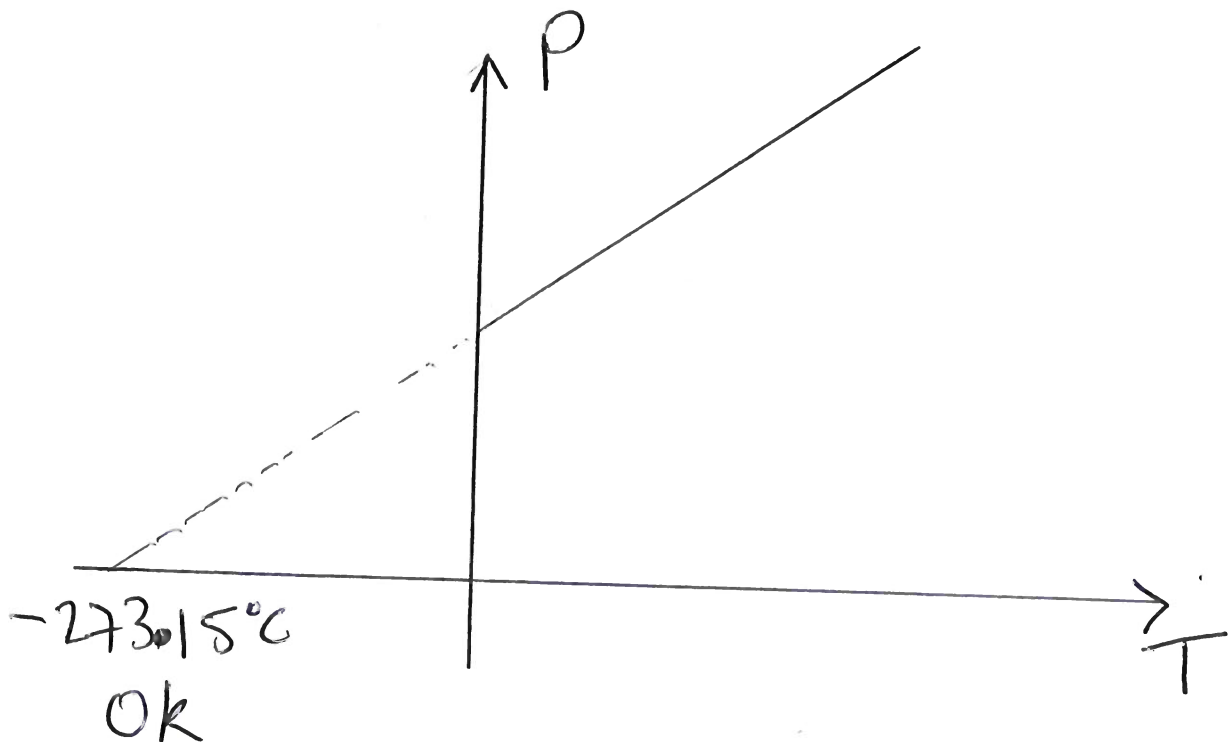
For a fixed mass of gas at Constant Volume, the Pressure is found to vary directly with the temperature.

$$\Rightarrow P \propto T$$

$$\Rightarrow P = kT$$

$$\Rightarrow \frac{P}{T} = k (\text{constant})$$

$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ etc.}$$



EXAMPLE

$$P_1 = 2 \times 10^5 \text{ Nm}^{-2} \quad P_2 = ?$$

$$T_1 = 273 \text{ K} \quad T_2 = 600 \text{ K}$$

Then from the pressure law

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{2 \times 10^5}{273} = \frac{P_2}{600}$$

$$\frac{2 \times 10^5 \times 600}{273} \text{ Nm}^{-2} = P_2$$

$$\underline{439560.44 \text{ Nm}^{-2} \text{ (2dp's)} = P_2}$$

The Combined Gas Law.

The 3 fundamental gas laws can all be combined into one single equation:

Boyles Law $PV = \text{Constant}$

Charles' Law $\frac{V}{T} = \text{Constant}$

Pressure Law $\frac{P}{T} = \text{Constant}$

Combining together gives:

$$\frac{PV}{T} = \text{Constant}$$

$$\text{or } \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

EXAMPLE

$$P_1 = 2 \times 10^5 \text{ (Pa)}$$

$$P_2 = ?$$

$$V_1 = 3 \text{ m}^3$$

$$V_2 = 4 \text{ m}^3$$

$$T_1 = 273 \text{ K}$$

$$T_2 = 373 \text{ K}$$

Then using :-

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{2 \times 10^5 \times 3}{273} = \frac{P_2 \times 4}{373}$$

$$\Rightarrow \frac{2 \times 10^5 \times 3 \times 373}{273 \times 4} = P_2$$

$$\underline{204945.055 \text{ Pa} = P_2}$$

The Ideal gas Equation for Any Amount of gas.

For Any amount of gas the
Ideal gas equation becomes: -

$$\underline{\underline{P V = n R T}}$$

Where $P \equiv$ The pressure of the gas

$V \equiv$ The volume occupied by the gas

$n \equiv$ no. of moles of the gas

$R \equiv$ The Molar gas Constant
($8.31 \text{ J K}^{-1} \text{ mol}^{-1}$)

$T \equiv$ The temperature in Kelvin.

EXAMPLE:

Find the Pressure of 16g of O_2 gas of Molar mass 32g occupying a Volume of $2m^3$ at a temperature of $30^\circ C$.

$$\text{No. of Moles} = \frac{\text{Mass}}{M_m} = \frac{16}{32} = \frac{1}{2}$$

Then using

$$PV = nRT$$

$$P \times 2 = \frac{1}{2} \times 8.31 \times (30 + 273)$$

$$2P = \frac{1}{2} \times 8.31 \times 303$$

$$P = \frac{1}{4} \times 8.31 \times 303 \text{ Pa}$$

$$\underline{P = 629.48 \text{ Pa (2dp's)}}$$

Adiabatic Processes.

