

# School of Engineering

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## Introduction to Energy Storage in Flywheels

Mark Sandford

# Rotation – Linear Parallels

## Linear Motion

## Rotational Motion

|                       |                              |                                              |                      |
|-----------------------|------------------------------|----------------------------------------------|----------------------|
| Position              | $x$                          | $\theta$                                     | Angular position     |
| Velocity              | $v$                          | $\omega$                                     | Angular velocity     |
| Acceleration          | $a$                          | $\alpha$                                     | Angular acceleration |
| Motion equations      | $x = \bar{v}t$               | $\theta = \bar{\omega}t$                     | Motion equations     |
|                       | $v = v_0 + at$               | $\omega = \omega_0 + \alpha t$               |                      |
|                       | $x = v_0t + \frac{1}{2}at^2$ | $\theta = \omega_0t + \frac{1}{2}\alpha t^2$ |                      |
|                       | $v^2 = v_0^2 + 2ax$          | $\omega^2 = \omega_0^2 + 2\alpha\theta$      |                      |
| Mass (linear inertia) | $m$                          | $I$                                          | Moment of inertia    |
| Newton's second law   | $F = ma$                     | $\tau = I\alpha$                             | Newton's second law  |
| Momentum              | $p = mv$                     | $L = I\omega$                                | Angular momentum     |
| Work                  | $Fd$                         | $\tau\theta$                                 | Work                 |
| Kinetic energy        | $\frac{1}{2}mv^2$            | $\frac{1}{2}I\omega^2$                       | Kinetic energy       |
| Power                 | $Fv$                         | $\tau\omega$                                 | Power                |

# Capri Accelerating

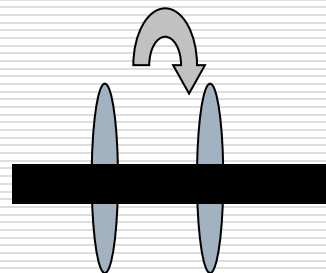
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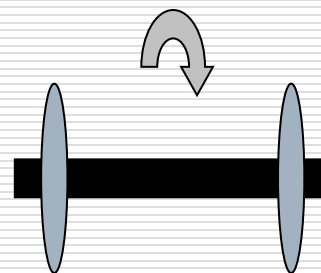
# moment of inertia

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- ❑ Force required to twist long dumbbell is higher than required for the short dumbbell
- ❑ Longer dumbbell has higher inertia
- ❑ Same force is required to lift both dumbbells
- ❑ Both dumbbells are the same mass
- ❑ Need to know mass and inertia properties to fully understand system behaviour



Short Dumbbell  
Easy to Twist



Long Dumbbell  
Hard to Twist

# Spinning Skater

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# Mass and Linear Accel

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☐ Vehicle energy

☐ Vehicle power

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# Moment of Inertia

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$$I = mr^2$$

For a point mass the moment of inertia is just the mass times the radius from the axis squared. For a collection of point masses (below) the moment of inertia is just the sum for the masses.

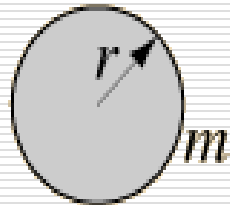
# Moment of Inertia

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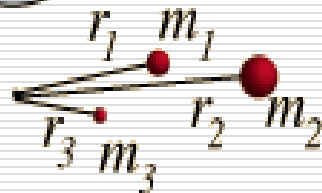
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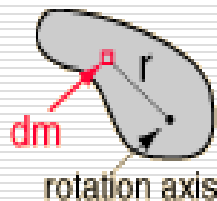
$$I = kmr^2$$

For an object with an axis of symmetry, the moment of inertia is some fraction of that which it would have if all the mass were at the radius  $r$ .



$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots$$

Sum of the point mass moments of inertia.



$$I = \int_0^M r^2 dm$$

Continuous mass distributions require an infinite sum of all the point mass moments which make up the whole. This is accomplished by an integration over all the mass.