In an adiabatic process, no heat enters or leaves the system under consideration, then from the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$\text{Then } \Delta Q = 0 \Rightarrow$$

$$0 = \Delta U + \Delta W$$

$$- \Delta U = \Delta W$$

$\Rightarrow$ Work is done at the expense of internal energy.

It can be shown that under such circumstances that:-

$$P V^\gamma = \text{Constant.}$$

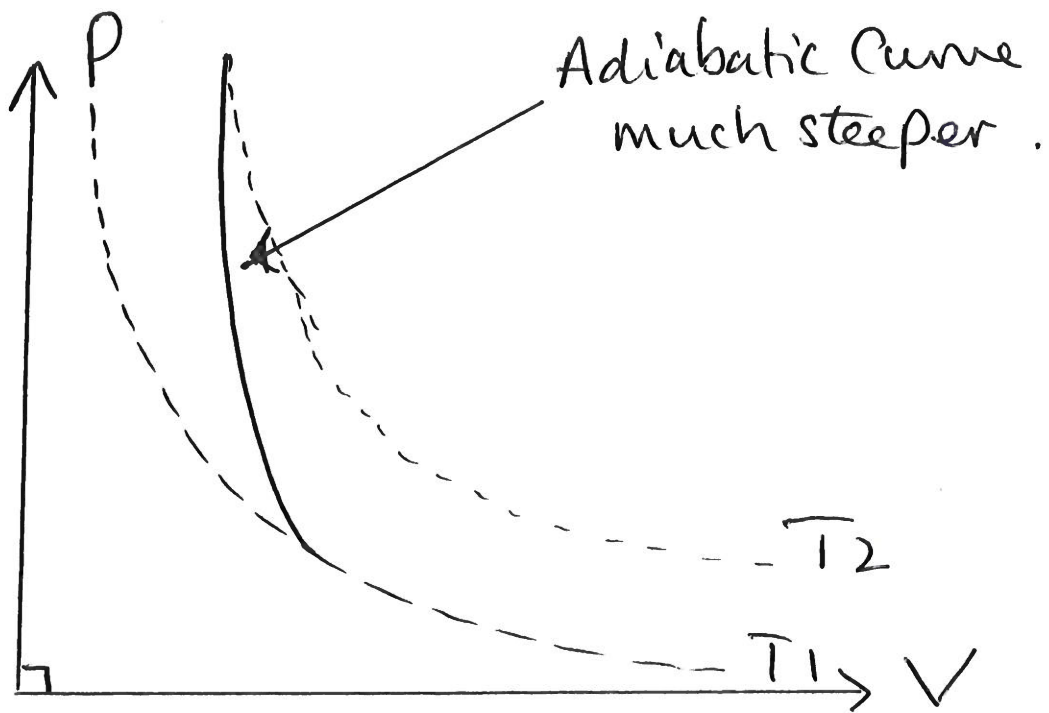
where γ takes on values in the range $1.3 \leq \gamma \leq 1.67$ depending on the gas.

Then as $P V^\gamma = \text{Constant}$

$$P = \frac{\text{Constant}}{V^\gamma}$$

$$P \propto \frac{1}{V^\gamma}$$

So for $\gamma > 1$ The Curve is steeper than those Isothermals generated from Boyle's Law



EXAMPLES

$$\text{If } P_1 = 2 \times 10^5 \text{ Pa}$$

$$P_2 = ?$$

$$V_1 = 2 \text{ m}^3$$

$$V_2 = 3.5 \text{ m}^3$$

$$\gamma = 1.4$$

$$\text{as } P V^\gamma = \text{Constant} \Rightarrow$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$2 \times 10^5 \times 2^{1.4} = P_2 \times (3.5)^{1.4}$$

$$\frac{2 \times 10^5 \times 2^{1.4}}{(3.5)^{1.4}} = P_2$$

$$\underline{91364.34 \text{ Pa (2dp's)} = P_2}$$

Also using $PV^\gamma = \text{Constant}$
and from the Ideal gas equation

$$\frac{PV}{T} = \text{Constant} \text{ then we}$$

Can derive another 2 useful equations
for adiabatic Processes.

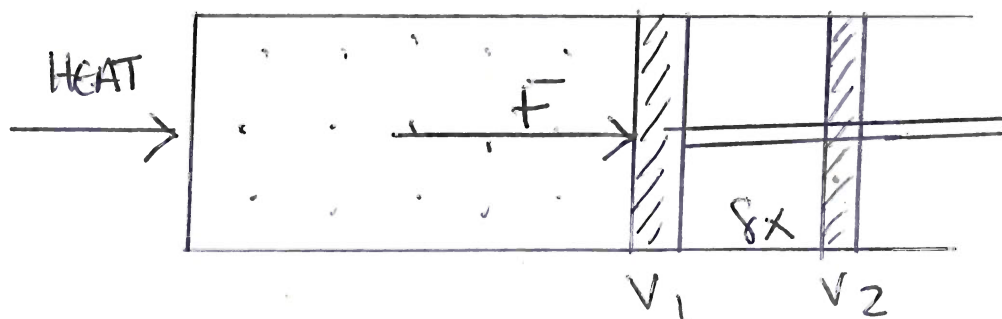
$$\Rightarrow TV^{\gamma-1} = \text{Constant}$$

$$\Rightarrow \underline{T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}}$$

and also: $P^{1-\gamma} T^\gamma = \text{Constant}$

$$\Rightarrow \underline{P_1^{1-\gamma} T_1^\gamma = P_2^{1-\gamma} T_2^\gamma}$$

Work Done by a gas in Expansion in a Closed System



Work Done = Force $\times$ Distance.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area.}}$$

$$\therefore W = PA \cdot \Delta x$$

$$W = P \Delta V$$

If P is Constant $W = P(V_2 - V_1) \text{ J}$

If P is Variable.

$$W = \int_{V_1}^{V_2} P \cdot dV$$

from Boyle's Law $PV = k$

$$W = \int_{V_1}^{V_2} \frac{k}{V} \cdot dV$$

$$= k [\ln V]_{V_1}^{V_2}$$

$$= k \ln\left(\frac{V_2}{V_1}\right) \text{ J}$$

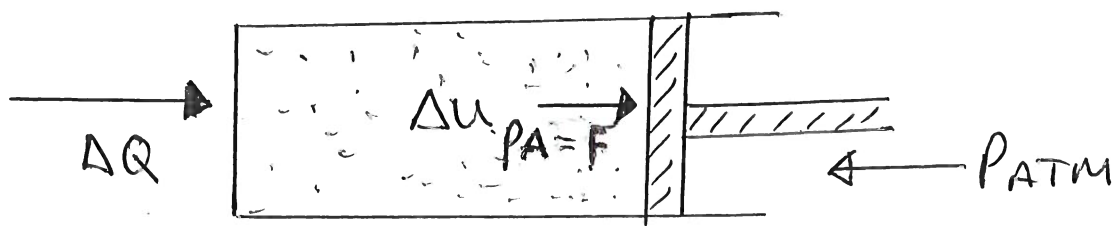
Heat Engines (Ideal Carnot Engine)

A device which Converts heat energy unto useful work is Called a heat Engine.

Examples of such are things like, the Steam Engine, the Internal Combustion engine.

For such engines to be useful, the engine must be able to do Continuous work and thus must work in Cyclical fashion.

Consider a gas enclosed in a cylinder by a piston which is frictionless.



If heat is Supplied to the gas it expands and pushes the piston backwards as work is done by the System. Thus heat energy has been converted into useful work. However, this stops when the pressure of the gas inside the cylinder equals the atmospheric pressure. Thus, before any further useful work to be done, the piston must be returned to its original position, i.e. the gas is then compressed, with work being done on the system. This process needs to be repeated in a Cyclical manner.

In such Cyclical processes the gas can be returned to its Initial State only if some of the heat energy it has originally taken in at the source is given up to a sink which must be at a lower temperature in order for heat transfer to take place.

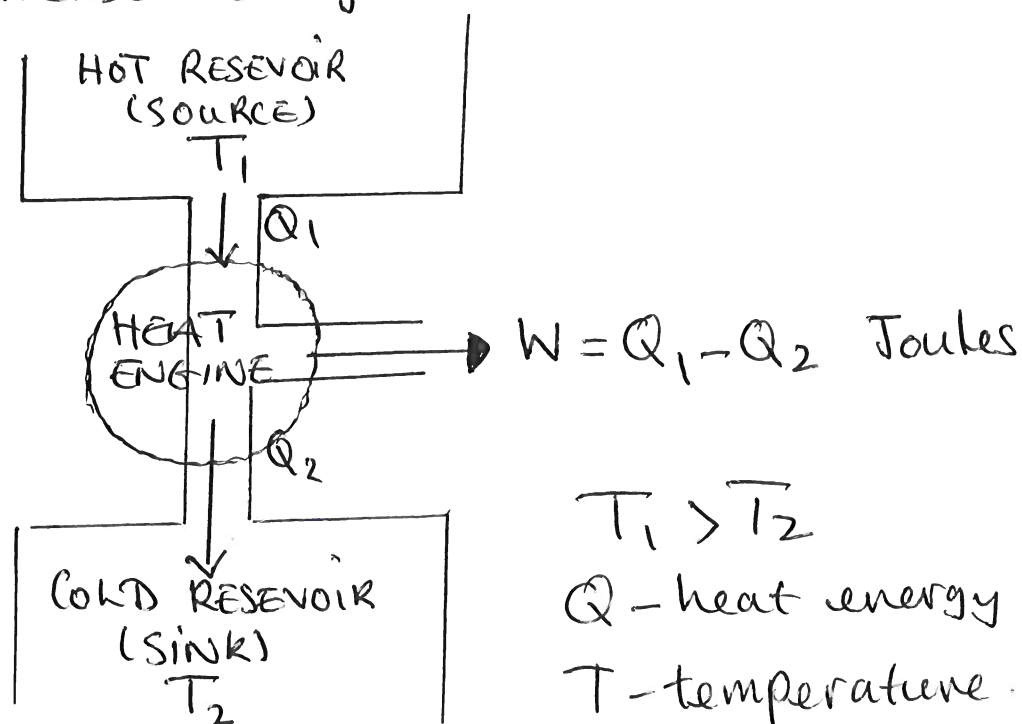
In general all heat engines work on this Cyclical principle; that is:

a) Heat is taken into System at the Source at a high temperature.

b) The System Converts heat into useful work

(c) Heat is rejected to a sink at a lower temperature

d) The Process is Cyclical.



Thermal Efficiency of the Ideal heat Engine.

The thermal efficiency ' η ' of the ideal heat engine is defined by:-

$$\eta = \frac{\text{Work Done over 1 Cycle}}{\text{Heat taken in at high temperature}}$$

$$\eta = \left[\frac{Q_1 - Q_2}{Q_1} \right] \times 100\%$$

$$\eta = \left[1 - \frac{Q_2}{Q_1} \right] \times 100\%$$

If the process is reversible it can be shown that a more useful result is:-

$$\eta = \left[\frac{T_1 - T_2}{T_1} \right] \times 100\%$$

$$\eta = \left[1 - \frac{T_2}{T_1} \right] \times 100\%$$

T_1, T_2 in Kelvin

No heat engine can be more efficient than the ideal Carnot engine, a reversible one, working between two temperatures T_1 and T_2 .

$$\text{As } \eta = \left[1 - \frac{T_2}{T_1} \right] \times 100\%$$

η can be maximised if T_2 is as small as practically possible and T_1 is as large as practically possible so $\frac{T_2}{T_1} \rightarrow 0$

For a real heat engine, the efficiency is usually significantly less than ' η ' due to frictional effects, fluid turbulence and heat being taken in at the source over a range of temperatures.

No heat engine working continuously in a cyclical fashion is able to convert all available heat into work.