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Linear  $F = ma$

Newton's Second  
Law

Angular  $\tau = I\alpha$

Linear  $p = mv$

Momentum

Angular  $L = I\omega$

Moment  
of Inertia  
 $I$

Linear  $KE = \frac{1}{2}mv^2$

Kinetic Energy

Angular  $KE = \frac{1}{2}I\omega^2$

Linear  $F_{net}d = \Delta \left( \frac{1}{2}mv^2 \right)$

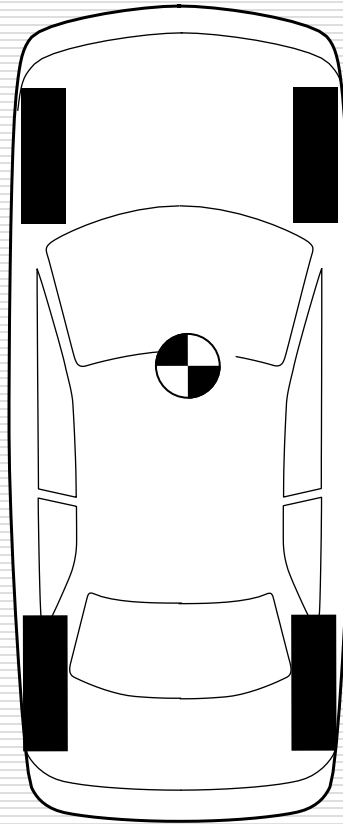
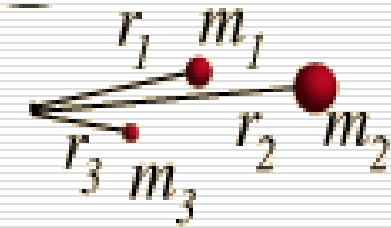
Work-Energy

Angular  $\tau_{net}\theta = \Delta \left( \frac{1}{2}I\omega^2 \right)$

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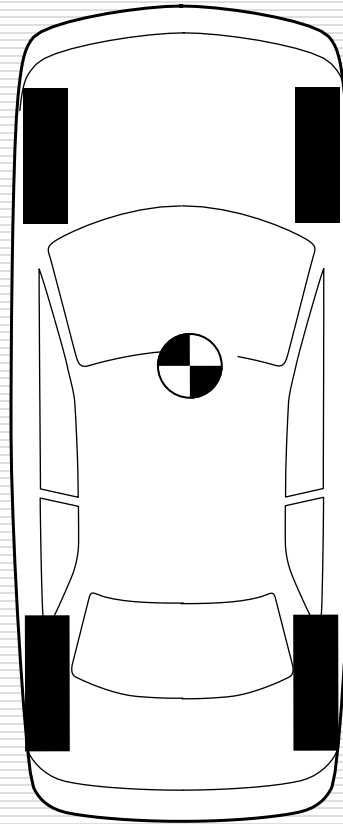
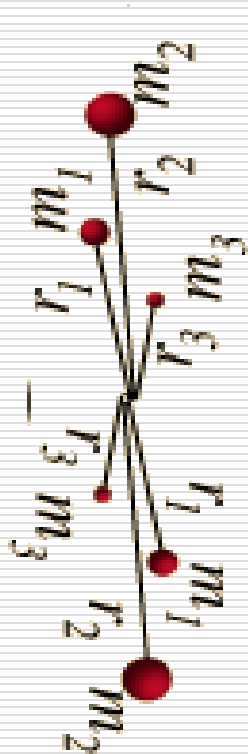
# Moment of Inertia

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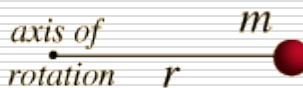
# Moment of Inertia

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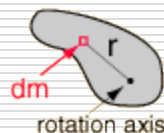
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Since the moment of inertia of an ordinary object involves a continuous distribution of mass at a continually varying distance from any rotation axis, the calculation of moments of inertia generally involves calculus, the discipline of mathematics which can handle such continuous variables. Since the moment of inertia of a [point mass](#) is defined by

$$I = mr^2$$


The diagram shows a horizontal line representing the axis of rotation. A point is marked on this line with the label "axis of rotation" above it. To the right of this point, a red sphere representing a point mass is shown. A horizontal line segment connects the axis of rotation to the mass, labeled with the variable "r". Above the mass is the letter "m".

then the moment of inertia contribution by an infinitesimal mass element  $dm$  has the same form. This kind of mass element is called a [differential element](#) of mass and its moment of inertia is given by

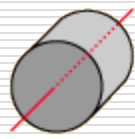


$$dI = r^2 dm$$

The "d" preceding any quantity denotes a vanishingly small or "differential" amount of it.