Example:

Find the efficiency of an Ideal 'Carnot' heat engine which takes in heat at a Constant temperature 727°C and rejects it to a sink at a Constant temperature of 227°C

$$\eta = \left[\frac{T_1 - T_2}{T_1} \right] \times 100\%$$

$$T_1 = 273 + 727 = 1000\text{ K}$$

$$T_2 = 273 + 227 = 500\text{ K}$$

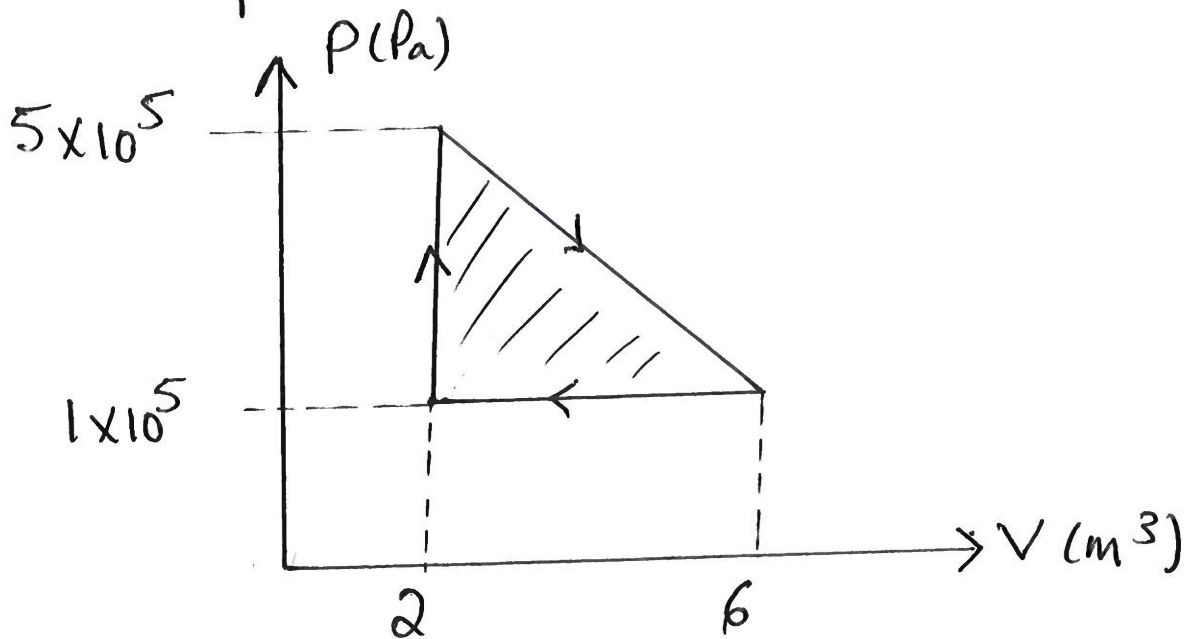
$$\eta = \left[\frac{1000 - 500}{1000} \right] \times 100\%$$

$$\eta = \frac{500}{1000} \times 100\%$$

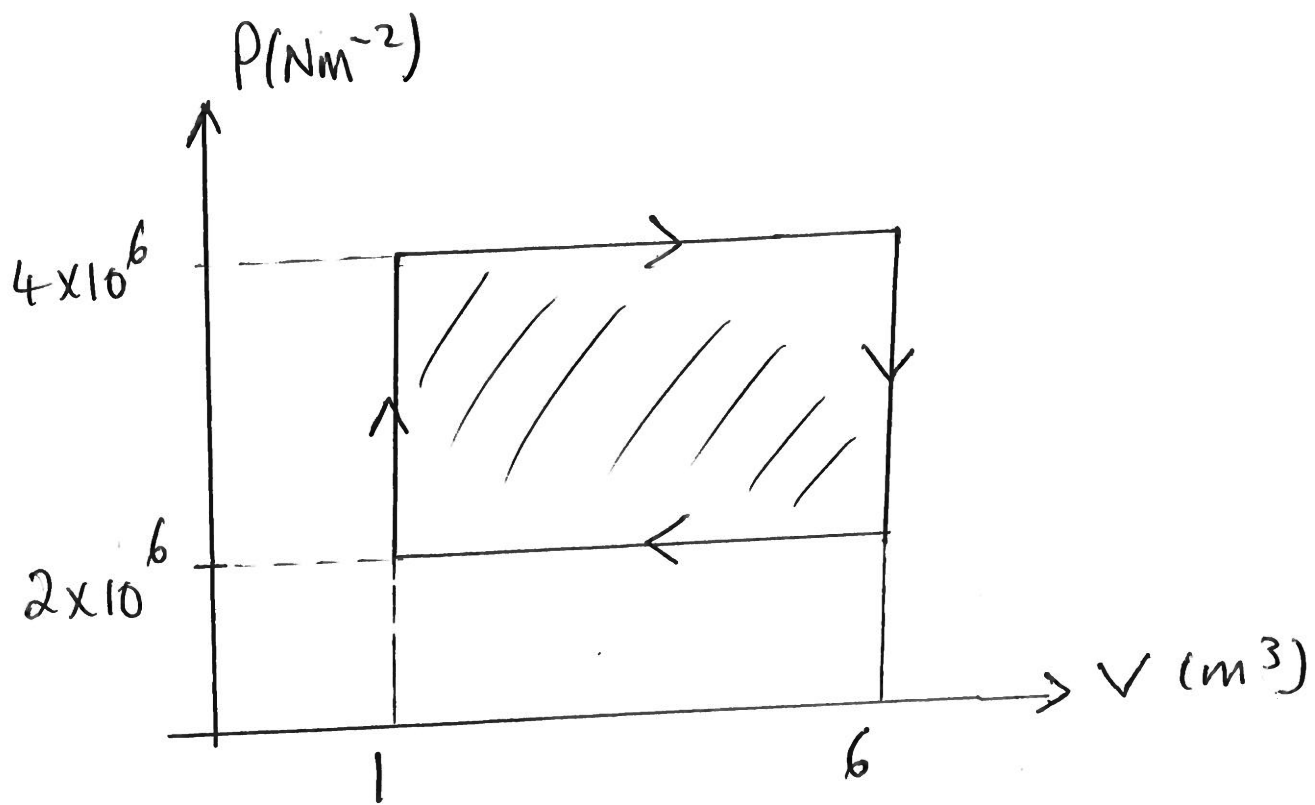
$$\underline{\eta = 50\%}$$

Calculation of Work Done over a Cycle on a P-V Diagram.

Example



$$\begin{aligned}\text{Work Done} &= \text{Area enclosed by P-V diagram} \\ &= \frac{1}{2} \times \text{base} \times \text{br height} \\ &= \frac{1}{2} \times (6-2) \times (5 \times 10^5 - 1 \times 10^5) \\ &= \frac{1}{2} \times 4 \times 4 \times 10^5 \\ &= \underline{8 \times 10^5 \text{ Joules}}\end{aligned}$$



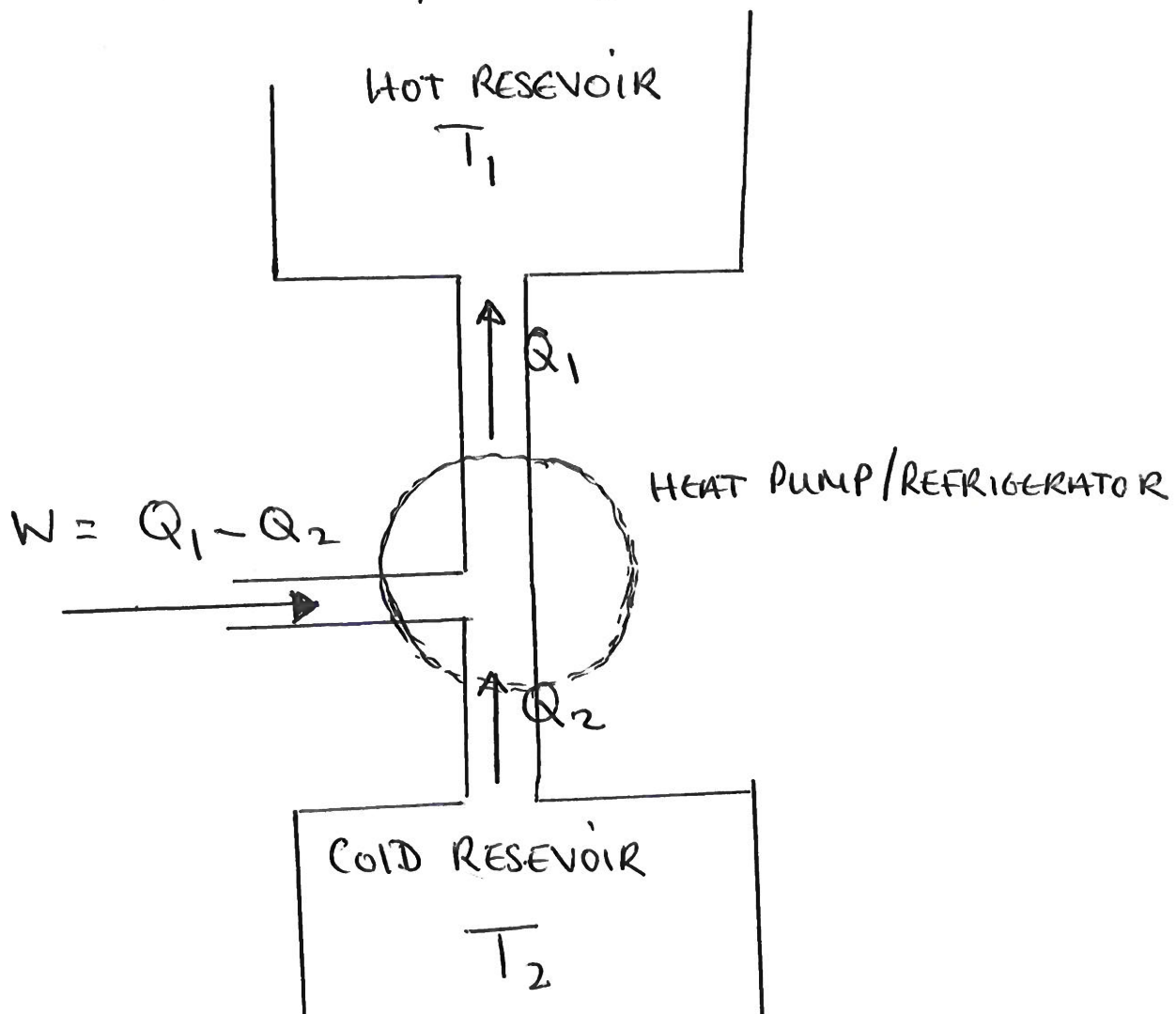
$$\begin{aligned}\text{Work Done over 1 Cycle} &= \text{Area under } P\text{-}V \text{ diagram} \\ &= \text{Area of rectangle} \\ &= (6-1) \times (4 \times 10^6 - 2 \times 10^6) \\ &= 5 \times 2 \times 10^6 \\ &= 10 \times 10^6 \\ &= \underline{10^7 \text{ Joules}}\end{aligned}$$

Heat Pumps & Refrigerators.

Heat pumps and refrigerators act similarly to heat engines but working in REVERSE.

That is, they take in heat at a low temperature and eject heat at a higher temperature.

In order to achieve this, an external agency has to do work on the working substance (fluid/gas).



Refrigerators aim to Cool whatever is inside it; that is to remove heat from the lower temperature reservoir. However, a heat pump's purpose is to supply heat to the higher temperature reservoir, i.e. to heat a room in a house.