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Solid cylinder or disc, symmetry axis



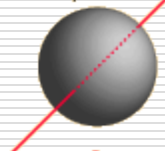
$$I = \frac{1}{2}MR^2$$

Hoop about symmetry axis



$$I = MR^2$$

Solid sphere



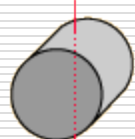
$$I = \frac{2}{5}MR^2$$

Rod about center



$$I = \frac{1}{12}ML^2$$

$$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$$



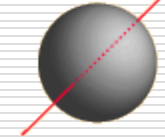
Solid cylinder, central diameter

$$I = \frac{1}{2}MR^2$$



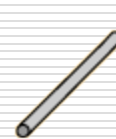
Hoop about diameter

$$I = \frac{2}{3}MR^2$$



Thin spherical shell

$$I = \frac{1}{3}ML^2$$



Rod about end

# Vehicle rotational inertia

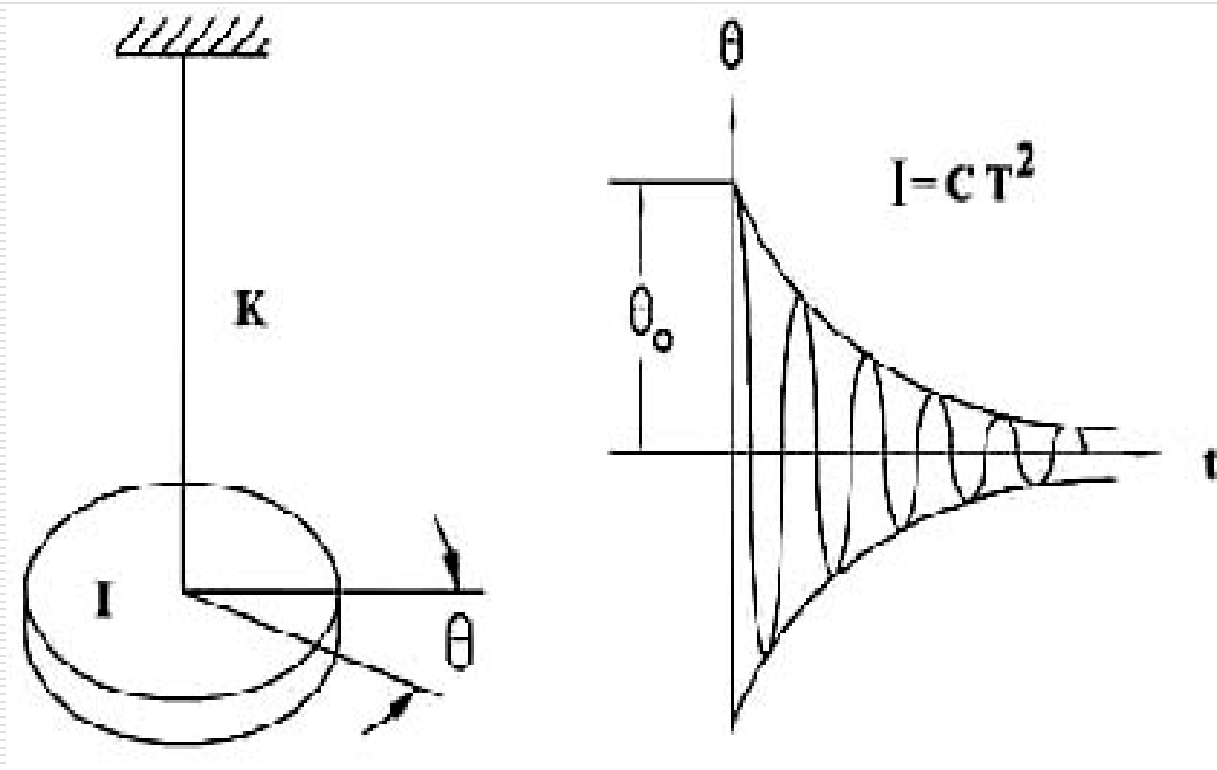
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- ☐ How can we ascertain vehicle rotational inertia
  - ☐ Put everything in CAD and ask the computer
  - ☐ Measure it
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# Inertia measurement

## Torsional pendulum

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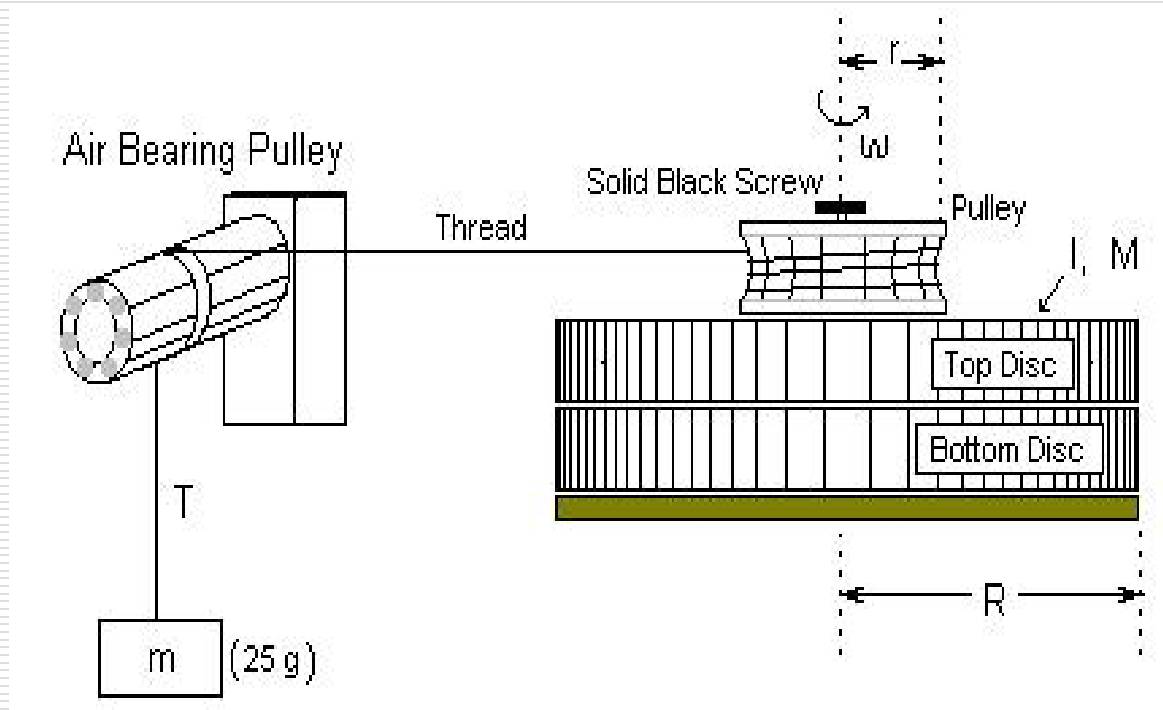




# Inertia measurement

## Torsional force

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Applied torque = Inertia x angular acceleration

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# Vehicle Inertial measurement

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CIC's Moment of Inertia Facilities provide measurement of:

- 3-Axis, Moment and Product of Inertias
- Principal Inertias and their Direction Cosines
- Total Mass, Centre of Gravity Location and Mass Distribution

|                 |                                                 |
|-----------------|-------------------------------------------------|
| Mass range      | From 1kg to 2,500kg                             |
| Dimension range | Nominally 6500 (L) x 2800 (W) x 2000 (H) mm max |