The efficiency / effectiveness of a heat pump or refrigerator is measured as the Coefficient of Performance (COP)

$$\begin{aligned} \text{COP for a Refrigerator} &= \frac{Q}{W} = \frac{Q}{Q_1 - Q_2} \\ &= \frac{T_2}{T_1 - T_2} \end{aligned}$$

$$\begin{aligned} \text{COP for a Heat pump} &= \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} \\ &= \frac{T_1}{T_1 - T_2} \end{aligned}$$

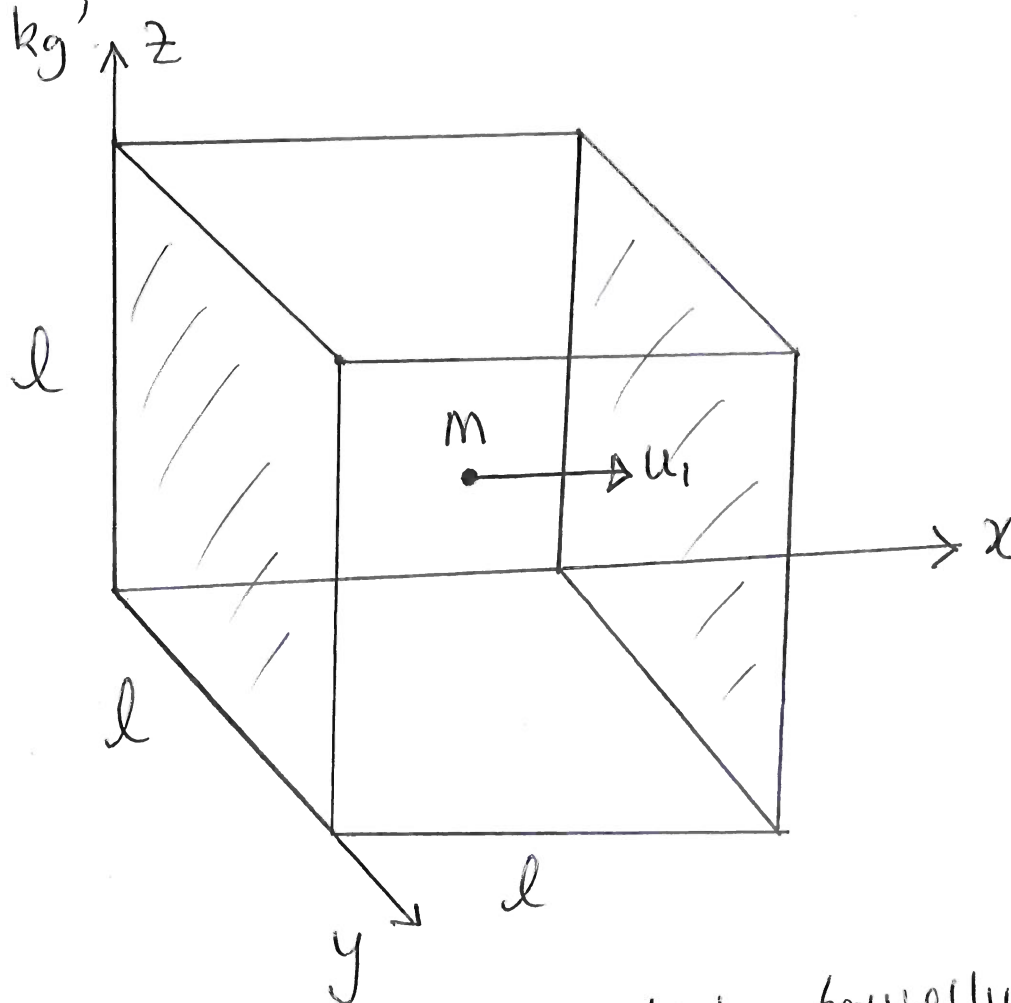
The Kinetic Theory of Gases

This is an attempt to explain the properties of an ideal gas, by considering the motion of the particles/atoms or molecules of the gas.

Assumptions

1. All gas particles are identical.
2. All collisions between the particles and the container are all perfectly elastic, so no energy is lost.
3. The particles exert no forces on each other except during collisions.
4. The particles motion are random and always move in straight lines.
5. There are sufficiently large number of molecules present for statistical analysis to be meaningfully applied.
6. The molecules/particles are treated as point masses.
7. Laws of Newtonian mechanics apply.

Consider a gas enclosed in a Cubical Container of side ' l 'm. let each gas particle have a mass ' m kg'



Consider Initially one gas particle travelling in the x direction with a velocity $u_1 \text{ ms}^{-1}$ as shown.

Time between collisions on parallel sides.

Distance = Speed $\times$ time.

$$2l = u_1 \times t.$$

$$\frac{2l}{u_1} \text{ seconds} = t.$$

$$\text{Initial momentum} \rightarrow +ve = mu_1 \text{ kg ms}^{-1}$$

$$\text{final momentum} = -mu_1 \text{ kg ms}^{-1}$$

$$\begin{aligned} \text{change in momentum} &= mu_1 - (-mu_1) \text{ kg ms}^{-1} \\ &= 2mu_1 \text{ kg ms}^{-1} \end{aligned}$$

$$\text{Rate of change of momentum} = \frac{\text{change in momentum}}{\text{Time between collisions}}$$

$$= \frac{2mu_1 \text{ kg ms}^{-1}}{\frac{2l}{u_1} \text{ s}}$$

$$= \frac{mu_1^2}{l} \text{ kg ms}^{-2}$$

Then Force $\propto$ Rate of change of momentum

$$\begin{aligned} \text{Force exerted by} &= \frac{mu_1^2}{l} \text{ N} \\ \text{1 particle on a} & \\ \text{wall} & \end{aligned}$$

$$\text{now Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$\begin{aligned} \text{Pressure due to} &= \frac{mu_1^2}{l} \\ \text{1 particle} &= \frac{mu_1^2}{l^2} \end{aligned}$$

$$\text{Pressure due to 1 particle} = \frac{mu_1^2}{l^3} \text{ Nm}^{-2}$$

Now Consider 'N' gas particles travelling in the x direction:

$$\text{Pressure due to N particles in x direction} = \frac{m}{l^3} [u_1^2 + u_2^2 + u_3^2 + \dots + u_N^2]$$

$$= \frac{mN}{l^3} \left[\frac{u_1^2 + u_2^2 + u_3^2 + \dots + u_N^2}{N} \right]$$

$$P_x = \frac{M}{l^3} \bar{u}^2 \quad [M = mN \text{ total mass}]$$

likewise performing the same analysis in the y and z direction $\Rightarrow$

$$P_y = \frac{M}{l^3} \bar{v}^2$$

$$P_z = \frac{M}{l^3} \bar{w}^2$$

Now assuming equal pressure in all 3 directions such that

$$P_x = P_y = P_z = P \Rightarrow$$

$$p_x + p_y + p_z = \frac{M}{l^3} \bar{u}^2 + \frac{M}{l^3} \bar{v}^2 + \frac{M}{l^3} \bar{w}^2$$

$$3p = \frac{M}{l^3} [\bar{u}^2 + \bar{v}^2 + \bar{w}^2]$$

$$3p = \frac{M}{l^3} \bar{c}^2$$

$$\underline{p = \frac{1}{3} \rho \cdot \bar{c}^2}$$

$$\text{where } \bar{c}^2 = \bar{u}^2 + \bar{v}^2 + \bar{w}^2$$

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{l^3}$$

The Relationship between Kinetic Energy and Temperature of an Ideal gas

$$\text{As } P = \frac{1}{3} \rho \bar{c}^2$$

and for an ideal gas:-

$$PV = nRT$$

$$\text{then: } P = \frac{1}{3} \frac{M}{V} \bar{c}^2$$

$$3PV = M \bar{c}^2$$

$$3nRT = M \bar{c}^2$$

$$\Rightarrow \frac{3nRT}{2} = \frac{1}{2} M \bar{c}^2$$

$$\Rightarrow \underline{T \propto KE}$$

Avogadro's Law

This states that equal volumes of all gases at the same temperature and pressure contain the same number of molecules.

i.e. Consider 2 ideal gases :

$$P_1 V_1 = n_1 R T_1$$

$$P_2 V_2 = n_2 R T_2$$

Then as the volumes are equal

$V_1 = V_2 = V$ and temperatures are the same $T_1 = T_2 = T$, also $P_1 = P_2 = P$

$$\Rightarrow PV = n_1 RT$$

$$PV = n_2 RT$$

$$\Rightarrow \cancel{n_1 RT} = \cancel{n_2 RT}$$

$$\Rightarrow \underline{n_1 = n_2} \quad (\text{equal no. of moles})$$

$\therefore$ Same no. of particles.

Dalton's Law of Partial Pressures

Suppose a volume V contains n_1 moles of one gas whose pressure is P_1 and n_2 moles of a second gas whose pressure is P_2 . If the temperature of both gases is T K

$$\text{Then as:- } P_1 V = n_1 RT$$

$$P_2 V = n_2 RT$$

$$\text{Then } \frac{\cancel{P_1 V}}{\cancel{P_2 V}} = \frac{\cancel{n_1 RT}}{\cancel{n_2 RT}}$$

$$\frac{P_1}{P_2} = \frac{n_1}{n_2}$$

Then as total pressure $P = P_1 + P_2$

And $\frac{n_2}{n_1} P_1 = P_2$ Then

$$P = P_1 + \frac{n_2}{n_1} P_1$$

$$P = P_1 \left[1 + \frac{n_2}{n_1} \right]$$

$$P = P_1 \left[\frac{n_1 + n_2}{n_1} \right]$$

$$\Rightarrow \frac{P \cdot n_1}{(n_1 + n_2)} = P_1$$

$$\text{likewise } P_2 = \frac{n_2 P}{(n_1 + n_2)}$$

The Speeds of 5 gas Molecules are
 100 ms^{-1} 200 ms^{-1} 300 ms^{-1} 400 ms^{-1} 500 ms^{-1}

find (i) The Mean Speed

(ii) The Mean Square Speed

(iii) The Root Mean Square Speed.

$$\begin{aligned} \text{(i)} \quad \bar{C} &= \frac{100 + 200 + 300 + 400 + 500}{5} \\ &= \frac{1500}{5} \\ &= \underline{300 \text{ ms}^{-1}} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \bar{C}^2 &= \frac{(100)^2 + (200)^2 + (300)^2 + (400)^2 + (500)^2}{5} \\ &= \frac{550,000}{5} \\ &= \underline{110,000} \end{aligned}$$

$$\text{(iii)} \quad \bar{C}_{\text{rms}} = \sqrt{110,000} = 741.62 \text{ ms}^{-1} \quad (\text{2dp's}).$$

EXAMPLE: If $\rho = \frac{1}{3} \rho \bar{c}^2$, find

$$\rho \quad \text{if} \quad \bar{c} = 500 \text{ m s}^{-1}$$

$$\rho = 900 \text{ kg m}^{-3}$$

$$\begin{aligned} \therefore \rho &= \frac{1}{3} \times 900 \times (500)^2 \\ &= 75 \times 10^6 \text{ Pa} \\ &= \underline{7.5 \times 10^7 \text{ Pa}} \end{aligned}$$

EXAMPLE: find $\bar{c}$, if $\rho = 3 \times 10^6 \text{ Nm}^{-2}$

$$\rho = 1000 \text{ kg m}^{-3}$$

Then as $\rho = \frac{1}{3} \rho \bar{c}^2$

$$\Rightarrow \sqrt{\frac{3\rho}{\rho}} = \bar{c}$$

$$\sqrt{\frac{3 \times 3 \times 10^6}{1000}} = \bar{c}$$

$$\underline{3\sqrt{10} \text{ m s}^{-1} = \bar{c}}$$

EXAMPLE: Find ρ if

$$P = 9 \times 10^5 \text{ Pa}, \quad \bar{C} = 450 \text{ m s}^{-1}$$

$$P = \frac{1}{3} \rho \bar{C}^2$$

$$9 \times 10^5 = \frac{1}{3} \times \rho \times (450)^2$$

$$\frac{3 \times 9 \times 10^5}{(450)^2} = \rho$$

$$\underline{13.3 \text{ kg m}^{-3} = \rho}$$

Some Consequences of the Kinetic Theory for an Ideal gas.

From Kinetic Theory $P = \frac{1}{3} \rho \bar{c}^2$

and from the Ideal gas equation

$$PV = nRT, \text{ Then: -}$$

$$\text{as } P = \frac{1}{3} \frac{M}{V} \bar{c}^2$$

$$\Rightarrow 3PV = M \bar{c}^2$$

$$\Rightarrow 3nRT = M \bar{c}^2$$

$$\Rightarrow \frac{3}{2} nRT = \frac{1}{2} M \bar{c}^2$$

$$\Rightarrow T \propto KE.$$

$$\Rightarrow \frac{KE}{T} = \text{Constant.}$$

EXAMPLE:

$$KE_1 = 1000 \text{ J}$$

$$KE_2 = ?$$

$$T_1 = 373 \text{ K}$$

$$T_2 = 500 \text{ K}$$

$$\Rightarrow \frac{KE_1}{T_1} = \frac{KE_2}{T_2}$$

$$\Rightarrow \frac{1000}{373} = \frac{KE_2}{500}$$

$$\Rightarrow \frac{500 \times 1000}{373} = KE_2$$

$$\underline{1340.4826 \text{ J} = K_2} \quad (4 \text{ dp's})$$

Diffusion Processes.

For an Ideal gas at Constant

$$\text{Pressure} \quad p = \frac{1}{3} \rho \bar{c}^2$$

$$\Rightarrow \text{Constant} = \rho \bar{c}^2.$$

$\therefore$ Consider 2 gases in a volume at
Constant pressure $\Rightarrow$

$$\rho_1 \bar{c}_1^2 = \rho_2 \bar{c}_2^2$$

$$\Rightarrow \frac{\bar{c}_1^2}{\bar{c}_2^2} = \frac{\rho_2}{\rho_1}$$

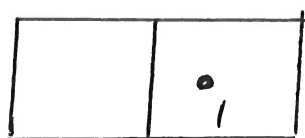
$$\frac{\bar{c}_1}{\bar{c}_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

ENTROPY

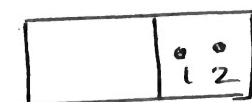
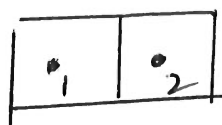
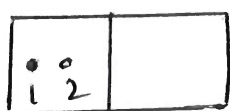
A Measure of how disordered a system is.

Arrangements.

Consider a number of particles and how they can be arranged into two compartments.



1 particle 2 ways.



2 particles in 4 ways

Similarly 3 particles can be arranged in 8 ways.

No. of Particles	No. of Arrangements W
1	$2 = 2^1$
2	$4 = 2^2$
3	$8 = 2^3$
$\vdots$	
n	2^n

$$\therefore W = 2^n, P(\text{1 particular Arrangement}) = \frac{1}{W}$$

When Energy is added to a Substance the number of ways W of arranging the "quanta" among the atoms increases.

In general if there are ' W ' ways in which ' q ' quanta can be distributed amongst N atoms, then the addition of one more quantum increases the number of possible ways W to W_1 , where:-

$$\frac{W_1}{W} = \frac{N+q}{q+1}$$

If N, q are very large then $q+1 \approx q$

$$\Rightarrow \frac{W_1}{W} \approx \frac{N+q}{q}$$

$$\Rightarrow \frac{W_1}{W} = 1 + \frac{N}{q}$$

This is a Boltzmann type distribution Curve, the slope of which is linked to the temperature of the Object.

Temperature and change in number of ways.

If every quantum of energy added to a solid increases W by the same amount $1 + \frac{N}{q} = 'a'$ say and depends only on the temperature of the object, then:-

$$\frac{W_n}{W} = a \times a \times a \dots \times a \quad (n \text{ times})$$

$$\frac{W_n}{W} = a^n$$

$$\Rightarrow \ln \left[\frac{W_n}{W} \right] = \ln [a^n]$$

$$\ln [W_n] - \ln [W] = n \ln a$$

$$\text{let } \delta \ln(W) = \ln [W_n] - \ln [W]$$

$$\Rightarrow \delta \ln(W) = n \ln a$$

The Quantity $\delta \ln(W) \propto \delta U$ (Internal Energy)

High temperature T is associated with a small change in W and so

$$\delta \ln W \propto \frac{1}{T} \cdot \delta u$$

$$\delta \ln W = C \cdot \frac{1}{T} \cdot \delta u$$

$$\text{If } C = \frac{1}{k}$$

$$\delta \ln W = \frac{1}{k} \cdot \frac{1}{T} \cdot \delta u$$

$$\Rightarrow k \cdot \delta \ln(W) = \frac{1}{T} \cdot \delta u$$

This expression defines temperature in terms of Energy and number of arrangements.

T defined in Kelvin, $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$
(Boltzmann's Constant).

The Quantity $k \delta \ln(W)$ is called the change in Entropy of the system δS

$$\Rightarrow \delta S = k \cdot \delta \ln(W) = \frac{\delta u}{T}$$

$$\delta S = k \cdot \delta \ln(W) = \frac{\delta Q}{T} \quad \begin{array}{l} \text{from 1st Law} \\ \text{thermodynamics} \\ \delta W = 0. \end{array}$$

Thus $\Delta S = \frac{\Delta Q}{T}$

$$\Delta S = \int_{T_1}^{T_2} \frac{\Delta Q}{T}$$

for substances $\Delta Q = mc dT$

$$\Delta S = \int_{T_1}^{T_2} mc \frac{dT}{T}$$

$$= mc [\ln T]_{T_1}^{T_2}$$

$$= mc [\ln T_2 - \ln T_1]$$

$$= mc \ln \left[\frac{T_2}{T_1} \right] \text{ JK}^{-1}$$